MUPAGE - Parameter Tuning

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1 Introduction

This document is intended to record the parameters which exists within MUPAGE and display the impact of variations in these parameters on some select observables in the KM3NeT detector.

The parametric equations within MUPAGE [1] are divided into four categories: the muon bundle flux, the lateral spread of the muon bundle, the energy spectrum of single muons and the energy spectrum of multiple muons. The same procedure is carried out for this investigation of the parameters: namely, run the full simulation chain within *data_processing* of

MUPAGE → JSirene → JTriggerEfficiency → J(ORCA)Reconstruction

for deviating values of the parameters within MUPAGE. Observables (physical parameters which can be measured by the detector) are then stored in a flat ntuple using the *ana* framework within *Aanet*. The same observables are then plotted: reconstructed zenith angle distribution θ, reconstructed energy E, the number of trigger overlays (i.e. the number of simultaneous triggers an event can produce) and the true muon bundle multiplicity (how many muons are in the bundles). The same procedure is applied to every parameter in order to be as systematic as possible - i.e. run 10,000 MUPAGE events and vary the parameter by a factor of ± 3 (except in the cases when the simulation takes too long to run). The ORCA-4 detector is used.

The most relevant formulae within MUPAGE for this parameter tuning exercise, and the parameters with the greatest impact on the observables, are noted.

2 Flux

2.1 Flux of muon bundles, given depth and zenith angle

From [2], the parametric formula for the flux of bundles is given by:

$$\phi(m; h, \theta) = \frac{K(h,\theta)}{m^{\nu(h,\theta)}} \text{ with } \nu = \frac{\nu_1}{(1 + \Lambda \dot{m})} \,. \tag{1}$$

In the context of MUPAGE, $\Lambda = 0$. For m=1, ϕ is the flux of **single** muons, at a vertical depth h and given zenith angle θ, in units of $m^{-2}s^{-1}sr^{-1}$.

2.2 The parameter K

So, for single muons,

$$\phi(m = 1; h, \theta) = K(h, \theta) = K_0(h) \cos\theta e^{K_1(h)\cdot \sec\theta} \,, \tag{2}$$

where

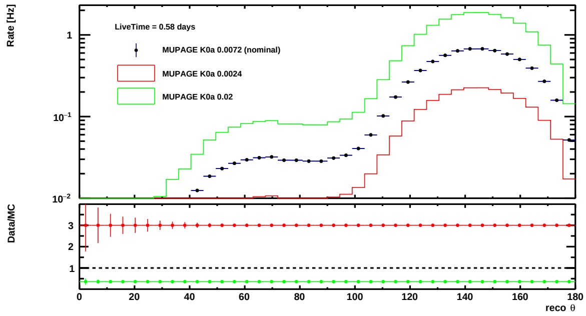
$$K_0(h) = K_{0a} \cdot h^{K_{0b}} \tag{3}$$

and

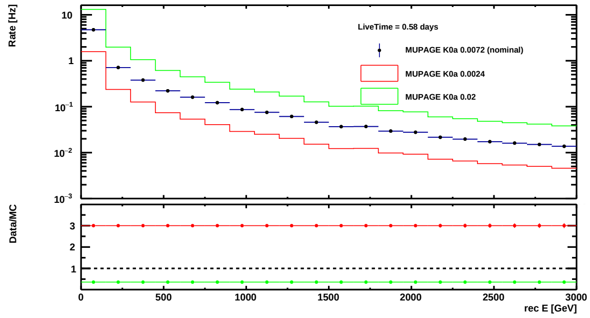
$$K_1(h) = K_{1a} \cdot h + K_{1b} \,. \tag{4}$$

The parameters K_{0a}, K_{0b}, K_{1a} and K_{1b} are varied. It is clear from the above formulae that these parameters affect the *single muon* flux as a function of depth. Nonetheless their impact on observables is investigated.

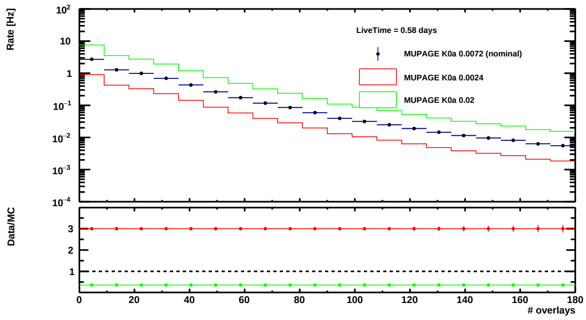
2.3 K_{0a}



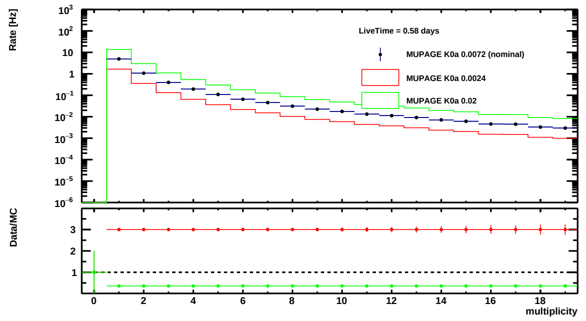
(a) K0a - reconstructed zenith angle



(b) K0a - reconstructed energy



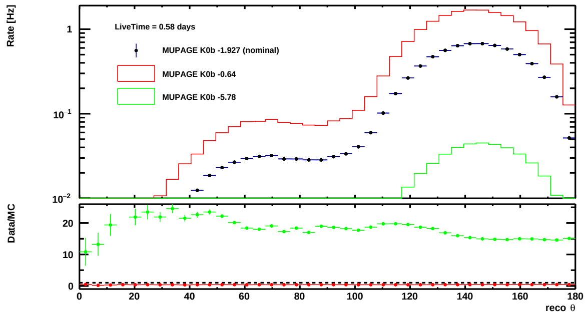
(c) K0a - overlays



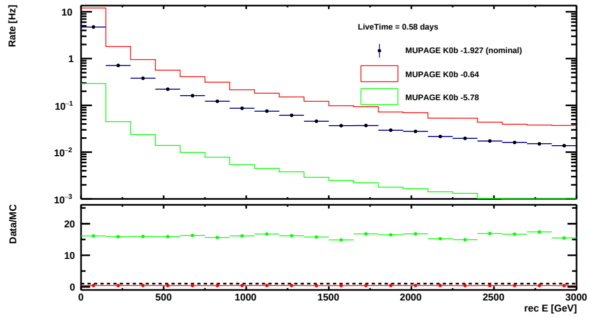
(d) K0a - multiplicity

Figure 1: K0a for different observables

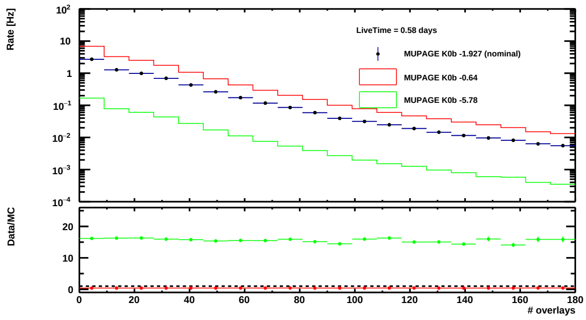
2.4 K_{0b}



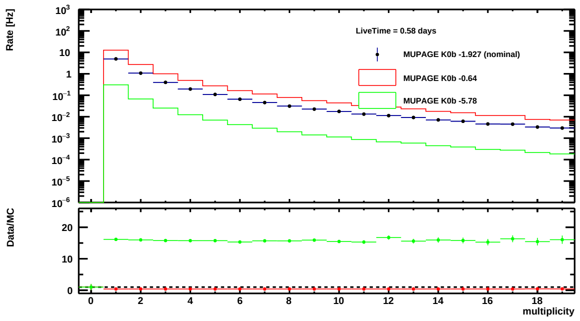
(a) K0b - reconstructed zenith angle



(b) K0b - reconstructed energy



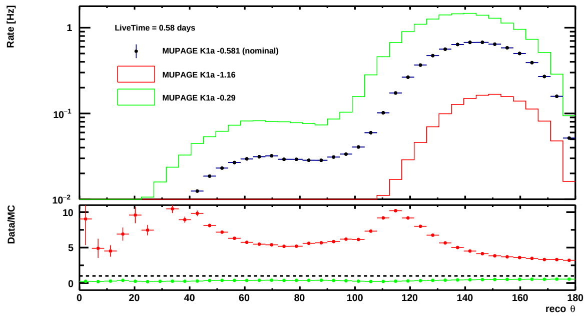
(c) K0b - overlays



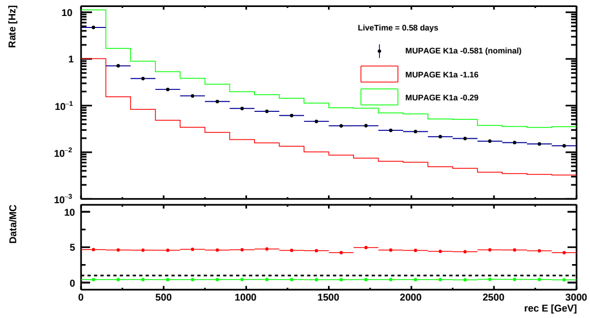
(d) K0b - multiplicity

Figure 2: K0b for different observables

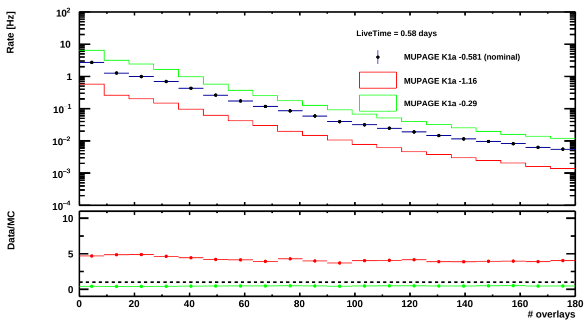
2.5 K_{1a}



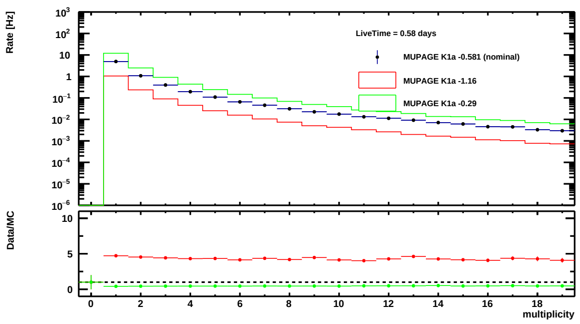
(a) K1a - reconstructed zenith angle



(b) K1a - reconstructed energy



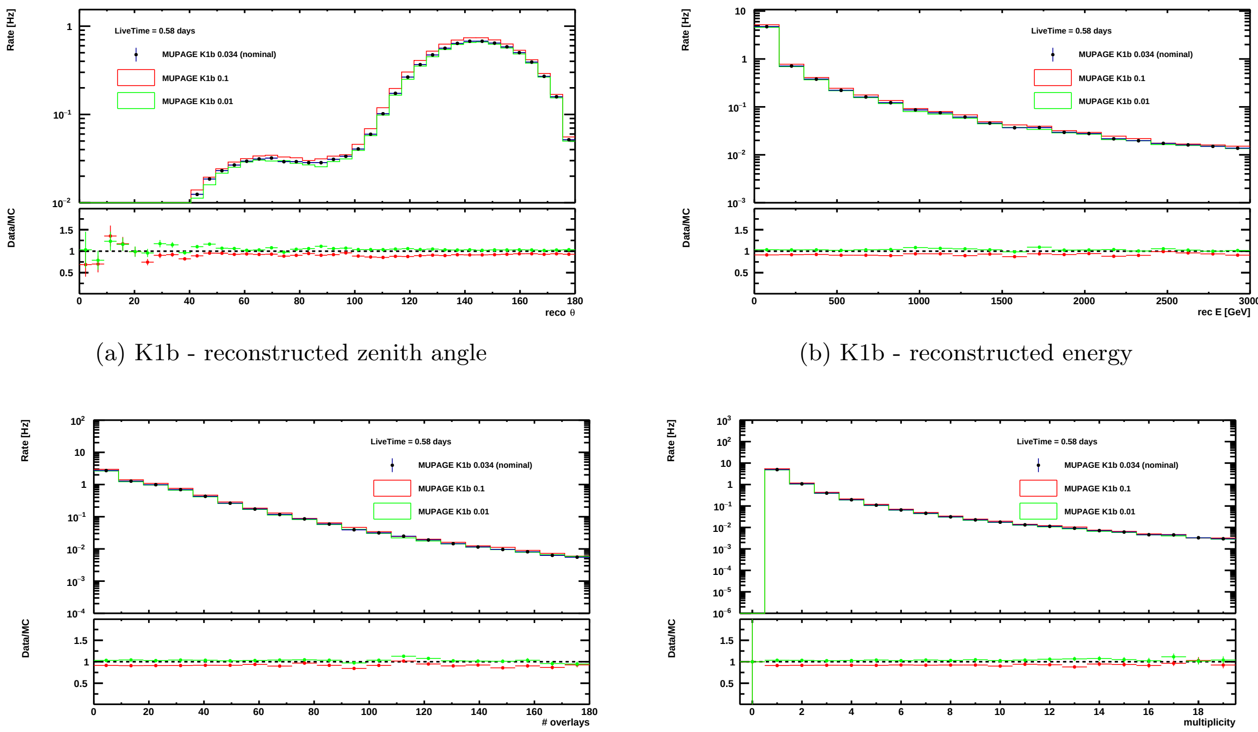
(c) K1a - overlays



(d) K1a - multiplicity

Figure 3: K1a for different observables

2.6 K_{1b}



(a) K1b - reconstructed zenith angle

(b) K1b - reconstructed energy

(c) K1b - overlays

(d) K1b - multiplicity

Figure 4: K1b for different observables

2.7 The parameter ν

The fraction of the multiple muon flux with respect to the single flux is given by:

$$\nu(h,\theta) = \nu_0(h) e^{\nu_1(h)\cdot\sec\theta}\,. \tag{5}$$

Again, there are parameters to vary within $\nu(h,\theta)$:

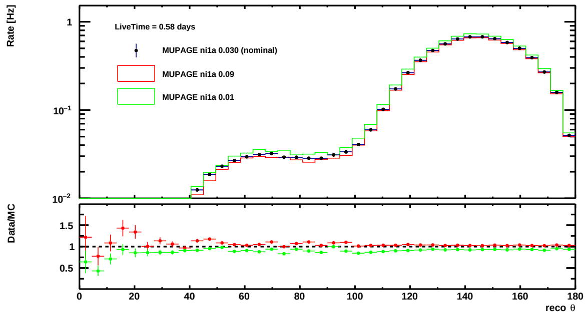
$$\nu_0(h) = \nu_{0a}\cdot h^2 + \nu_{0b}\cdot h + \nu_{0c} \tag{6}$$

and

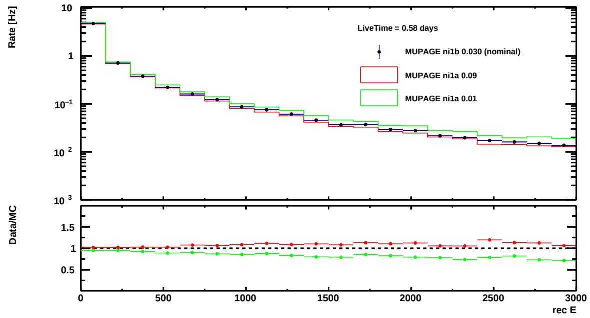
$$\nu_1(h) = \nu_{1a}\cdot e^{\nu_{1b}\cdot h}\,. \tag{7}$$

Altering ν_{0b} does appear to have some impact on the shape of the plotted observables.

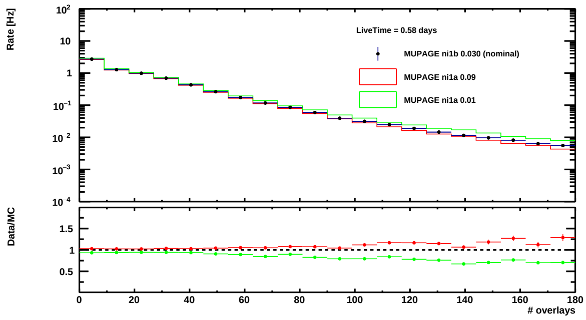
2.8 ν_{1a}



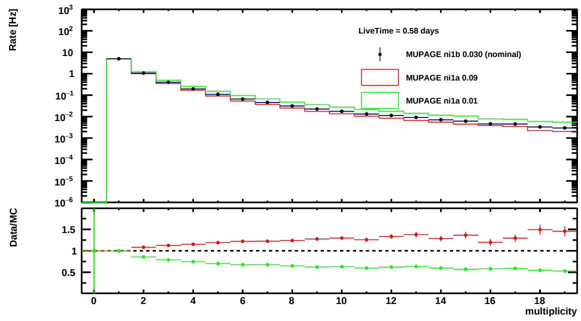
(a) ni1a - reconstructed zenith angle



(b) ni1a - reconstructed energy



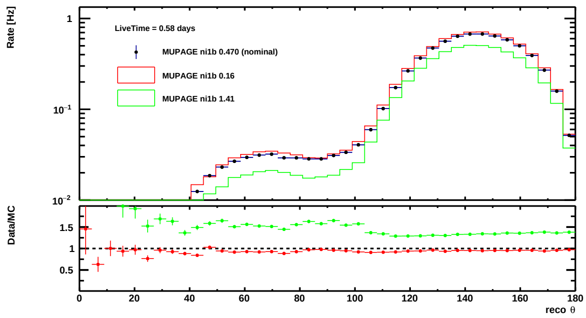
(c) ni1a - overlays



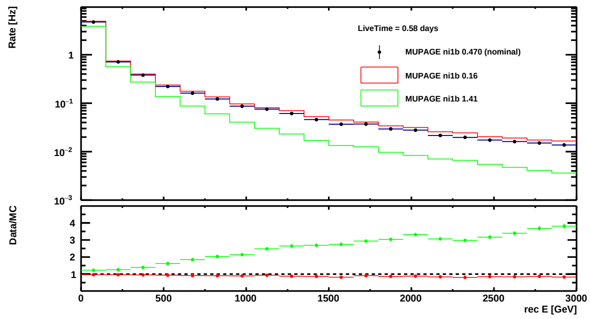
(d) ni1a - multiplicity

Figure 5: ni1a for different observables

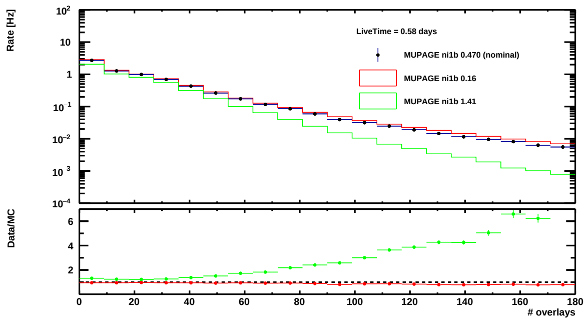
2.9 ν_{1b}



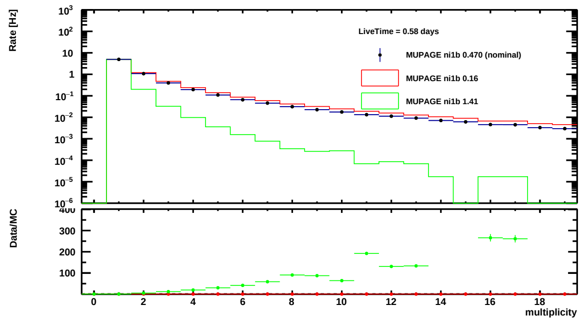
(a) ni1b - reconstructed zenith angle



(b) ni1b - reconstructed energy



(c) ni1b - overlays



(d) ni1b - multiplicity

Figure 6: ni1b for different observables

3 Lateral Spread

In muon bundles, the most energetic muons are expected to arrive closer to the axis of the shower, as muons produced in the decay of secondary mesons tend to follow the energy distribution of the parent mesons. Thus, one can parametrise the energy of muons in a bundle using the *lateral spread* of muons in a bundle: their radial distance to the shower axis R.

The muon lateral distribution - in plane perpendicular to the shower axis - can be described by

$$\frac{dN}{dR} = C\frac{R}{(R+R_0)^\alpha}\,. \tag{8}$$

C represents the normalisation factor, $C = (\alpha-1)(\alpha-2)\cdot R_0^{\alpha-2}$. In [2], the variable M for multiplicity is defined as

$$\begin{array}{c} M = m, \quad \text{if } m \leq 3 \\ M = 4, \quad \text{if } m \leq 4 \end{array}\,. \tag{9}$$

The average radial distance $\langle R\rangle = 2R_0/(\alpha-3)$ (in metres) is also used in the parametrisation:

$$\langle R\rangle = \rho(h,\theta,M) = \rho_0(M)\cdot h^{\rho_1}\cdot F(\theta)\,, \tag{10}$$

where

$$\rho_0(M) = \rho_{0a}\cdot M + \rho_{0b} \tag{11}$$

and

$$F(\theta) = \frac{1}{e^{(\theta-\theta_0)\cdot f}+1}\,. \tag{12}$$

The parameter α depends on the depth and multiplicity.

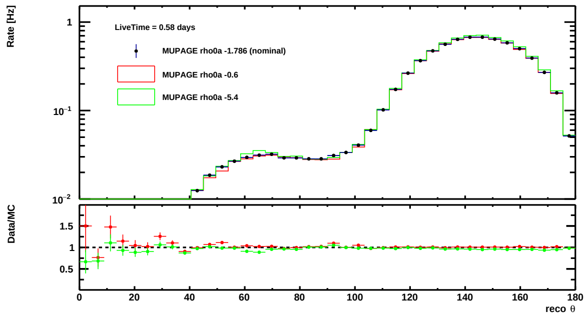
$$\alpha = \alpha(h,M) = \alpha_0(M)\cdot e^{\alpha_1(M)\cdot h}\,, \tag{13}$$

where

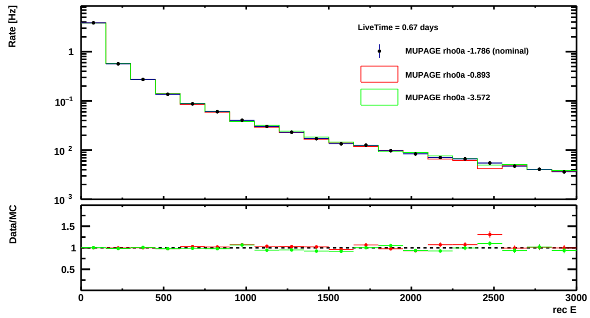
$$\alpha_0(M) = \alpha_{0a}\ cdot M + \alpha_{0b} \tag{14}$$

$$\alpha_1(M) = \alpha_{1a}\ cdot M + \alpha_{1b}\,. \tag{15}$$

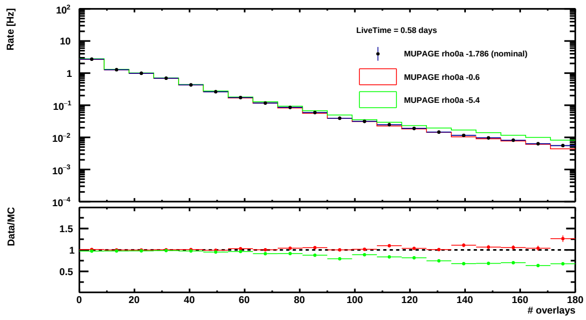
3.1 ρ_{0a}



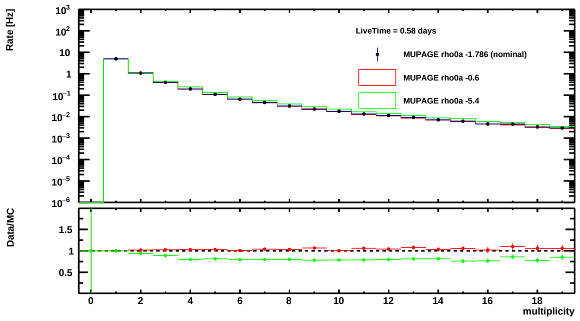
(a) rho0a - reconstructed zenith angle



(b) rho0a - reconstructed energy



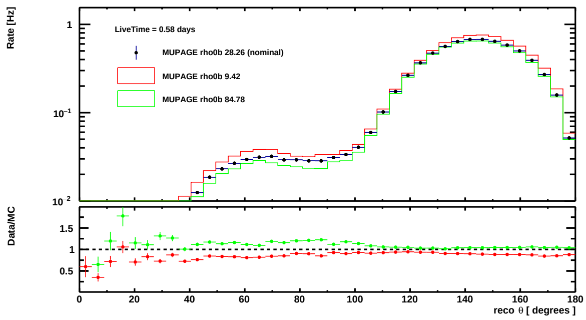
(c) rho0a - overlays



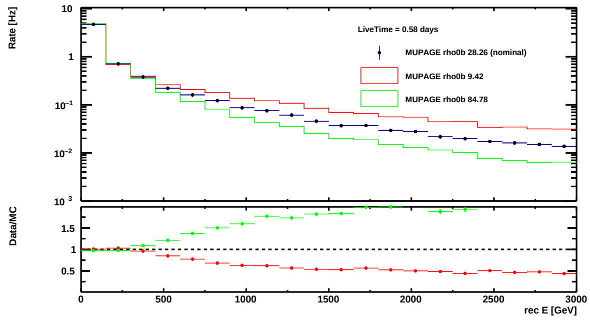
(d) rho0a - multiplicity

Figure 7: rho0a for different observables

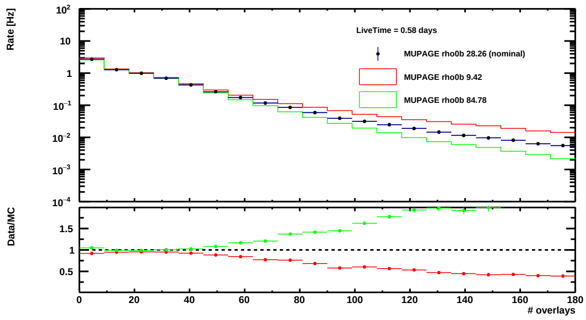
3.2 ρ_{0b}



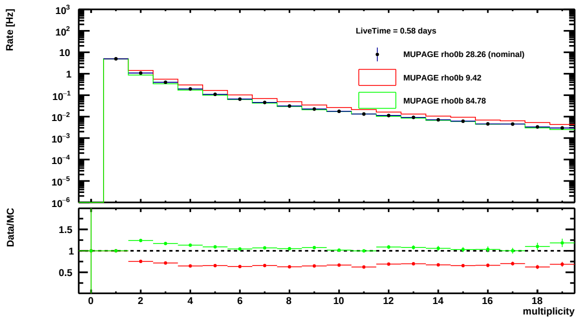
(a) rho0b - reconstructed zenith angle



(b) rho0b - reconstructed energy



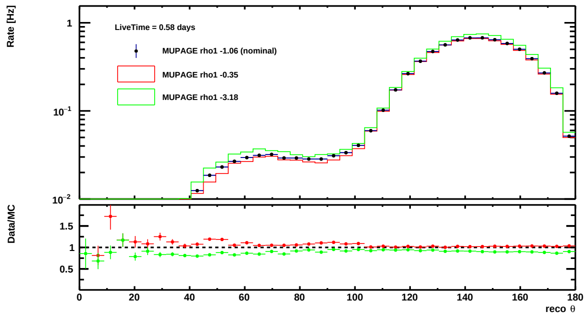
(c) rho0b - overlays



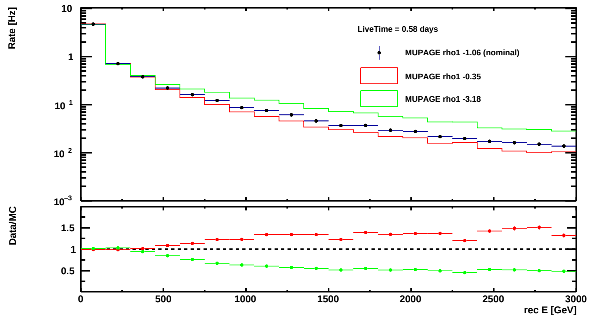
(d) rho0b - multiplicity

Figure 8: rho0b for different observables

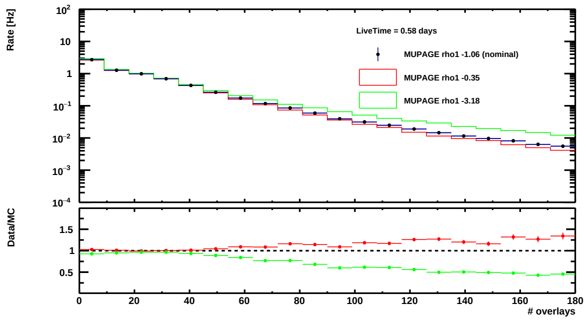
3.3 ρ_1



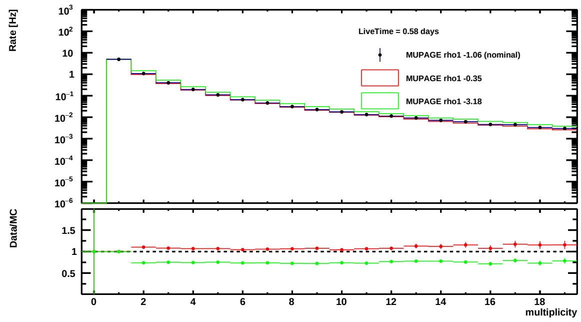
(a) rho1 - reconstructed zenith angle



(b) rho1 - reconstructed energy



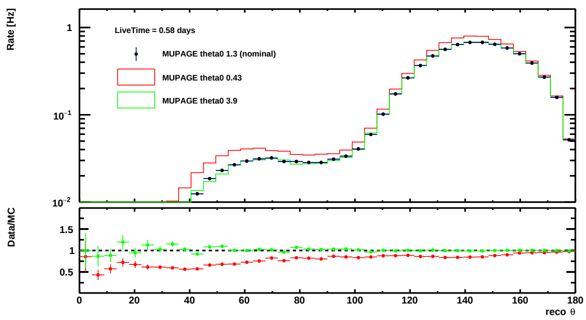
(c) rho1 - overlays



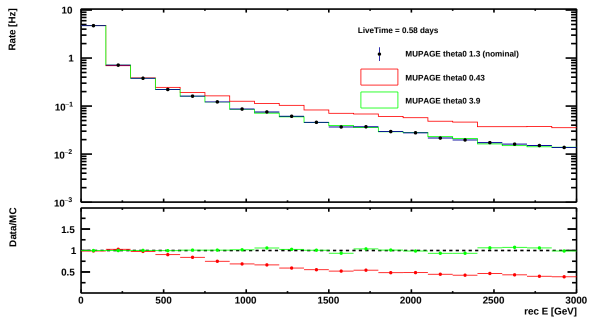
(d) rho1 - multiplicity

Figure 9: rho1 for different observables

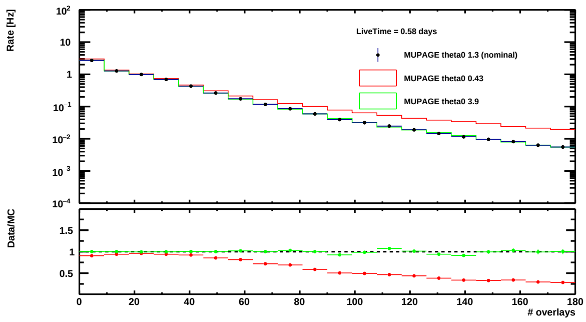
3.4 θ_0



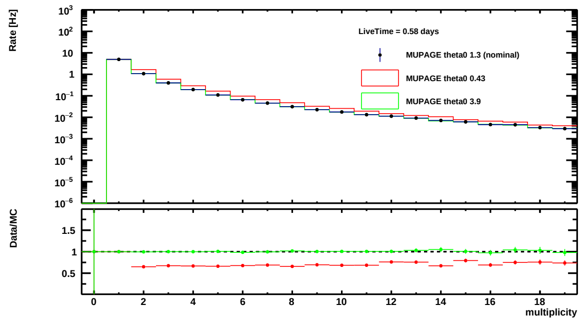
(a) theta0 - reconstructed zenith angle



(b) theta0 - reconstructed energy



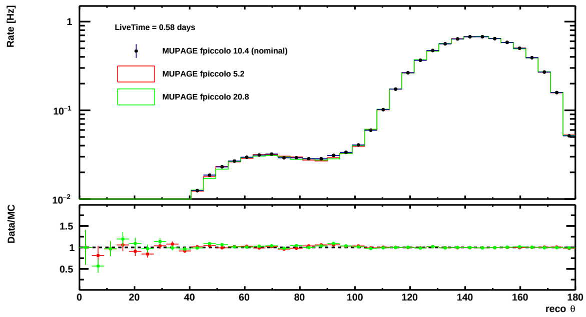
(c) theta0 - overlays



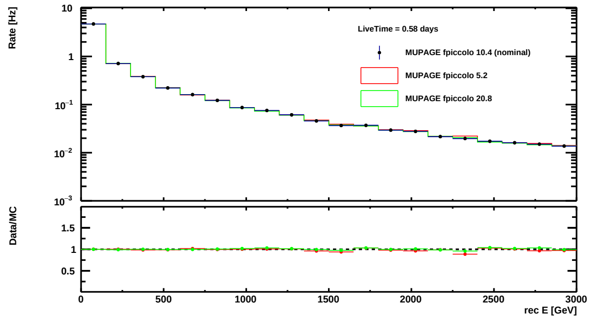
(d) rho1 - multiplicity

Figure 10: theta0 for different observables

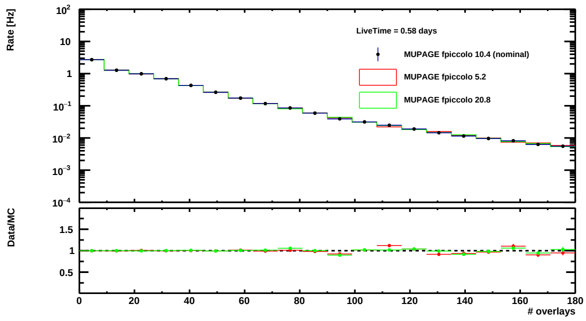
3.5 f



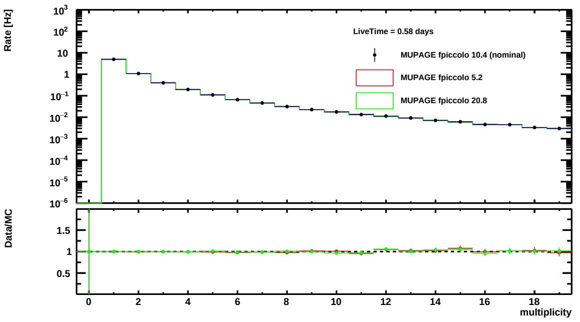
(a) f - reconstructed zenith angle



(b) f - reconstructed energy



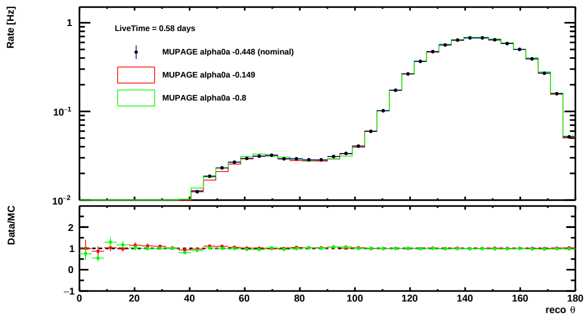
(c) f - overlays



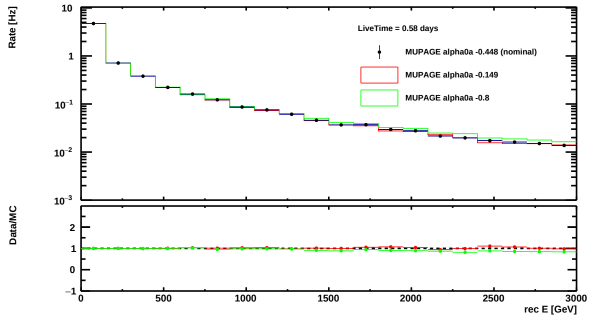
(d) f - multiplicity

Figure 11: f for different observables

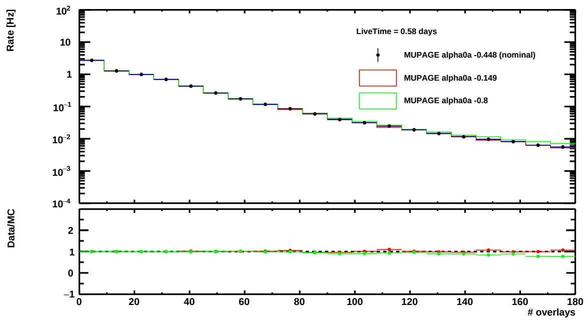
3.6 α_{0a}



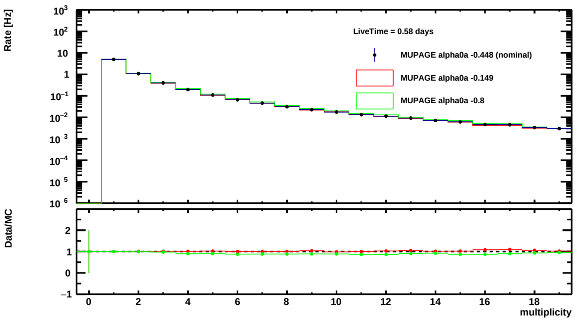
(a) alpha0a - reconstructed zenith angle



(b) alpha0a - reconstructed energy



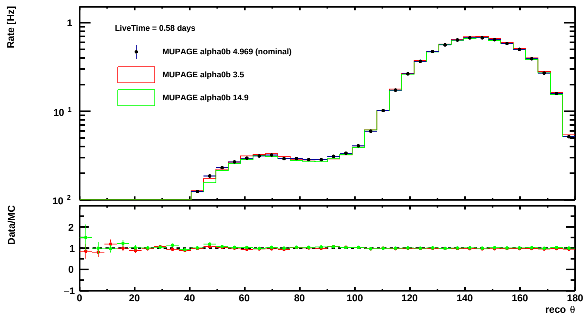
(c) alpha0a - overlays



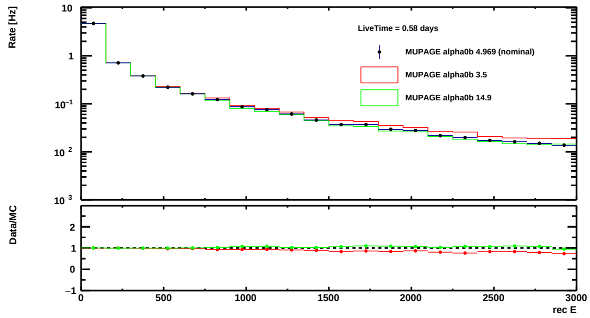
(d) alpha0a - multiplicity

Figure 12: alpha0a for different observables

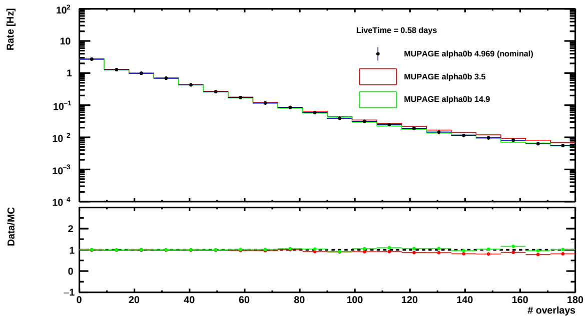
3.7 α_{0b}



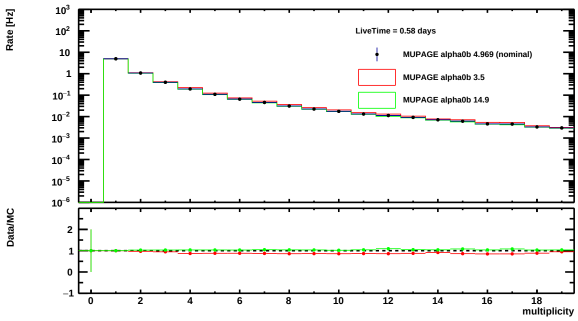
(a) alpha0b - reconstructed zenith angle



(b) alpha0b - reconstructed energy



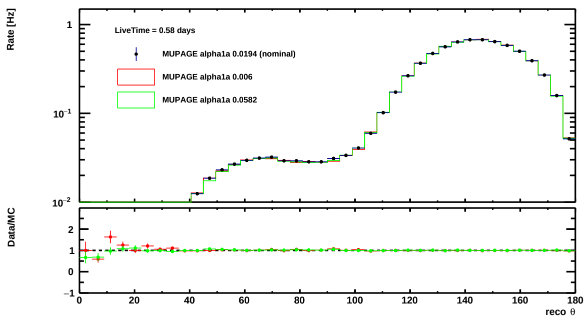
(c) alpha0b - overlays



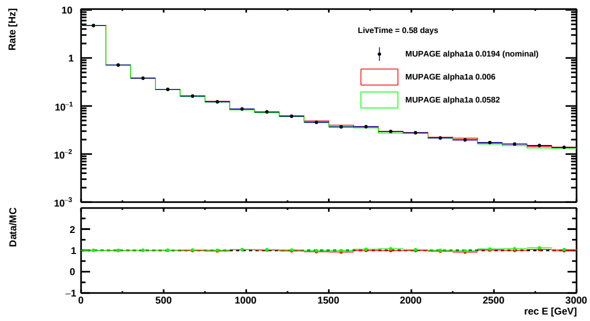
(d) alpha0b - multiplicity

Figure 13: alpha0b for different observables

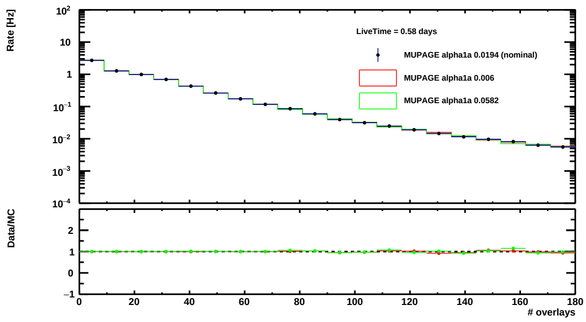
3.8 α_{1a}



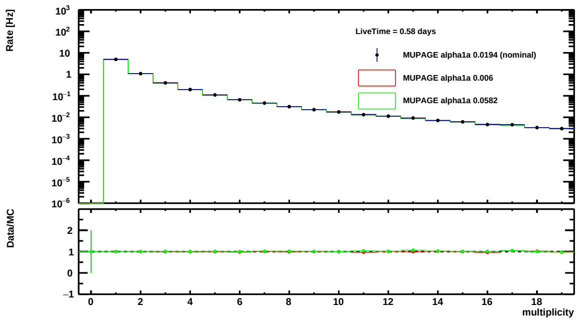
(a) alpha1a - reconstructed zenith angle



(b) alpha1a - reconstructed energy



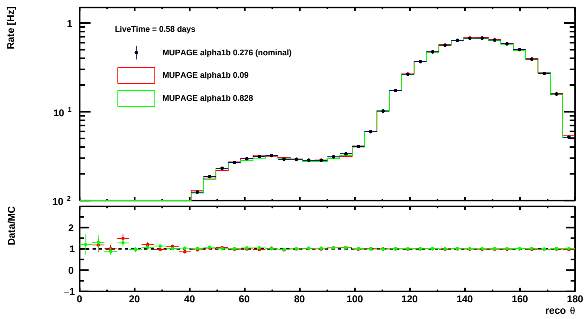
(c) alpha1a - overlays



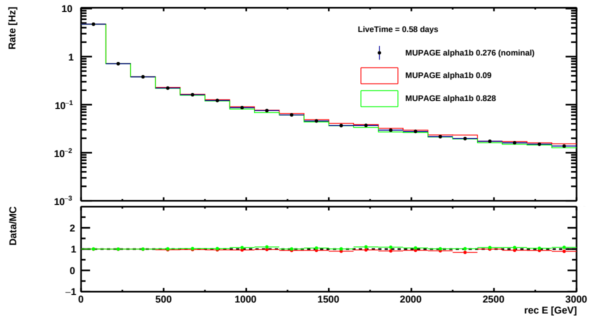
(d) alpha1a - multiplicity

Figure 14: alpha1a for different observables

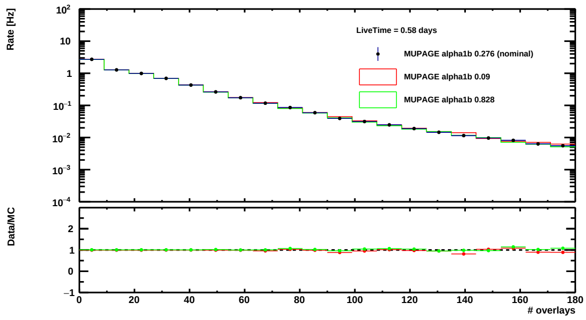
3.9 α_{1b}



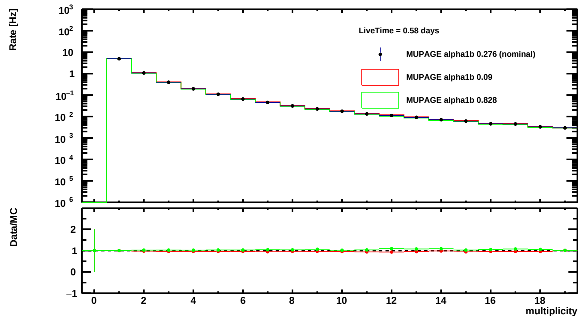
(a) alpha1b - reconstructed zenith angle



(b) alpha1b - reconstructed energy



(c) alpha1b - overlays



(d) alpha1b - multiplicity

Figure 15: alpha1b for different observables

4 Energy Spectrum (single muons)

The energy spectrum of the muons which reach the detector is separated for single muons and muon bundles. The energy loss processes per unit depth $X = h/cos\theta$ is described in terms of the sum of continuous energy loss α (different from the one encountered for the lateral spread) and catastrophic energy loss β:

$$-\left\langle \frac{dE(E_\mu)}{dX} \right\rangle = \alpha + \beta E_\mu \,. \tag{16}$$

Assuming a power-law for X, the expected energy distribution is written as:

$$\frac{dN}{d(\log_{10} E_\mu)} = G \cdot E_\mu e^{\beta X(1-\gamma)}[E_\mu + \epsilon(1 + e^{-\beta X})]^{-\gamma} \,, \tag{17}$$

where γ is the spectral index of the primary mesons and $\epsilon = \alpha/\beta$. Note that in this context these parameters have no direct physical meaning. The normalization factor is defined as

$$G(\gamma, \beta, epsilon) = 2.30 \cdot (\gamma - 1) \cdot \epsilon^{\gamma-1} \cdot e^{(\gamma-1)\cdot\beta\cdot X} \cdot (1 - e^{-\beta\cdot X})^{\gamma-1} \,. \tag{18}$$

The units of ϵ are TeV, γ is dimensionless, and the muon energy is in TeV.

Within MUPAGE the parameters γ and ϵ are defined as:

$$\gamma = \gamma(h) = \gamma_0 \cdot \ln(h) + \gamma_1 \tag{19}$$

and

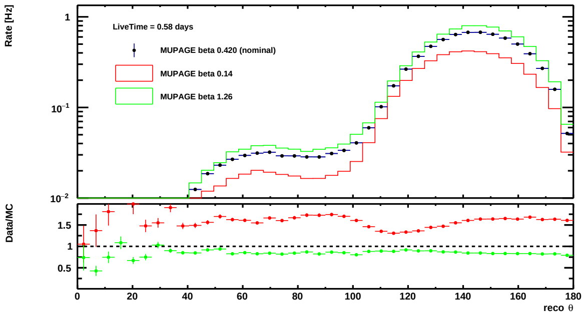
$$\epsilon = \epsilon(h, \theta) = \epsilon_0(h) \cdot \sec\theta + \epsilon_1 h \,, \tag{20}$$

where

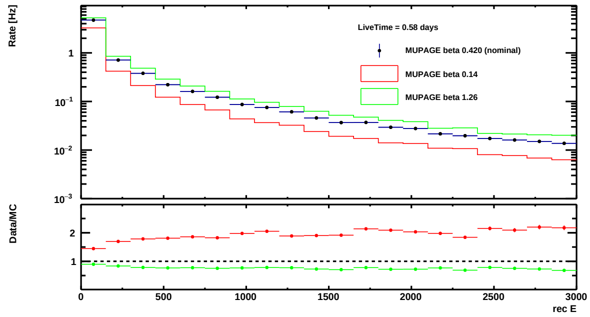
$$\epsilon_0(h) = \epsilon_{0a} \cdot e^{\epsilon_{0b}\cdot h} \,, \tag{21}$$

$$\epsilon_1(h) = \epsilon_{1a} \cdot h + \epsilon_{1b}\,. \tag{22}$$

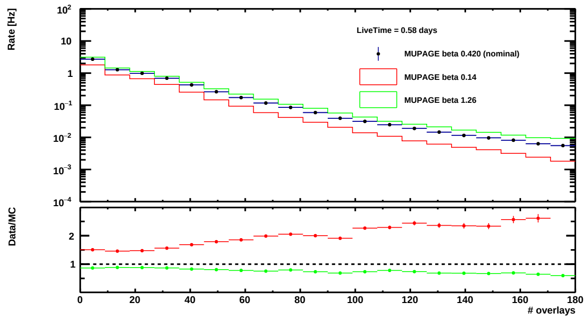
4.1 β



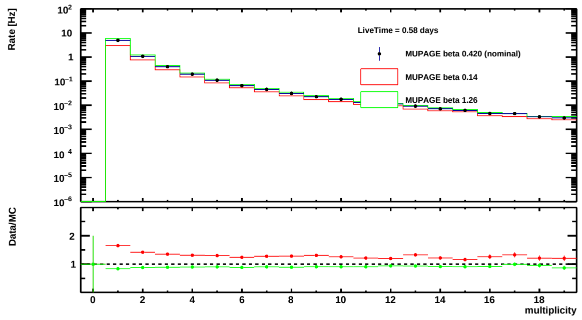
(a) beta - reconstructed zenith angle



(b) beta - reconstructed energy



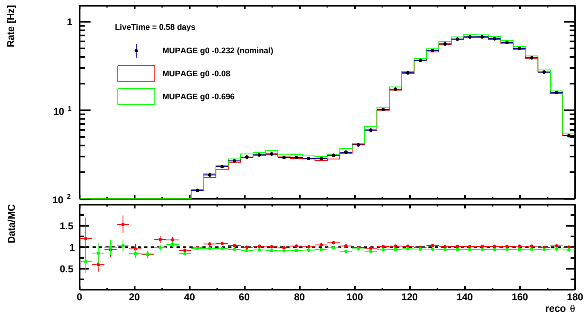
(c) beta - overlays



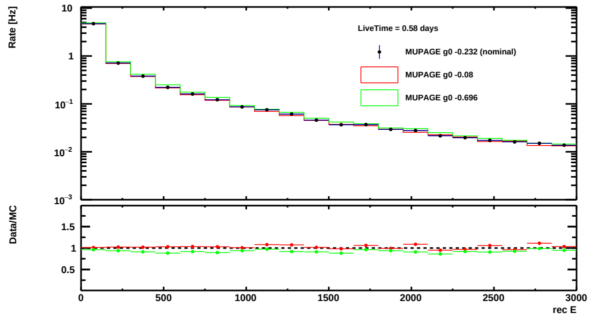
(d) alpha1b - multiplicity

Figure 16: beta for different observables

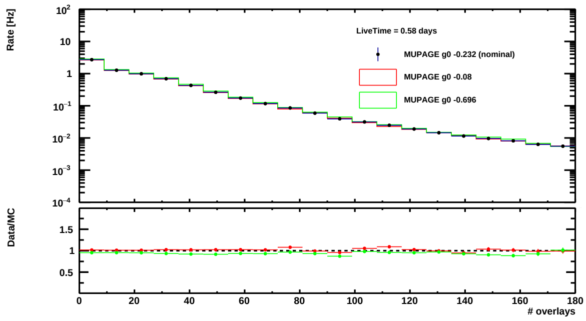
4.2 γ_0



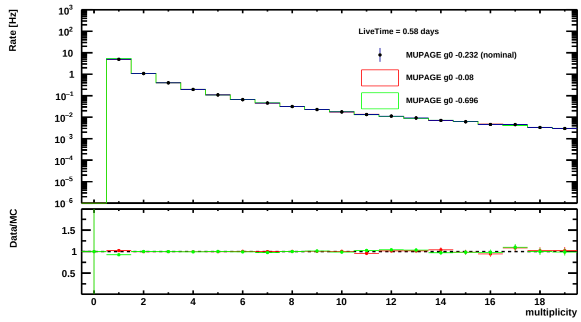
(a) g0 - reconstructed zenith angle



(b) g0 - reconstructed energy



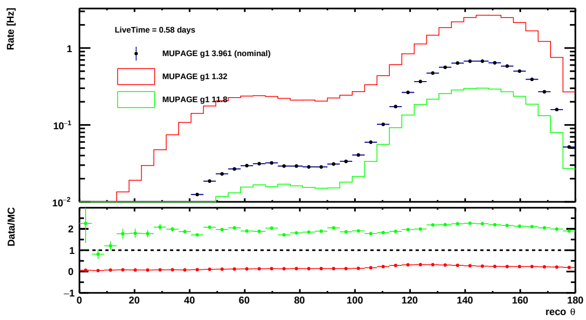
(c) g0 - overlays



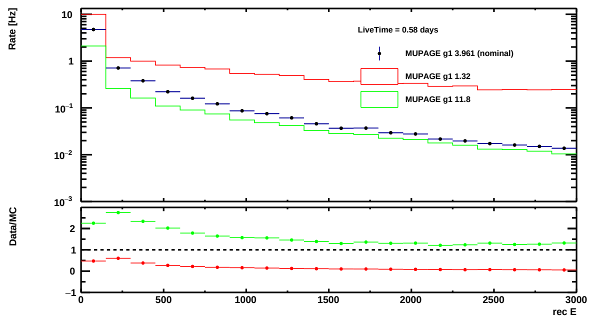
(d) g0 - multiplicity

Figure 17: g0 for different observables

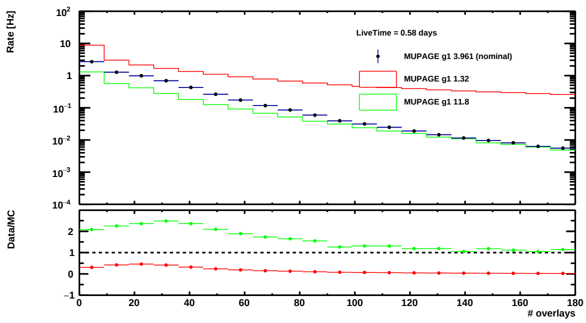
4.3 γ_1



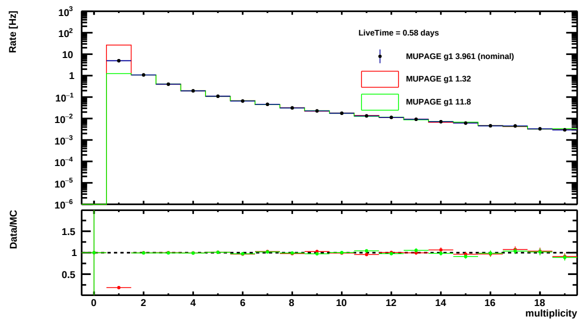
(a) g1 - reconstructed zenith angle



(b) g1 - reconstructed energy



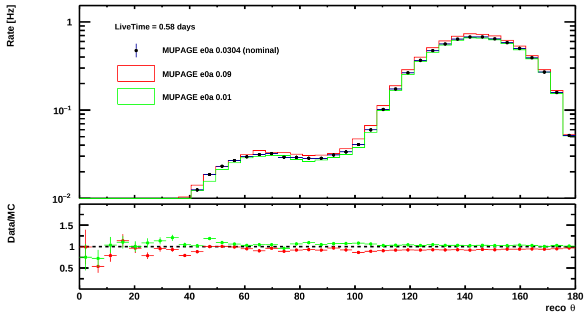
(c) g1 - overlays



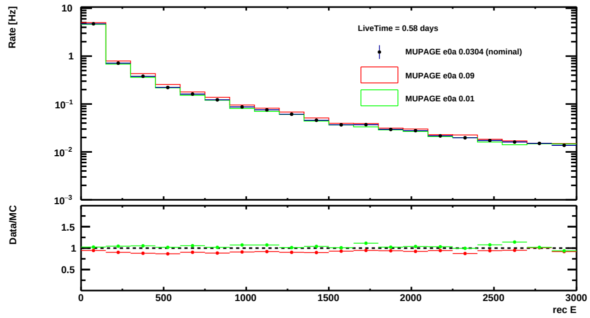
(d) g1 - multiplicity

Figure 18: g1 for different observables

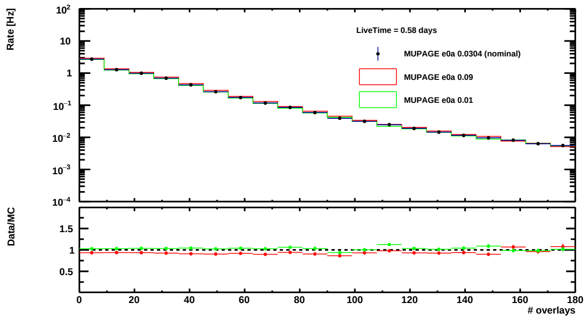
4.4 ϵ_{0a}



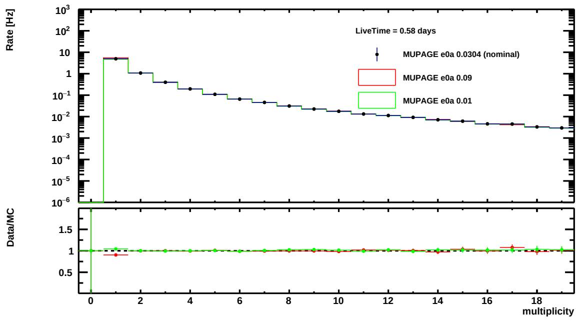
(a) e0a - reconstructed zenith angle



(b) e0a - reconstructed energy



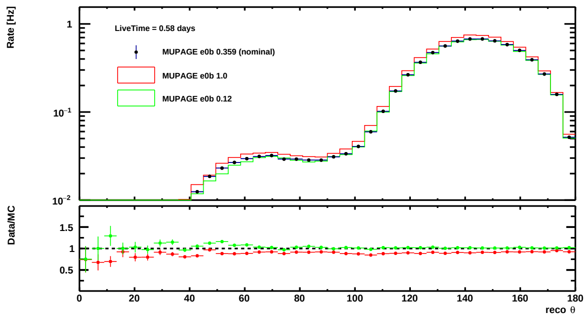
(c) e0a - overlays



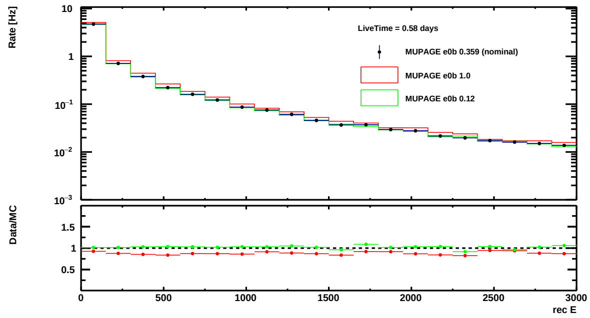
(d) e0a - multiplicity

Figure 19: e0a for different observables

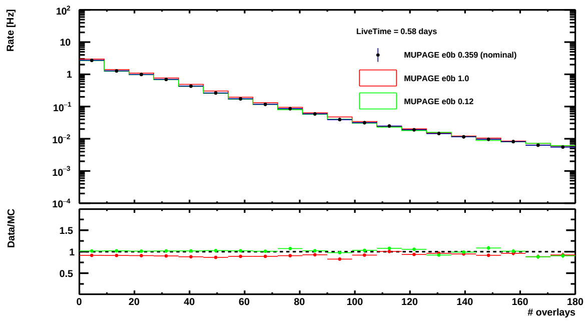
4.5 ϵ_{0b}



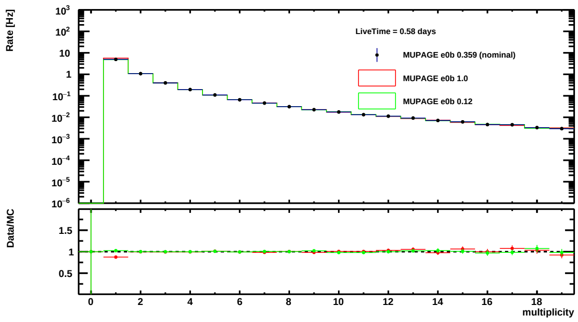
(a) e0b - reconstructed zenith angle



(b) e0b - reconstructed energy



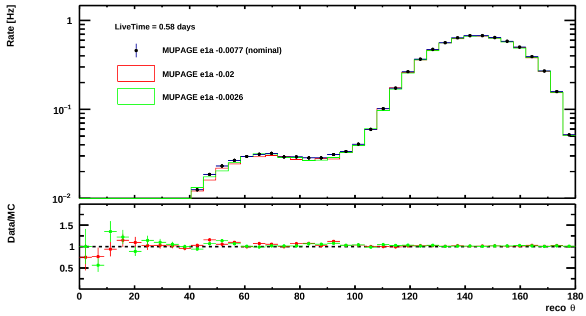
(c) e0b - overlays



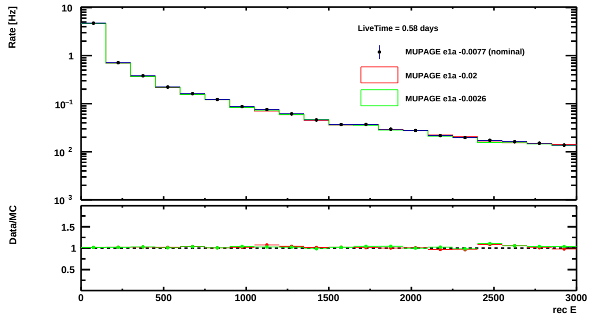
(d) e0b - multiplicity

Figure 20: e0b for different observables

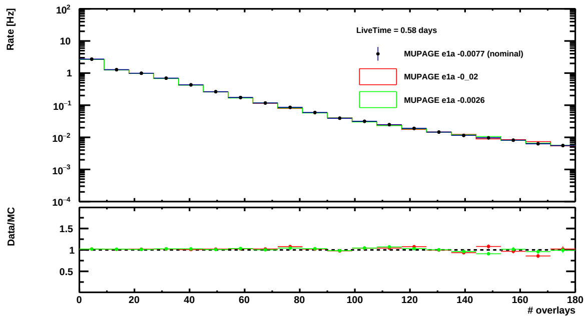
4.6 ϵ_{1a}



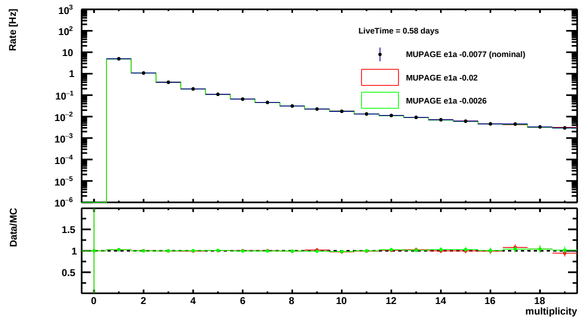
(a) e1a - reconstructed zenith angle



(b) e1a - reconstructed energy



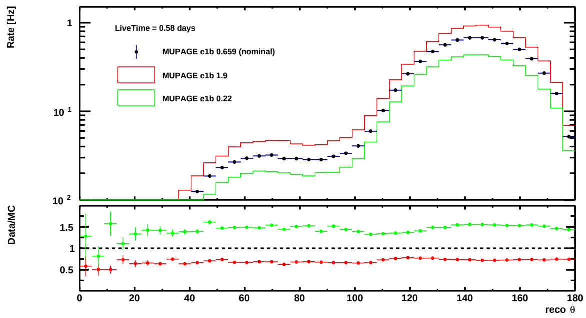
(c) e1a - overlays



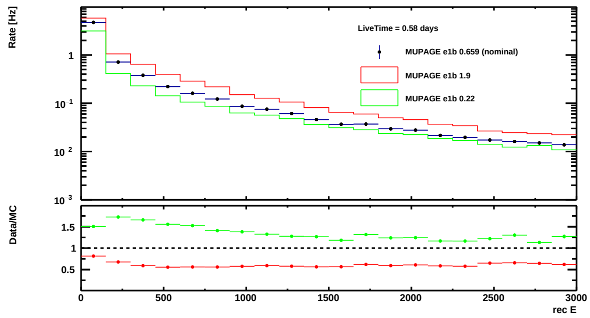
(d) e1a - multiplicity

Figure 21: e1a for different observables

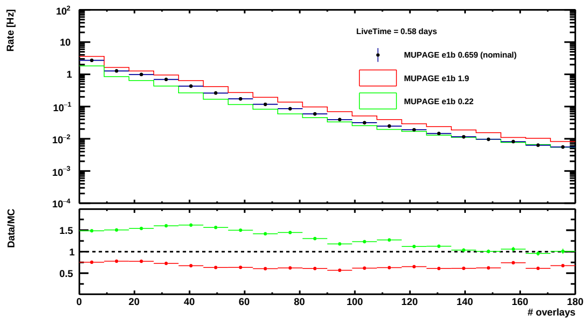
4.7 ϵ_{1b}



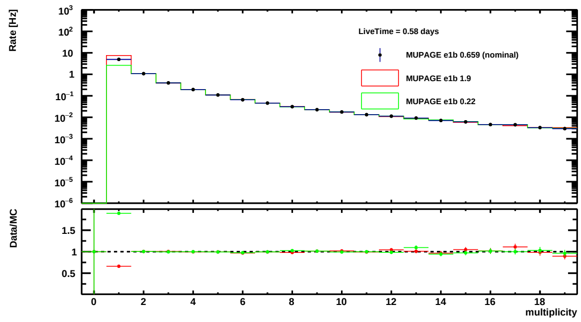
(a) e1b - reconstructed zenith angle



(b) e1b - reconstructed energy



(c) e1b - overlays



(d) e1b - multiplicity

Figure 22: e1b for different observables

5 Energy Spectrum (multiple muons)

Equation (17) for single muons applies to the bundle energy spectrum, with $\gamma^*, \beta^*, \epsilon^*$ (to distinguish from the single muon case) now dependent on the multiplicity M and the radial distance to the shower axis R.

The parameter γ^* is defined as follows in MUPAGE to parametrise the energy of the bundles.

$$\gamma^* = \gamma^*(R, h, M) = a(h) \cdot R + b(h, M) \cdot \left(1 - \frac{1}{2} e^{q(h) \cdot R}\right) , \tag{23}$$

where

$$a(h) = a_0 \cdot h + a_1 , \tag{24}$$

$$b(h, M) = b_0(M) \cdot h + b_1(M) , \tag{25}$$

with

$$b_0 = b_{0a} \cdot M + b_{0b} , \tag{26}$$

$$b_1 = b_{1a} \cdot M + b_{1b} . \tag{27}$$

The correction factor of eq. (23) has an additional parameter q:

$$q(h) = q_0 \cdot h + q_1 . \tag{28}$$

Finally, the parameter ϵ^* is described by

$$\epsilon^* = \epsilon^*(R, h, \theta) = c(R, h) \cdot \theta + d(R, h) \tag{29}$$

with

$$c(R, h) = c_0(h) \cdot e^{c_1 \cdot R} , \tag{30}$$

$$d(R, h) = d_0(h) \cdot R^{d_1} . \tag{31}$$

The variables within these equations are only a function of the depth h:

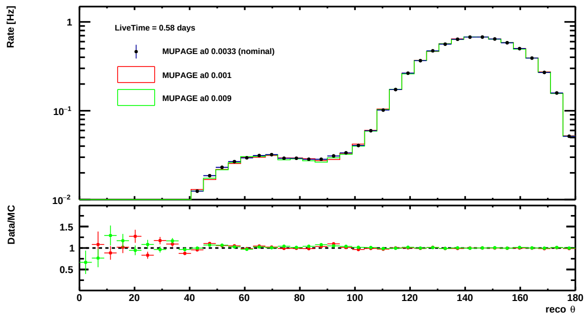
$$c_0 = c_{0a} \cdot h + c_{0b} , \tag{32}$$

$$d_0 = d_{0a} \cdot h + d_{0b} , \tag{33}$$

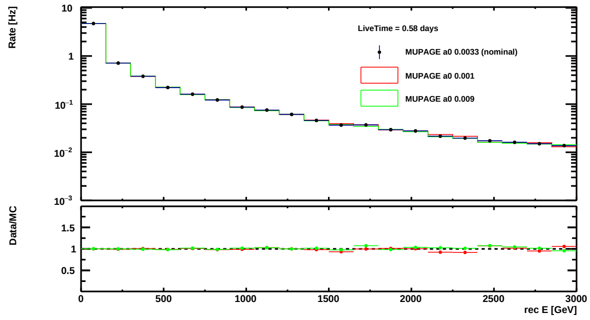
$$d_1 = d_{1a} \cdot h + d_{1b} , \tag{34}$$

while c_1 is a constant.

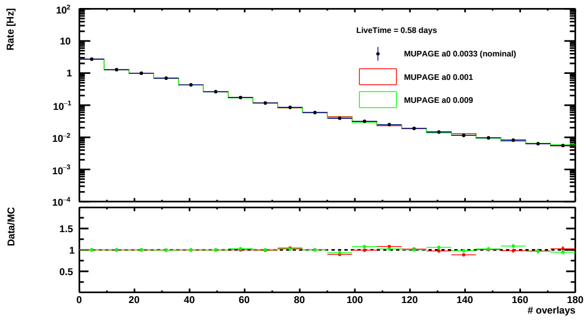
5.1 a_0



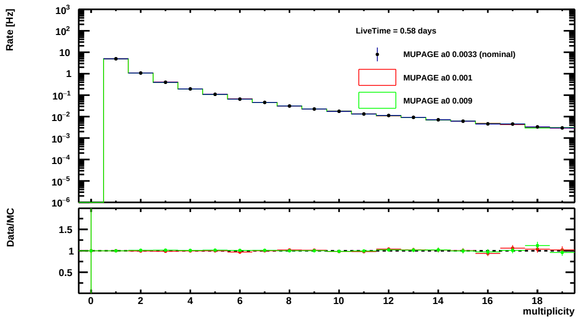
(a) a0 - reconstructed zenith angle



(b) a0 - reconstructed energy



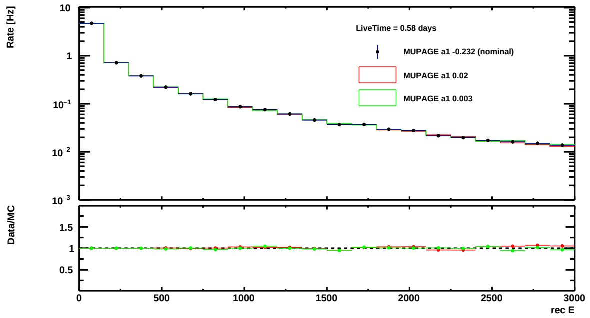
(c) a0 - overlays



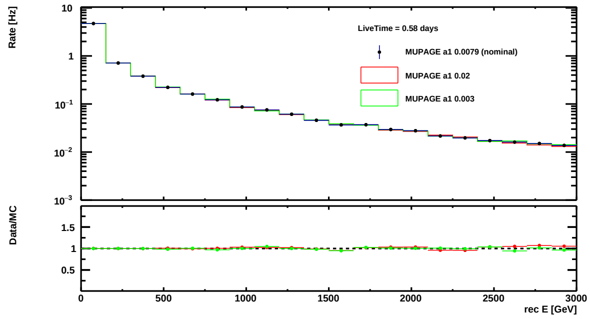
(d) a0 - multiplicity

Figure 23: a0 for different observables

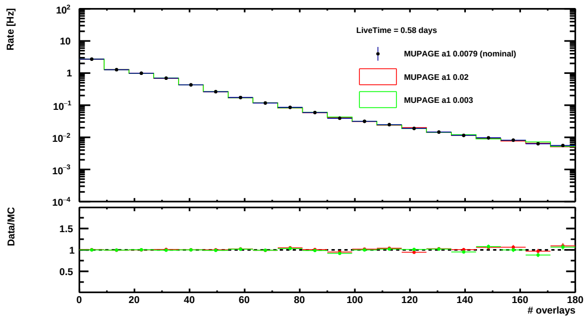
5.2 a_1



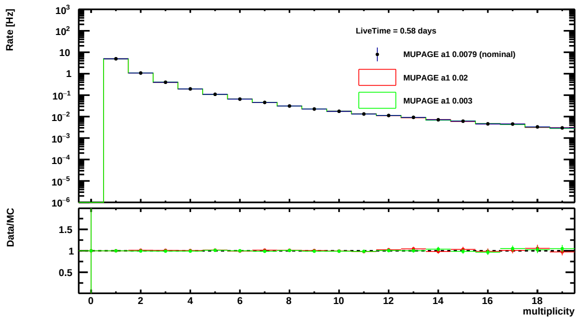
(a) a1 - reconstructed zenith angle



(b) a1 - reconstructed energy



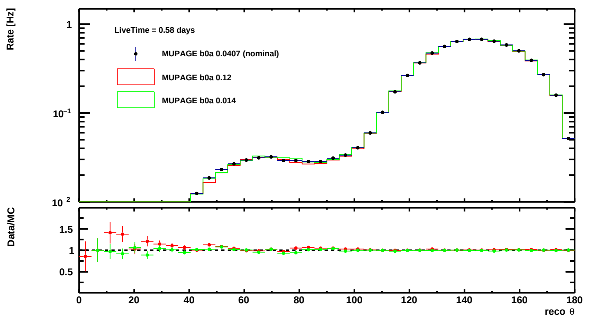
(c) a1 - overlays



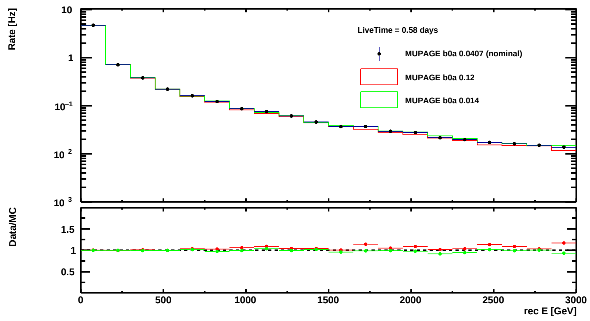
(d) a1 - multiplicity

Figure 24: a1 for different observables

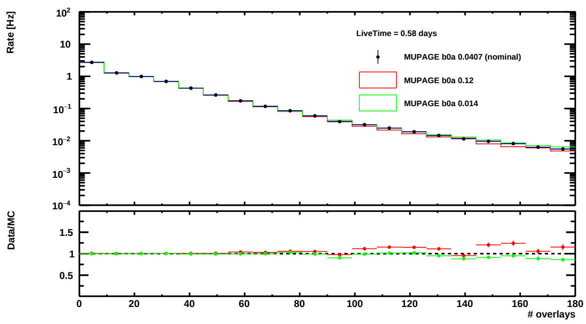
5.3 b_{0a}



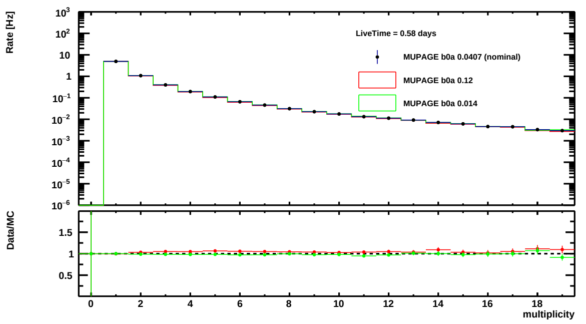
(a) b0a - reconstructed zenith angle



(b) b0a - reconstructed energy



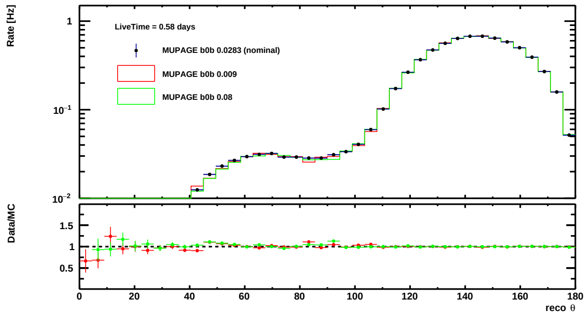
(c) b0a - overlays



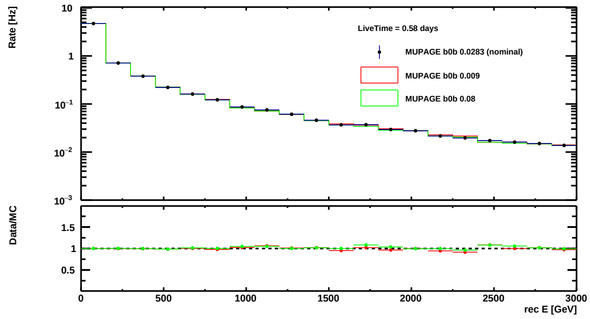
(d) b0a - multiplicity

Figure 25: b0a for different observables

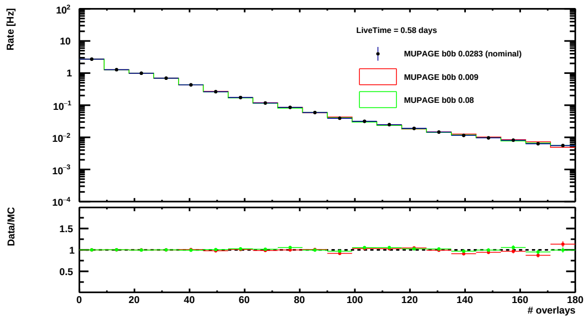
5.4 b_{0b}



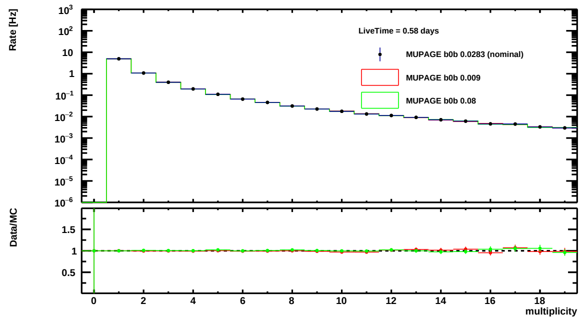
(a) b0a - reconstructed zenith angle



(b) b0b - reconstructed energy



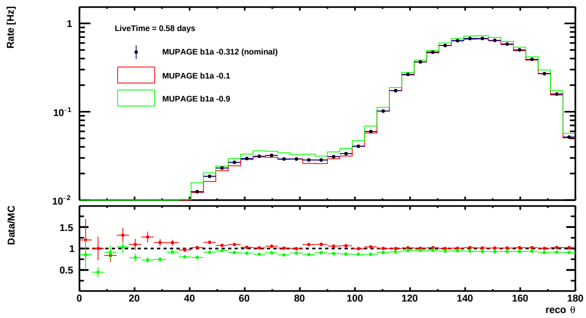
(c) b0b - overlays



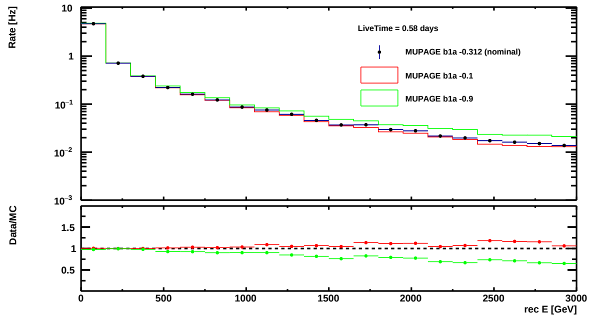
(d) b0b - multiplicity

Figure 26: b0b for different observables

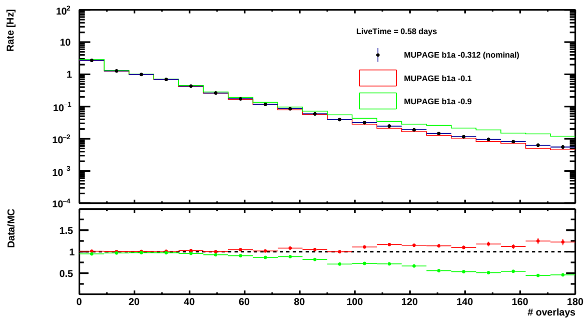
5.5 b_{1a}



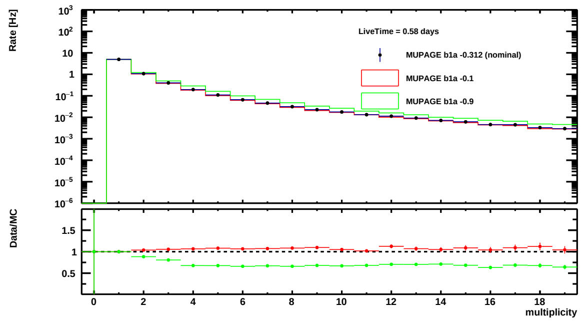
(a) b1a - reconstructed zenith angle



(b) b1a - reconstructed energy



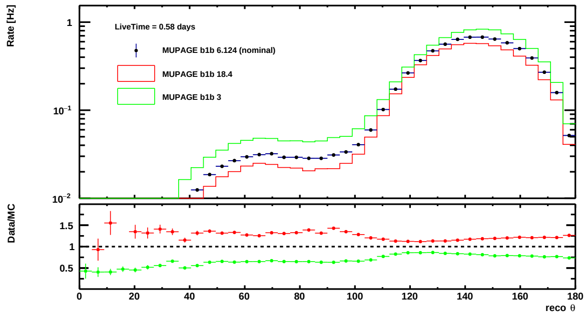
(c) b1a - overlays



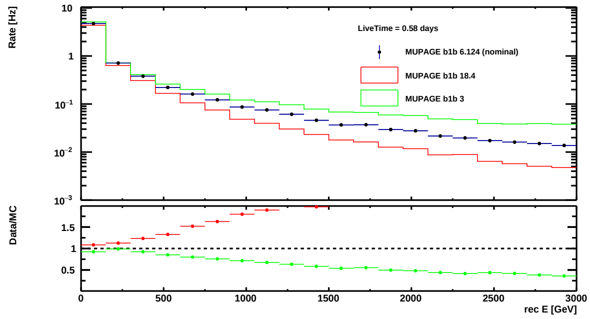
(d) b1a - multiplicity

Figure 27: b1a for different observables

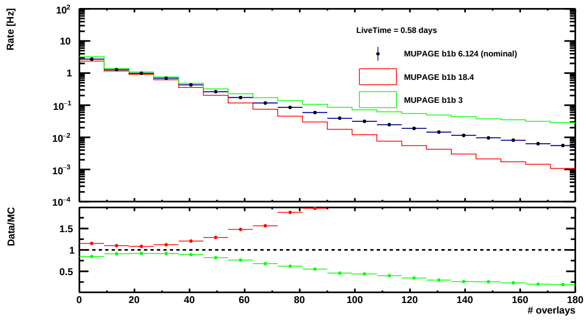
5.6 b_{1b}



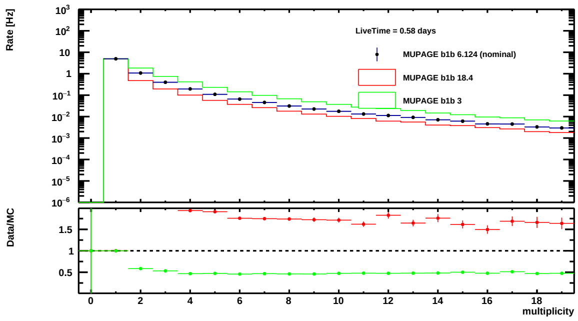
(a) b1b - reconstructed zenith angle



(b) b1b - reconstructed energy



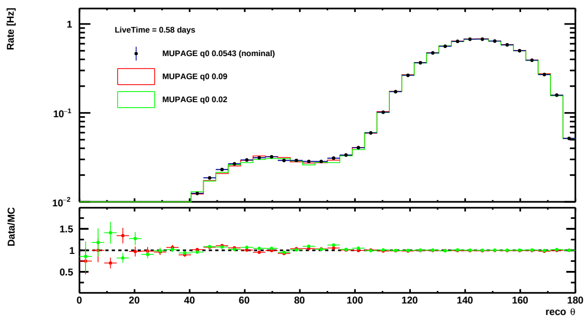
(c) b1b - overlays



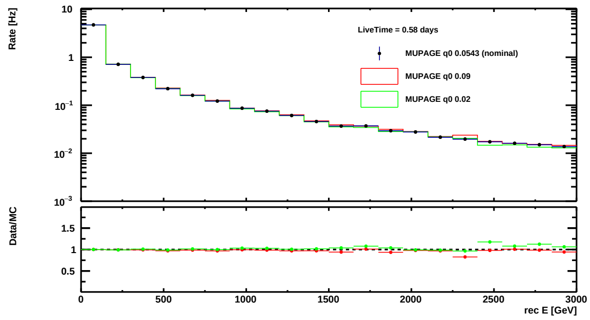
(d) b1b - multiplicity

Figure 28: b1b for different observables

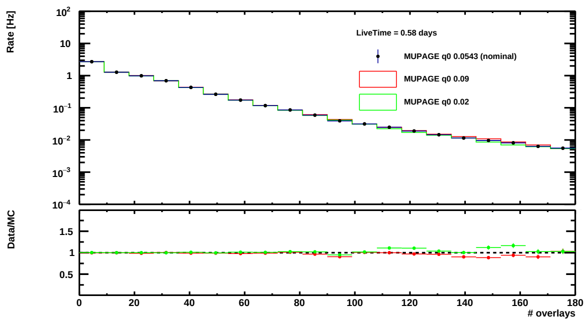
5.7 q_0



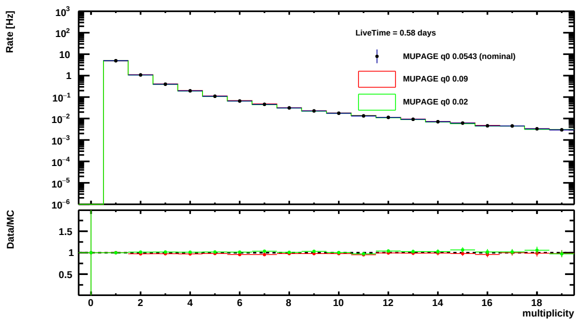
(a) q0 - reconstructed zenith angle



(b) q0 - reconstructed energy



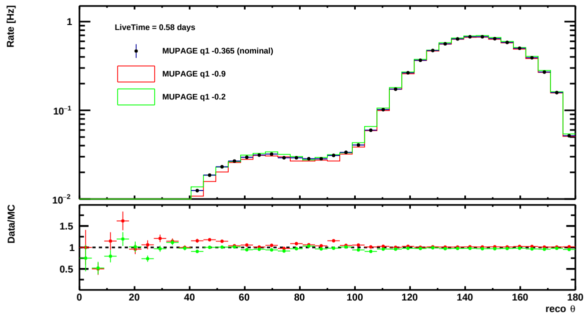
(c) q0 - overlays



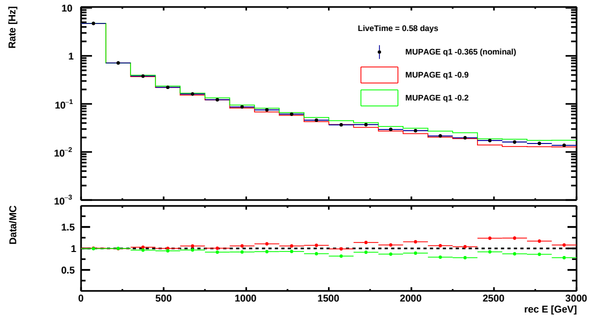
(d) q0 - multiplicity

Figure 29: q0 for different observables

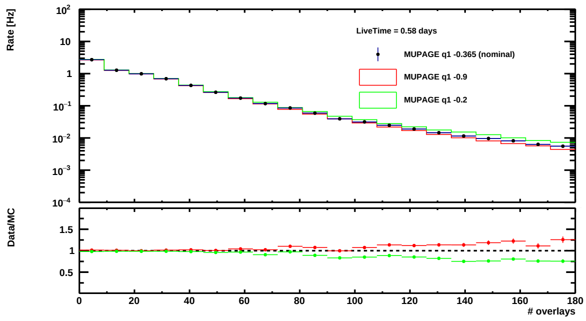
5.8 q_1



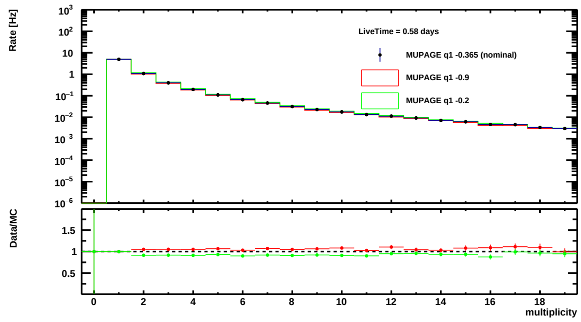
(a) q1 - reconstructed zenith angle



(b) q1 - reconstructed energy



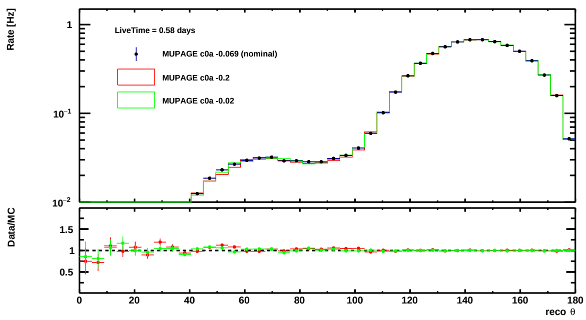
(c) q1 - overlays



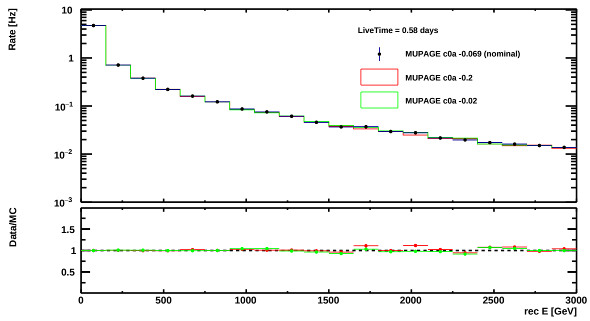
(d) q1 - multiplicity

Figure 30: q1 for different observables

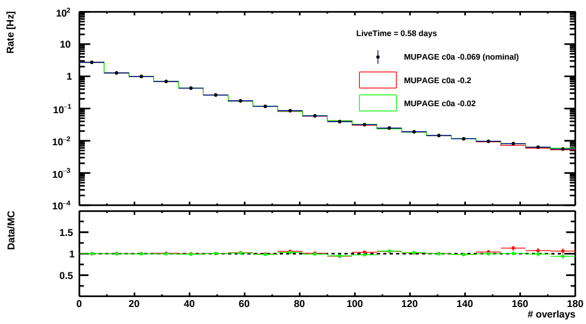
5.9 c_{0a}



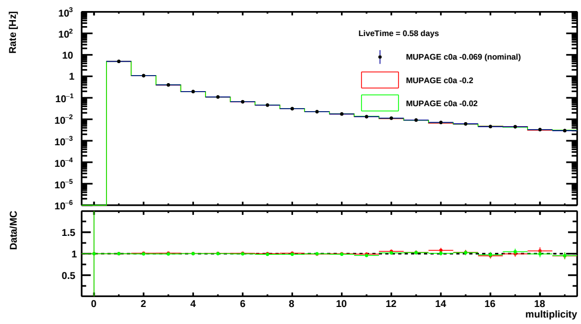
(a) c0a - reconstructed zenith angle



(b) c0a - reconstructed energy



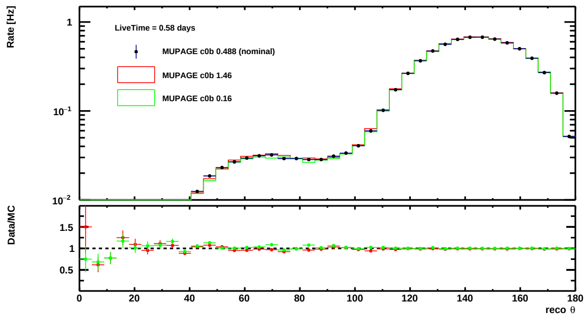
(c) c0a - overlays



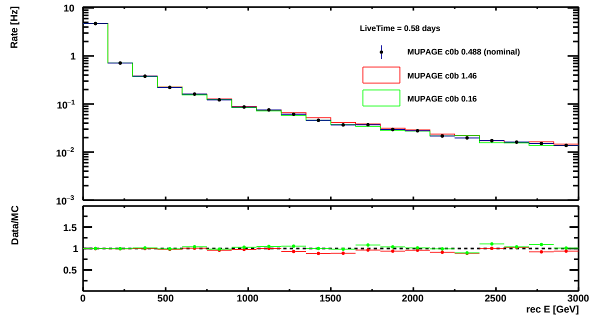
(d) c0a - multiplicity

Figure 31: c0a for different observables

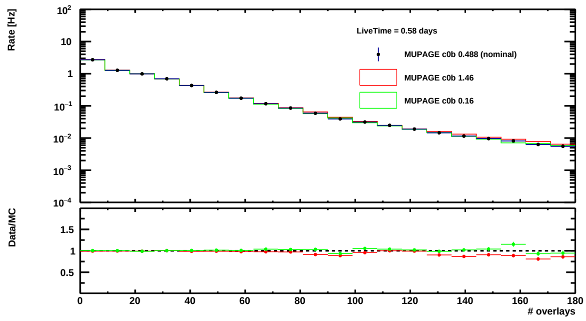
5.10 c_{0b}



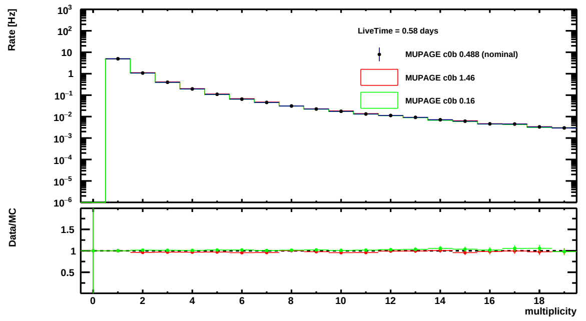
(a) c0b - reconstructed zenith angle



(b) c0b - reconstructed energy



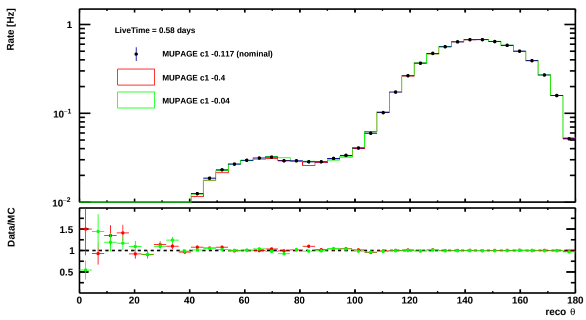
(c) c0b - overlays



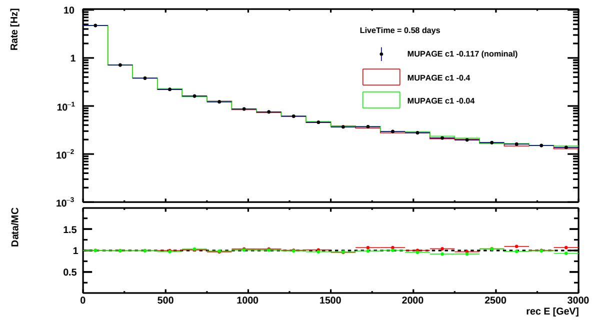
(d) c0b - multiplicity

Figure 32: c0b for different observables

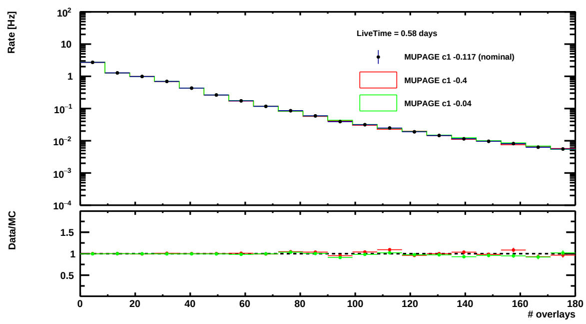
5.11 c_1



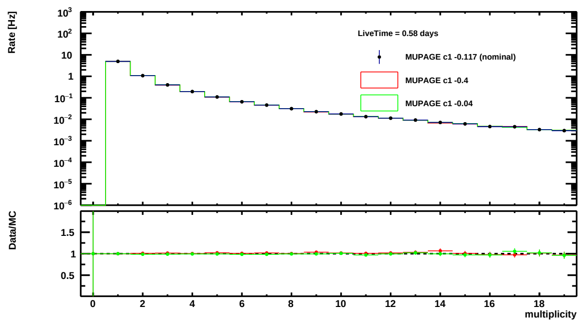
(a) c1 - reconstructed zenith angle



(b) c1 - reconstructed energy



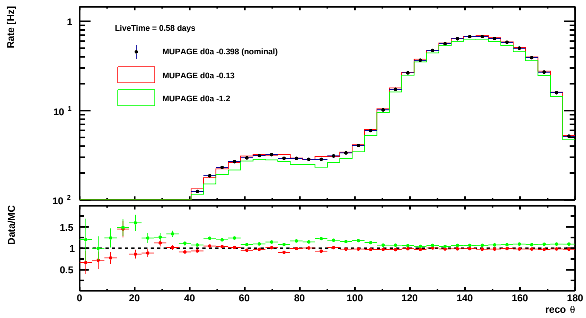
(c) c1 - overlays



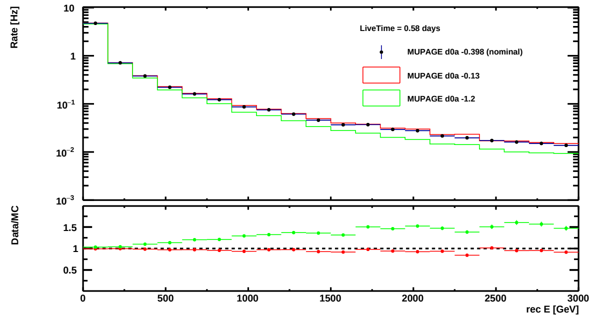
(d) c1 - multiplicity

Figure 33: c1 for different observables

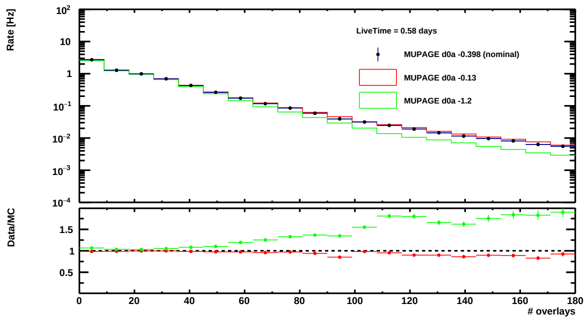
5.12 d_{0a}



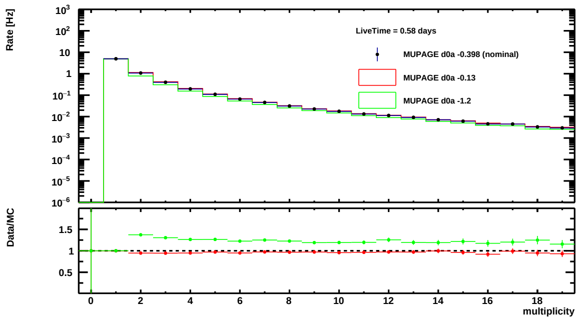
(a) d0a - reconstructed zenith angle



(b) d0a - reconstructed energy



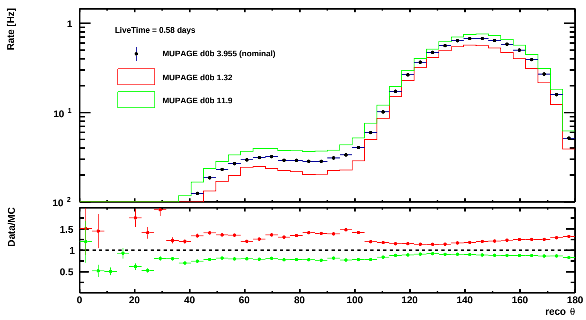
(c) d0a - overlays



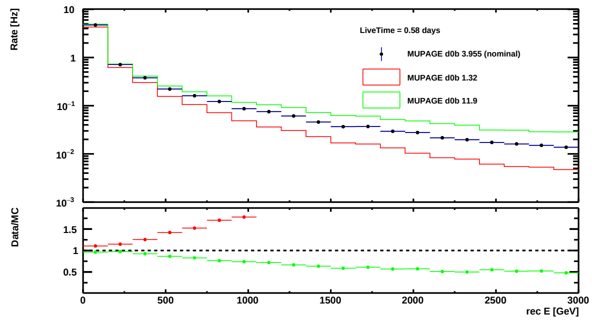
(d) d0a - multiplicity

Figure 34: d0a for different observables

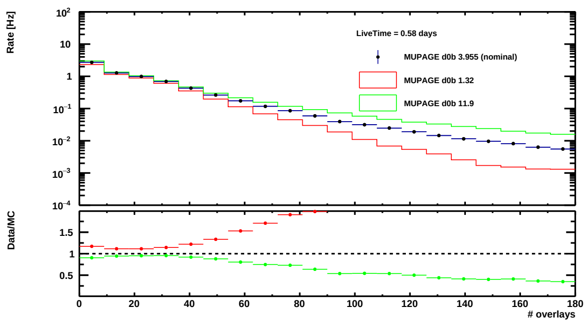
5.13 d_{0b}



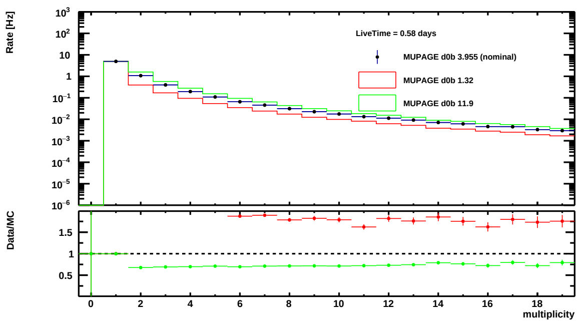
(a) d0b - reconstructed zenith angle



(b) d0b - reconstructed energy



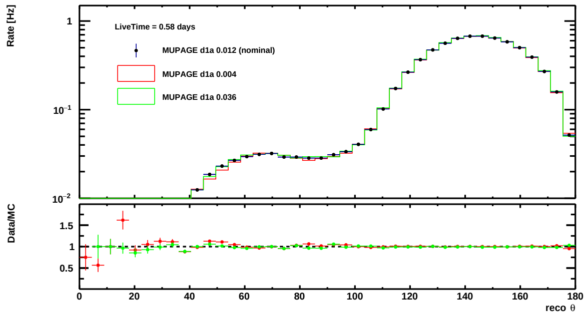
(c) d0b - overlays



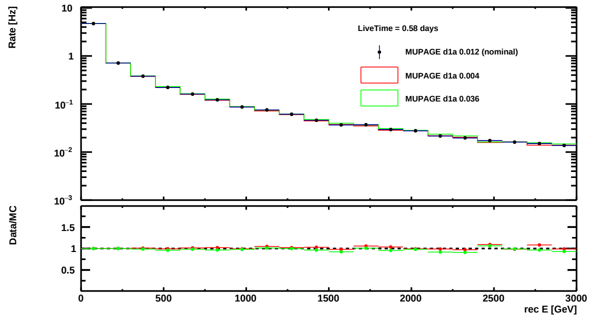
(d) d0b - multiplicity

Figure 35: d0b for different observables

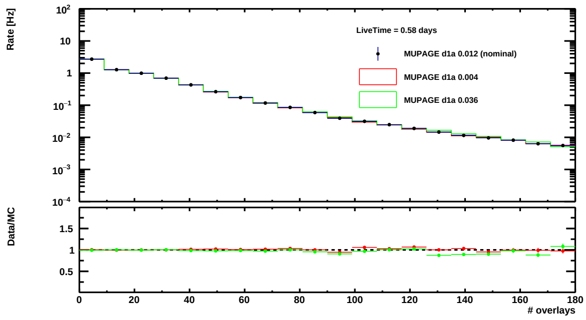
5.14 d_{1a}



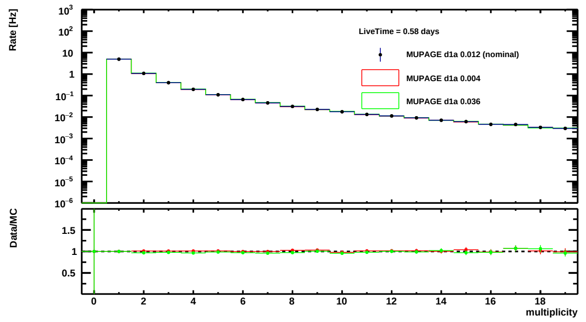
(a) d1a - reconstructed zenith angle



(b) d1a - reconstructed energy



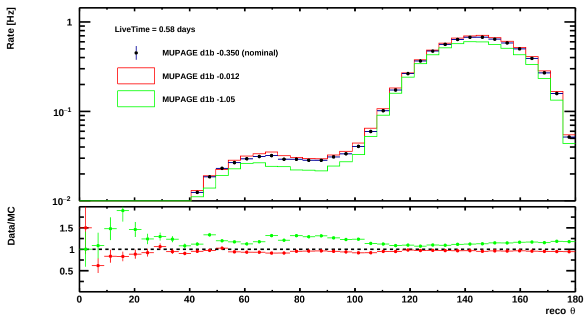
(c) d1a - overlays



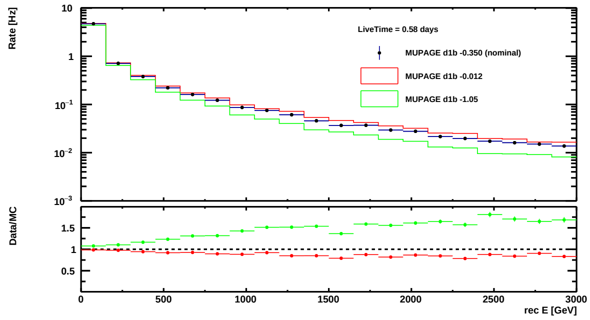
(d) d1a - multiplicity

Figure 36: d1a for different observables

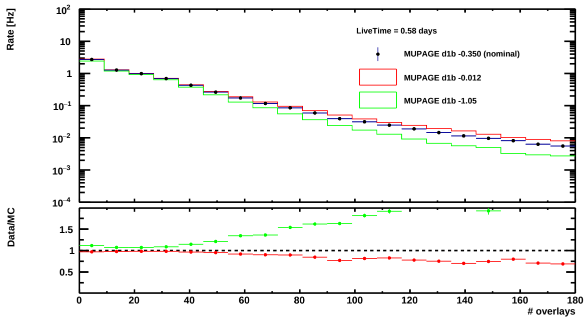
5.15 d_{1b}



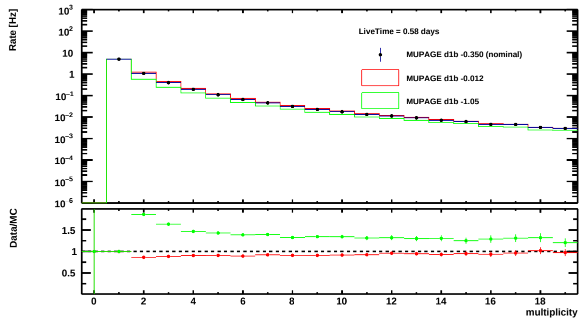
(a) d1b - reconstructed zenith angle



(b) d1b - reconstructed energy



(c) d1b - overlays



(d) d1b - multiplicity

Figure 37: d1b for different observables

6 Discussion & Conclusions

This document provides a first look at the 40 different parameters within MUPAGE, and their impact on physical observables when varied. The plots are normalised to the livetime, giving the rate, however not to the total number of events, which allows for a better check of the shape. Also, the parameters are varied individually; it is clear that varying more than one parameter will cause further changes to the distributions - e.g. parameters pertaining to the lateral spread **and** the muon bundle energy spectrum simultaneously. It is clear by eye that some parameters have a large effect on the observables, and others have (significantly) less effect. With the intention to tune these parameters to correctly describe the data, it is important to understand the impact of all these parameters individually on the physical observables from the detector.

References

[1] G. Carminati et al. "Atmospheric MUons from PArametric formulas: a fast GEnerator for neutrino telescopes (MUPAGE)". In: *Computer Physics Communications* 179.12 (Dec. 2008), pp. 915–923. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2008.07.014. URL: http://dx.doi.org/10.1016/j.cpc.2008.07.014.

[2] Y. Becherini et al. "A parameterisation of single and multiple muons in the deep water or ice". In: *Astroparticle Physics* 25.1 (2006), pp. 1–13. ISSN: 0927-6505. DOI: https://doi.org/10.1016/j.astropartphys.2005.10.005. URL: http://www.sciencedirect.com/science/article/pii/S092765050500157X.