# Informal intro SSB/Higgs mechanism

## Topical Lectures June 2020

**Avanish & Solange**

First of all..

# Feel free to ask questions!

# Index

- Shortcomings of EW model
- Breaking a global continuous symmetry
- Breaking a local continuous symmetry

# Literature

- Based mostly on “Particle Physics 1” by Marcel Merk, Ivo van Vulpen and Wouter Hulsbergen
- “Quantum Field Theory and the Standard Model” by Matthew D. Schwartz (modern and very comprehensive)

  Also useful to learn more on QFT:
- “Quantum Field Theory” by Mark Srednicki (self consistent modular format)
- “An Introduction to Quantum Field Theory” by Peskin and Schroeder (classic)
- “The Quantum Theory of Fields (vol. I, II and III)” by Weinberg (sacred scripts of a theorist)

# Shortcomings Electroweak Model

- Massive gauge bosons are forbidden
- Massive fermions are forbidden in EW theory
- Unitarity violations

# Forbidden massive gauge bosons

- QED is locally gauge invariant:
  the Lagrangian is invariant under $\phi' \rightarrow e^{i\alpha(x)}\phi$ U(1)

- To this end we introduced the covariant derivative $D_\mu$ and vector field $A_\mu$ (gauge boson/photon field):

$$\partial_\mu \rightarrow D_\mu - ieA_\mu, \quad A'_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\mathscr{L}_{QED} = \mathscr{L}_{free} + \mathscr{L}_{int} + \mathscr{L}_{Maxwell}$$

# Forbidden massive gauge bosons

- QED is locally gauge invariant:
  the Lagrangian is invariant under $\phi' \to e^{i\alpha(x)}\phi$ U(1)

- To this end we introduced the covariant derivative $D_\mu$ and vector field $A_\mu$ (gauge boson/photon field):

$$\partial_\mu \to D_\mu - ieA_\mu, \;\; A'_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\mathscr{L}_{QED} = \mathscr{L}_{free} + \mathscr{L}_{int} + \mathscr{L}_{Maxwell}$$

Propagation of $e$

# Forbidden massive gauge bosons

- QED is locally gauge invariant:
  the Lagrangian is invariant under $\phi' \to e^{i\alpha(x)}\phi$ U(1)
- To this end we introduced the covariant derivative $D_\mu$ and vector field $A_\mu$ (gauge boson/photon field):

$$\partial_\mu \to D_\mu - ieA_\mu, \quad A'_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha$$

$$\mathcal{L}_{QED} = \boxed{\mathcal{L}_{free}} + \boxed{\mathcal{L}_{int}} + \mathcal{L}_{Maxwell}$$

Propagation of $e$

Interaction $2e$ and $\gamma$

# Forbidden massive gauge bosons

- QED is locally gauge invariant:
  the Lagrangian is invariant under $\phi' \to e^{i\alpha(x)}\phi$   U(1)

- To this end we introduced the covariant derivative $D_\mu$ and vector field $A_\mu$ (gauge boson/photon field):

$$\partial_\mu \to D_\mu - ieA_\mu, \quad A'_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\mathscr{L}_{QED} = \boxed{\mathscr{L}_{free}} + \boxed{\mathscr{L}_{int}} + \boxed{\mathscr{L}_{Maxwell}}$$

Propagation of $e$

Interaction $2e$ and $\gamma$

Maxwell eqs in vacuum

# Forbidden massive gauge bosons

- Notice that the $\gamma$ is massless (no mass term in the Lagrangian)
- So let's add one?

$$\mathscr{L}_{QED}^{new} = \mathscr{L}_{QED} + \frac{1}{2} m_\gamma^2 A_\mu A^\mu$$

- We require this Lagrangian to be locally U(1) invariant, but is it?

$$\frac{1}{2} m_\gamma^2 A_\mu A^\mu = \frac{1}{2} m_\gamma^2 \left( A_\mu + \frac{1}{e} \partial_\mu \alpha \right) \left( A^\mu + \frac{1}{e} \partial^\mu \alpha \right) \neq \frac{1}{2} m_\gamma^2 A_\mu A^\mu$$

# Forbidden massive gauge bosons

- Thus we see that massive gauge bosons aren’t allowed in QED
- This is fine for photons
- Similar arguments hold for EW theory, where we know that the W and Z bosons are massive!
- Spontaneous symmetry breaking (SSB) will fix this!

# Forbidden massive fermions in EW

- EW requires covariant derivatives of the form
  $D_\mu = \partial_\mu + \frac{ig}{2}\vec{\tau} \cdot \overrightarrow{W}_\mu + \frac{ig'}{2} Y B_\mu$
- Here, isospin singlets ($\psi_R$) and doublets ($\psi_L$) transform as
  $\psi'_R \to e^{i\beta(x)Y}\psi_R$

  $\psi'_L \to e^{i\alpha(x)T + i\beta(x)Y}\psi_L$

# Forbidden massive fermions in EW

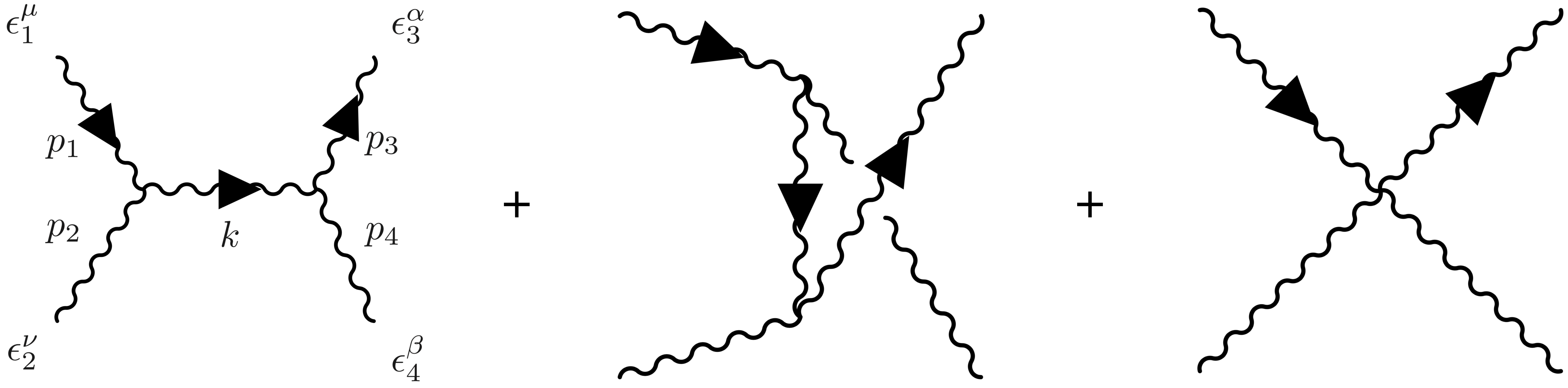
- A mass term expressed in helicity states would be
$$-m_f\bar{\psi}\psi = -m_f\left(\bar{\psi}_R + \bar{\psi}_L\right)\left(\psi_L + \psi_R\right)$$
$$= -m_f\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$$
since $\bar{\psi}_i\psi_i = \frac{1}{4}\bar{\psi}\left(1 - \left(\gamma^5\right)^2\right)\psi = 0$, with $i = R$ or $L$ and $\left(\gamma^5\right)^2 = 1$

- After applying the transformations from the previous slide, we don't obtain the same mass term :-(

- Spontaneous symmetry breaking will fix this!

# Unitarity violations

- Say the W and Z obtained masses through SSB. Let's consider $\sigma\left(W_L^+ Z_L \to W_L^+ Z_L\right)$; three contributing diagrams:



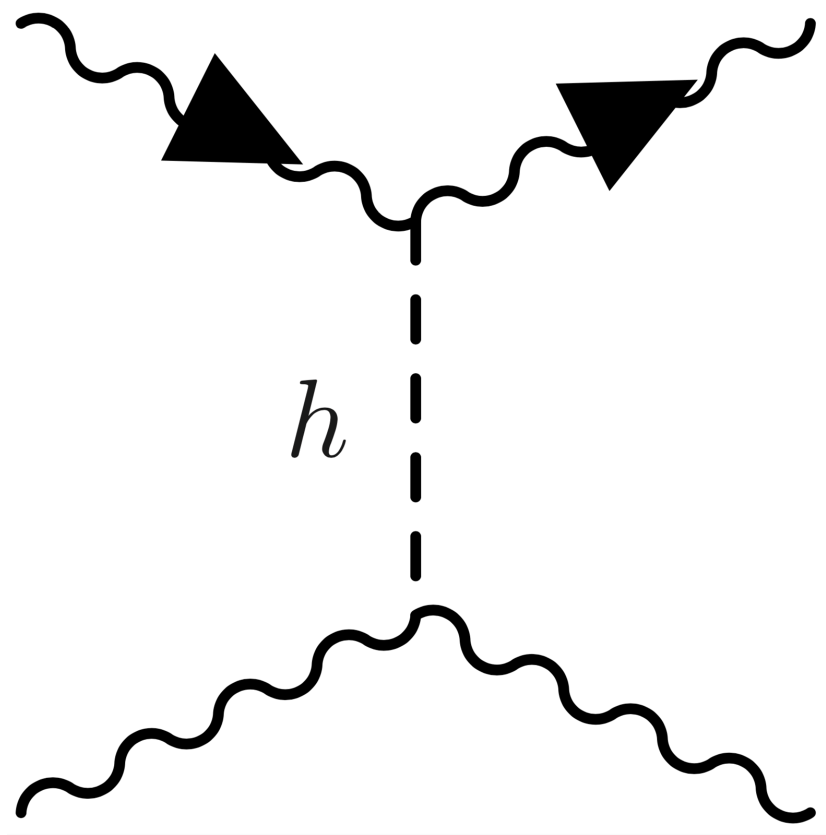
# Unitarity violations

- Resulting in an amplitude $\mathscr{M} \propto \frac{E_{CM}^2}{m_W^2} + O(1)$

- ***Perturbative*** unitarity bounds violated at $E_{CM} \approx 2.5$ TeV, meaning that we cannot use perturbative QFT (i.e. Feynman rules) above this scale

- Including the Higgs exchange diagram, this behavior is exactly canceled:

  $$\mathscr{M}_h \propto -\frac{E_{CM}^2}{m_W^2} + O(1)$$



# Excited? Let’s get into it!

# But first… a Lagrangian mechanics refresher

# Lagrangian Mechanics

## Field equations

- $\mathscr{L} = T\,(\text{kin}) - V\,(\text{pot})$
- Euler-Lagrange e.o.m. $\dfrac{d}{dt}\left(\dfrac{\partial \mathscr{L}}{\frac{\partial q_i}{\partial t}}\right) - \dfrac{\partial \mathscr{L}}{\partial q_i} = 0$ for each type $i$
- Klein-Gordon $\left(\partial_\mu \partial^\mu + m^2\right)\phi = 0$ from $\mathscr{L} = \dfrac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \dfrac{1}{2} m^2 \phi^2$
- Dirac eq $\left(i\gamma_\mu \partial^\mu - m\right)\psi = 0$ from $\mathscr{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$

# Lagrangian Mechanics
## Terminology

- $\mathscr{L} = \partial_\mu \phi \partial^\mu \phi + a_0 + a_1 \phi + a_2 \phi^2 + a_3 \phi^3 + \dots$
- First term is the ***kinetic*** term
- $a_0$ is irrelevant if one considers the EL equations, thus can be neglected
- $a_1$ terms are absent, since the vacua of the potentials are usually located at $\langle \phi \rangle = 0$ or the fields are shifted to force this
- $a_2$ is the ***mass*** term
- All higher order terms are ***interaction*** terms

# Symmetry breaking

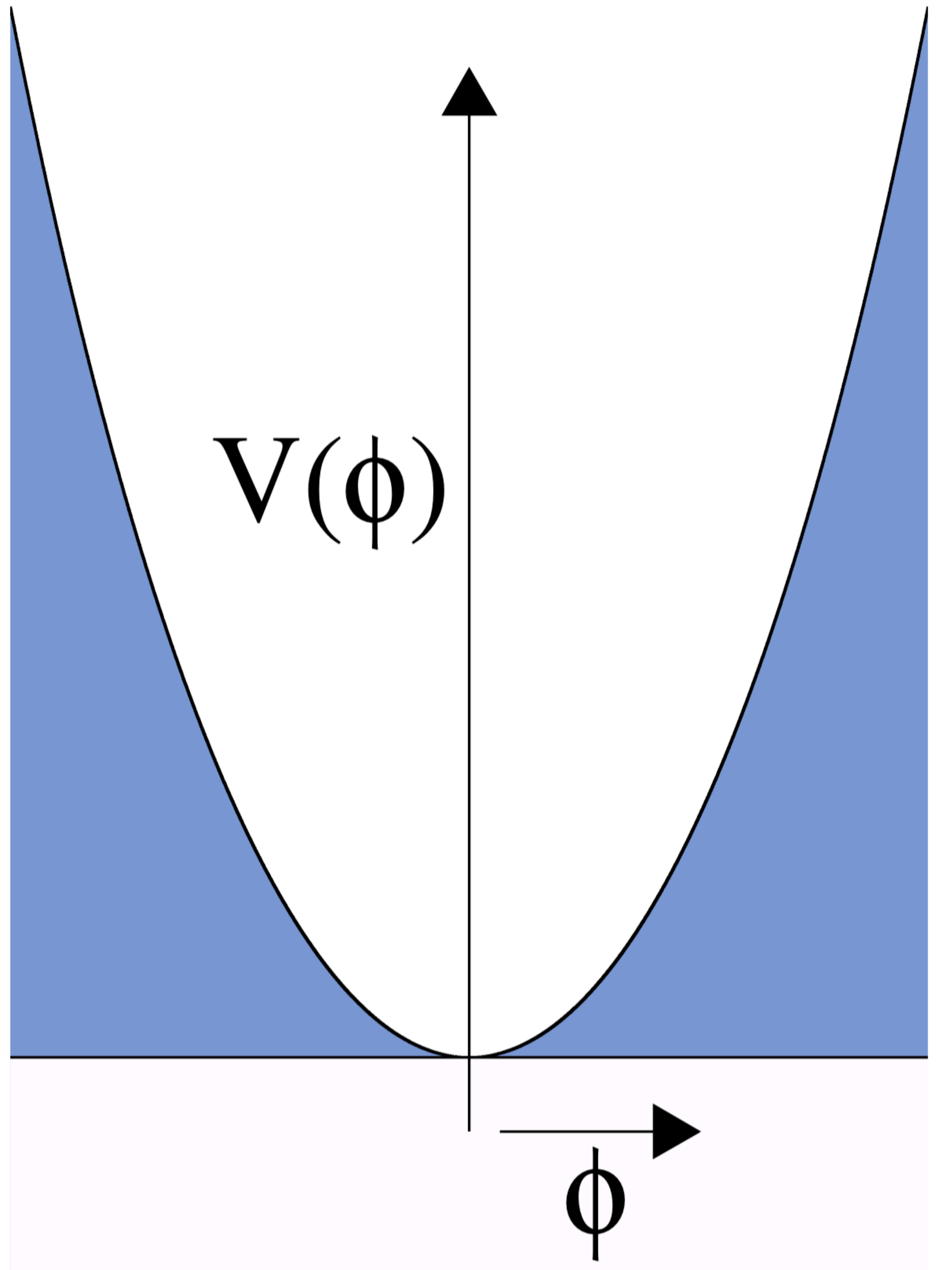
## Simple model

- $$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V\left(\phi\right)$$
  $$= \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$
- Two things to notice:
  1. Symmetric under $\phi \to -\phi$
  2. Potential has extrema at $|\phi_0| = 0 \vee |\phi_0| = \sqrt{\frac{-\mu^2}{\lambda}}$
- The first extremum holds for all $\mu$, the latter for $\mu^2 < 0$

# Symmetry breaking

## Simple model

- In perturbative QFT we treat the fields $\phi$ as perturbations around a stable minimum $|\phi_0|$, called the vacuum
- For $\mu^2 > 0$ the vacuum lies at $|\phi_0| = 0$, thus it was justified to put $a_1 = 0$

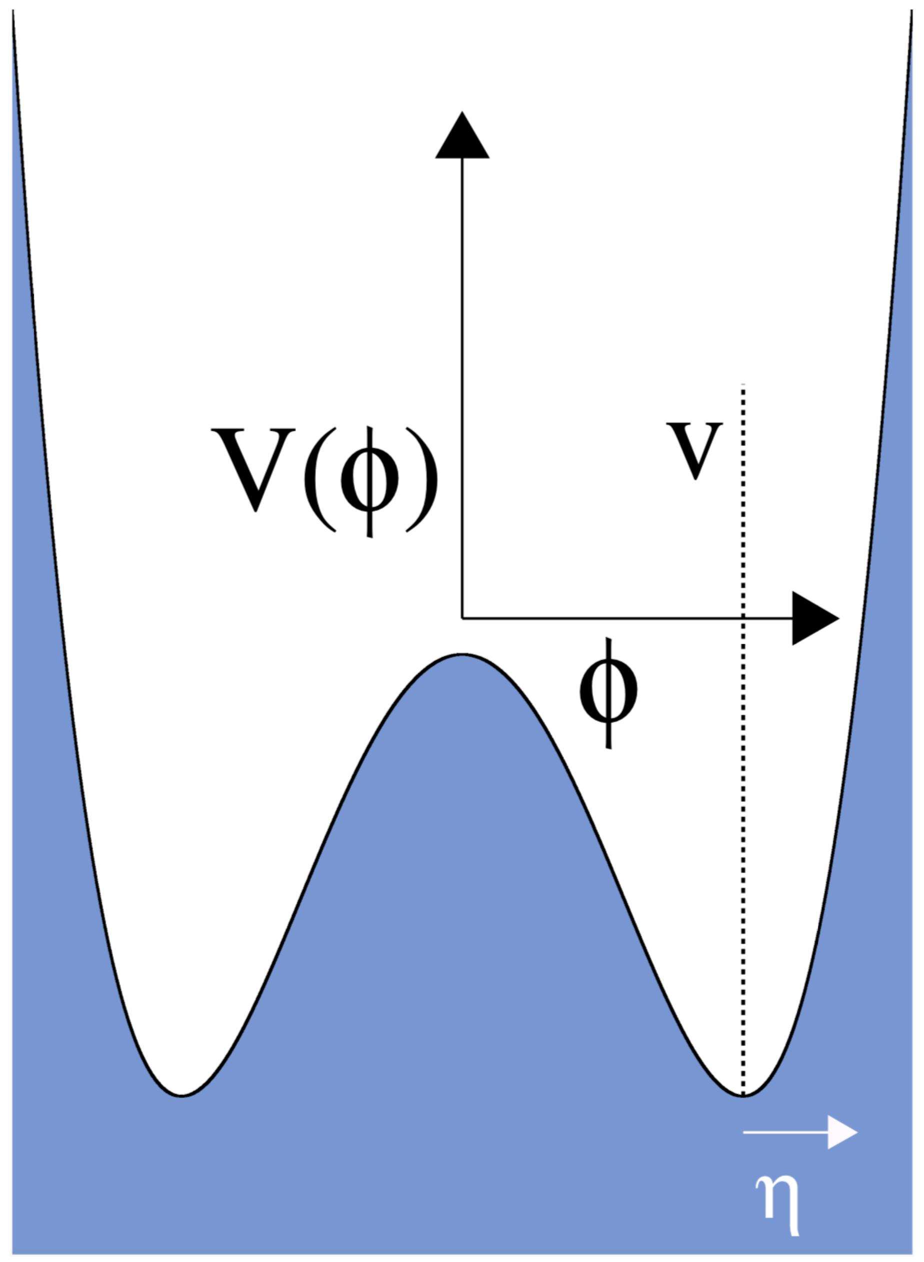


# Symmetry breaking

## Simple model

- In perturbative QFT we treat the fields $\phi$ as perturbations around a stable minimum $|\phi_0|$, called the vacuum
- For $\mu^2 > 0$ the vacuum lies at $|\phi_0| = 0$, thus it was justified to put $a_1 = 0$
- For $\mu^2 < 0$ we have an unstable maximum at $|\phi_0| = 0$

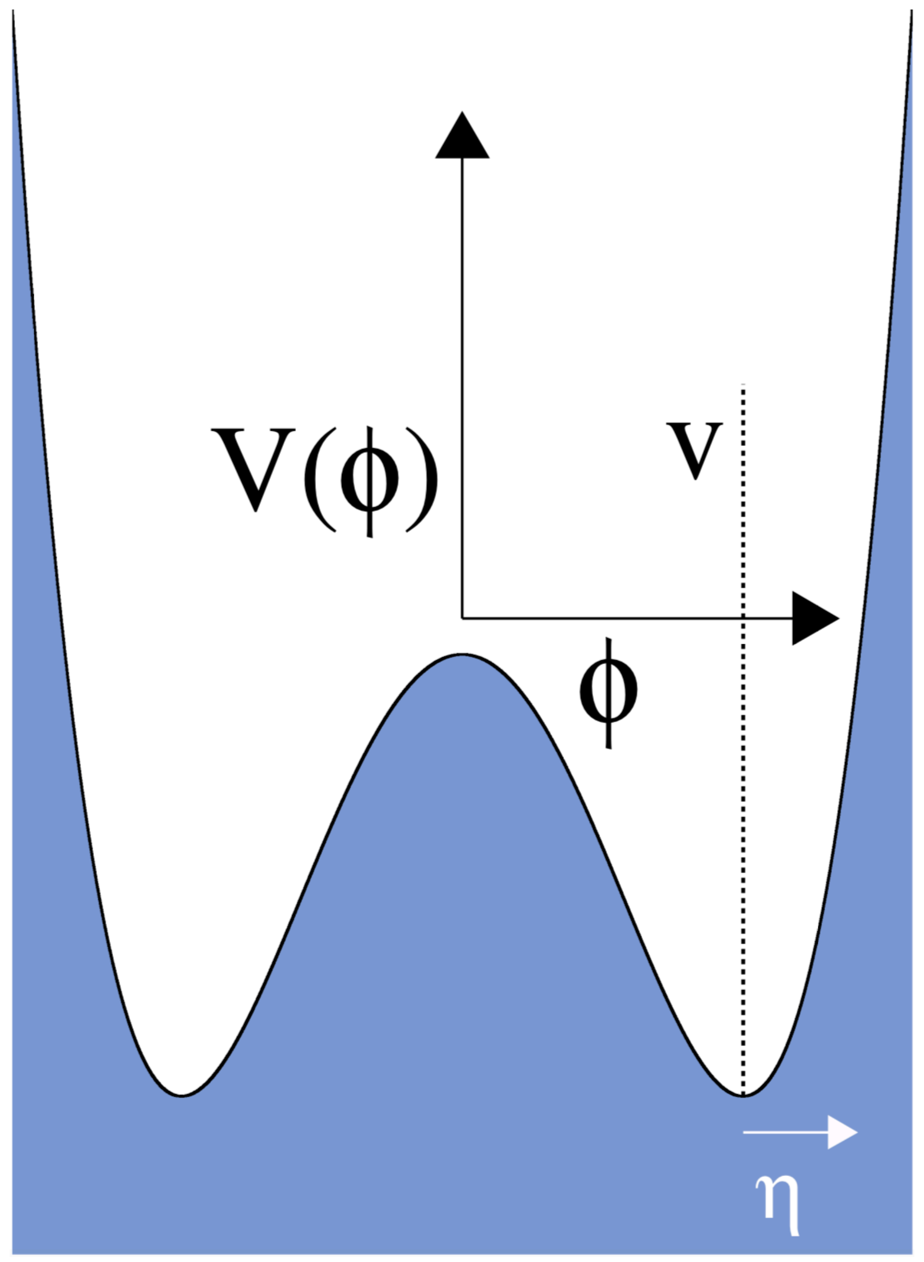
  and stable minima at $|\phi_0| = \sqrt{\dfrac{-\mu^2}{\lambda}}$



# Symmetry breaking

## Simple model

- Let us pick the vacuum to be $\phi_0 = \sqrt{\frac{-\mu^2}{\lambda}} \equiv v$
- Shift the fields by this constant $\eta = \phi - v$, such that $\eta_0 = 0$



# Symmetry breaking
## Simple model

$$\mathcal{L}_{kin} = \frac{1}{2}\left(\partial_\mu\left(\eta + v\right)\partial^\mu\left(\eta + v\right)\right) = \frac{1}{2}\left(\partial_\mu \eta\right)^2$$

$$V = \frac{\mu^2}{2}\left(\eta + v\right)^2 + \frac{\lambda}{4}\left(\eta + v\right)^4$$

$$= \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4 - \frac{1}{4}\lambda v^4$$

$$\mathcal{L}_{full} = \frac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda v^4$$

# Symmetry breaking
## Simple model

$$\mathscr{L}_{full} = \frac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda v^4$$

- Second term determines the mass: $\frac{1}{2}m_\eta^2 = \lambda v^2 \rightarrow m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$
  remember that $\mu^2 < 0$, thus the mass is positive
- Lagrangian for massive scalar particle with interactions
- No symmetry in $\eta \rightarrow -\eta$, although $\phi \rightarrow -\phi$ still present in original Lagrangian. In other words, the symmetry is broken in the vacuum, but still present in the original theory: ***Spontaneous Symmetry Breaking***

# Symmetry breaking

## U(1) global symmetry

- Let's alter the model by adding a complex scalar field $\phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$

  $$\mathscr{L} = \left(\partial_\mu \phi\right)^* \left(\partial^\mu \phi\right) - \mu^2 \left(\phi^* \phi\right) - \lambda \left(\phi^* \phi\right)^2$$

- Respects a U(1) global symmetry $\phi' \to e^{i\alpha}\phi$

- In terms of components

  $$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2 \left(\phi_1^2 + \phi_2^2\right) - \frac{1}{4}\lambda \left(\phi_1^2 + \phi_2^2\right)^2$$

Let us play the same game

# Symmetry breaking
## U(1) global symmetry

- Extremal values of the potential at

$$\sqrt{2}\,|\phi_0| = \sqrt{\phi_1^2 + \phi_2^2} = 0 \vee \sqrt{2}\,|\phi_0| = \pm\sqrt{\phi_1^2 + \phi_2^2} = \pm\sqrt{\frac{-\mu^2}{\lambda}} \equiv \pm v$$

- As in the previous case, the first minimum is the vacuum for $\mu^2 > 0$ and it is an unstable extremum for $\mu^2 < 0$

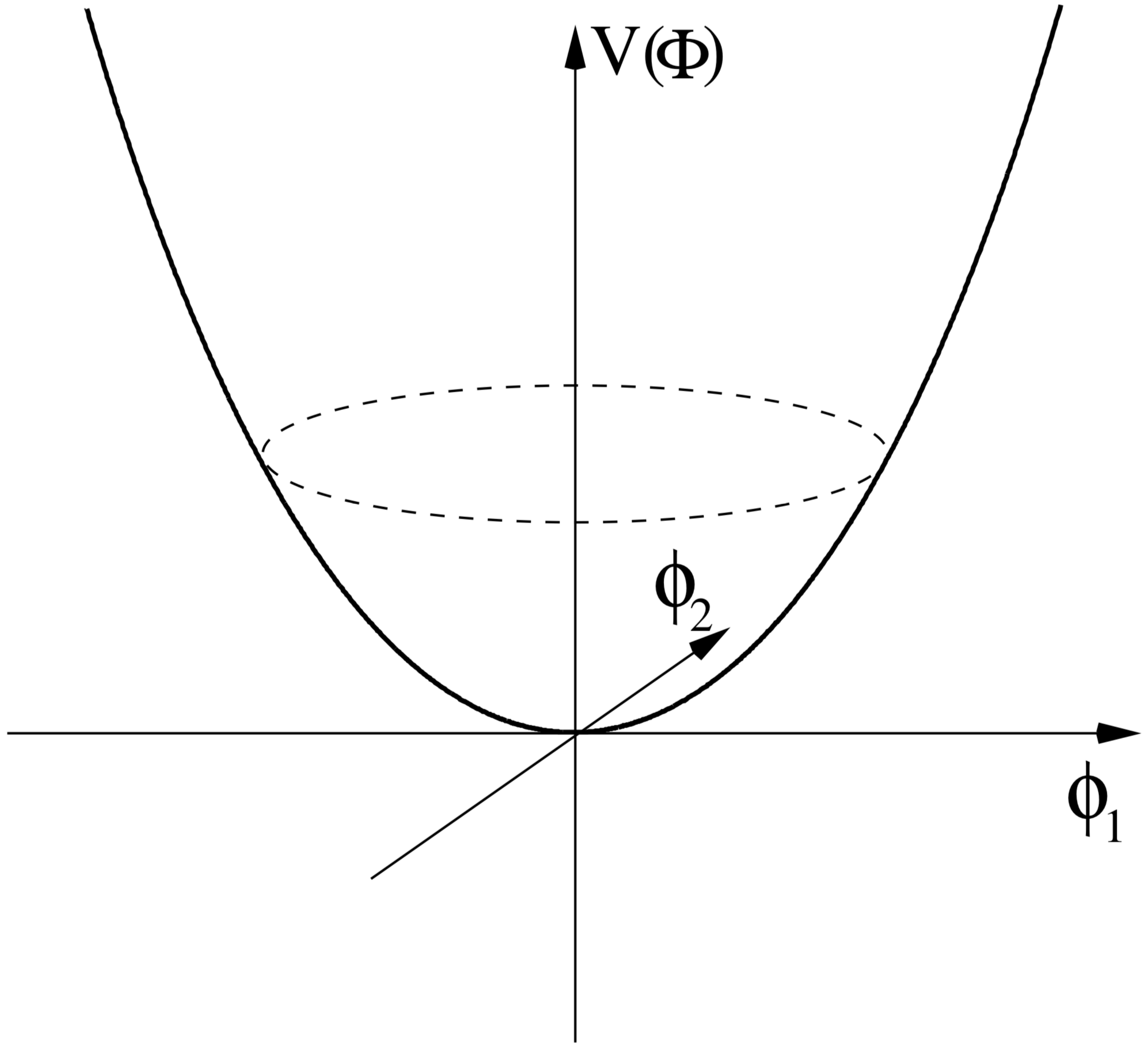
- However, the second minimum now corresponds to an infinite amount of vacua for $\mu^2 < 0$ constrained to the value of $v$

# Symmetry breaking

## U(1) global symmetry

$\mu^2 > 0$

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 - \frac{1}{2}\mu^2\phi_1^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2\phi_2^2 + \mathscr{L}_{int}$$




Particle 1 with mass $\mu$

Particle 2 with mass $\mu$

**Single vacuum at (0,0)**

# Symmetry breaking

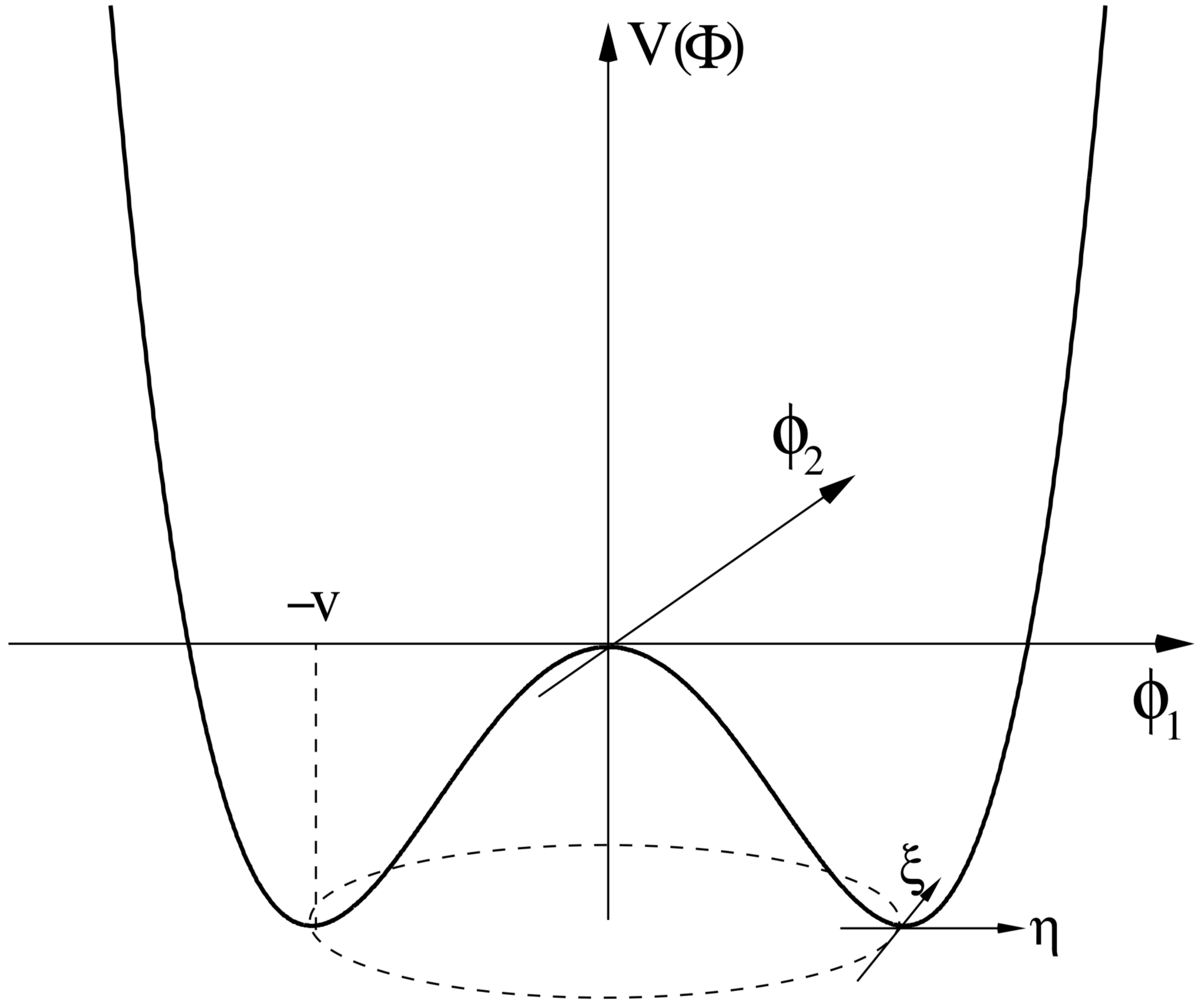
## U(1) global symmetry

$\mu^2 < 0$

Unstable extremum at (0,0)

**Infinite vacua constrained to** $\sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\dfrac{-\mu^2}{\lambda}} \equiv v$

Choose a vacuum state $\phi_0$ as $\phi_1 = v \wedge \phi_2 = 0$

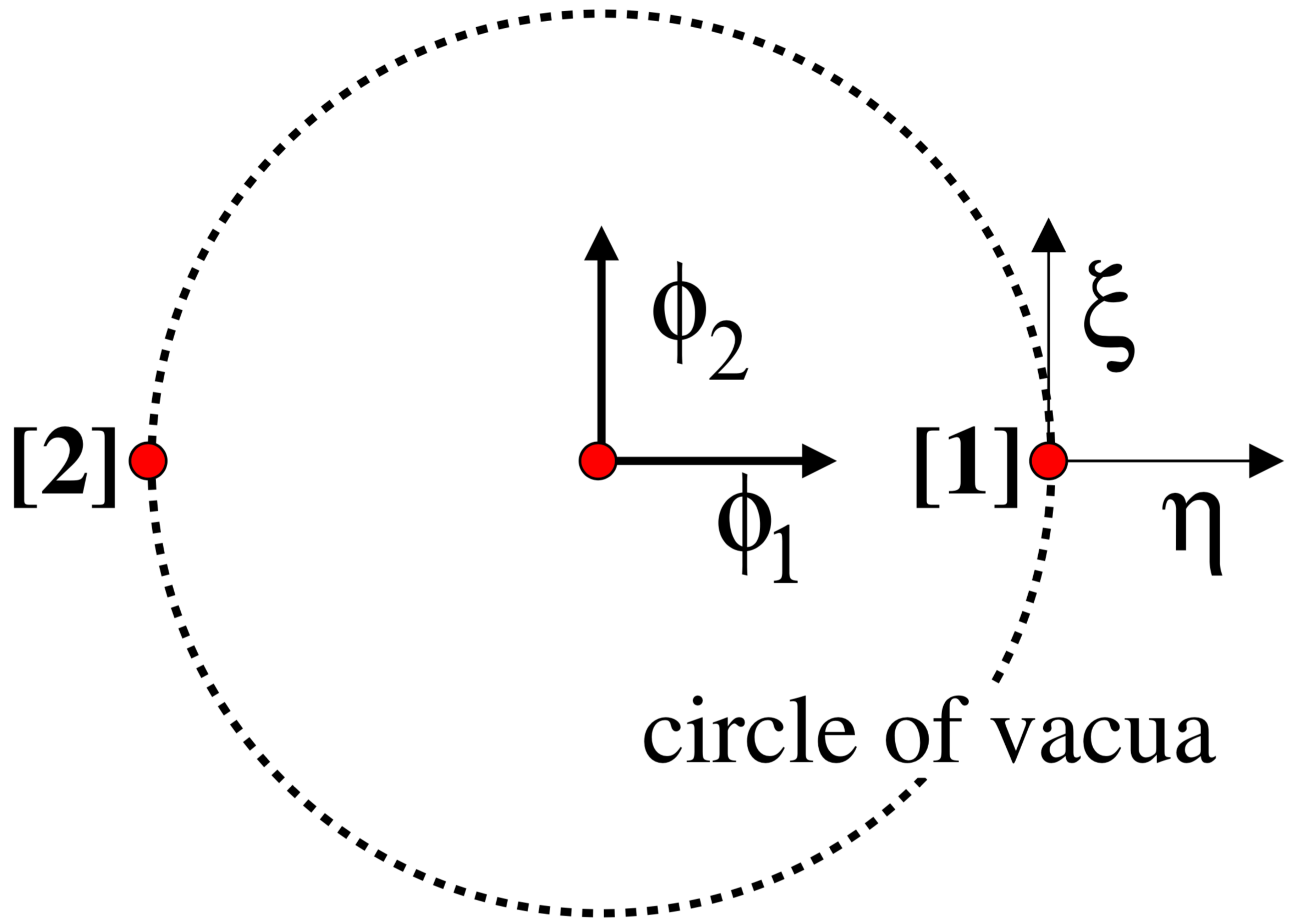


# Symmetry breaking

## U(1) global symmetry

$\mu^2 < 0$

- Let us shift the fields again to have a zero vacuum expectation value:
  $\eta = \phi_1 - v$ and $\xi = \phi_2$ such that
  $$\phi = \frac{1}{\sqrt{2}}\left(\eta + v + i\xi\right)$$
- Subsequently we want to express our Lagrangian in terms of the shifted fields



# Symmetry breaking

## U(1) global symmetry

$\mu^2 < 0$

$$\mathscr{L}_{kin} = \frac{1}{2}\left(\partial_\mu\left(\eta + v - i\xi\right)\partial^\mu\left(\eta + v + i\xi\right)\right) = \frac{1}{2}\left(\partial_\mu \eta\right)^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2$$

$$V = -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2$$

# Symmetry breaking

## U(1) global symmetry

$\mu^2 < 0$

$$\mathscr{L}_{kin} = \frac{1}{2}\left(\partial_\mu\left(\eta + v - i\xi\right)\partial^\mu\left(\eta + v + i\xi\right)\right) = \frac{1}{2}\left(\partial_\mu \eta\right)^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2$$

$$V = -\frac{1}{4}\lambda v^4 + \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda \xi^4 + \lambda v \eta \xi^2 + \frac{1}{2}\lambda \eta^2 \xi^2$$

Massive scalar particle $\eta$ with mass

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0$$

# Symmetry breaking

## U(1) global symmetry

$$\mu^2 < 0$$

$$\mathscr{L}_{kin} = \frac{1}{2}\left(\partial_\mu\left(\eta + v - i\xi\right)\partial^\mu\left(\eta + v + i\xi\right)\right) = \frac{1}{2}\left(\partial_\mu \eta\right)^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2$$

$$V = -\frac{1}{4}\lambda v^4 + \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda \xi^4 + \lambda v \eta \xi^2 + \frac{1}{2}\lambda \eta^2 \xi^2$$

Massive scalar particle $\eta$ with mass $m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0$

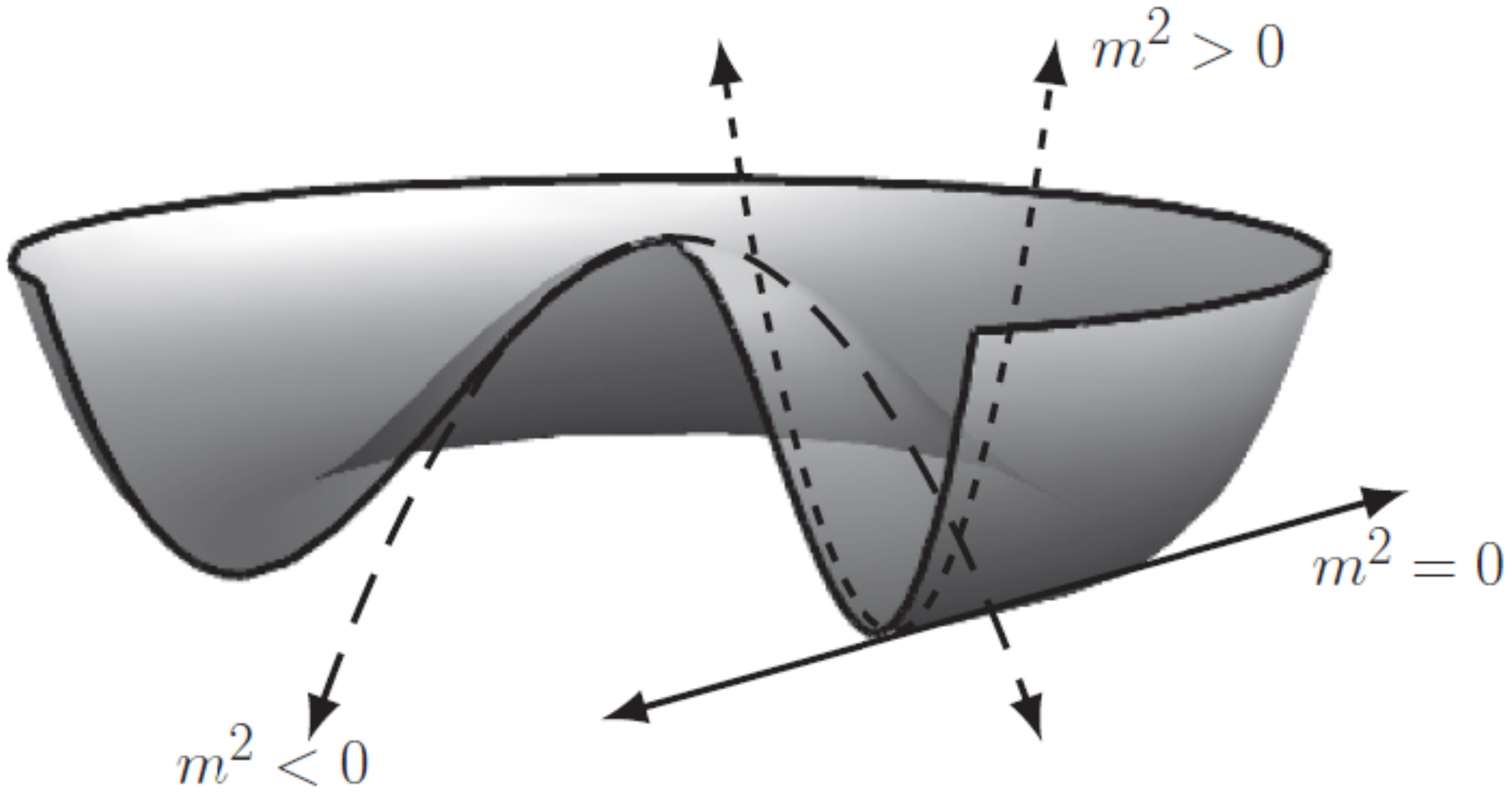
Massless scalar particle $\xi$

# Symmetry breaking
## U(1) global symmetry

- The vacuum is not U(1) invariant anymore, no $\eta' \to e^{i\alpha}\eta$ invariance
- At leading order a shift in $\xi$ as $\xi' \to \xi + \alpha$ leaves the Lagrangian invariant
- Can be shown that this is a symmetry at all orders if we were to parametrize the vacuum differently, $\phi = \left(v + c_1\eta\right) e^{ic_2\xi}$ for constants $c_1$ and $c_2$
- This is a remnant of the global U(1) symmetry and the massless spin-zero particle $\xi$ is the ***Goldstone boson***
- Each spontaneously broken continuous global symmetry implies the existence of a massless particle (Goldstone boson)

# Symmetry breaking

## U(1) global symmetry




The Mexican hat potential: expansions around the minimum with $m^2 > 0$ correspond to radial excitations ($\eta$), $m^2 = 0$ corresponds to excitations around the symmetry direction where the potential is flat ($\xi$).

# So what about local gauge symmetries?

# Symmetry breaking
## Local U(1) gauge symmetry

- Lagrangian invariant under $\phi' \to e^{i\alpha(x)}\phi$

- To this end we introduced the covariant derivative $D_\mu$ and vector field $A_\mu$ (gauge boson/photon field):

$$\partial_\mu \to D_\mu - ieA_\mu, \;\; A'_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\mathscr{L}_{QED} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - \frac{1}{4}F^2_{\mu\nu} - \mu^2\left(\phi^* \phi\right) - \lambda\left(\phi^* \phi\right)^2$$

# Symmetry breaking
## Local U(1) gauge symmetry

- $\mu^2 > 0$ is the same as in the previous model, but with an additional massless photon
- For $\mu^2 < 0$ we again obtain an infinite amount of vacua, bounded as
  $$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} \equiv v^2$$
- Define shifted fields as usual, such that $\phi = \frac{1}{\sqrt{2}}\left(\eta + v + i\xi\right)$

# Symmetry breaking

## Local U(1) gauge symmetry

$$\mathscr{L}_{kin} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right)$$

$$= \left(\partial_\mu + \boxed{ieA_\mu}\right)\phi^*\left(\partial^\mu - \boxed{ieA^\mu}\right)\phi$$

New terms

$$V = \lambda v^2 \eta^2 + \dots$$

# Symmetry breaking

## Local U(1) gauge symmetry

$$\mathscr{L}_{kin} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right)$$

$$= \left(\partial_\mu + ieA_\mu\right)\phi^* \left(\partial^\mu - ieA^\mu\right)\phi$$

New terms

$$V = \lambda v^2 \eta^2 + \ldots$$

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda v^2 \eta^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2 - \frac{1}{4}\left(F_{\mu\nu}\right)^2 + \frac{1}{2}e^2 v^2 A_\mu^2 - evA_\mu\left(\partial^\mu \xi\right) + \ldots$$

# Symmetry breaking

## Local U(1) gauge symmetry

$$\mathscr{L}_{kin} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right)$$
$$= \left(\partial_\mu + \boxed{ieA_\mu}\right)\phi^* \left(\partial^\mu - \boxed{ieA^\mu}\right)\phi$$

New terms

$$V = \lambda v^2 \eta^2 + \ldots$$

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda v^2 \eta^2 + \frac{1}{2}\left(\partial_\mu \xi\right)^2 - \frac{1}{4}\left(F_{\mu\nu}\right)^2 + \frac{1}{2}e^2 v^2 A_\mu^2 - evA_\mu\left(\partial^\mu \xi\right) + \ldots$$

Massive $\eta$

Massless $\xi$

Photon

Weird

# Symmetry breaking
## Local U(1) gauge symmetry

- The weird term complicates the interpretation of the Lagrangian
- Luckily there is a way around!
- Remember that the gauge fields are defined up to a (partial) derivative $A'_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$, let us focus on the following three terms in $\mathscr{L}$
- $\frac{1}{2}\left(\partial_\mu \xi\right)^2 + \frac{1}{2}e^2v^2A_\mu^2 - evA_\mu\left(\partial^\mu \xi\right) = \frac{1}{2}e^2v^2\left(A_\mu - \frac{1}{ev}\left(\partial_\mu \xi\right)\right)^2$

# Symmetry breaking
## Local U(1) gauge symmetry

- Here we recognize a transformed gauge field with $\alpha = -\xi/v$, such that it becomes $\frac{1}{2}e^2v^2\left(A'_\mu\right)^2$. This choice is called the ***unitary gauge***

- The scalar field becomes $\phi' \to e^{-i\frac{\xi}{v}}\phi = e^{-i\frac{\xi}{v}}\frac{1}{\sqrt{2}}\left(\eta + v + i\xi\right)$

  expand the exponential as $e^{i\frac{\xi}{v}} = 1 + i\frac{\xi}{v} + O(\xi^2)$, then
  $e^{-i\frac{\xi}{v}}\frac{1}{\sqrt{2}}\left(\eta + v + i\xi\right) = e^{-i\frac{\xi}{v}}\frac{1}{\sqrt{2}}\left(\eta + v\right)e^{i\frac{\xi}{v}}$, where we neglected the $O(\xi\eta)$ term

- Here we recognize $\eta$ as the Higgs scalar field $h$, so $\phi' \to \frac{1}{\sqrt{2}}\left(v + h\right)$

# Symmetry breaking

## Local U(1) gauge symmetry

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu h\right)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu^2 + e^2 v A_\mu^2 h + \frac{1}{2}e^2 A_\mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

$$-\frac{1}{4}F_{\mu\nu}^2$$

# Symmetry breaking

## Local U(1) gauge symmetry

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu h\right)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu^2 - \frac{1}{4}F_{\mu\nu}^2 + e^2 v A_\mu^2 h + \frac{1}{2}e^2 A_\mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

Massive $h$

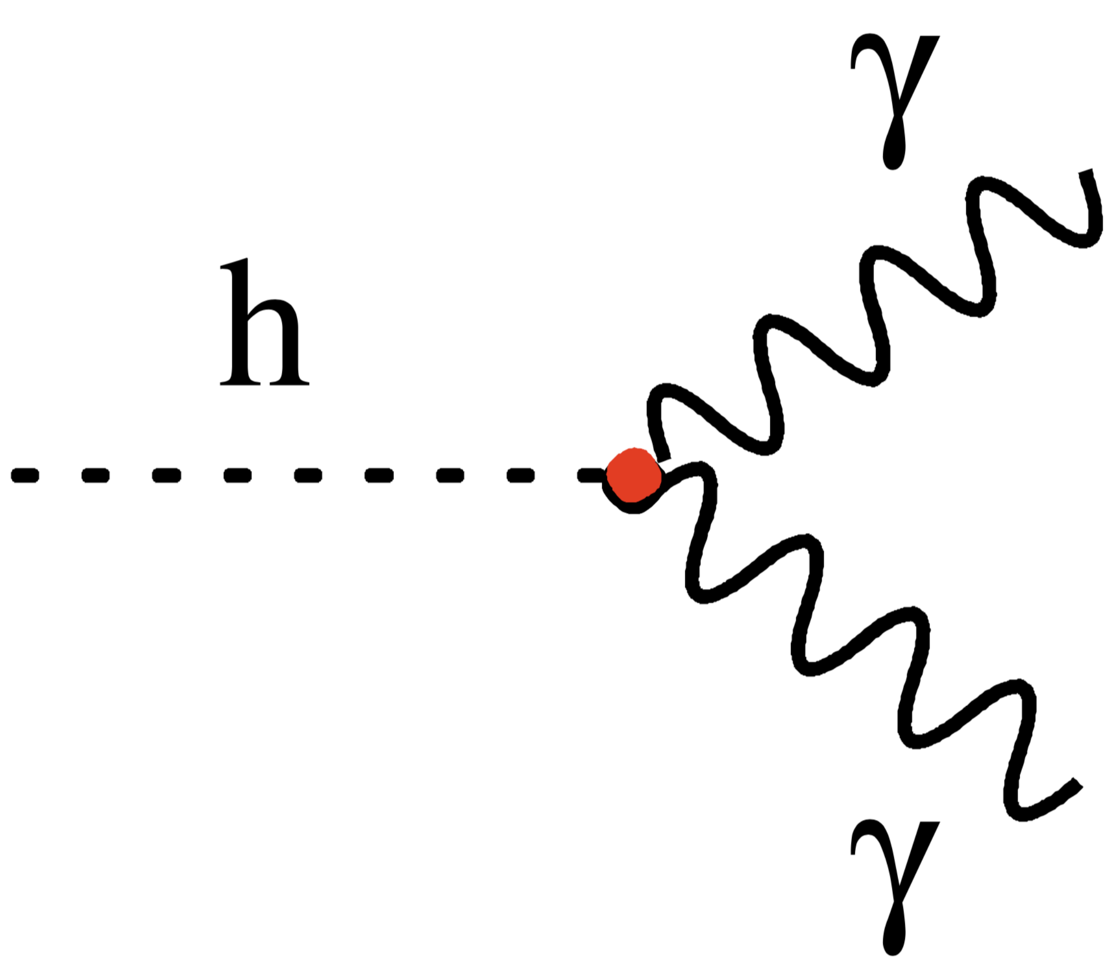
Massive $\gamma$

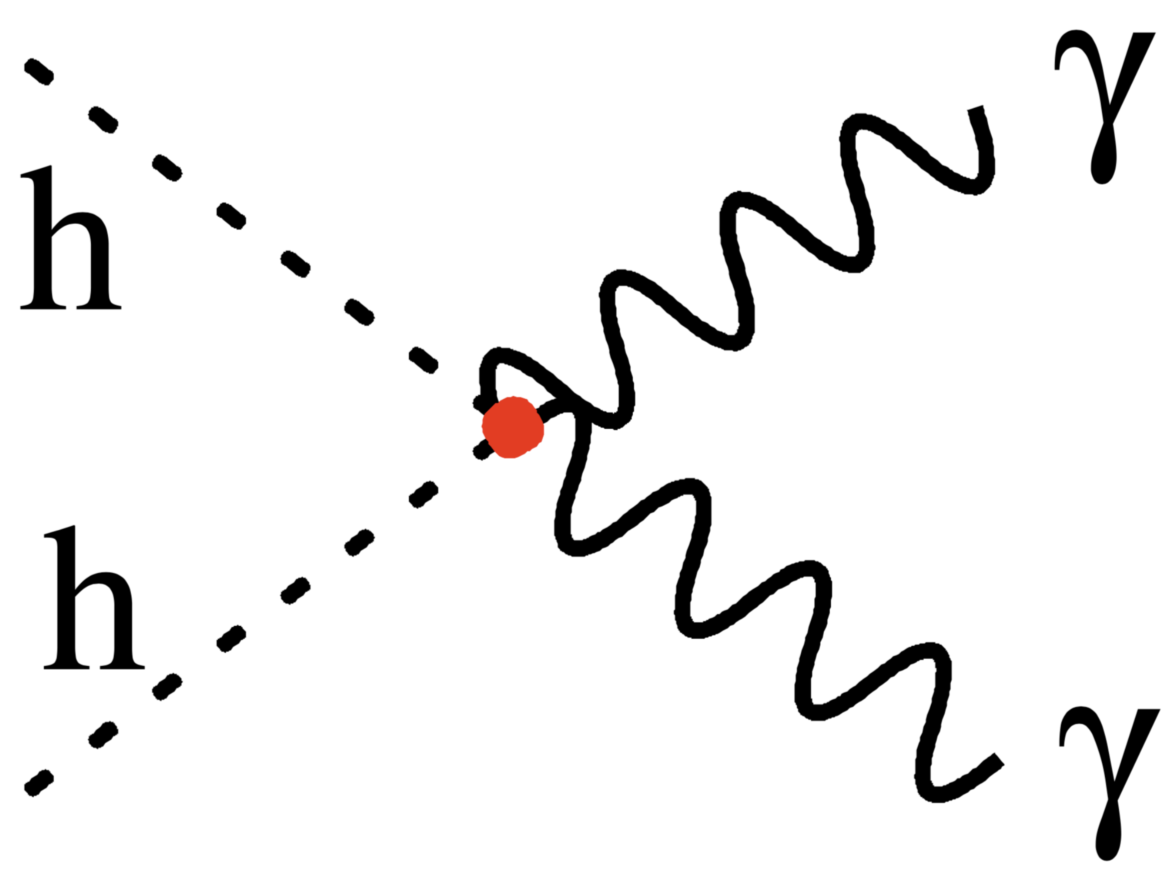
$h - \gamma$ interactions

$h$ self-interactions

# Symmetry breaking
## Local U(1) gauge symmetry

- The gauge boson $A_\mu$ has eaten the Goldstone boson $\xi$ to obtain a mass $m_A = ev$, this is the ***Higgs mechanism***
- We identified interactions between the massive gauge boson and the massive Higgs scalar and also Higgs *self-interactions*



# Take away message

- A ***spontaneously broken symmetry*** is broken in the true vacuum, but still present in the original Lagrangian
- Each broken global continuous symmetry results in a massless particle, ***Goldstone boson***
- When breaking a local gauge symmetry, the Goldstone gets eaten by the gauge particle to become massive, ***Higgs mechanism***

# Extra: Superconductivity

# Ginzburg-Landau model

- Near $T_C$ with order parameter $\phi$
  $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m^2|\phi|^2 - \frac{1}{4}|\phi|^4$ where $m^2 \propto (T - T_C)$
- For $T < T_C \rightarrow$ abelian Higgs model, U(1) spontaneously broken
- Effective Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_A^2 A_\mu^2$$

# Ginzburg-Landau model

- Current density $\vec{j} = -\dfrac{\partial \mathscr{L}}{\partial \overrightarrow{A}} = -m_A^2 \overrightarrow{A}$, second London equation
- Also $\nabla \times \vec{j} = -m_A^2 \nabla \times \overrightarrow{A} = -m_A^2 \overrightarrow{B}$ and $\nabla \times \overrightarrow{B} = \vec{j}$
- Then

$$\begin{aligned} \nabla \times (\nabla \times \overrightarrow{B}) &= \nabla(\nabla \cdot \overrightarrow{B}) - \nabla^2 \overrightarrow{B} \\ &= \nabla \times \vec{j} = -m_A^2 \overrightarrow{B} \end{aligned}$$

- Remember that $\nabla \cdot \overrightarrow{B} = 0$

# Ginzburg-Landau model

- First London equation $(m_A^2 - \nabla^2)\overrightarrow{B} = 0$
- For $m_A^2 \neq 0$ there is no constant solution
- The Meissner effect: a superconductor cannot possess a magnetic field
- A solution

$$\overrightarrow{B}(x) \propto \overrightarrow{B}_0 e^{-\frac{x}{\lambda}}$$

$$\lambda = \frac{1}{m_A}$$

where $\lambda$ is the penetration depth