

# LPM EFFECT

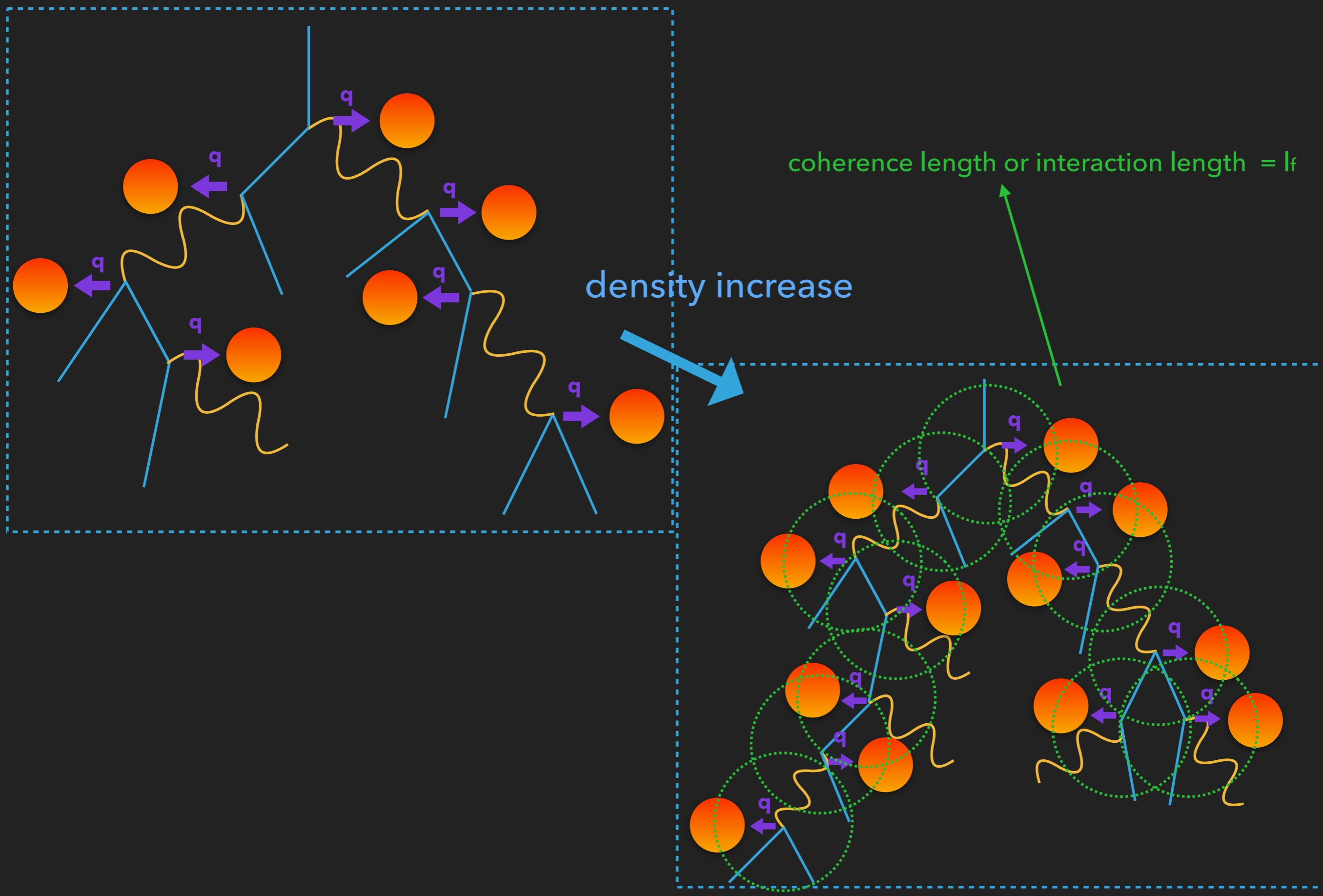
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KM3NET JUNIOR GROUP MEETING - 5/3/2020

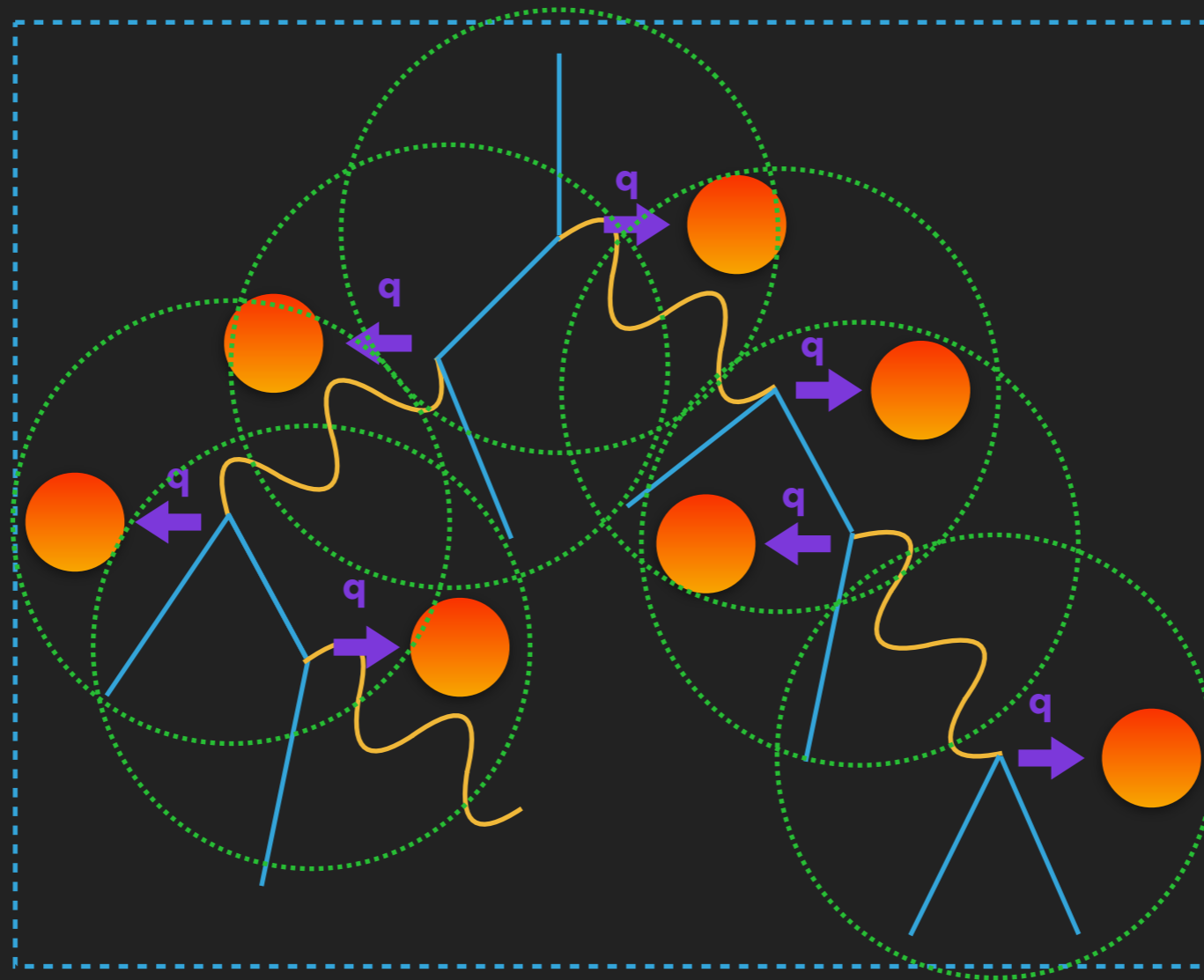
CLARA GATIUS

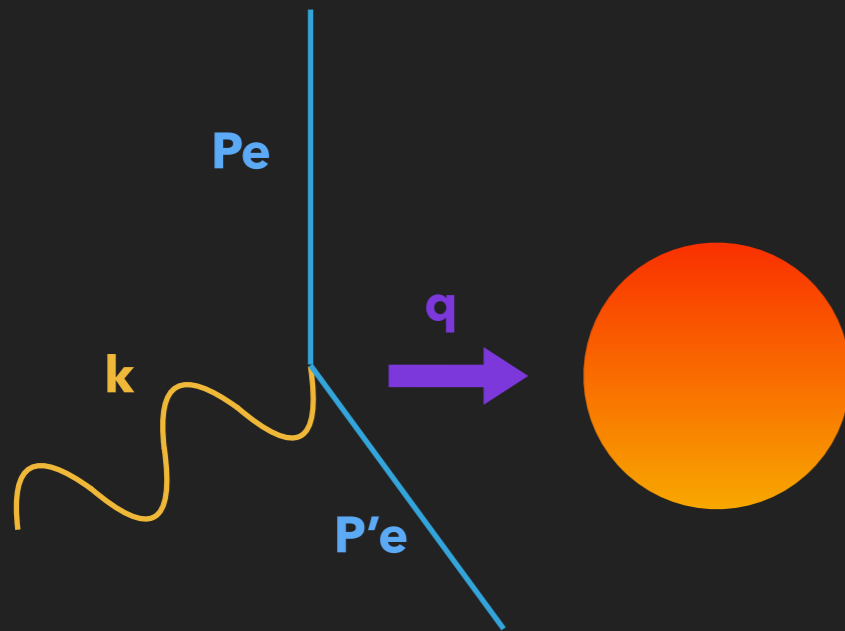
## **LANDAU-POMERANCHUK-MIGDAL EFFECT:**

Suppression of the Bremsstrahlung and Pair Production cross-section at high densities or high energies.



increase particles energy  $\rightarrow$  increase coherence length or interaction length





Bremsstrahlung interaction

$$q_{\parallel} = p_e - p'_e - p_{\gamma} = \sqrt{(E/c)^2 - (mc)^2} - \sqrt{[(E-k)/c]^2 - (mc)^2} - k/c$$

$$q_{\parallel} \sim \frac{m^2 c^3 k}{2E(E-k)}$$

$$\sqrt{1-y} \sim 1 - y/2$$

as higher E → smaller momentum transfer  
 as lower k → smaller momentum transfer

Some numbers to illustrate....

$$E = 1 \text{ EeV}, k = 100 \text{ PeV} \rightarrow q = 10^{-9} \text{ eV}/c$$



**Uncertainty principle becomes relevant!!!**

## Uncertainty principle

$$\Delta x \Delta p \geq \hbar$$

$$l_{f0} = \frac{\hbar}{q_{\parallel}} = \frac{2\hbar E(E-k)}{m^2 c^3 k}$$

coherence length or interaction length:

space the interactions need to take place

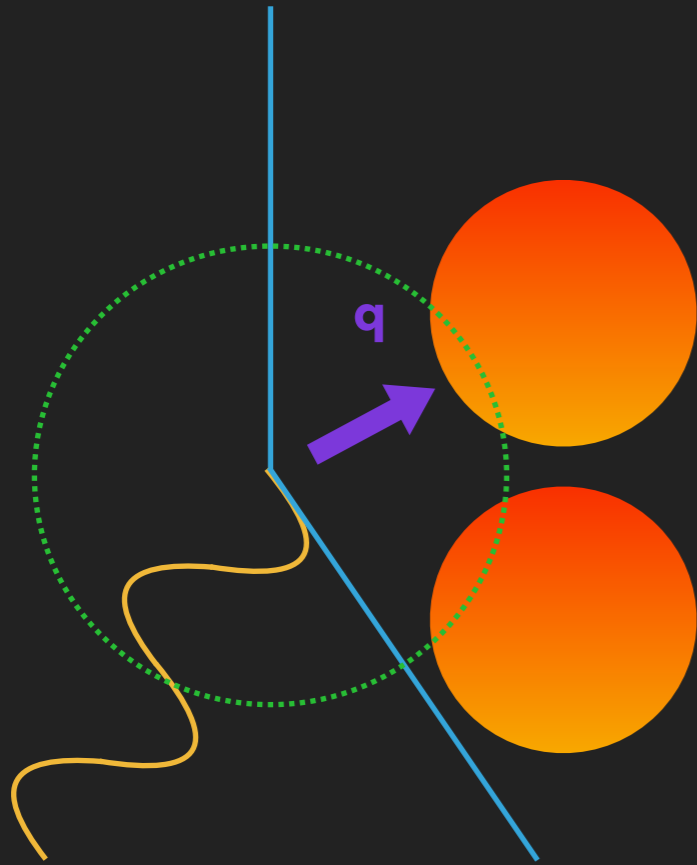
as higher  $E \rightarrow$  higher  $l_f$

Gives room for other interactions to take place!

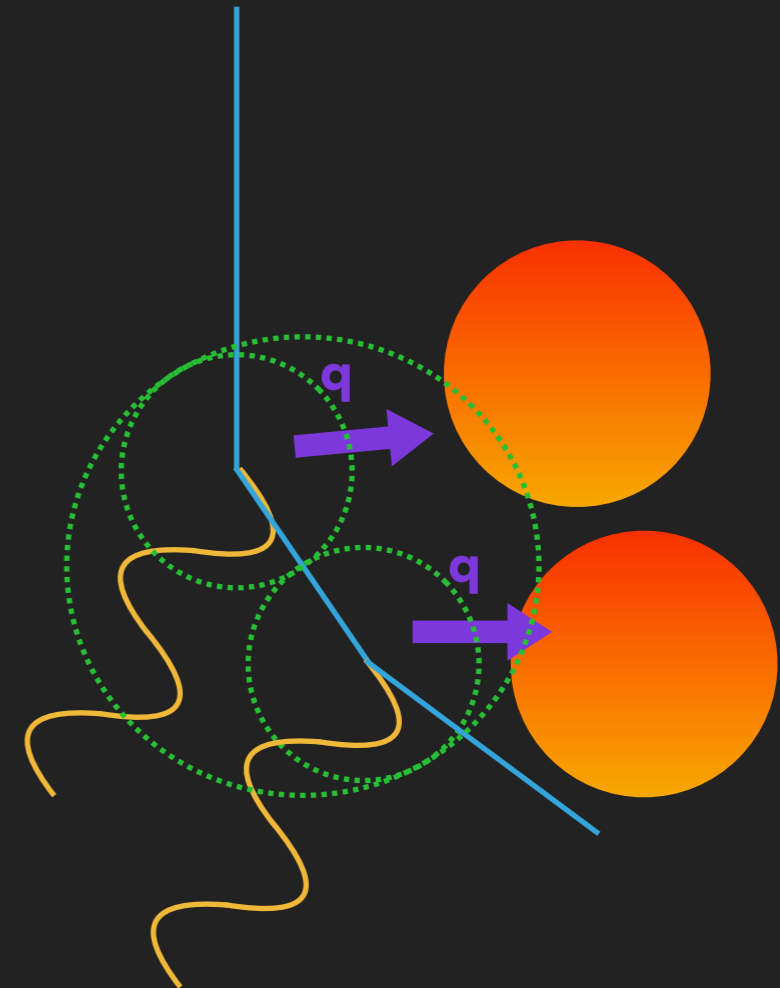
Continuing the "some numbers to illustrate..."

$E = 1 \text{ EeV}, k = 100 \text{ PeV} \rightarrow$

$l_f = 200 \text{ m}$



Bremsstrahlung interaction



Double Bremsstrahlung interaction

increasing the number of interactions...

Radiation intensity  $\propto l_f$

$$q \rightarrow 2q$$

$$l_f \rightarrow l_f / 2$$

$$\text{Radioation intensity: } \sim l_f^2 \rightarrow \sim 2 \cdot (l_f/2)^2$$

## 3 sources of suppression:

1. Multiple scattering of the e-
2. Interaction of the emitted photon with the medium (dielectric effect)
3. Pair creation by the emitted photon



Multiple scattering of the e-

$$q_{\parallel} = \sqrt{(E \cos \theta_{MS/2}/c)^2 - (mc)^2} - \sqrt{[(E-k) \cos \theta_{MS/2}/c]^2 - (mc)^2} - k/c,$$

$$q_{\parallel} = \frac{km^2c^3}{2E(E-k)} + \frac{k\theta_{MS/2}^2}{2c} \rightarrow l_f = l_{f0} \sqrt{\frac{kE_{LPM}}{E(E-k)}}$$

Suppression:

$$S = \frac{d\sigma/dk}{d\sigma_{BH}/dk} = \frac{l_f}{l_{f0}} = \sqrt{\frac{kE_{LPM}}{E(E-k)}}$$

as bigger its contribution → smaller  $l_f$  → more suppressed cross-section

Condition for suppression:

$$\theta_{MS/2} > 1/\gamma.$$

higher E → higher limit for photon energy

$$k < k_{LPM} = \frac{E(E-k)}{E_{LPM}}$$

medium dependent

$$E_{LPM} = \frac{m^2c^3 X_0 \alpha}{4\pi\hbar} \approx 7.7 \text{ TeV/cm} \cdot X_0$$

| Material                | $Z$ | $\rho$ (g/cm <sup>3</sup> ) | $X_0$ (cm) | $E_{LPM}$ (TeV) | $y_{die}$            | $E_p$   |
|-------------------------|-----|-----------------------------|------------|-----------------|----------------------|---------|
| Hydrogen (liquid at BP) | 1   | 0.07                        | 865        | 6.6 PeV         | $1.4 \times 10^{-5}$ | 4.6 PeV |
| Helium (liquid at BP)   | 2   | 0.125                       | 755        | 5.8 PeV         | $1.5 \times 10^{-5}$ | 4.3 PeV |
| Carbon                  | 6   | 2.2                         | 19.6       | 151             | $5.5 \times 10^{-5}$ | 410 TeV |
| Aluminum                | 13  | 2.70                        | 8.9        | 68              | $6.0 \times 10^{-5}$ | 205 TeV |
| Iron                    | 26  | 7.87                        | 1.76       | 13.6            | $1.0 \times 10^{-4}$ | 67 TeV  |
| Lead                    | 82  | 11.35                       | 0.56       | 4.3             | $1.1 \times 10^{-4}$ | 24 TeV  |
| Tungsten                | 74  | 19.3                        | 0.35       | 2.7             | $1.5 \times 10^{-4}$ | 20 TeV  |
| Uranium                 | 92  | 18.95                       | 0.35       | 2.7             | $1.4 \times 10^{-4}$ | 19 TeV  |
| Gold                    | 79  | 19.32                       | 0.33       | 2.5             | $1.5 \times 10^{-4}$ | 19 TeV  |
| Air (STP)               | -   | 0.0012                      | 30400      | 234 PeV         | $1.3 \times 10^{-6}$ | 15 PeV  |
| Water                   | -   | 1.00                        | 36.1       | 278             | $3.9 \times 10^{-5}$ | 540 TeV |
| Std. rock               | 11  | 2.65                        | 10.0       | 77              | $6.0 \times 10^{-5}$ | 230 TeV |

Dielectric effect

$$q_{\parallel} = \frac{k}{2c\gamma^2} + \frac{(\hbar\omega_p)^2}{2ck} \quad \longrightarrow \quad l_f = \frac{2\hbar ck\gamma^2}{k^2 + k_p^2}$$

Suppression:  $S = \frac{k^2}{k^2 + k_p^2}$

as bigger its contribution  $\rightarrow$  smaller  $l_f$   $\rightarrow$  more suppressed cross-section

Condition for suppression:

$$k < k_p \quad , \quad k_p = \gamma \hbar \omega_p \quad , \quad \epsilon(k) = 1 - (\hbar\omega_p)^2 / k^2$$

← plasma frequency medium  
← permittivity

$$y = k/E < y_{die} \quad , \quad y_{die} = \hbar\omega_p / mc^2$$

| Material                | $Z$ | $\rho$ (g/cm <sup>3</sup> ) | $X_0$ (cm) | $E_{LPM}$ (TeV) | $y_{die}$            | $E_p$   |
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Pair production

$$l_{f0} > X_0$$

Suppression:

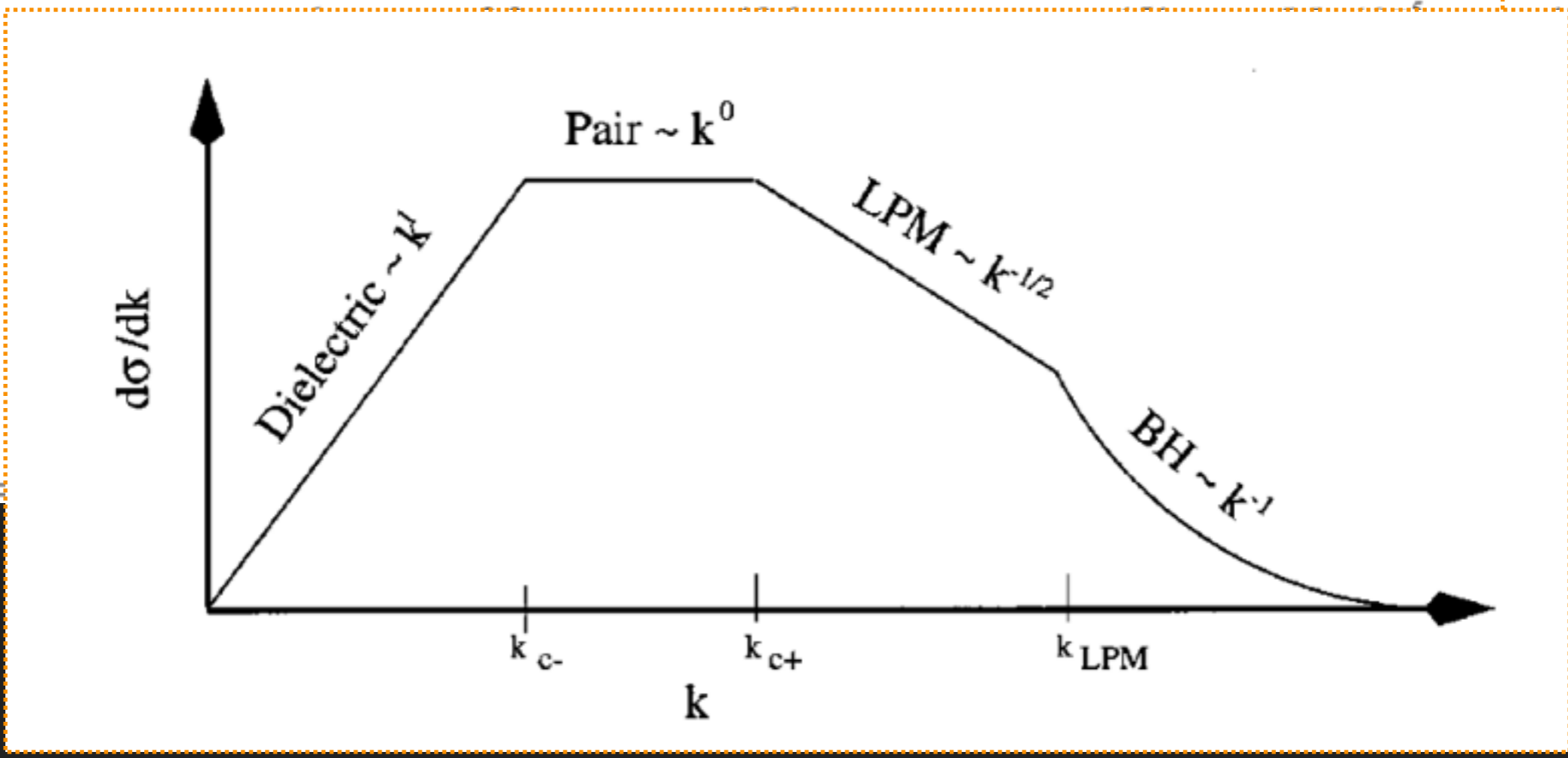
$$S = \frac{X_0}{l_{f0}} = \frac{m^2 c^3 k X_0}{2 \hbar E (E - k)}$$

Condition for supression:

$$E > E_p = \frac{X_0 \omega_p E_s}{\sqrt{2} c}, \quad E_s = mc^2 \sqrt{4 \pi / \alpha}$$

| Material                | $Z$ | $\rho$ (g/cm <sup>3</sup> ) | $X_0$ (cm) | $E_{LPM}$ (TeV) | $y_{die}$            | $E_p$   |
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| Material                | Z | $\rho$ (g/cm <sup>3</sup> ) | $X_0$ (cm) | $E_{LPM}$ (TeV) | $y_{die}$            | $E_p$   |
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| Carbon                  |   |                             |            |                 |                      | 10 TeV  |
| Aluminum                |   |                             |            |                 |                      | 05 TeV  |
| Iron                    |   |                             |            |                 |                      | 7 TeV   |
| Lead                    |   |                             |            |                 |                      | 4 TeV   |
| Tungsten                |   |                             |            |                 |                      | 0 TeV   |
| Uranium                 |   |                             |            |                 |                      | 9 TeV   |
| Gold                    |   |                             |            |                 |                      | 9 TeV   |
| Air (STP)               |   |                             |            |                 |                      | 5 PeV   |
| Water                   |   |                             |            |                 |                      | 40 TeV  |
| Std. rock               |   |                             |            |                 |                      | 30 TeV  |



## Total suppression...

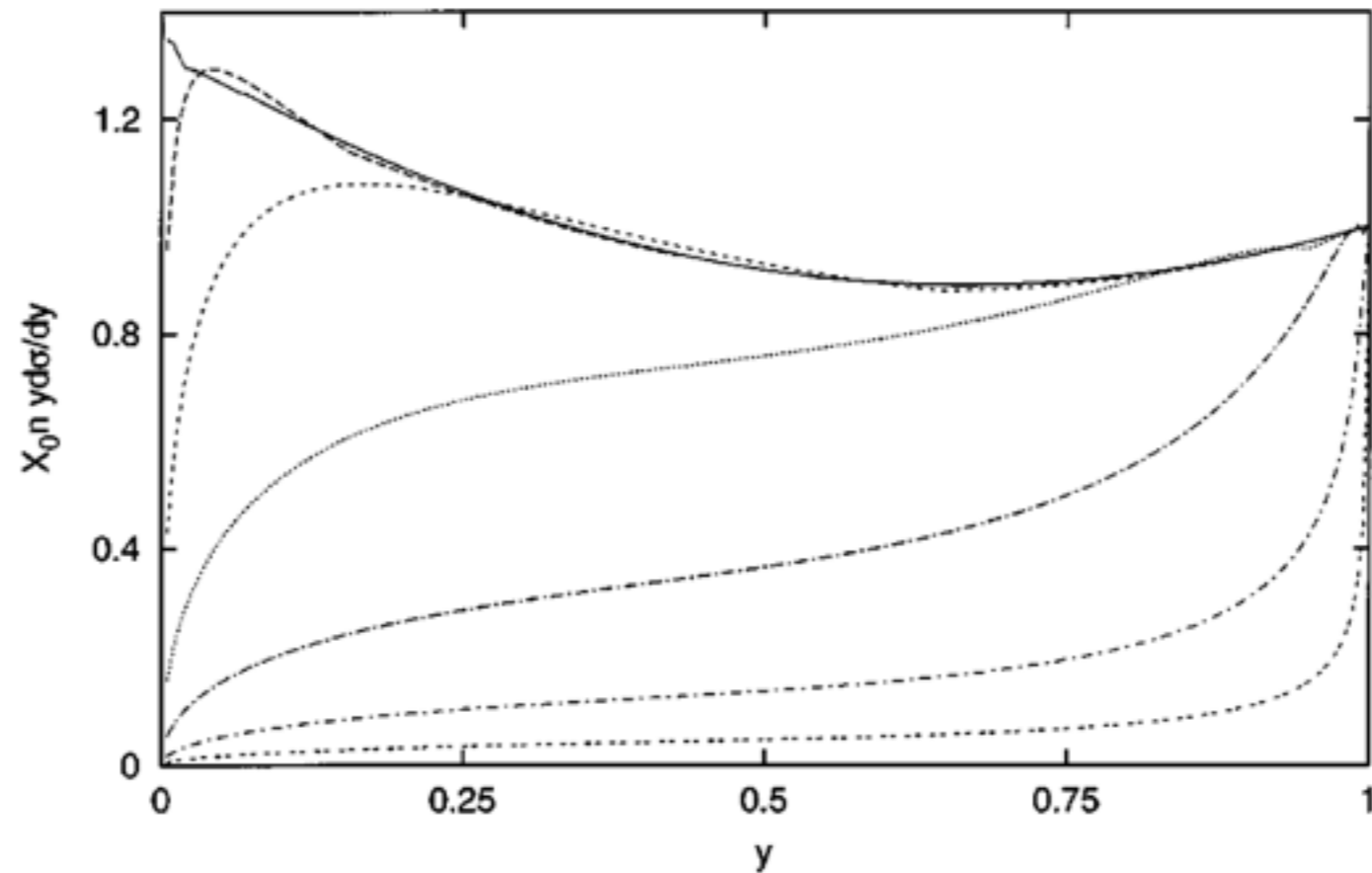


FIG. 9. Energy-weighted differential cross section for bremsstrahlung,  $X_0 n y d\sigma_{LPM}/dy$ , for various electron energies in a lead target, showing how the spectral shape changes: top curve, electrons of energies 10 GeV; remaining curves, 100 GeV, 1 TeV, 10 TeV, 100 TeV, 1 PeV, and 10 PeV (bottom curve). The units are fractional energy per radiation length.



source of suppression: Bremsstrahlung.

Partially produced e- and e+ undergo Bremsstrahlung.

Identical approach as for multi-scattering, but now the contribution to the momentum transfer is given by the Bremsstrahlung of e- or e+

$$\rightarrow l_f = l_{f0} \sqrt{\frac{k E_{LPM}}{E(k-E)}}$$

Suppression:  $S = \sqrt{\frac{E_{LPM} k}{E(k-E)}}$

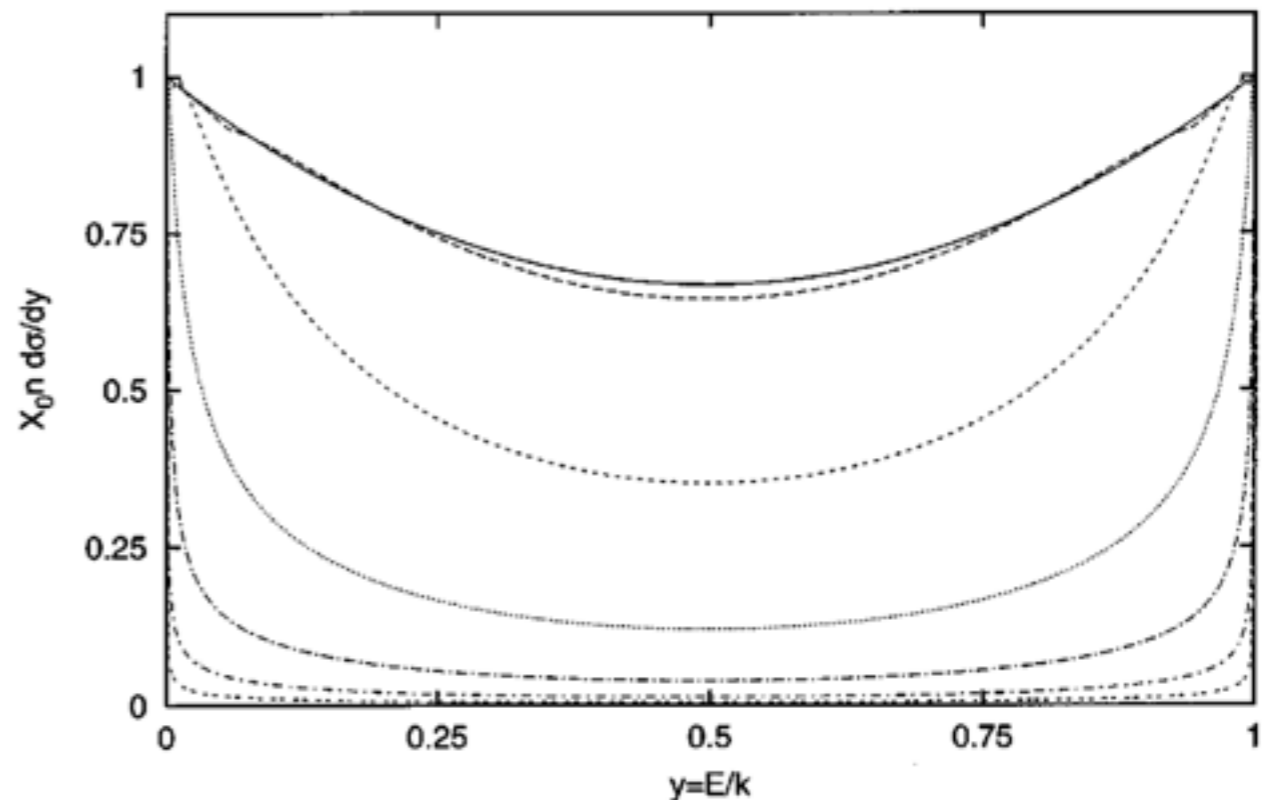


FIG. 10. Differential cross section for pair production,  $X_0 n d\sigma_{LPM}/dy$ , in lead for various photon energies, showing how the spectral shape changes. Cross sections are plotted for photons of energies 1 TeV (top curve), 10 TeV, 100 TeV, 1 PeV, 10 PeV, 100 PeV, and 1 EeV (bottom curve).

**+ not only effect on particle spectrum, also effect on angular distribution !**  
**(by all types of suppressions)**

$I_{f_0} \propto 1/\theta$   $\rightarrow$  less room for interference of other interactions



suppression small emission angles



increase angular spread

### Improvements by Migdal:

- No use of an average scattering angle
- Addition of quantum effects: electron spin and photon polarization

**Approach:** calculation of averaged radiation per interaction and the following interference between radiation from different collisions.

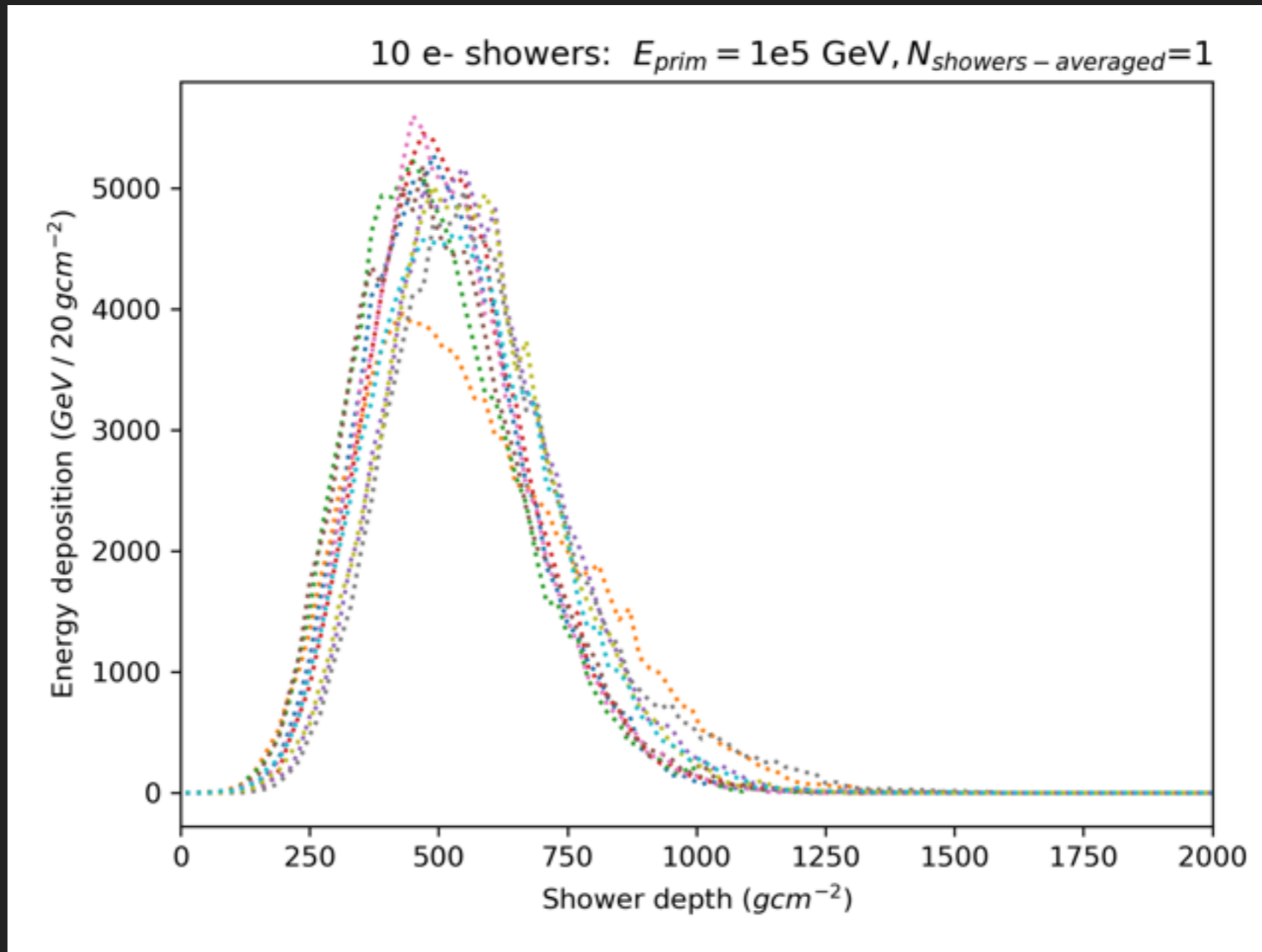
**Bremsstrahlung:**

$$\frac{d\sigma_{LPM}}{dk} = \frac{4\alpha r_e^2 \xi(s)}{3k} \{y^2 G(s) + 2[1 + (1-y)^2] \phi(s)\} Z^2 \times \ln\left(\frac{184}{Z^{1/3}}\right),$$

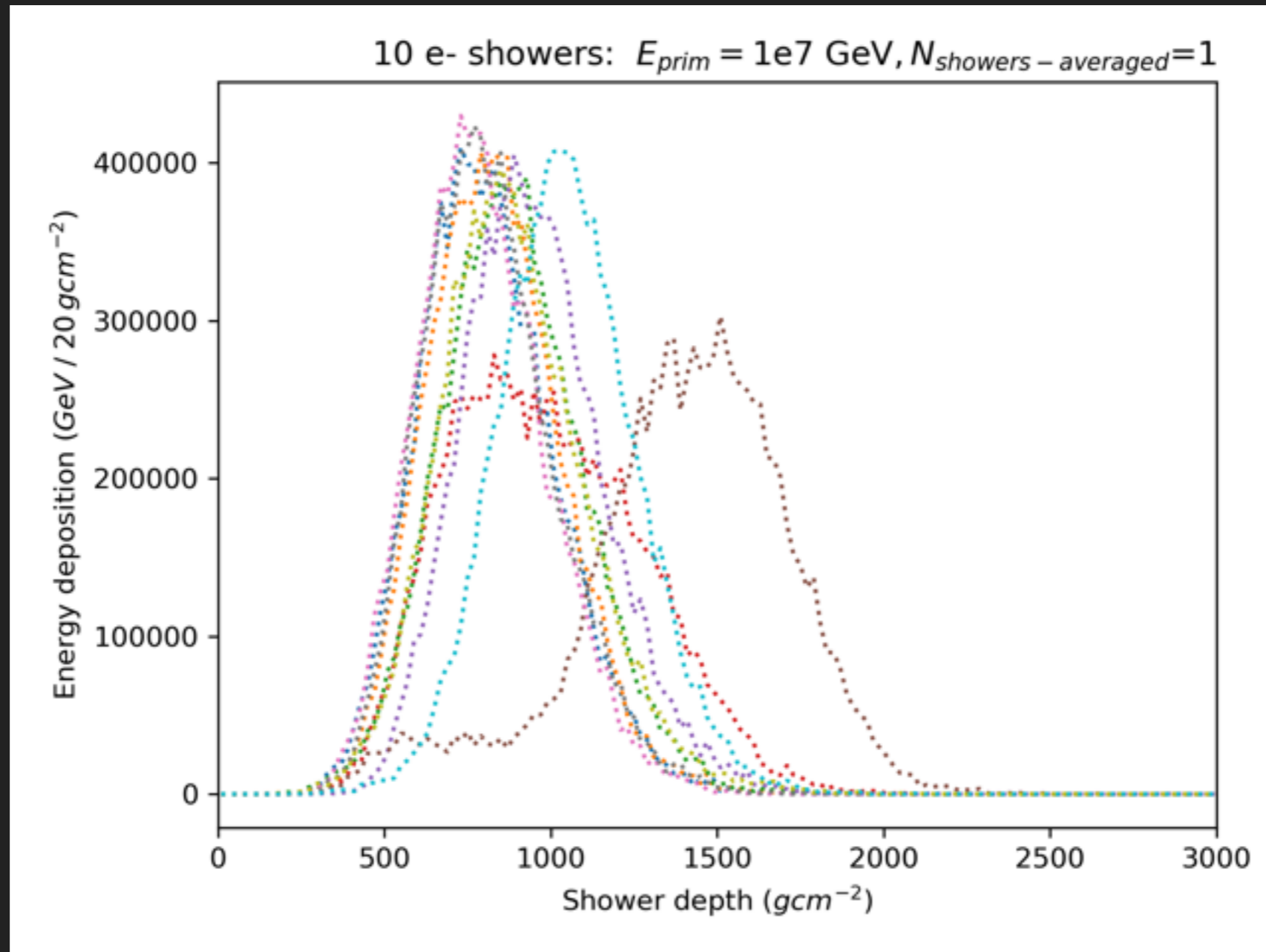
**Pair production:**

$$\frac{d\sigma_{LPM}(\gamma \rightarrow e^+ e^-)}{dE} = \frac{4\alpha r_e^2 \xi(\tilde{s})}{3k} \left\{ G(\tilde{s}) + 2 \left[ \frac{E^2}{k^2} + \left(1 - \frac{E}{k}\right)^2 \right] \phi(\tilde{s}) \right\}$$

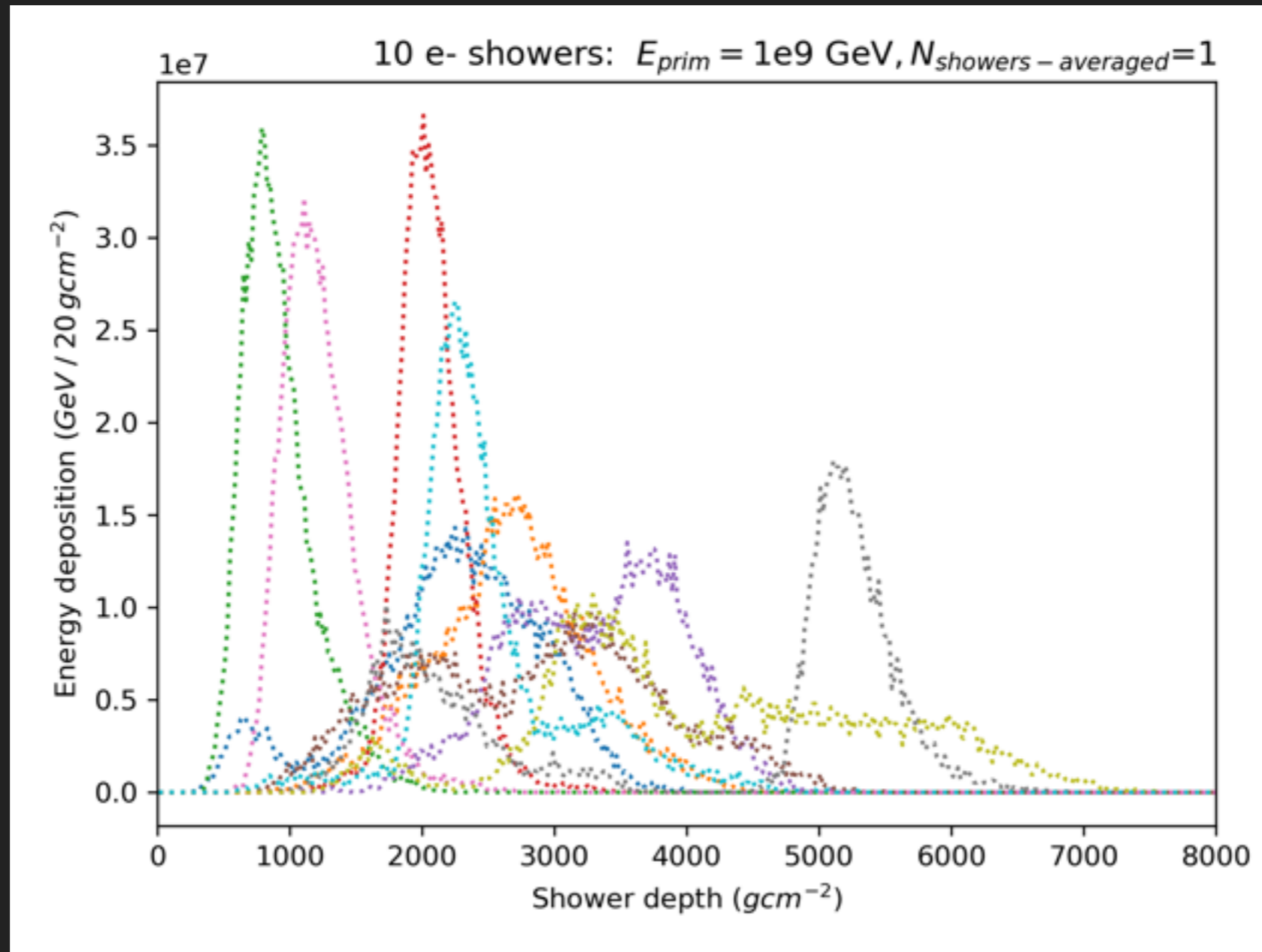
# LPM EFFECT ON ELECTROMAGNETIC SHOWERS IN WATER:



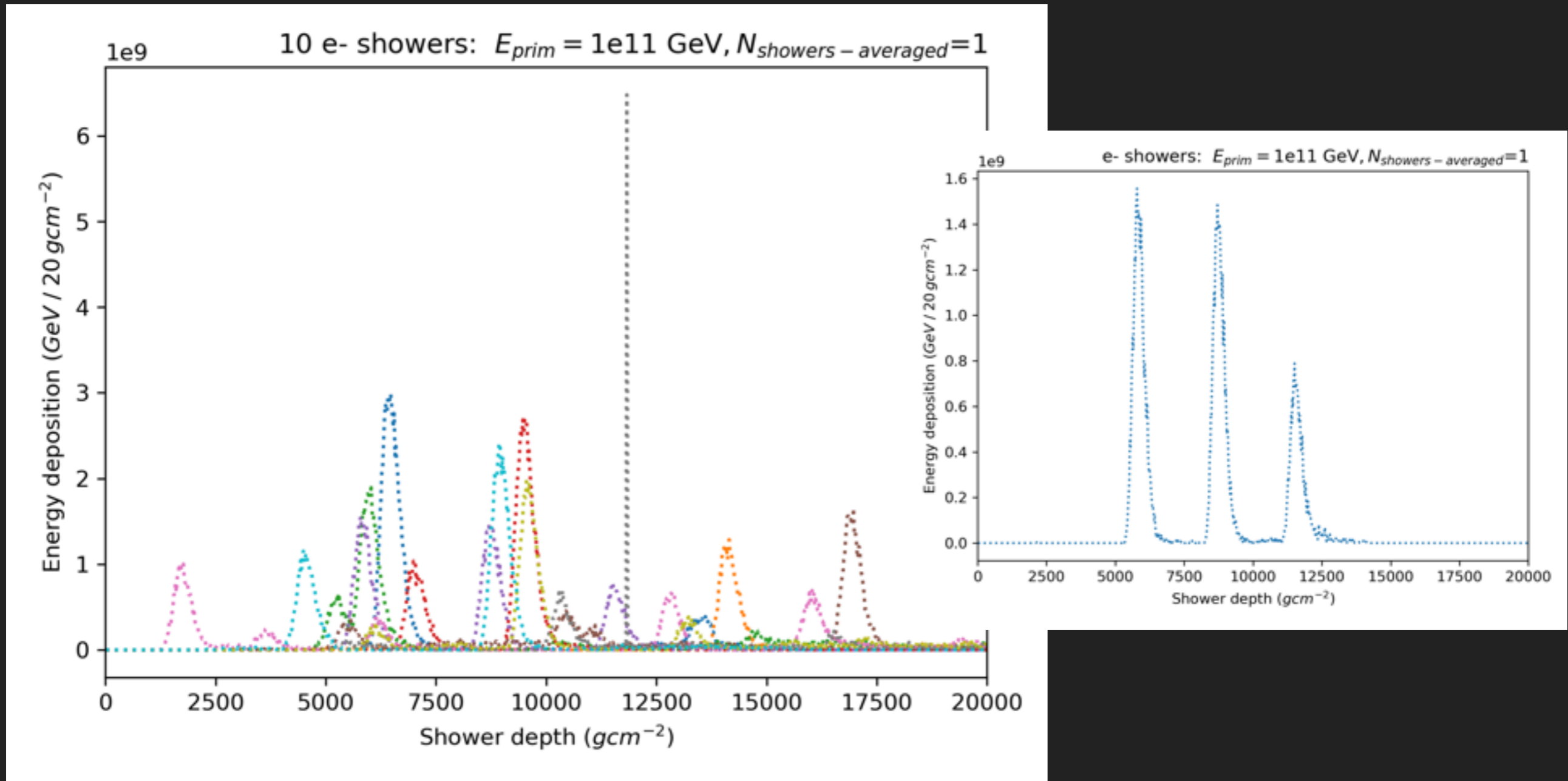
# LPM EFFECT ON ELECTROMAGNETIC SHOWERS IN WATER:



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1. Klein, S. Suppression of bremsstrahlung and pair production due to environmental factors. *Rev. Mod. Phys.* 71, 1501–1538 (1999).
  2. Klein, S. R. Bremsstrahlung and Pair Creation: Suppression Mechanisms and How They Affect EHE Air Showers. (2000).
  3. Migdal, A. B. Bremsstrahlung and pair production in condensed media at high energies. *Phys. Rev.* 103, 1811–1820 (1956).