

(part 2)

Higgs in SM

We will discuss 2 main topics

- Gauge boson masses
- Fermion masses
- (◦ interactions Higgs with physical fields?)

Quick background EW-sector

$$SM : \text{SU}(3)_c \times \underbrace{\text{SU}(2)_L \times \text{U}(1)_Y}_{\text{electroweak}}$$

gauge bosons: $w^{a,\mu}$ and B^μ , where $a=1, 2, 3$

fermion fields: $\psi = \psi_L + \psi_R$

$$\uparrow \quad \quad \quad = \frac{(\gamma^5)^-}{2} \psi + \frac{(\gamma^5)^+}{2} \bar{\psi}$$

transform in
the fundamental
representation
of $\text{SU}(2) \times \text{U}(1)_L$

$$\text{with } \gamma^5_- = \gamma^5 \quad ; \quad \gamma^5_+ = \gamma^5$$

$$\bar{\psi} = \psi^+ \gamma^0$$

\Rightarrow impose $\text{SU}(2)_L$ symmetry on left-handed fermions only.

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

doublet
of Dirac
spinors

problems

1) no boson masses

→ we know Z^0 , W^\pm are massive

→ photon massless.

Use Higgs Mechanism!

2) no fermion masses

→ not allowed (by invariance under gauge sym)

Use Yukawa coupling with Higgs!

3) not unitary

→ some $\sigma \propto E^2$

we will not discuss how introducing the Higgs solves this.

Masses for gauge bosons

We have 4 gauge bosons: $W^{\mu,a}$, B^μ

must be
massless
to respect
gauge sym.

$$L_{\text{bosons}} = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^a B_a^{\mu\nu} + \text{coupling to fermions} \\ + D_\mu H^\dagger D^\mu H - V(H)$$

where $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$ $\gamma = \frac{1}{2}$
4 dof.

$$H(x) = \frac{e^{ig_1 \beta(x)} Y}{c u(x)_Y} \frac{e^{ig_2 \alpha(x)} T^a}{c s u(2)_L} H(x)$$

$$T^a = \frac{1}{2} \tau^a$$

$$D_\mu H(x) = \partial_\mu H(x) - ig_2 w_\mu^a(x) T_a H(x) - \frac{ig_1}{2} B_\mu(x) H(x)$$

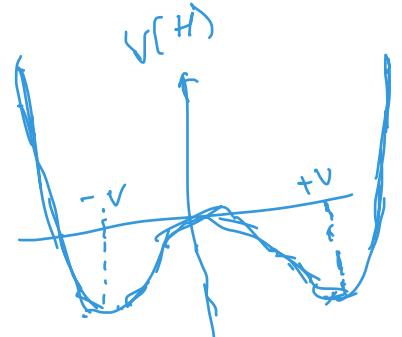
$\downarrow \mu^2 < 0$

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

used $\gamma_H = \frac{1}{2}$

\curvearrowleft H will obtain a non-zero VEV

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ v \end{pmatrix}$$



$$v^2 = \frac{-\mu^2}{2\lambda} \quad \begin{matrix} (\text{v from muon decay}) \\ \rightarrow \text{see later} \end{matrix}$$

\rightarrow Higgs VEV: source of EWSB

$$SU(2)_L \times U(1)_Y \xrightarrow{\quad} U(1)_{EM}$$

\curvearrowleft gauge boson: photon

linear combination of initial gauge fields stays massless

how do we see/know that there is a symmetry left after expanding around the vacuum?

\rightarrow subgroup of $SU(2)_L \times U(1)_Y$ leaves $\langle H \rangle$ invariant.

$$e^{ig_2 T^3(x)} e^{ig_1 \gamma_H B(x)} \langle H \rangle = \langle H \rangle$$

$$e^{i(\frac{1}{2}g_2 \alpha^3(x) + \frac{1}{2}g_1 \beta(x))} \quad \left(\begin{array}{c} e^{\frac{i}{\sqrt{2}} \alpha^3(x)} \\ 0 \\ e^{\frac{i}{\sqrt{2}} \beta(x)} \end{array} \right)$$

$$\xi_2(x) \propto -g_2 \alpha^3(x) + g_1 \beta(x)$$

since we want $\xi_{EM}(x)$ to leave vacuum invariant.

$$\Rightarrow \alpha^3(x) = \cos \theta_W \xi^z(x) + \sin \theta_W \xi^{EM}(x)$$

$$\beta(x) = \cos \theta_W \xi^{EM}(x) - \sin \theta_W \xi^z(x)$$

$$(-g_2 \cos \theta_W - g_1 \sin \theta_W) \xi^z$$

$$+ (-g_2 \sin \theta_W + g_1 \cos \theta_W) \xi^{EM}$$

$$= 0 \text{ if } \tan \theta_W = \frac{g_1}{g_2}$$

in accordance with this mixing we also redefine

$$B^r = \cos \theta_W A_\mu - \sin \theta_W Z_\mu$$

$$W_\mu^3 = \cos \theta_W Z_\mu + \sin \theta_W A_\mu$$

one can prove that

$$A^\mu' = A^r + \partial^\mu \xi^{EM}$$

$$Z^\mu' = Z^r + \partial^\mu \xi^z$$

$$T_3 g_2 \alpha^3(x) + Y g_1 \beta(x) = (T_3 g_2 \cos \theta_W - Y g_1 \sin \theta_W) \xi^z$$

$$+ (T_3 g_2 \sin \theta_W + Y g_1 \cos \theta_W) \xi^{EM}$$

$$\Rightarrow \xi^{EM} \underbrace{g_1 \cos \theta_W}_{e} \underbrace{(T_3 + Y)}_{Q}$$

$$\begin{aligned}
 D_\mu H|_{VEV} &= [\partial_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu + i g_1 Y_H \beta_\mu] \left(\begin{smallmatrix} 0 \\ \frac{v}{\sqrt{2}} (+h) \end{smallmatrix} \right) \\
 &= \frac{i}{\sqrt{8}} \left(g_2 \tau^1 W^1 + g_2 \tau^2 W^2 + g_2 \tau^3 W^3 + g_1 \beta^0 \right) \left(\begin{smallmatrix} 0 \\ v \end{smallmatrix} \right) \xrightarrow{\text{to get interaction terms.}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{i}{\sqrt{8}} \begin{pmatrix} g_2 W^0 + g_1 \beta^0 & g_2 (w_1 - iw_2) \\ g_2 (w_1 + iw_2) & -g_2 w^0 + g_1 \beta^0 \end{pmatrix} \left(\begin{smallmatrix} 0 \\ v \end{smallmatrix} \right) \\
 &= \frac{iv}{\sqrt{8}} \begin{pmatrix} g_2 (w_1 - iw_2) \\ -g_2 w_3 + g_1 \beta^0 \end{pmatrix}
 \end{aligned}$$

$$(D_\mu H)^+ D^\mu H = \frac{v^2}{8} \left[g_2^2 (w_1^2 + w_2^2) + (-g_2 w_3^0 + g_1 \beta^0)^2 \right]$$

$\nwarrow w^-, w^+$ $\nearrow \gamma, Z^0$

Massive \bar{W} , W

generators $\tau_1 \propto \tau_2$ mix the components of left handed fermion doublets.

→ interaction piece from dirac h

$$L_{int} = -g_2 \bar{\psi}_L \gamma^\mu w_\mu^\alpha T_\alpha \psi_L$$

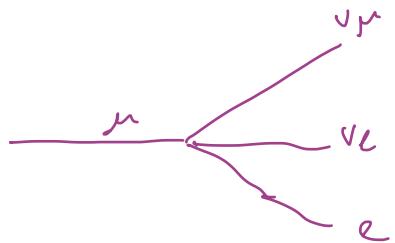
→ weak force only couples to left handed fermions

$$\bar{\psi}_L \left(\begin{smallmatrix} 0 & \underbrace{w_1 - iw_2}_{w^-} \\ \underbrace{w_1 + iw_2}_{w^+} & 0 \end{smallmatrix} \right) \psi_L$$

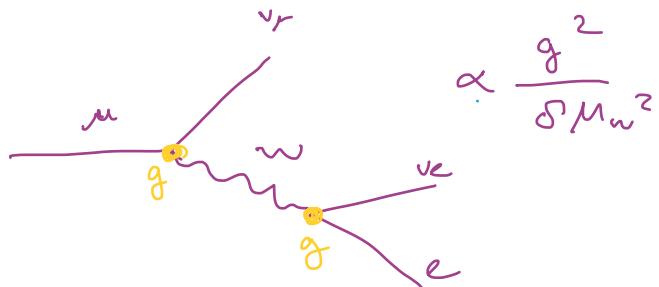
going back to our mass term we see

$$w_1^2 + w_2^2 = w^-^2 + w^+^2$$

and we deduce that $M_W = M_{W^\pm} = \frac{g_2 v}{2}$



$$\propto \frac{G_F}{\sqrt{2}}$$



$$v = \frac{1}{\sqrt{\sqrt{2} G_F}} = 246 \text{ GeV} \quad (\text{EW scale})$$

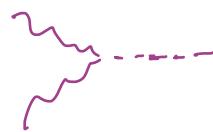
coupling to H :

$D_\mu H^+ D^\mu H$ upon expanding $\frac{i}{\sqrt{2}}(v_{th})$

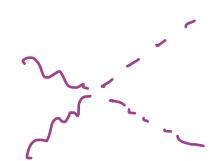
τ_{W^\pm}

$$\boxed{\frac{1}{8} w_\mu^+ (h + v) g_2^2 w^\mu_- (h + v)}$$

$$g_2 v w^- w^+ h \sim \frac{M_W^2}{\sqrt{2}} g^{\mu\nu}$$



$$\frac{g_2^2}{2} w^- w^+ H H \sim \frac{M_W^2}{2\sqrt{2}} g^{\mu\nu}$$



Massive Z^0 , massless γ

$$\frac{v^2}{8} (-g_2 w_3^r + g_1 B^r)^2 = \frac{v^2}{8} (w_3^r B^r) \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} w_3^r \\ B^r \end{pmatrix}$$

$2Y_H$

So only because $Y_H \neq 0$ the fields mix

eigenvectors of the mass matrix represent physical fields

$$\lambda = 0 \rightarrow \frac{1}{\sqrt{g_1^2 + g_2^2}} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

$$\lambda = g_1^2 + g_2^2 \rightarrow \frac{1}{\sqrt{g_1^2 + g_2^2}} \begin{pmatrix} g_1 \\ -g_2 \end{pmatrix}$$

one can now write: (since $\det = 0$)

$$\frac{1}{\sqrt{g_1^2 + g_2^2}} \begin{pmatrix} g_1 & g_1 \\ g_2 & -g_2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & g_1^2 + g_2^2 \end{pmatrix} \begin{pmatrix} g_1 & g_2 \\ g_1 & -g_2 \end{pmatrix} \frac{1}{\sqrt{g_1^2 + g_2^2}}$$

check that it reproduces $\begin{pmatrix} g_1^2 - g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix}$

now we see that our mass eigenstate is given by the fields

$$A^\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 W_3^\mu + g_2 B^\mu) \Rightarrow M_1 = 0$$

$$\gamma^\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 W_3^\mu - g_2 B^\mu) \Rightarrow M_2 = \frac{\sqrt{g_1^2 + g_2^2}}{2}$$

where we can rewrite $M_2 = \frac{\sqrt{g_2}}{2 \cos \theta_W} = \frac{\sqrt{g_2}}{2} \sqrt{1 + \tan^2 \theta_W}$

We can determine $\frac{M_W}{M_2} = \cos \theta_W$ from experiment

and even put bounds on $M_2 \geq M_W$

note: $e, \theta_W \Leftrightarrow g_1, g_2$

SM makes no predictions for M_W, M_2

Fermion masses

$$Q_L = \begin{pmatrix} v_L \\ e_L \end{pmatrix} \quad \leftarrow Y = -\frac{1}{2}$$

$$L_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \leftarrow Y = \frac{1}{6}$$

\uparrow weak isospin doublets

$$e_R, v_R, u_R, d_R$$

$$Y = -1, Y = 0, Y = \frac{2}{3}, Y = -\frac{1}{3}$$

\uparrow weak isospin singlets

$$Q = Y + T_3$$

$$D_\mu \Psi_L = \partial_\mu \Psi_L - i q_1 Y_\Psi \beta_\mu \Psi_L - i q_2 \vec{T} \cdot \vec{W} \Psi_L$$

We are not allowed to write a mass term for the fermions, since we are considering a chiral theory:

$$m_f \bar{\Psi} \Psi = m [(\bar{\Psi}_L + \bar{\Psi}_R) (\Psi_L + \Psi_R)]$$

$$= m [\cancel{\bar{\Psi}} \cancel{\Psi}_L + \cancel{\bar{\Psi}} \cancel{\Psi}_R + \underline{\bar{\Psi}_L \Psi_R} + \underline{\bar{\Psi}_R \Psi_L}]$$

$$\begin{aligned} \bar{\Psi}_L \Psi_L &= \bar{\Psi} \gamma^0 (-\gamma^{5+}) \gamma^0 (-\gamma^5) \Psi \\ &= \bar{\Psi} (1 + \gamma^5) (1 - \gamma^5) \Psi \\ &= \bar{\Psi} (1 - \gamma^{52}) \Psi \\ &= 0 \end{aligned}$$

'forbidden by gauge invariance'

How do fermions get mass?

\rightarrow Yukawa coupling with the H complex scalar field

We are allowed to add terms as

$\lambda \bar{\psi}_L H \psi_R$, since it is invariant as long as we make sure $Y_\psi + Y_h + Y_{\psi R} = 0$

yukawa coupling

for example:

$$\lambda_e \bar{L}_L H e_R + \lambda_{e\bar{e}} \bar{H}^+ L_L$$

$$\text{EWSB} \rightarrow \frac{\lambda_e v}{\sqrt{2}} \underbrace{[\bar{e}_L e_R + \bar{e}_R e_L]}_{\bar{e}e} + \frac{\lambda_e}{\sqrt{2}} [h \bar{e}e] \underbrace{\text{eeh int}}_{\propto M_e}$$

with this same construction we can give masses to down-type quarks. However, up-type quarks and neutrino's require a different coupling to the higgs to obtain mass

$$Y = -\frac{1}{6} - \frac{1}{2} + \frac{2}{3} = 0$$

$$\lambda_u \bar{Q}_L \tilde{H}^c u_R + \text{n.c.}$$

$$\hookrightarrow \tilde{H}^c = -i \tau_2 H^* \Rightarrow \frac{-i}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}$$

note: all yukawa couplings λ are free parameters

What about yukawa couplings that mix flavors?

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} ; \begin{pmatrix} T_3 = \frac{+1}{2} \\ T_3 = \frac{-1}{2} \end{pmatrix}$$

$$u_R, i \leftarrow y = \frac{2}{3} \quad d_R, i \leftarrow y = -\frac{1}{3}$$

We can now make $(\bar{u}_L \bar{d}_L) H S_R$ ✓ inv under $U(1)$
 $-\frac{1}{6} + \frac{1}{2} - \frac{1}{3} = 0$ ✓ inv under $SU(2)$

this means that the states we have grouped together in doublets are not eigenstates of the mass matrix

CKM-matrix

→ we will focus on the mixing between quark flavors
(similar for neutrino's)

most general : $\mathcal{L} = -Y_{ij}^d Q_{L,i}^{I\text{ interaction}} H d_{R,j}^I - Y_{ij}^u Q_{L,i}^{I\text{ interaction}} H u_{R,j}^I + \text{h.c}$

$Y_{\text{ Yukawa}}$

+ leptons

Y_{ij} in general complex matrices

indeed if $i \neq j$ we get terms that are hard to interpret

$$Y_{ij}^d Q_{L,i}^I H d_{R,j}^I \Big|_{\text{VEV}} = (d_L^I, s_L^I, b_L^I) \frac{v}{\sqrt{2}} \begin{pmatrix} Y_{11}^d & Y_{12}^d & Y_{13}^d \\ Y_{21}^d & Y_{22}^d & Y_{23}^d \\ Y_{31}^d & Y_{32}^d & Y_{33}^d \end{pmatrix} \begin{pmatrix} d_R^I \\ s_R^I \\ b_R^I \end{pmatrix}$$

similarly one can construct the mass matrix for u -type

M_{ij}^d

It is possible to diagonalize H to find the physical (massive) quark fields.

$$M^D = D_L^+ M_{\text{diag}}^D D_R$$

with the requirement that D_L & D_R are unitary
($D_L^+ D_L = \mathbb{1}$, $D_R^+ D_R = \mathbb{1}$)

$$\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} = D_R \begin{pmatrix} d_L^I \\ s_L^I \\ b_L^I \end{pmatrix} \quad \text{and}$$

↪ $d_{R,i} = D_{R;j} d_{L,j}^I$

$$\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = D_L \begin{pmatrix} d_R^I \\ s_R^I \\ b_R^I \end{pmatrix}$$

↪ $d_{L,i} = D_L^{-1} d_{R,j}^I$

The same can be done for u, c, t
 $\rightarrow U_L, U_R$

diagonalize M^u
 $M^u = U_L^\dagger M^u_{\text{diag}} U_R$

since $U_L \neq D_L$ we will find
mixing between generations/families
 $\hookrightarrow u \& d$ in same doublet

to see this explicitly one can consider the interaction terms in $\mathcal{L} \mapsto$ switching to mass eigenstates will mix interaction terms.

$$\frac{g^2}{\sqrt{2}} \bar{u}_L^I \gamma^\mu w_\mu^- d_L^I \propto \bar{\psi}_L i \not{D} \psi_L$$

↓ rotate to physical fields

These is no mixing for right handed fermions

$$\sum_i \frac{g^2}{\sqrt{2}} \bar{u}_L U_L^{\pm i} W^- D_L^{\mp i}$$

including all families in a single notation

$$\sum_{ijk} \frac{g^2}{\sqrt{2}} \bar{u}_{L,i} U_L^{ij} W^- D_L^{jk} d_{L,k}$$

$$V_{CKM} = U_L D_L^\dagger$$

by convention,

$$u_i^I = u_i$$

$$d_i^I = V_{CKM,ij} d_j$$



source of CP-violation
 $CP(\bar{\psi}_L H \psi_R) = \bar{\psi}_R H^+ \psi_L$
so CP violated if $y_{ij} \neq y_{ji}^*$

So the price we pay is the mixing between families, e.g.-



→ similar thing going on for neutrino's
(no time)

CKM matrix important consequence :

→ mesons involving s are unstable & therefore decay

for example $V_{us} \approx 0.22$

+ --

$$U(x) \in SU(2)_L \rightarrow \psi' = U(x)\psi$$

$$U(x) = e^{i\alpha_a T^a} ; \text{ where } T^a = \frac{\gamma^a}{2}$$

$$\text{such that } w_\mu^a = w_\mu^a + \frac{1}{g} \partial_\mu \alpha^a - g^{abc} \alpha^b(x) W_\mu^c(x)$$

$$W^{\mu\nu} = \frac{i}{g} [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig [W_\mu, W_\nu]$$

$$\text{since } D_\mu = \partial_\mu - ig w_\mu$$

$$D_\mu \psi = \partial_\mu \psi - ig T^a w_a^\mu \psi$$

$$G(x) \in U(1)_Y$$

$$G(x) = e^{i Y \beta(x)}$$

$$\mathcal{L}_{\text{disc}} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi \rightarrow (i \gamma^\mu \partial_{\mu+m}) \psi = 0$$

$$D_\mu \psi = \partial_\mu \psi - ig Y B_\mu \psi$$

$$B_\mu = B_\mu + \frac{1}{g} \partial_\mu \beta(x)$$

$$\hat{Q} = \hat{Y} + \hat{T}_3$$

$$\tau^1 w^+ + \tau^2 w^- = \frac{1}{2} \begin{pmatrix} 0 & \omega_1 - i\omega_2 \\ \omega_1 + i\omega_2 & 0 \end{pmatrix} \cdot \begin{pmatrix} \tau^- w^+ & \tau^+ w^- \\ w^+ & 0 \end{pmatrix}$$

$$\boxed{\tau = \frac{\gamma}{2}}$$

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$i\tau^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau^- = \frac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau^+ = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\text{Now } \text{we see}$

$$\text{since } w^+ = \frac{i}{\sqrt{2}}(\omega_1 + i\omega_2)$$

$$w^- = \frac{1}{\sqrt{2}}(\omega_1 - i\omega_2)$$

$$\tau^1 w^+ + \tau^2 w^- = (\tau^+ w^- + \tau^- w^+) \frac{1}{\sqrt{2}}$$

Sent in
notes
Milne

$$(\bar{u}_L \bar{d}_L) \begin{pmatrix} -\omega & \omega^2 \\ \omega^2 & \omega \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\stackrel{?}{=} (\bar{u}_L \bar{d}_L) \begin{pmatrix} 0 & \omega - i\omega_2 \\ \omega + i\omega_2 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\hookrightarrow (\bar{u}_L \bar{d}_L) \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \omega^- \\ \frac{1}{\sqrt{2}} \omega^+ & 0 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

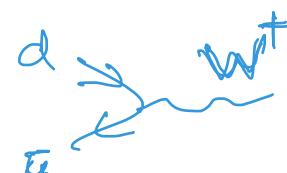
$$\hookrightarrow (\bar{u}_L \bar{d}_L) \left\{ \begin{array}{l} \frac{1}{\sqrt{2}} \omega^- d_L \\ \frac{1}{\sqrt{2}} \omega^+ u_L \end{array} \right\}$$

$$\hookrightarrow \frac{1}{\sqrt{2}} \bar{u}_L \cancel{\omega^+} d_L + \frac{1}{\sqrt{2}} \bar{d}_L \cancel{\omega^+} u_L$$

$$Q(\bar{u}) = -\frac{2}{3}$$

$$Q(\bar{d}) = -\frac{1}{3}$$

$$Q(\bar{u}) + Q(\bar{d}) = -1$$



$$(\bar{v} \bar{e}) \begin{pmatrix} \omega^- \\ \omega^+ \end{pmatrix} \begin{pmatrix} v \\ e \end{pmatrix} \xrightarrow{y = -\frac{1}{2}}$$

$$\bar{v} \bar{w}^+ e \not\propto \bar{e} w^+ v$$

$$Q(v) + Q(e) = -1$$

