Deflections of Neutrinos in the Earth

Jordan Seneca

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In this document, the idea that deflection of point source neutrinos in the Earth could result in a change of flux at the detector will be tested. It is found that deflections have a negligible effect on the flux at the detector under realistic conditions. With extreme deflection angles, one effect due to deflections could yield an increase or decrease of the flux depending on the position of the detector and details of the deflection, while another effect due to deflections and absorptions would yield a decrease in the flux with respect to a situation without deflections.

Situation

We imagine a plane of source neutrinos with positions $\mathbf{p} \in (0, [-R_S, R_S], [-R_S, R_S])$, all with their directions going in the $\mathbf{v} = (1, 0, 0)$ direction. The detector is at the $\mathbf{p} = (2R_\oplus, 0, 0)$ position, see Fig.1.

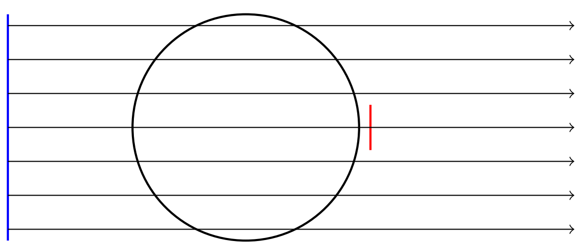

Figure 1: Schematic of plane source of neutrinos (blue) onto the Earth (black) and detector (red).

We will concentrate on the case given by Table 1. We'll inject 1 M neutrinos for every case.

R_S	10 km
R_D	0.5 km
$\langle\alpha\rangle$	10^{-5}
$\langle\Delta\rangle$	$R_\oplus$
$\langle A\rangle$	$2R_\oplus$

Table 1: Setup for the toy simulations, where R_S is the size of the source, R_D is the radius of the detector, $\langle\alpha\rangle$ is the mean deflection angle, $\langle\Delta\rangle$ is the mean deflection free path, and $\langle A\rangle$ is the mean absorption free path.

Absent Earth

Let's remove the Earth for now, to see what kind of flux we expect from empty space. In Fig.2, we clearly see that the initial flux and survival flux is identical. Indeed, there was nothing to absorb or deflect the neutrinos, so they all went in a straight line to the detector

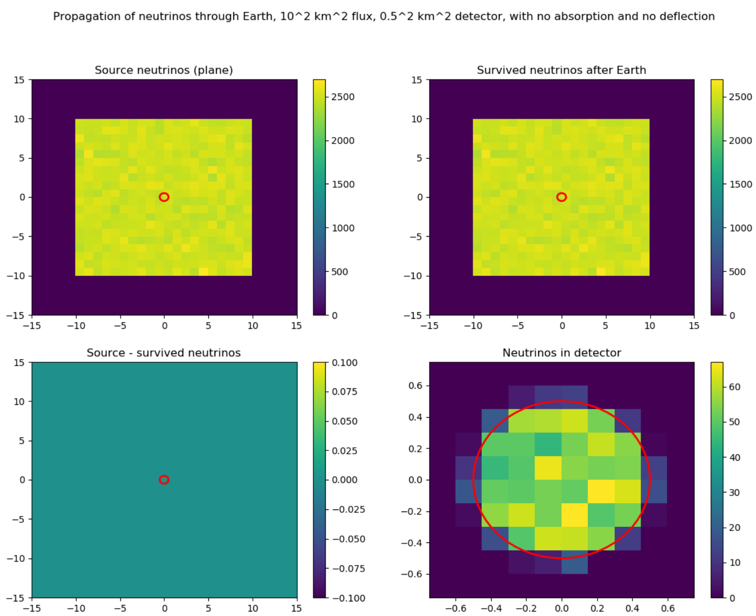

Figure 2: Without absorption nor deflections taken into account, the in-going and out-going flux is identical. About 2000 events are recorded in the detector.

plane. 1947 of those made it into the detector, which is roughly $\frac{1}{500}$ of the injected events, which makes sense since the area of the source is about 500 times greater than that of the detector.

Opaque Earth

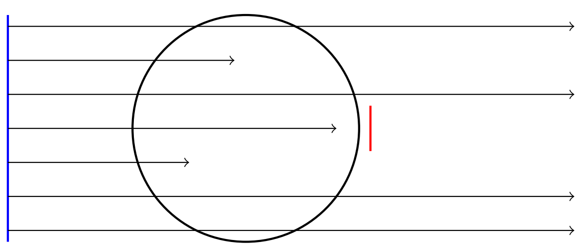

Figure 3: Schematic of plane source of neutrinos (blue) onto the Earth (black) and detector (red), with some neutrinos absorbed.

For our second case, neutrinos can only be absorbed but not deflected. In Fig.4, we see that roughly half of the neutrinos were absorbed, and 729 neutrinos made it through to the detector. For an exponential decay of the flux of neutrinos, we expect $1 - e^{-x/\lambda} \simeq 37\%$ of neutrinos to survive at a distance equal to the mean free path, which is what we see here, give or take 10 neutrinos.

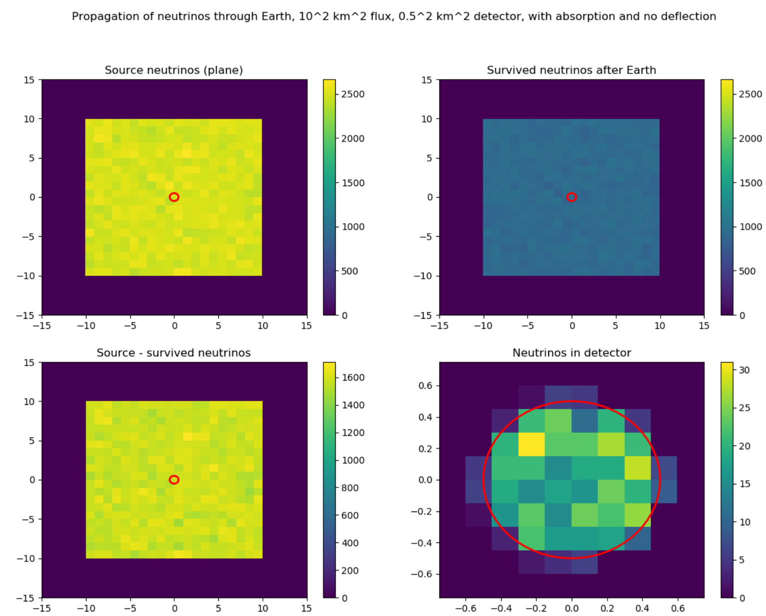

Figure 4: When absorption is taken into account, a large fraction of the in-going flux is missing.

Transparent Earth

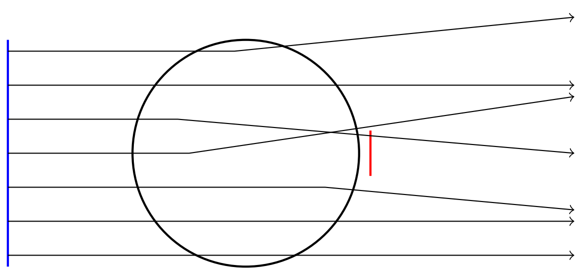

Figure 5: Schematic of plane source of neutrinos (blue) onto the Earth (black) and detector (red), with some neutrinos deflecting in the Earth.

In this third case, we make the Earth transparent to neutrinos, meaning that neutrinos cannot be absorbed by it, but can deflect in it. As can be seen in Fig.6, the survived neutrinos look a lot like the original neutrinos, and we get 1938 neutrinos in the detector. Notice however that they have been scrambled, and some of them have been scattered outside of the original area of the source. This effect is seen more evidently when a larger scattering angle was chosen, see Fig. 7.

We find ourselves in a situation where on the other side of the Earth, we have a plane of neutrinos, each of which have been deflected according to some probability density function (p.d.f.). Assuming that the Earth is both much larger than the size of the deflection and the detector, we can locally assume that each neutrino

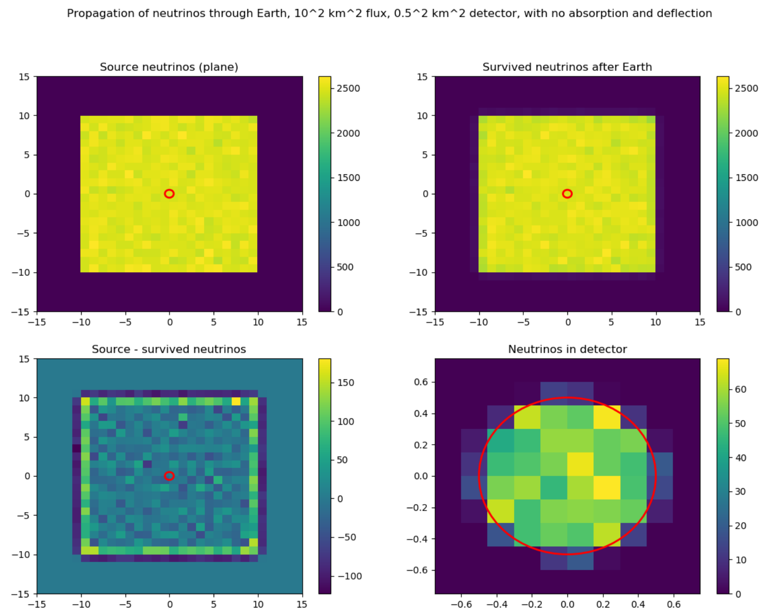

Figure 6: When only small deflections are taken into account, the flux is scrambled but its overall magnitude unchanged, except on the edge of the out-going flux, where a slight smearing can be observed.

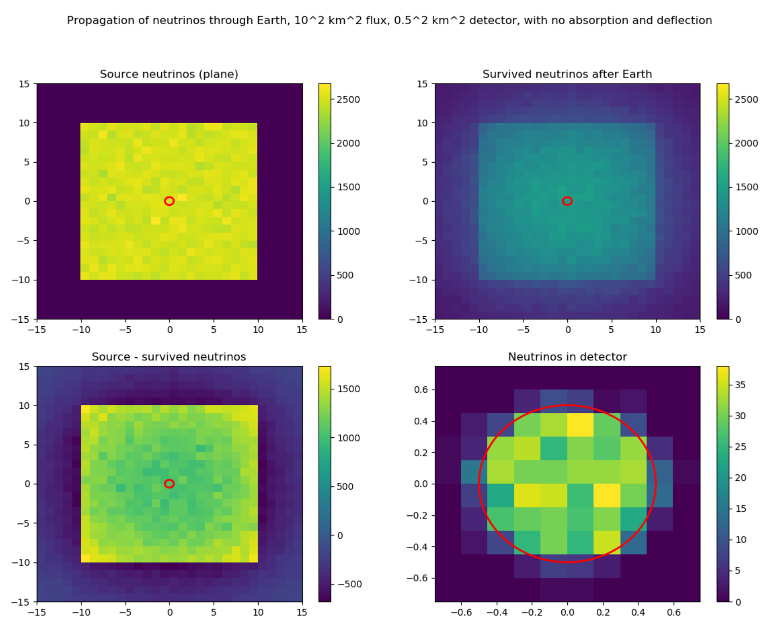

Figure 7: Flux of neutrinos with only deflections taken into account, with a sprea of 10^{-3} radians.

is deflected according to the same p.d.f. centered on their original direction, and that neutrinos can come from an infinite plane source. We propose that these conditions imply that any p.d.f. (including a δ-distribution, i.e. no deflections) will yield the same expected number of neutrinos in the detector.

Proposition 1. *Let f be a p.d.f. which is continuous and translation invariant over all space. The expected number of particles obtained within a region R lying in a uniform and infinite plane of particles which are spread by f is independent on f.*

Proof. In general, expected number of particles to land in R, N_R is

$$E[N_R] = \iint\limits_S n(\mathbf{x_0}) P_{\in R}(\mathbf{x_0}) \mathrm{d}\mathbf{x_0}, \tag{1}$$

where n is the density in the plane, $P_{\in R}$ is the probability to end up in R, and $\mathbf{x_0}$ is the position at which the contribution towards the particles landing in R is being evaluated. The probability for a particle to end up in R is the integral over R of the p.d.f., $f(\mathbf{x_0}, \mathbf{x_R})$, of the particle to end up at a certain point in the plane,

$$P_{\in R} = \iint\limits_R f(\mathbf{x_0}, \mathbf{x_R}) \mathrm{d}\mathbf{x_R}, \tag{2}$$

where $\mathbf{x_R}$ is the position being evaluated in R. The general expression for $E[N_R]$ is then

$$E[N_R] = \iint\limits_S n(\mathbf{x_0}) \iint\limits_R f(\mathbf{x_0}, \mathbf{x_R}) \mathrm{d}\mathbf{x_R} \mathrm{d}\mathbf{x_0}, \tag{3}$$

If the number density is constant across the plane, $n(\mathbf{x_0}) \equiv n_0$. If f is translation invariant, it depends only on the difference between the position of contribution and evaluation in R, and takes the form $f(\Delta\mathbf{x})$, where $\Delta\mathbf{x} \equiv \mathbf{x_0} - \mathbf{x_R}$. We make the substitution for $\mathbf{x_0} = \Delta\mathbf{x} + \mathbf{x_R}$, yielding a differential element $\mathrm{d}\mathbf{x_0} = \mathrm{d}\Delta\mathbf{x}$, and bounds $\lim_{\mathbf{x_0}\to\pm\infty} \mathbf{x_0} = \lim_{\Delta\mathbf{x}\to\pm\infty} \Delta\mathbf{x}$[1]. Finally, when solving the first step of this integral, the choice has to be made between fixing the position of the particle to be spread and integrate over R, or fixing the position of the point in R to be evaluated, and integrate over all particle contributions[2]. We choose the latter and arrive to our final expression,

$$E[N_R] = n_0 \iint\limits_R \iint\limits_S f(\Delta\mathbf{x}) \mathrm{d}\Delta\mathbf{x} \mathrm{d}\mathbf{x_R}. \tag{4}$$

By the definition of a p.d.f., $f(\Delta\mathbf{x})$ integrates to unity over all space, and the expression simplifies to

$$E[N_R] = n_0 \iint\limits_R \mathrm{d}\mathbf{x_R} = n_0 A_R, \tag{5}$$

[1] Note that $\mathbf{x_R}$ is fixed when performing the integral over $\mathrm{d}\mathbf{x_0}$

[2] This permutation is allowed for iterated integrals where the integrated function is continuous over the integrated region, which is required for the proposition.

where A_R is the area of R. Eqn. 5 implies that the expected number of particles in any region of the plane depends only on the number density of the plane and the area of the region. □

Note that had the function for n or f been dependent on $\mathbf{x_0}$, it would not have been possible to obtain the inner direct integral of the p.d.f. over all space, and therefore no statement could have been made about the general independence of $E[N_R]$ on f.

Let's now think about the case that the Earth is finite and spherical. We will also assume that neutrinos spread according to a Gaussian-like function, i.e. we let f be symmetric and peaked at 0, and let the spread of this function be related to the amount of matter traversed. As the total angle of deflection[3] increases, so does the area of the source plane from which the neutrinos contribute towards the final detector flux. Neutrinos that would deflect inwards towards the detector to compensate for the flux lost by the outwards deflection of neutrinos bound to the detector, will now have a smaller chance to deflect due to the smaller amount of matter encountered in the shorter Earth chord lengths, see a schematic of this situation in Fig. 8.

[3] The angle determined by the difference between the initial direction of the neutrino and the vector between the original and final position

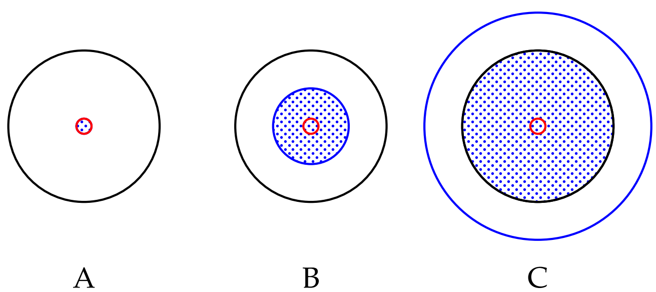

Figure 8: View of the Earth (black), detector (red), area of influence from the source (blue line) and are of contribution of from the source (dots), for A, no deflections, B, small deflections, and C, large deflections. Since some of the neutrinos in the dotted area of B travel through less matter than in A, and thus have a smaller chance of deflecting, situation B sees fewer neutrinos in the detector than A. The situation is exacerbated in C, where some neutrinos that could have deflected into the detector have no chance of hitting the Earth, and thus no chance of deflecting into the detector.

On the other hand, a detector in outer space will experience an increase in flux close to the Earth compared to far away from the Earth, since some neutrinos deflecting in the Earth will end up in the detector. Depending on the details of the deflections, there could also exist a region on the outskirts of the Earth for which a detector will also experience an enhancement in the flux. Both of these effects are readily seen when the deflection magnitude is cranked up, see Fig. 10

Real(ish) Earth

In our fourth and final case, we take into account both absorption and deflections. The final flux is shown in Fig. 11 and is very similar to that of Fig. 4, when only absorption was taken into account. We obtain 743 neutrinos in the detector.

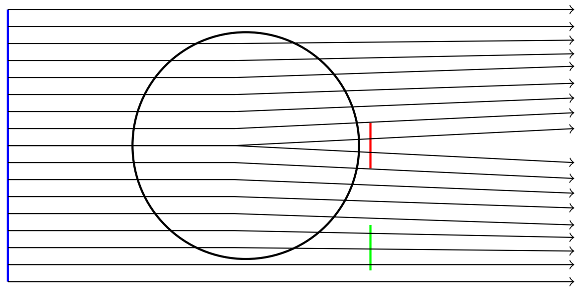

Figure 9: Detector red sees a decrease in flux with respect to no deflections, while detector green sees a potential increase, depending on the details of the deflections.

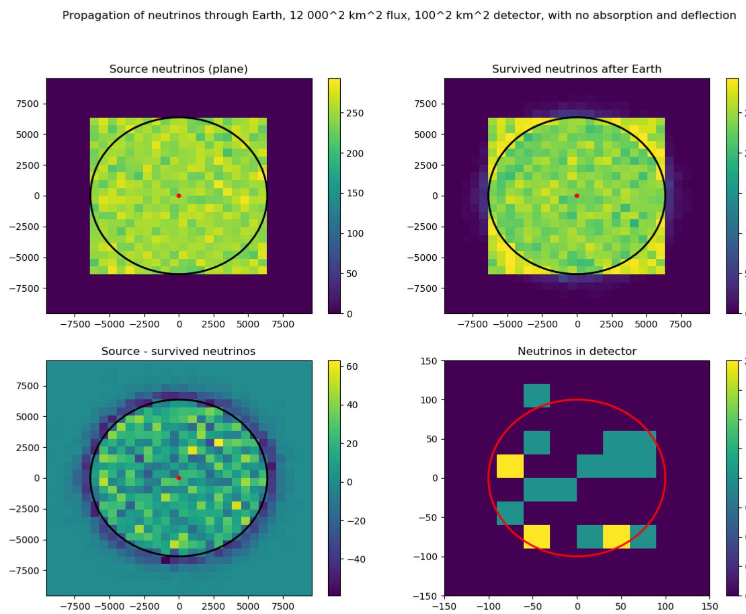

Figure 10: Neutrino flux for a source the size of the Earth, and a 100 km detector. With the average deflection increased to 0.1 radians, the large deflections yield an overall decreased flux inside the Earth, and an increased flux outside the Earth. It is apparent here that the Earth acts as a neutrino disperser.

If we assume the chances of absorption to be equal for the bulk of neutrinos which can end up in the detector, the assumptions for Proposition 1 hold, and we expect deflections to have no effect on the final flux observed at the detector, which is what is observed. If we use more realistic absorption, deflected neutrinos will have a higher chance of absorbing since they will have a longer path through the Earth. Similarly to the arguments in the previous section, this will result in a smaller flux at the detector, since the contribution from neutrinos deflecting into the detector will be lessened.

Conclusion

In this document, four scenarios were explored for the combinations of turning absorptions and deflections of neutrinos on and off. De-

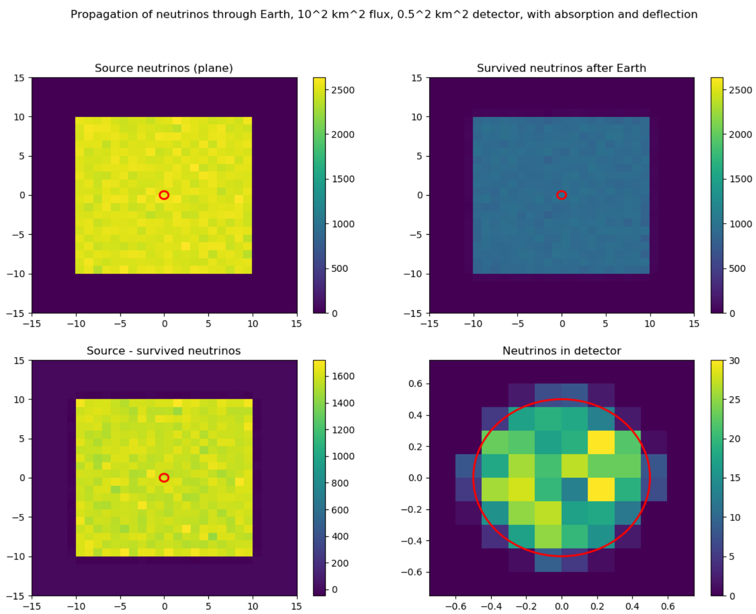

Figure 11: The combined effect of absorption and small deflections is nearly indistinguishable from that of absorption alone.

flections were not found to have any significant effect on the flux encountered in the detector for realistic parameters, both in reasoning and in simulations. However, deflections do cause a negligible change in flux at the detector, due to the shape of the Earth, and increased path lengths for deflected neutrinos. Further work should be pursued for situations in which neutrinos are found to have large deflection angles, and where these effects could potentially be significant.