On the linearization of model fit to acoustic data

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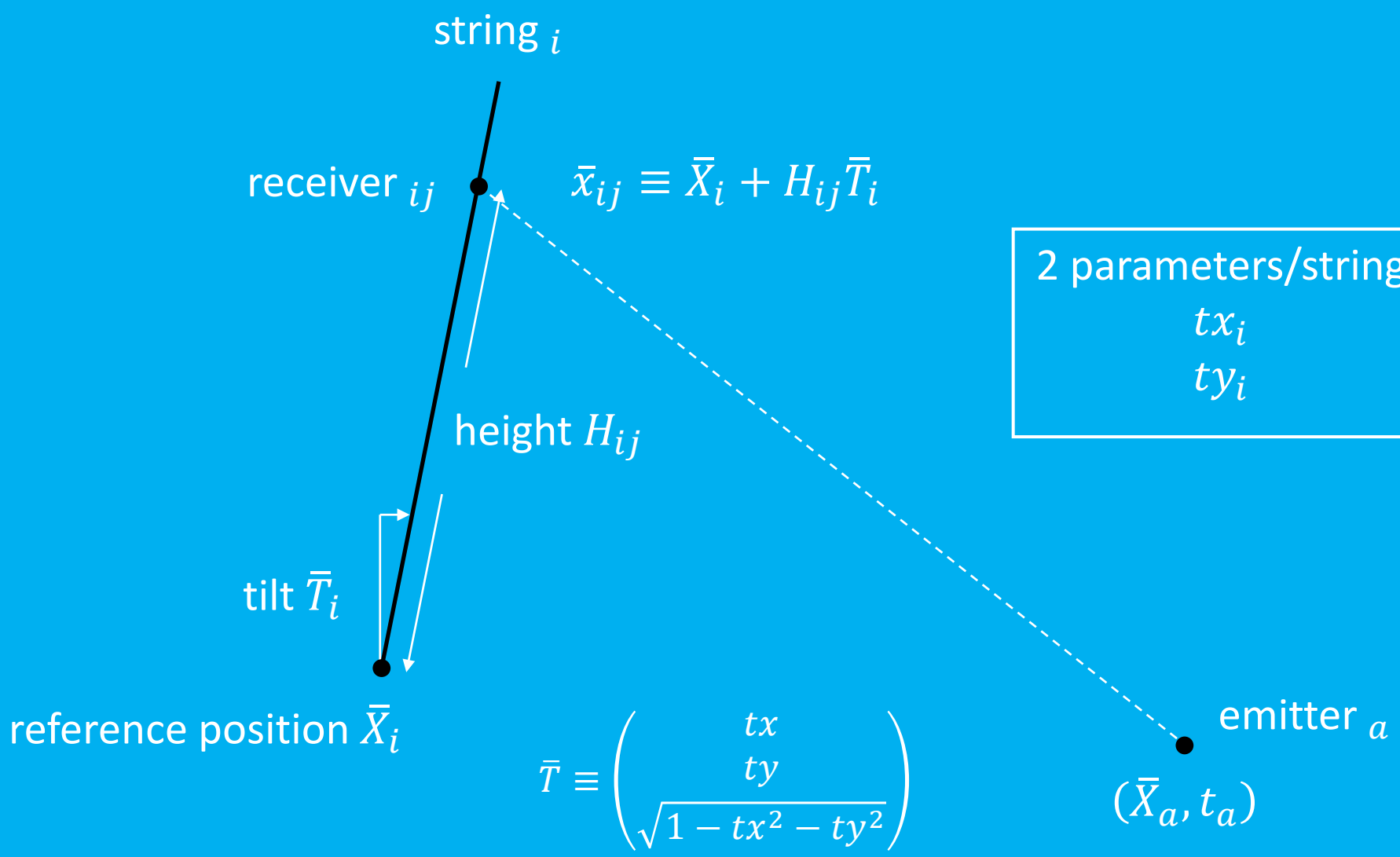
Introduction

- Assuming straight strings (and possibly other shapes), can dependence of arrival time on model parameters be linearized?

- If so, shapes of <u>all</u> strings can be determined in one step by simple matrix inversion

Model parameters

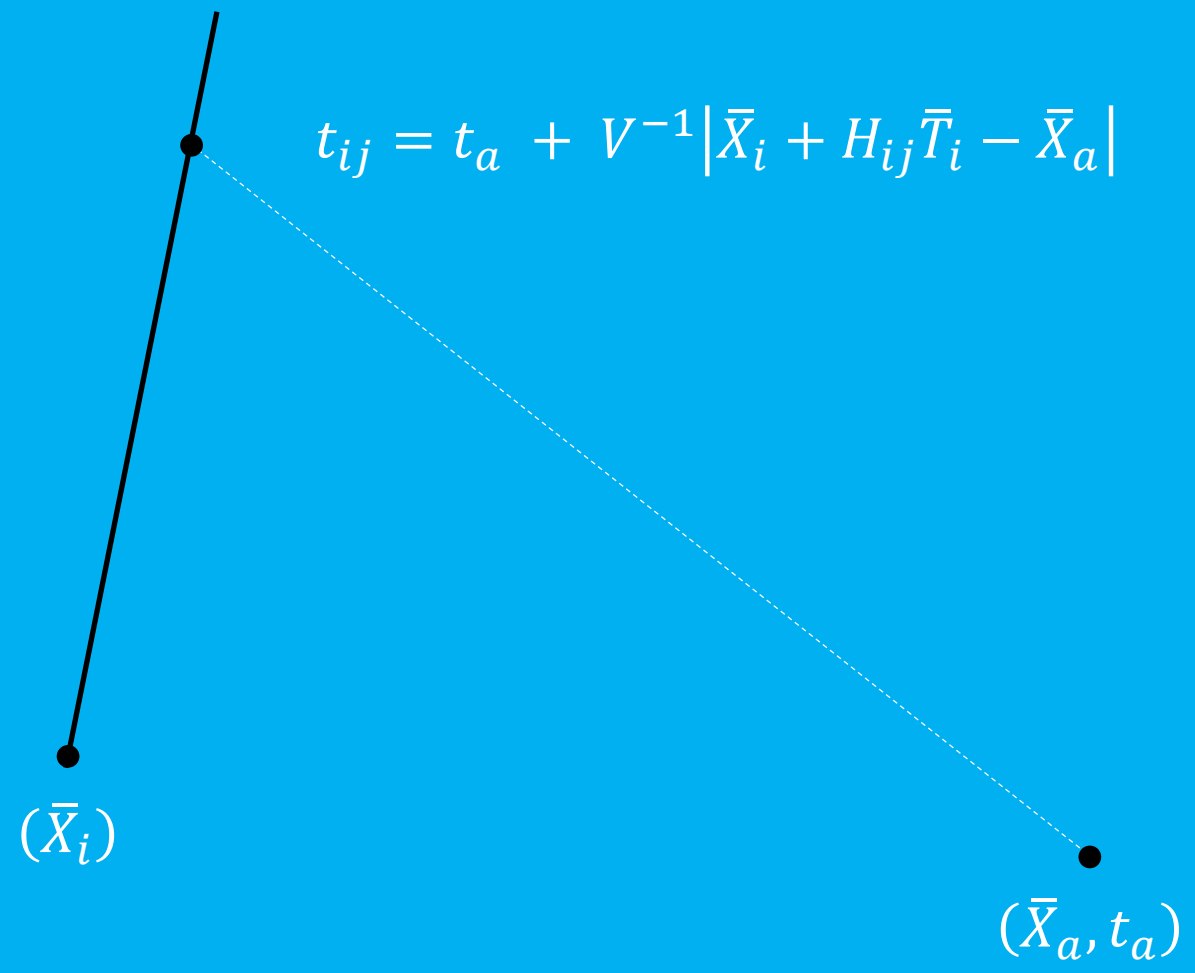


2 parameters/string	1 parameter/ping
tx_i	t_a
ty_i	

time-of-arrival

$$t_{ij} = t_a + V^{-1}\left|\bar{X}_i + H_{ij}\bar{T}_i - \bar{X}_a\right|$$

V = velocity of sound



approximations

$$\sqrt{1+x} \cong 1 + \frac{1}{2}x$$

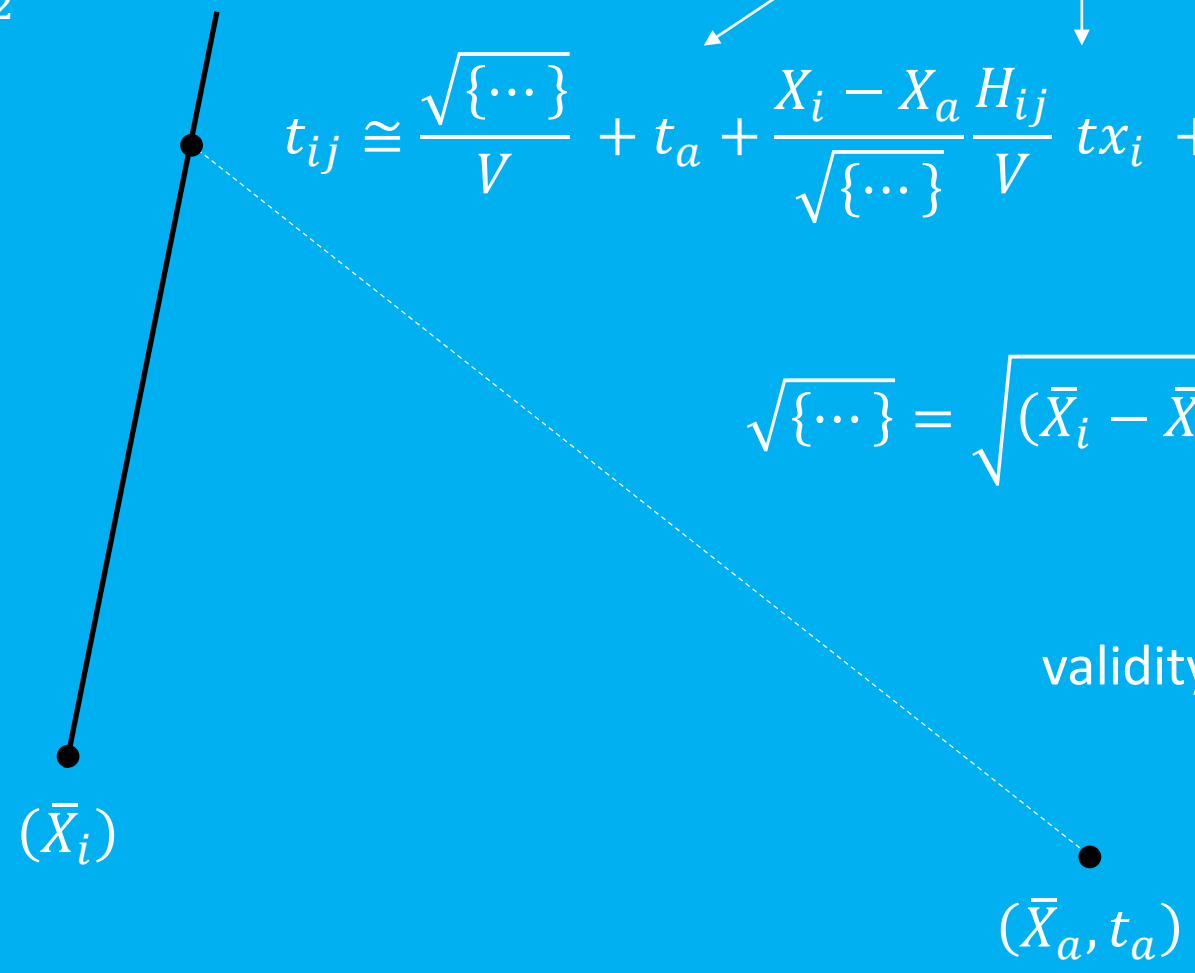
$$\bar{T} \cong \begin{pmatrix} tx \\ ty \\ 1 \end{pmatrix}$$

linear dependence

$$t_{ij} \cong \frac{\sqrt{\{\cdots\}}}{V} + t_a + \frac{X_i - X_a}{\sqrt{\{\cdots\}}}\frac{H_{ij}}{V}\, tx_i + \frac{Y_i - Y_a}{\sqrt{\{\cdots\}}}\frac{H_{ij}}{V}\, ty_i$$

$$\sqrt{\{\cdots\}} = \sqrt{(\bar{X}_i - \bar{X}_a)^2 + H_{ij}^2 + 2(Z_i - Z_a)H_{ij}}$$

validity $\sqrt{\{\cdots\}} \gg H_{ij}\, tx_i\,,\, H_{ij}\, ty_i$



Solution

$$\bar{\theta} \equiv (t_a\,, t_b\,, \cdots, tx_i\,, ty_i\,, tx_{i+1}\,, ty_{i+i}\,, \cdots)$$

$$\bar{H} \equiv \begin{pmatrix} 1 \cdots & \cdots & \cdots \\ 1 \cdots & \dfrac{X_i - X_a}{\sqrt{\{\cdots\}}}\dfrac{H_{ij}}{V} \cdots & \dfrac{Y_i - Y_a}{\sqrt{\{\cdots\}}}\dfrac{H_{ij}}{V} \cdots \\ 1 \cdots & \cdots & \cdots \end{pmatrix} \quad \Downarrow \quad \bar{Y} \equiv \begin{pmatrix} \cdots \\ t_{ij} - \dfrac{\sqrt{\{\cdots\}}}{V} \\ \cdots \end{pmatrix}$$

$$\hat{\theta} = (\bar{H}^T \bar{H})^{-1}\, \bar{H}^T \bar{Y}$$

Conclusions

- Assuming straight line, dependence of arrival time on model parameters can accurately be linearized

- As a consequence, shapes of <u>all</u> strings can be determined in one step by simple matrix inversion