Probability Density Function of arrival time of light from neutrino interaction vertex

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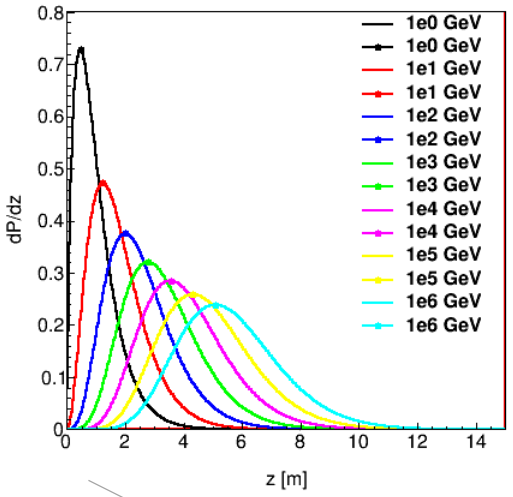
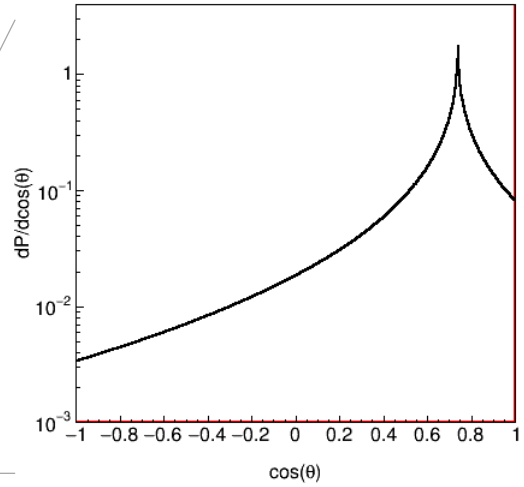
longitudinal shower profile



light emission profile



$u \equiv$ longitudinal shower profile

$v \equiv$ light propagation

longitudinal shower profile

$$u \sin\theta_0 = v \sin\theta_1$$
$$u \cos\theta_0 + v \cos\theta_1 = D$$

$$\Rightarrow \quad v^2 = D^2 + u^2 - 2Du\cos\theta_0$$
$$\Rightarrow v = \sqrt{D^2 + u^2 - 2Du\cos\theta_0}$$

$$ct_1 = ct_0 + u + nv \quad \Rightarrow \quad \left(\frac{ct_1 - ct_0 - u}{n}\right)^2 = D^2 + u^2 - 2Du\cos\theta_0$$

- for given $\cos\theta_0$ and arrival time t_1, u and v fixed

light emission profile

$$u + v\cos\theta_s = D\cos\theta_0 \quad \Rightarrow \quad \cos\theta_s = \frac{D^2 - u^2 - v^2}{2u}$$

- $\cos\theta_s$ also fixed

PMT angular acceptance

$$\hat{v} = \begin{pmatrix} -\sin\theta_1 \cos\phi_0 \\ -\sin\theta_1 \sin\phi_0 \\ -\cos\theta_1 \end{pmatrix} \qquad \hat{w} = \begin{pmatrix} \sin\theta_{PMT} \\ 0 \\ \cos\theta_{PMT} \end{pmatrix}$$

- PMT angular acceptance depends on $\hat{v} \cdot \hat{w}$
- due to isotropy, integral over ϕ_0 is trivial

arrival time

$$ct_1 = ct_0 + u + nv \quad \Leftrightarrow \quad ct_1 = ct_0 + nD$$

$$\Delta ct_1 = n\left(D - \sqrt{D^2 + u^2 - 2Du\cos\theta_0}\right) + u$$

- (non-Gaussian) tails to either side!

conclusions & outlook

- It is possible to include longitudinal shower profile in isotropic PDF
  - (only) two-dimensional integral over λ and $\cos\theta_0$
  - (non-Gaussian) tails in the arrival time distribution