Top-quark pair production at hadron colliders

Guoxing Wang

Based on:

W. L. Ju, Guoxing Wang, X. Wang, X. Xu, Y. Xu and L. L. Yang, arXiv:1908.02179,

Guoxing Wang, X. Xu, L. L. Yang and H. X. Zhu, JHEP, **1806**, 013 (2018), arXiv:1804.05218,

and some preliminary results

National Institute for Subatomic Physics(Nikhef)

5/12/2019

Catalog

- Introduction to top quark and top-quark pair production
- Top-quark pair production in threshold limit
- Top-quark pair production in soft limit—Soft Function
- Top-quark pair production in soft limit—Hard Function
- Summary and outlook

Basic facts about the top quark

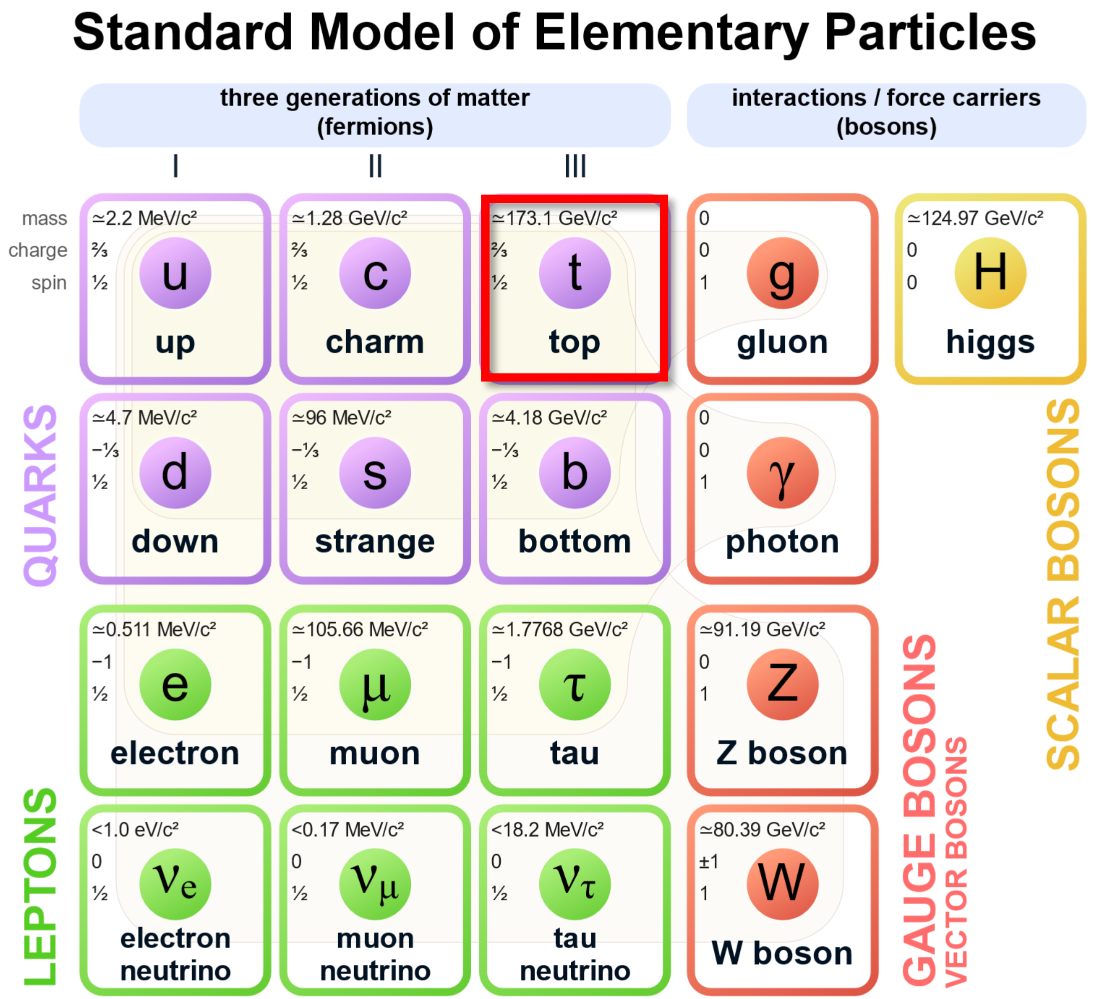
- Large mass: $m_t \sim 173\ \text{GeV}$
- Strong Yukawa coupling: $y_t \sim 1$

Fermion mass origin
Hierarchy problem
Vacuum stability

- Decay width: $\Gamma_t \sim 1.4\text{GeV}$

Decays before hadronization:

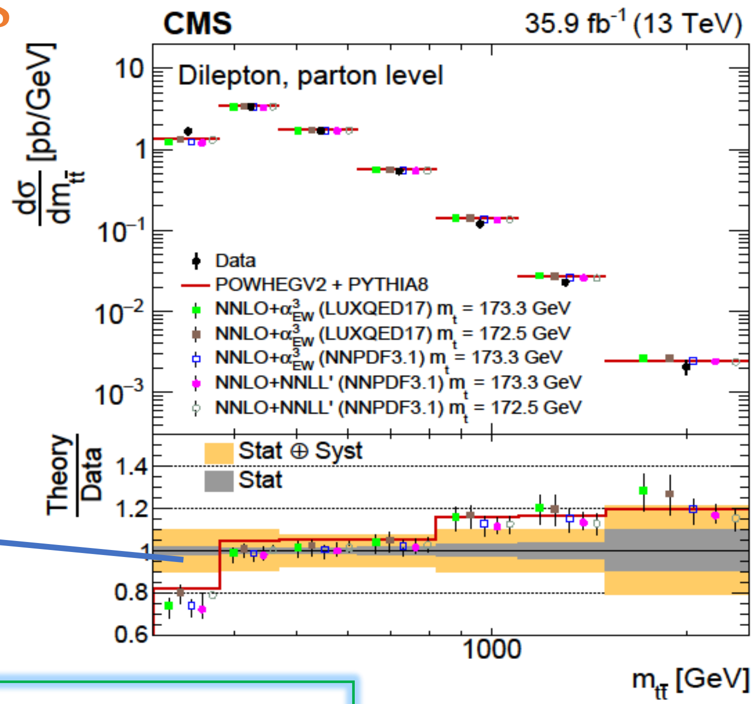
pQCD dominates



Top-quark pair production

A standard candle for the LHC and future colliders

- Test of the SM at the energy frontier
- Possible signals of new physics
- Major background to many searches



The first bin:
[300, 380] GeV

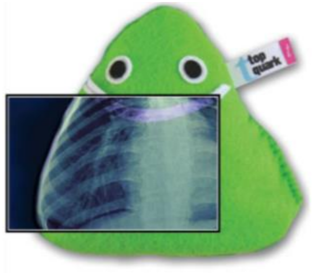
JHEP 1902(2019)149

Many important measurements

Threshold: sensitive to top quark mass.

Tail: influenced by some signals of new physics.

- Total cross section: $\sigma_{t\bar{t}}$
- Differential cross sections: $\boxed{M_{t\bar{t}}}$, $p_{T,t\bar{t}}$, $y_{t\bar{t}}$, $p_{T,t}$, $\cdots$



Top-quark pair production

Multi-scale process

- Threshold region:

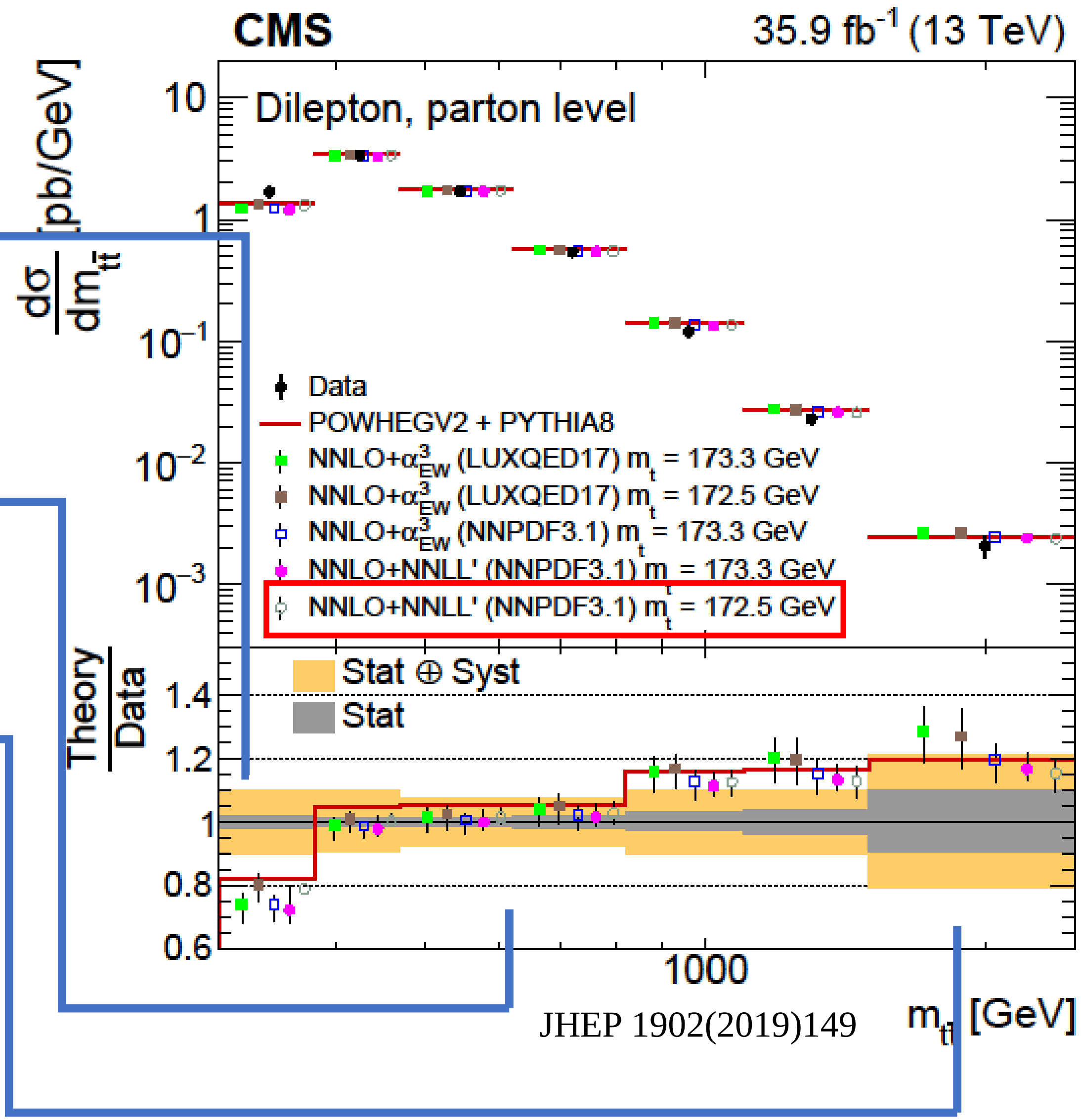
  fix-order + Coulomb + *soft*

- Intermediate region:

  fix-order + *soft*

- Boosted region:

  fix-order + *soft* + quasi-collinear



Factorization

- Kinematics: $N(P_1)+N(P_2)\to t(p_3)+\bar{t}(p_4)+X_{(s)}(p_{X_{(s)}})$

$$s=(P_1+P_2)^2\,,\quad \hat{s}=(p_1+p_2)^2\,,\quad M^2=(p_3+p_4)^2\,,$$

$$\boxed{z=\frac{M^2}{\hat{s}}\,,}\quad \tau=\frac{M^2}{s}\,,\quad \boxed{\beta=\sqrt{1-\frac{4m_t^2}{M^2}}\,,}\quad t_1=-\frac{M^2}{2}(1-\beta\cos\theta_t)\,.$$

- QCD factorization theorm: Collins, Soper and Sterman. arXiv:hep-ph/0409313

$$\frac{d\sigma}{dM}=\sum_{i,j}\int_\tau^1\frac{dz}{z}\,\frac{\tau}{z}\int d\Theta\,\frac{d\hat{\sigma}_{ij}(z,\mu_f)}{dM\,d\Theta}\,ff_{ij}(\tau/z,\mu_f)$$

- Threshold limit $\beta\to 0$ Ju, GXW, Wang, Xu, Xu and Yang: 1908.02179 $E=M-2m_t+\boxed{i\Gamma_t}$

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{NLP}}}{dM\,d\Theta}=\frac{16\pi^2\alpha_s^2(\mu_r)}{M^5}\sqrt{\frac{M+2m_t}{2M}}\sum_\alpha c_{ij,\alpha}(\cos\theta_t)\,H_{ij,\alpha}(z,M,Q_T,Y,\mu_r,\mu_f)\,J^\alpha(E)+\mathcal{O}(\beta^3)$$

- Soft limit $z\to 1$ Ahrens, Ferroglia, Neubert, Pecjak and Yang: 1003.5827

$$\frac{d\hat{\sigma}_{ij}}{dM\,d\cos\theta_t}=\frac{8\pi\beta}{3\hat{s}M}\mathrm{Tr}\Big[\boldsymbol{H}_{ij}(M,m_t,\cos\theta_t,\mu_f)\,\boldsymbol{S}_{ij}\Big(\sqrt{\hat{s}}(1-z),M,m_t,\cos\theta_t,\mu_f\Big)\Big]+\mathcal{O}(1-z)$$

Top-quark pair production near threshold-Review

- Total cross section:

Moch, Uwer: 0804.1476
Beneke, Falgari, Schwinn: 1007.5414
Beneke, Falgari, Klein, Schwinn: 1109.1536
Piclum, Schwinn: 1801.05788

$\beta \to 0 \ and \ z \to 1$

- Differential cross section:

Hagiwara, Sumino, Yokoya: 0804.1014

Petrelli, Cacciari, Greco, Maltoni, Mangano: hep-ph/9707223
Kiyo, Kuhn, Moch, Steinhauser, Uwer: 0812.0919

$\beta \to 0$

Please forgive me for missing your works

Top-quark pair production near threshold-Review

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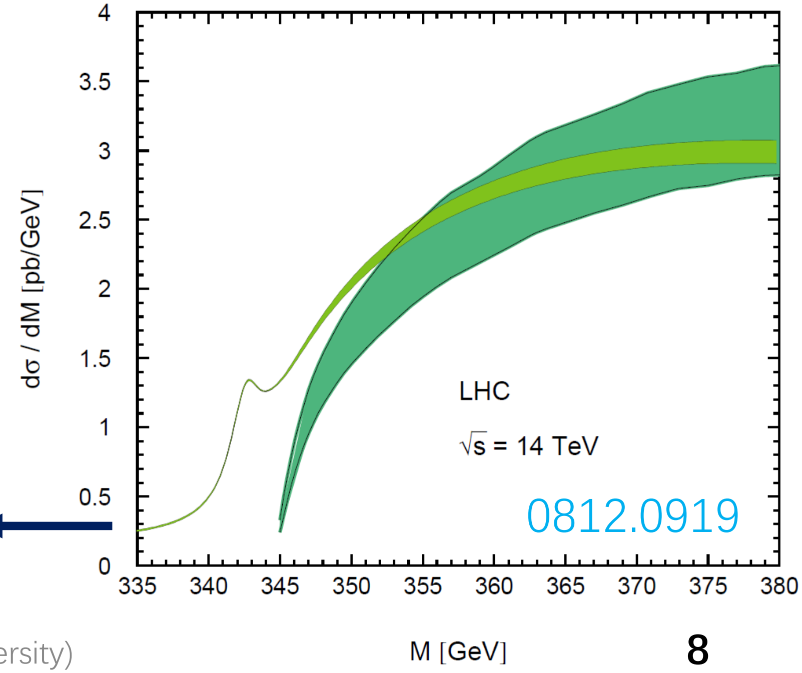
$\beta \to 0$

$$\frac{d\hat{\sigma}_{ij}}{dM} \sim \boxed{H_{ij,\alpha}} \otimes J^{\alpha}\,, \quad (\beta \to 0)$$

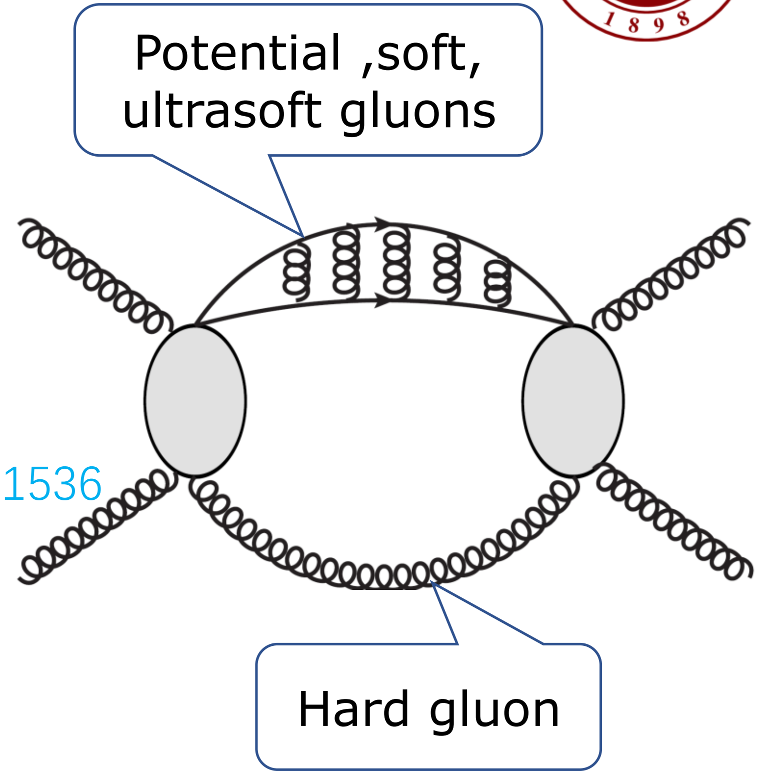
Soft gluon resummation and matching

“The formula is applicable for M up 360 GeV at most”



Top quark pair production near threshold

- Threshold limit: $\beta \to 0\ (M \to 2m_t),\ \alpha_s \sim \beta$
- Scale separation: $M \gg M\beta \gg M\beta^2$
- Factorization:
  - Potential function: $J^{\alpha}(E)$ Beneke, Falgari, Klein, Schwinn: 1109.1536
  - hard function: $H_{ij,\alpha}$
- Dynamic scale: $H_T = \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_t^2} \quad (\sim M)$
- Matching: Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang and Yang, arXiv:1803.07623



$$\frac{d\sigma^{\mathrm{(N)NLO+NLP}}}{dM} = \frac{d\sigma^{\mathrm{NLP}}}{dM} + \frac{d\sigma^{\mathrm{(N)NLO}}}{dM} - \frac{d\sigma^{\mathrm{(n)nLO}}}{dM}$$

Rescale factor

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{NLP}}}{dM\,d\Theta} = \frac{16\pi^2\alpha_s^2(\mu_r)}{M^5}\sqrt{\frac{M+2m_t}{2M}}\sum_{\alpha} \boxed{c_{ij,\alpha}(\cos\theta_t)} \times H_{ij,\alpha}(z, M, Q_T, Y, \mu_r, \mu_f)\, J^{\alpha}(E) + \mathcal{O}(\beta^3)$$

Resummation and fixed-order expansion

$$\frac{d\hat{\sigma}_{ij}^{\text{NLP}}}{dM\,d\Theta} = \frac{16\pi^2\alpha_s^2(\mu_r)}{M^5}\sqrt{\frac{M+2m_t}{2M}}\sum_{\alpha} c_{ij,\alpha}(\cos\theta_t)\,H_{ij,\alpha}(z,M,Q_T,Y,\mu_r,\mu_f)\,J^{\alpha}(E) + \mathcal{O}(\beta^3)$$

- **NLP kernel:** $K_{ij,\alpha}^{\text{NLP}}(z,M,m_t,Q_T,Y,\mu_r,\mu_f) = H_{ij,\alpha}^{(0)}(J_0^{\alpha}(E)+J_1^{\alpha}(E)) + \frac{\alpha_s(\mu_r)}{4\pi}H_{ij,\alpha}^{(1)}J_0^{\alpha}(E)$

$$= \frac{M^2}{8\pi}\sqrt{\frac{2E}{M}}\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n K_{ij,\alpha}^{(n)}$$

$$D_1 = -C_F,\; D_8 = \frac{1}{2N_c}$$

$$E = M - 2m_t + \boxed{i\Gamma_t}$$

LO

$$K_{ij,\alpha}^{(0)} = H_{ij,\alpha}^{(0)},$$

nLO

$$K_{ij,\alpha}^{(1)} = -2\pi^2 D_\alpha\sqrt{\frac{M}{2E}}H_{ij,\alpha}^{(0)} + H_{ij,\alpha}^{(1)},$$

n^2LO

$$K_{ij,\alpha}^{(2)} = \boxed{\frac{4\pi^4 D_\alpha^2}{3}\frac{M}{2E}H_{ij,\alpha}^{(0)}}$$

Divergence in $\beta\to 0$

$$+2\pi^2 D_\alpha\sqrt{\frac{M}{2E}}\left[(\beta_0 L_r - a_1)H_{ij,\alpha}^{(0)} - H_{ij,\alpha}^{(1)}\right],$$

n^3LO

⋮

n^4LO

$$H_{q\bar{q},1}^{(0)} = 0,$$

$$H_{q\bar{q},8}^{(0)} = \frac{C_A C_F}{9}\delta(Q_T^2)\,\delta(1-z)\,\delta(Y),$$

$$H_{gg,1}^{(0)} = \frac{C_F}{32}\delta(Q_T^2)\,\delta(1-z)\,\delta(Y),$$

$$H_{gg,8}^{(0)} = \frac{(C_A^2-4)C_F}{64}\delta(Q_T^2)\,\delta(1-z)\,\delta(Y).$$

Fixed-order expansion

$$\frac{d\hat{\sigma}_{ij}^{\rm NLP}}{dM\,d\Theta} = \frac{16\pi^2\alpha_s^2(\mu_r)}{M^5}\sqrt{\frac{M+2m_t}{2M}}\sum_\alpha c_{ij,\alpha}(\cos\theta_t)H_{ij,\alpha}(z,M,Q_T,Y,\mu_r,\mu_f)\,J^\alpha(E)+\mathcal{O}(\beta^3)$$

- NLP kernel:

$$K_{ij,\alpha}^{\rm NLP} = H_{ij,\alpha}^{(0)}\left(J_0^\alpha(E)+J_1^\alpha(E)\right)+\frac{\alpha_s(\mu_r)}{4\pi}H_{ij,\alpha}^{(1)}J_0^\alpha(E)$$

$$D_1=-C_F,\ D_8=\frac{1}{2N_c} \qquad = \frac{M^2}{8\pi}\sqrt{\frac{2E}{M}}\sum_{n=0}^\infty\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n$$

$$E=M-2m_t+\boxed{i\Gamma_t}$$

LO $$K_{ij,\alpha}^{(0)} = H_{ij,\alpha}^{(0)}\,,$$

nLO $$K_{ij,\alpha}^{(1)} = -2\pi^2 D_\alpha\sqrt{\frac{M}{2E}}H_{ij,\alpha}^{(0)}+H_{ij,\alpha}^{(1)}\,,$$

n^2LO $$K_{ij,\alpha}^{(2)} = \boxed{\frac{4\pi^4D_\alpha^2}{3}\frac{M}{2E}H_{ij,\alpha}^{(0)}}$$

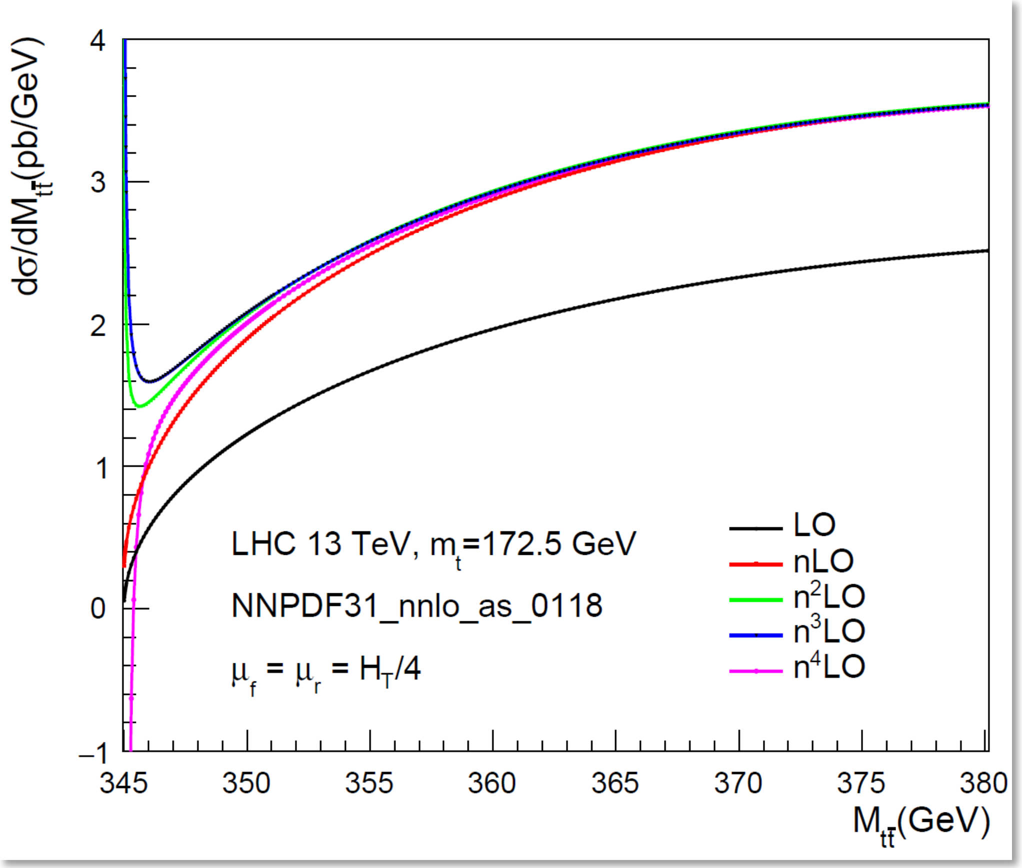
Divergence in $\beta\to0$

$$+2\pi^2D_\alpha\sqrt{\frac{M}{2E}}\left[(\beta_0L_r-a_1)H_{ij,\alpha}^{(0)}-H_{ij,\alpha}^{(1)}\right],$$

n^3LO

⋮

n^4LO



Various approximations

- Threshold limit $\beta \to 0$ with $c_{ij,\alpha}$
  - Our formula is indeed valid,
    and supports the resummation.

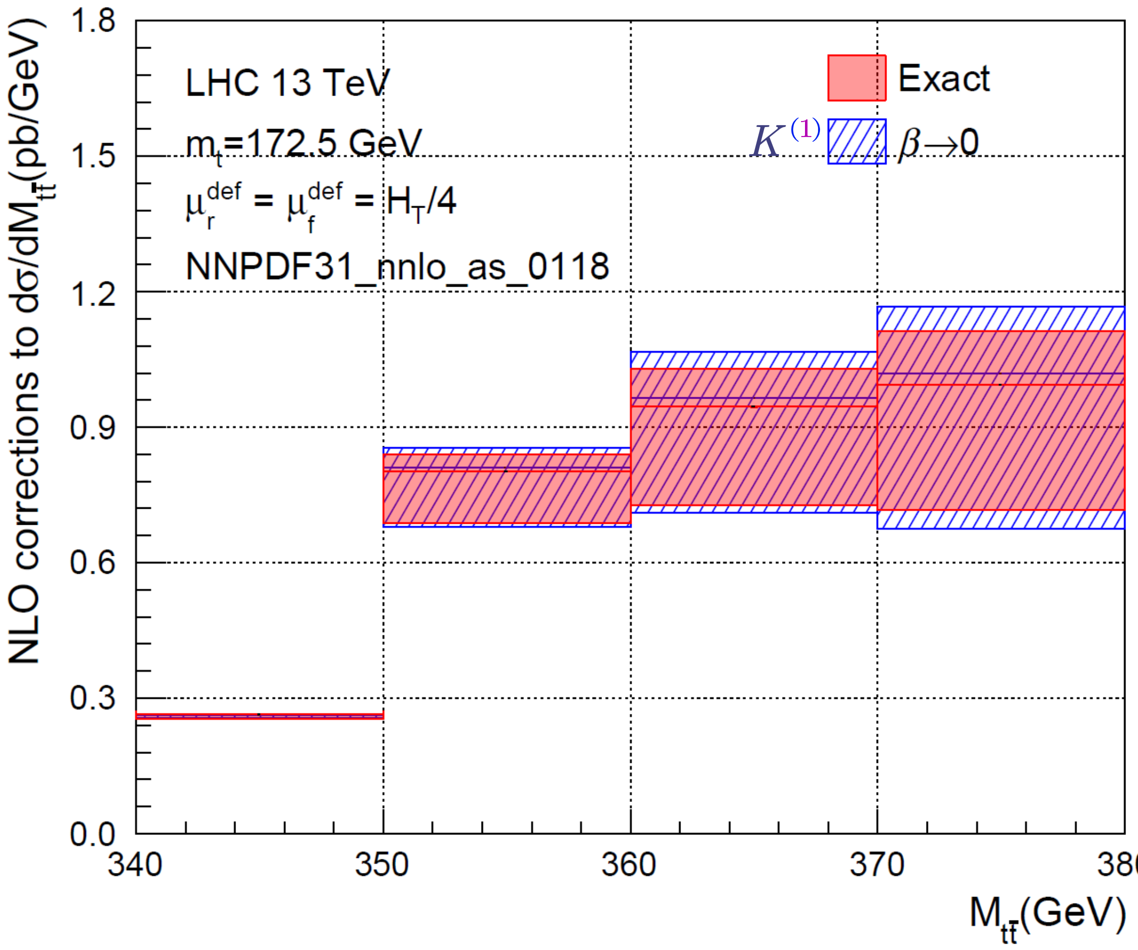


FIG.4. The exact NLO correction and its small β approximation in the range [340, 380]GeV

Various approximations

- Threshold limit $\beta \to 0$ with $c_{ij,\alpha}$
  - Our formula is indeed valid, and supports the resummation.
- Soft limit $z \to 1$
  - Soft limit provides a reasonable approx. to the full results in low M .

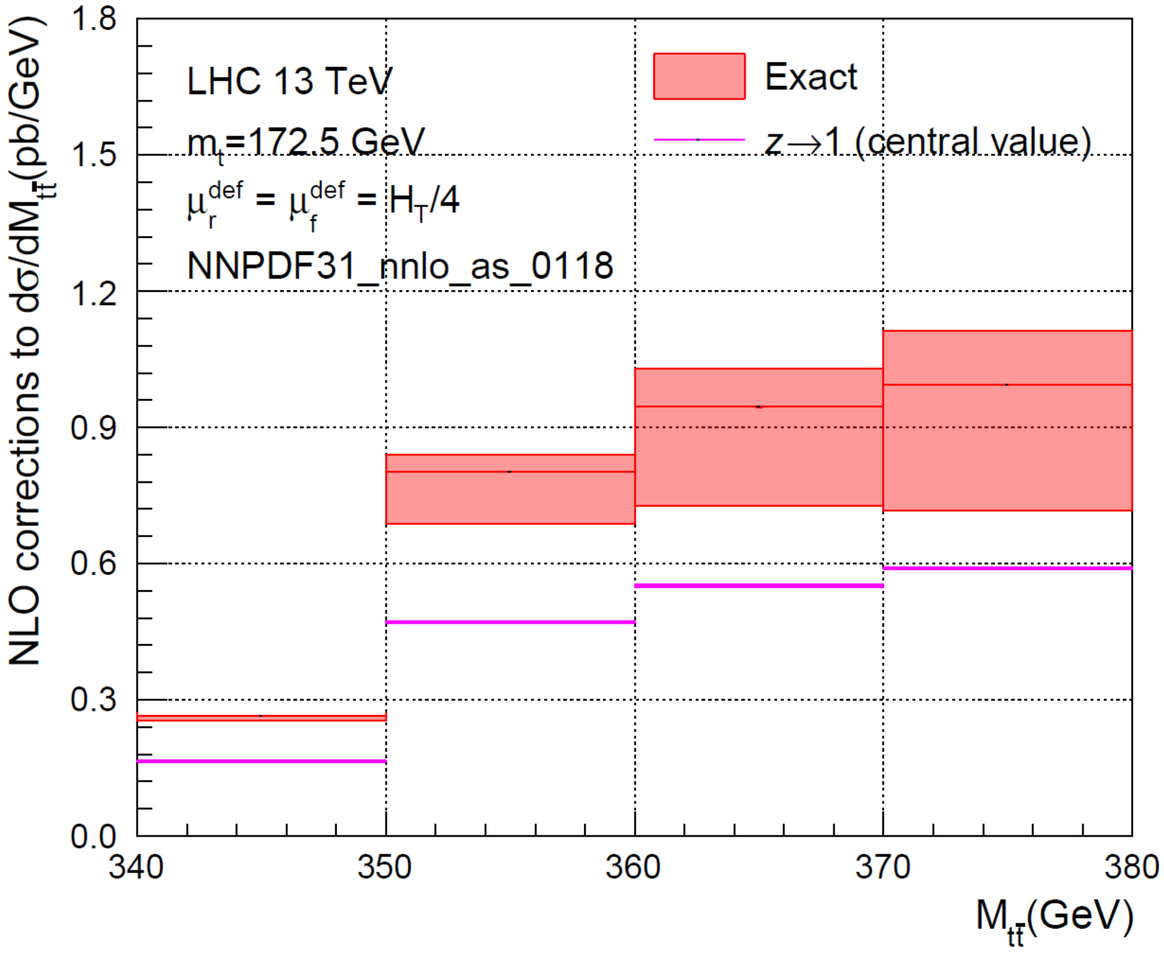


FIG.5. The exact NLO correction and its soft approximation in the range [340, 380]GeV

Various approximations

- Threshold limit $\beta \to 0$ with $c_{ij,\alpha}$
  - Our formula is indeed valid, and supports the resummation.
- Soft limit $z \to 1$
  - Soft limit provides a reasonable approx. to the full results in low M .
- Double limit $\beta \to 0$ & $z \to 1$
  - Double limit does not capture the dominant contribution at NLO.
  - The exact reason why we do not consider such soft limit.

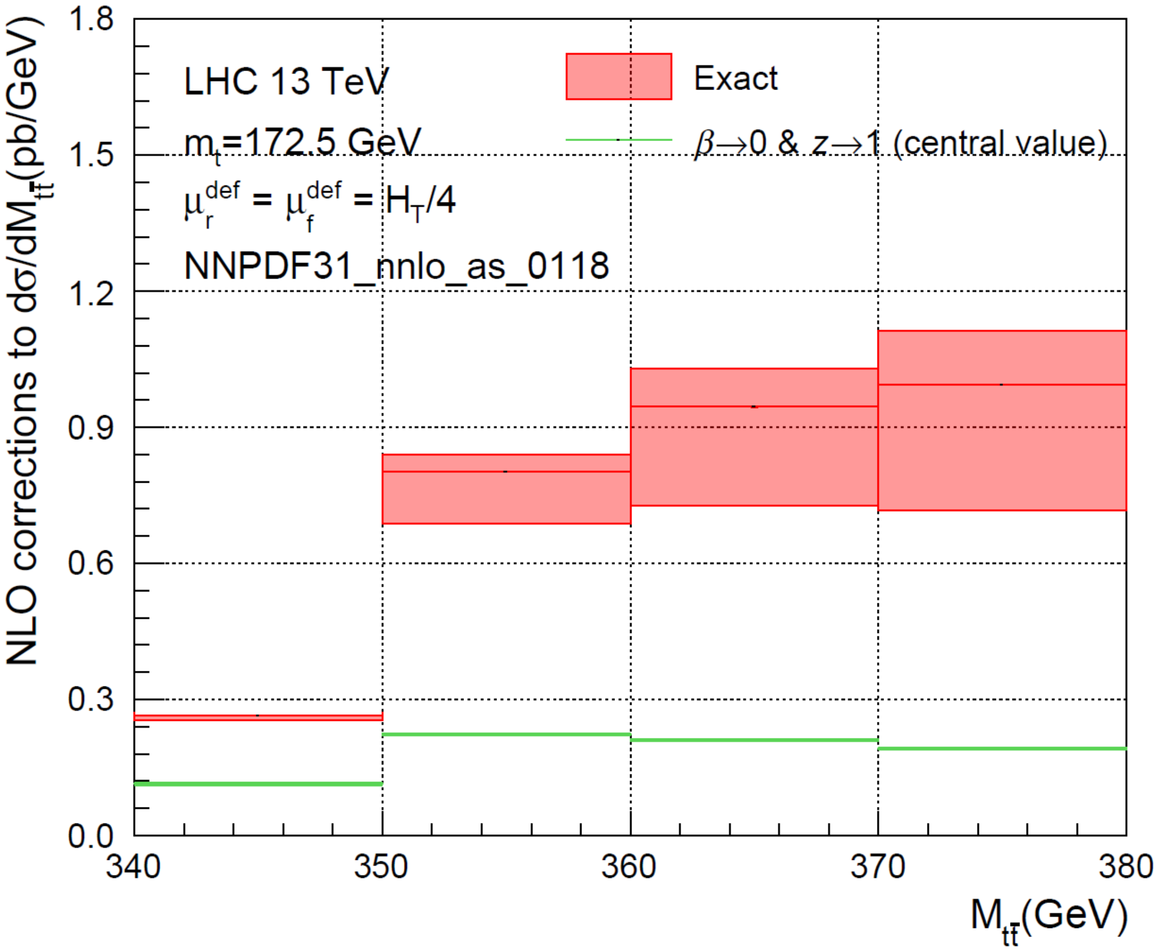


FIG.6. The exact NLO correction and its small beta and soft approximation in the range [340, 380]GeV

Various approximations

- Threshold limit $\beta \to 0$ with $c_{ij,\alpha}$
  - Our formula is indeed valid, and supports the resummation.
- Soft limit $z \to 1$
  - Soft limit provides a reasonable approx. to the full results in low M.
- Double limit $\beta \to 0$ & $z \to 1$
  - Double limit does not capture the dominant contribution at NLO.
  - The exact reason why we do not consider such soft limit.

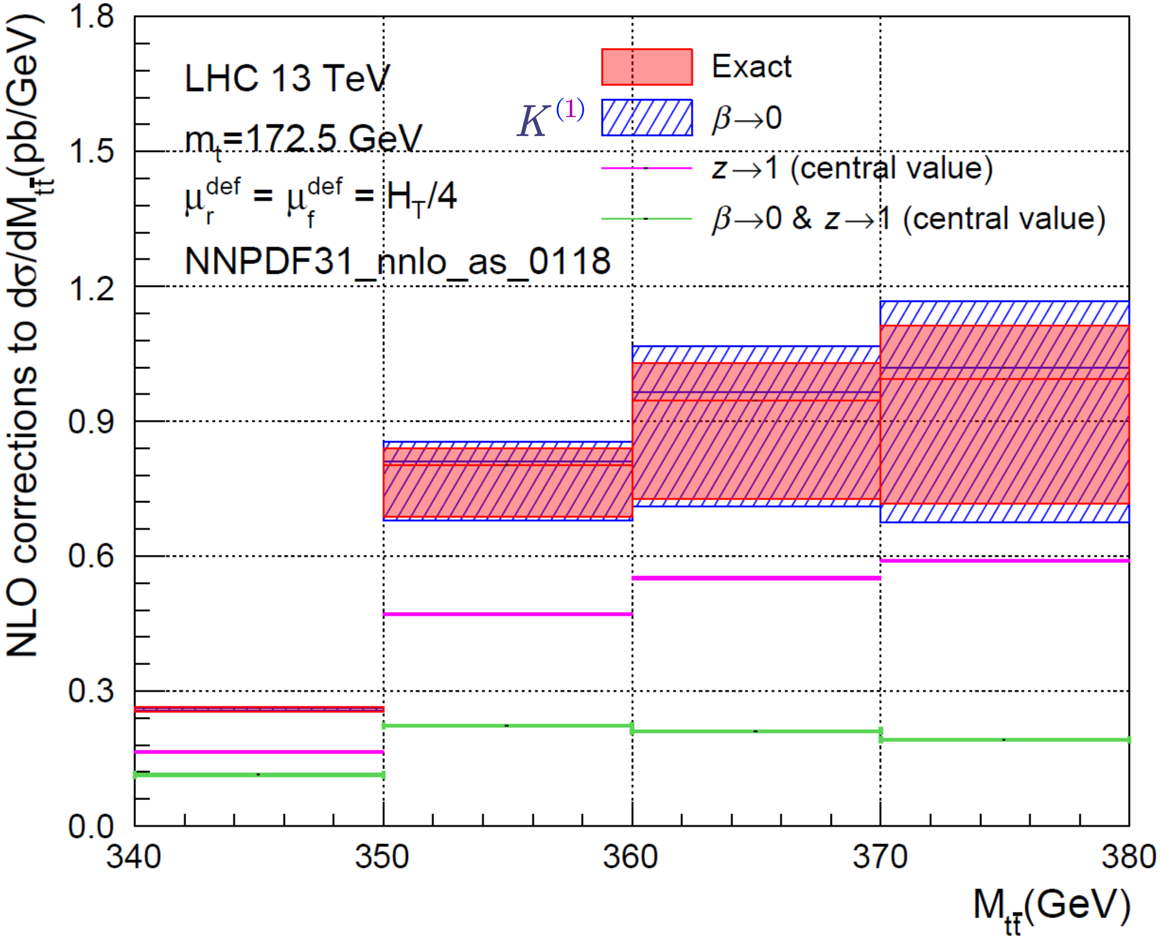


FIG.7. The exact NLO correction and its various approximations in the range [340, 380]GeV

Resummation v.s. fixed-order expansion

- Divergent behavior at threshold region is regularized .
- Finite prediction after NLP resummation.
- Fast convergence NLP vs. n^kLO for $M > 360\,\mathrm{GeV}$.
  - Dominant beyond-NNLO correction comes from the region $M < 350\,\mathrm{GeV}$.
  - pNRQCD is perfectly applicable $\beta < 0.17$.

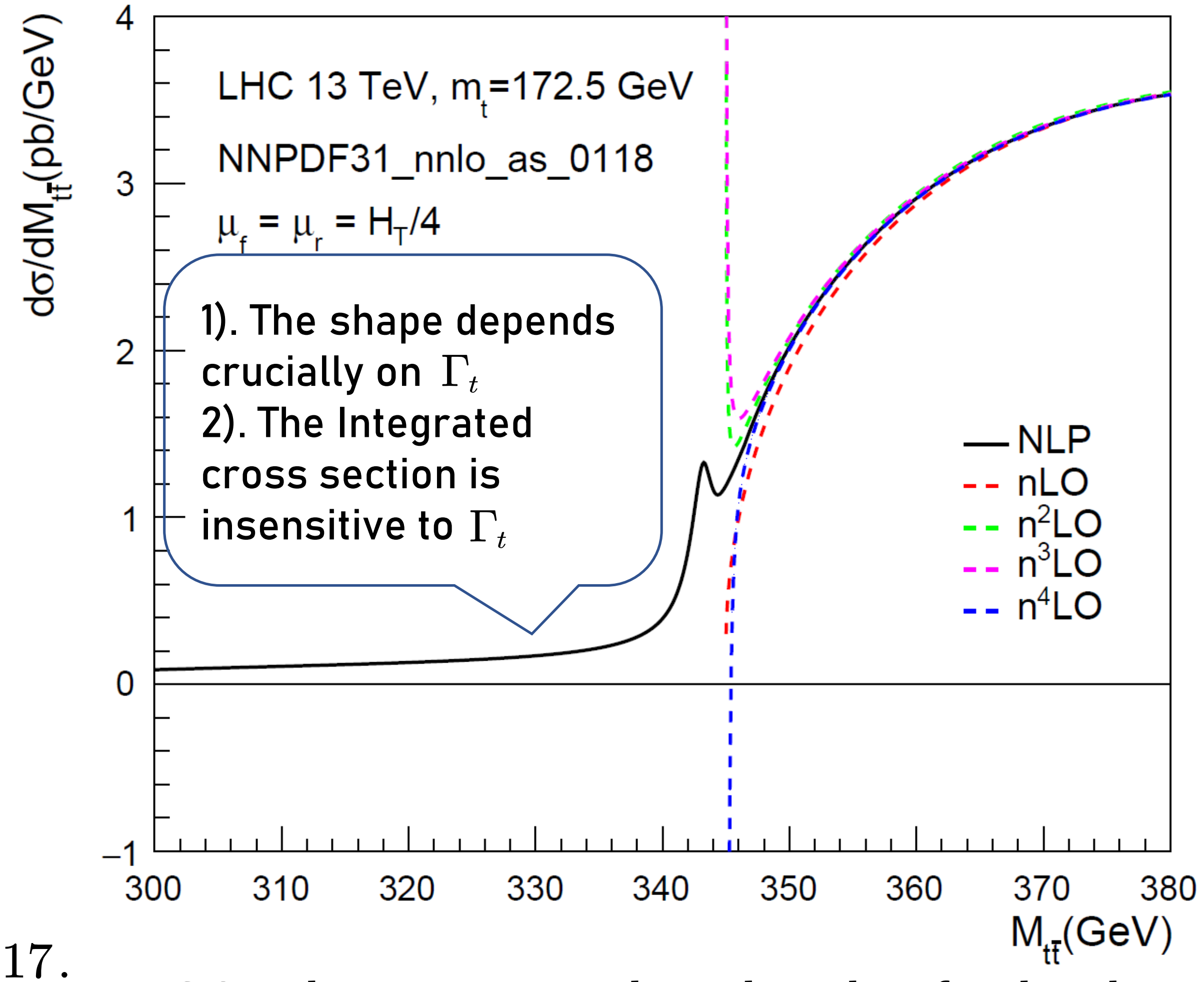


FIG.8. The NLP resumed result and its fixed-order expansion

Final results and conclusion

- Our best prediction: NNLO+NLP.
- Resummation effect enhances the NNLO results by about 9%.

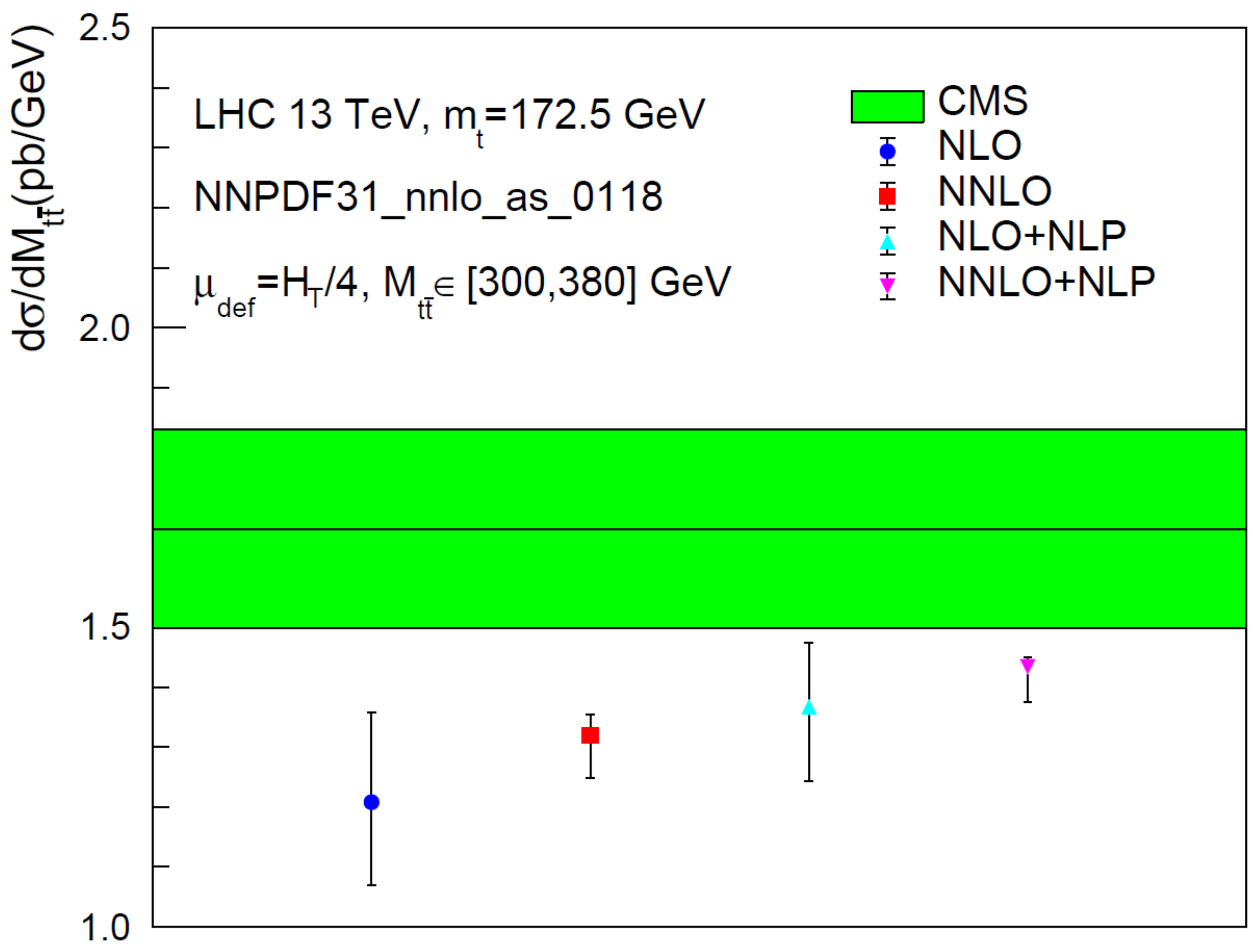
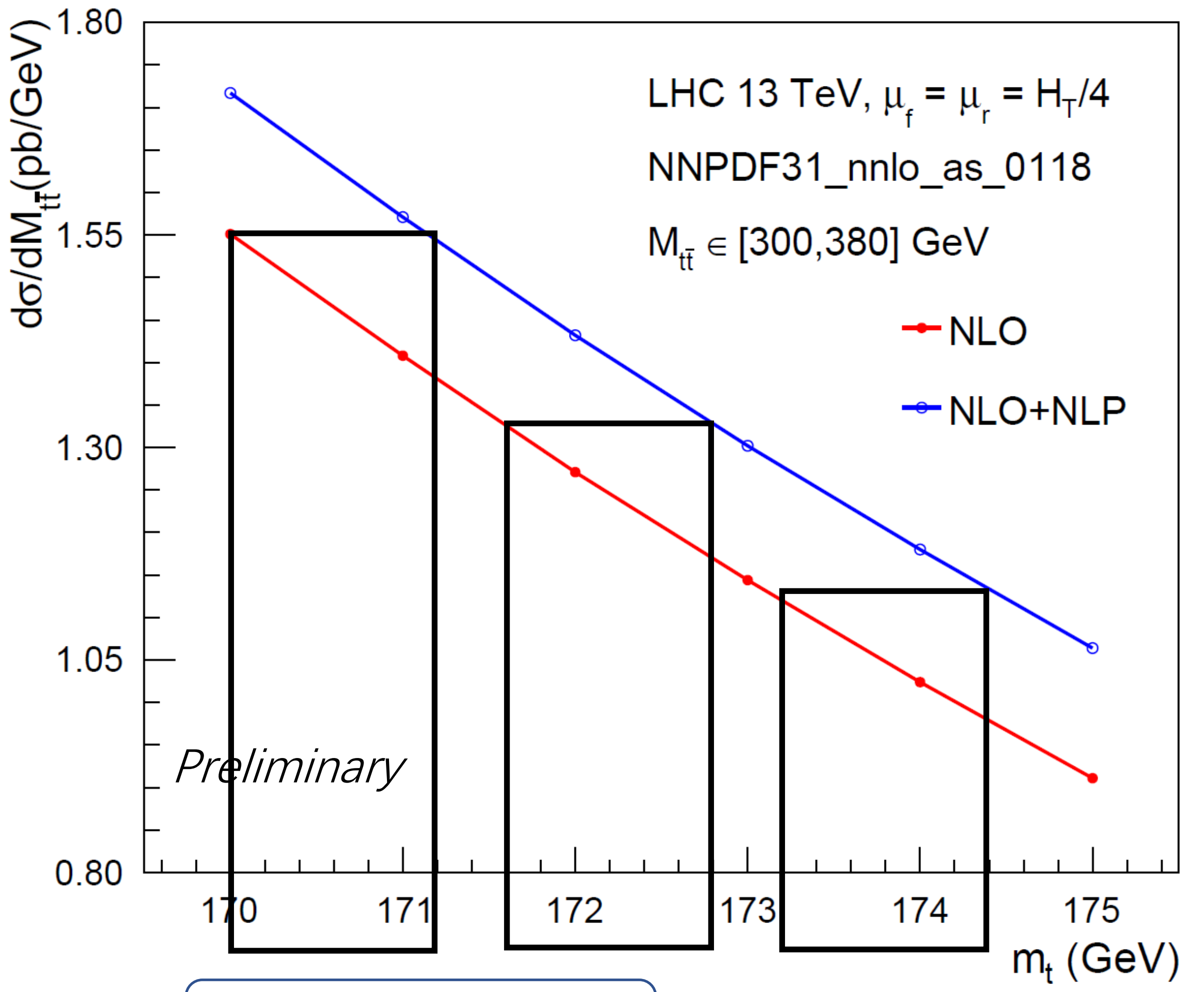


FIG.9. The average $t\bar{t}$ invariant mass distribution in [300, 380] GeV

Final results and conclusion

- Our best prediction: NNLO+NLP.
- Resummation effect enhances the NNLO results by about 9%.
- NNLO+NLP ⟷ NLO

  172.5 GeV ⟷ ~171 GeV
- We show the top mass dependence of NLO and NLO+NLP
  - The difference is about 1.2 GeV and is insensitive to top quark mass.
- Our results can be further combined with, e.g., soft gluon resummation via a matching procedure.



NLO was used to fit

$m_t^{\text{pole}} = 170.5 \pm 0.8\ \text{GeV}$ CMS,arXiv:1904.05237

Soft function-Review

- IR anomalous dimension

Becher, Neubert: 0904.1021; Ferroglia, Neubert, Pecjak, Yang: 0907.4791, 0908,3676.

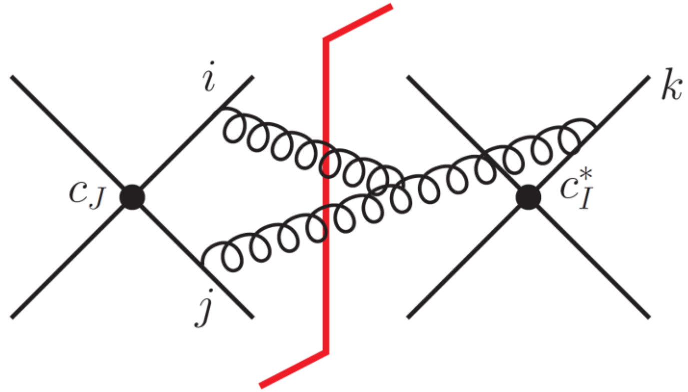
$$\Gamma = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \gamma_{\rm cusp}(\alpha_s) \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) - \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2} \gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s)$$
$$+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j \gamma_{\rm cusp}(\alpha_s) \ln\frac{m_I \mu}{-s_{Ij}} + \boxed{\sum_{(I,J)} \sum_k i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_k^c f_2\left(\beta_{IJ}, \ln\frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\right)}$$
$$+ \boxed{\sum_{(I,J,K)} i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_K^c F_1(\beta_{IJ},\beta_{JK},\beta_{KI})} + \mathcal{O}(\alpha_s^3)$$

3-parton correlations

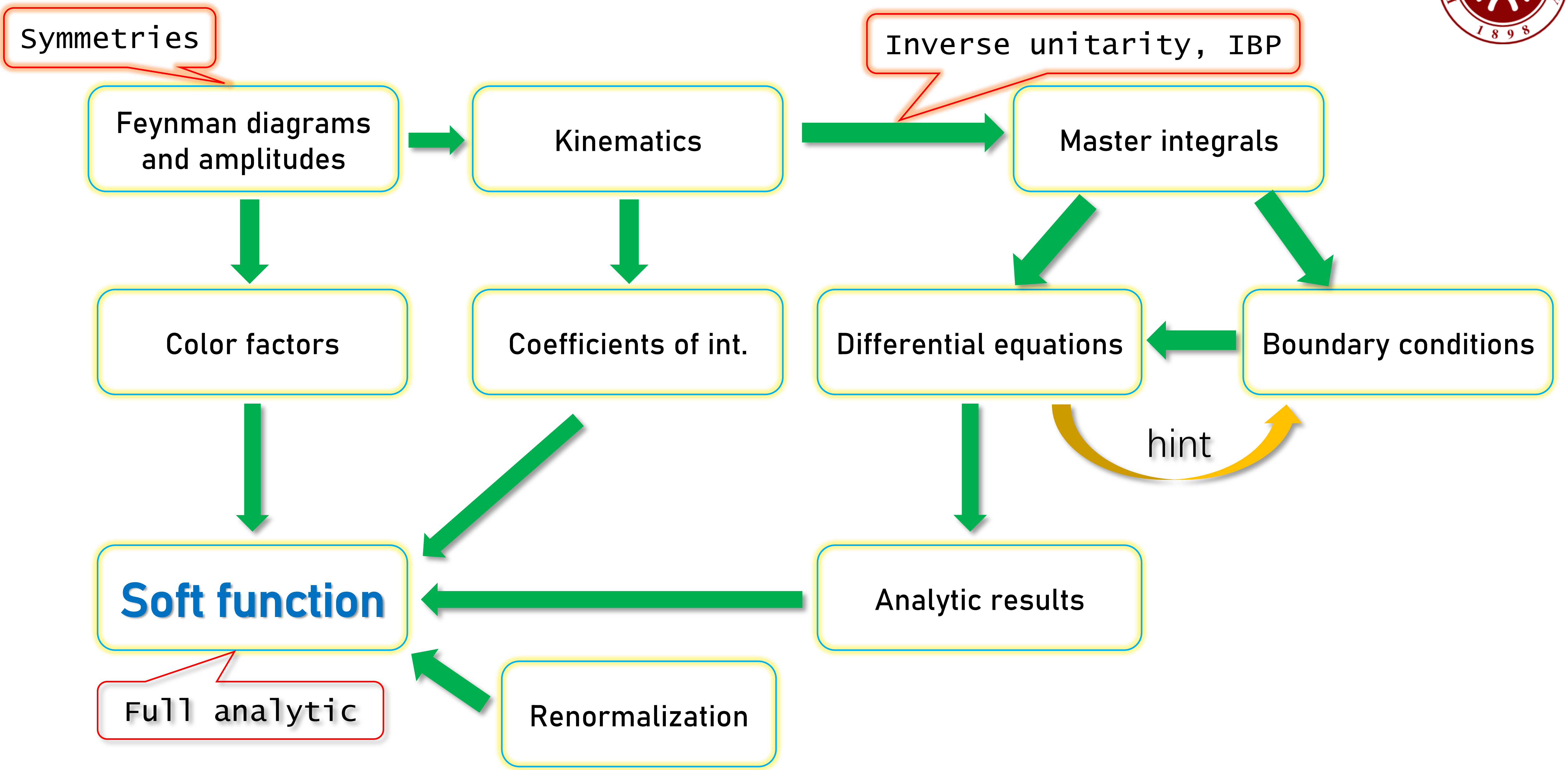
- NLO massive soft function Ahrens, Ferroglia, Neubert, Pecjak, Yang: 1003.5827
- NNLO massless soft function Ferroglia, Pecjak, Yang: 1207.4798

  (except an off-diagonal 3-parton piece)
- NNLO massive soft function GXW, Xu, Yang, Zhu: 1804.05218

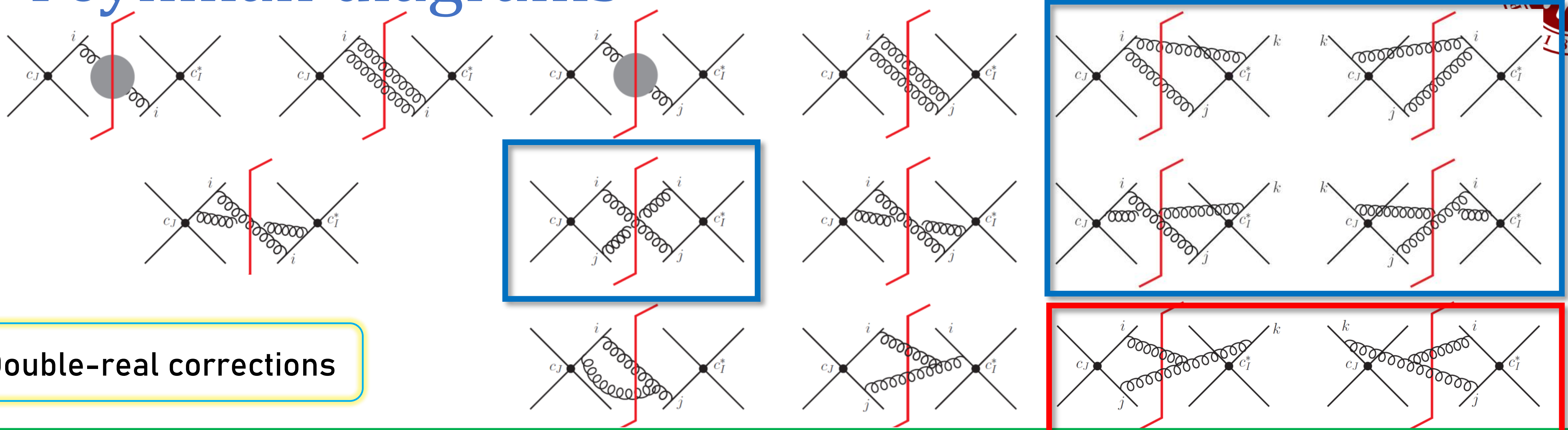
  (include the off-diagonal 3-parton piece)



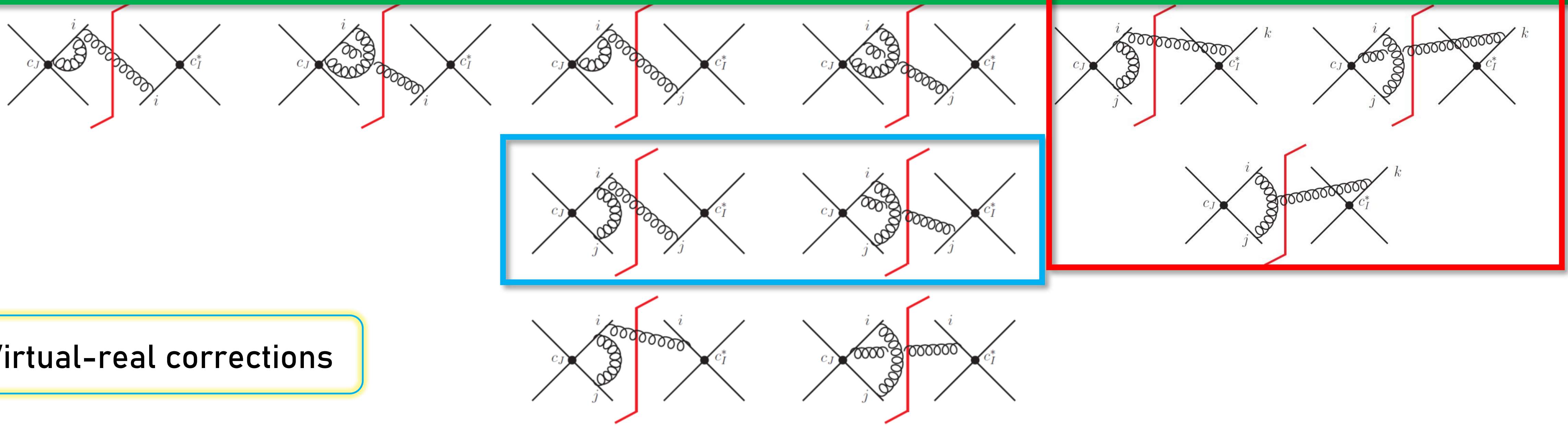
NNLO soft function-work flow



Feynman diagrams



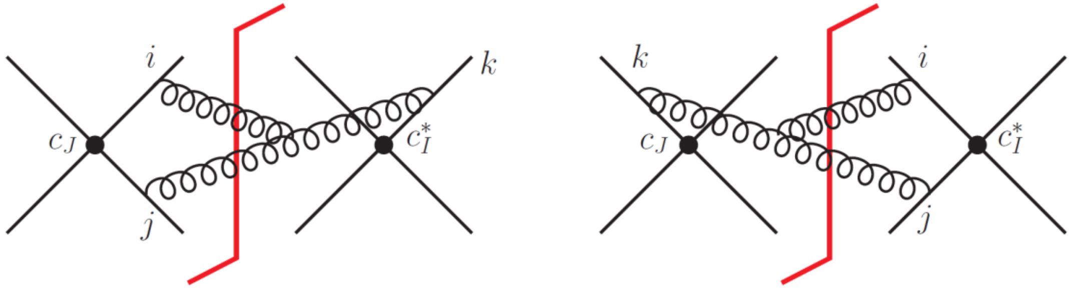
Double-real corrections



Virtual-real corrections

Three-parton correlations

Double-Real corrections



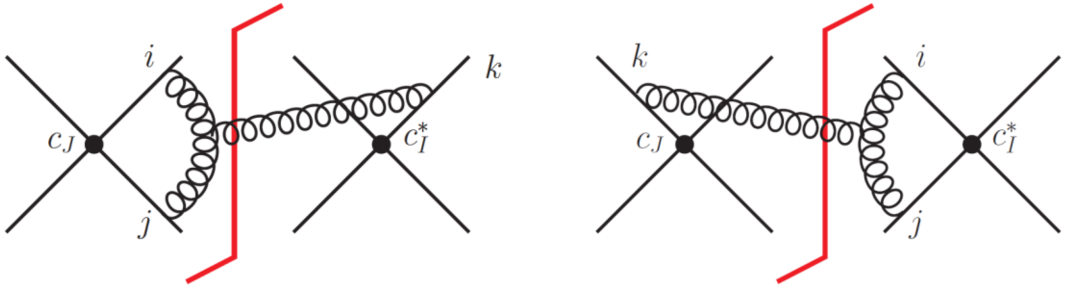
(a) (b)

$$D^{(a)} \to +if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,I_1\,, \quad D^{(b)} \to -if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,I_1^*$$

$$D^{(a)} + D^{(b)} \to if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,2\,\mathrm{Im}\,(I_1) \equiv 0$$

$$I_1 = \int [dk_1]\,[dk_2]\,\frac{v_i^\mu\,v_j^\nu\,v_k^\rho\,[(2k_1+k_2)_\nu\,g_{\mu\rho}+\cdots]}{(k_1+k_2)^2\,v_i\cdot k_1\;v_j\cdot k_2\;v_k\cdot(k_1+k_2)}$$
$$\times\delta[\omega - v_0\cdot(k_1+k_2)]\,.$$

Real-virtual corrections



(c) (d)

$$D^{(c)} \to +if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,I_2\,, \quad D^{(d)} \to -if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,I_2^*$$

$$D^{(a)} + D^{(b)} \to if^{abc}\,\boldsymbol{T}_i^a\,\boldsymbol{T}_j^b\,\boldsymbol{T}_k^c\,2\,\mathrm{Im}\,(I_2) \neq 0$$

- The non-trivial 3-parton correlations

$$\sum_{(I,J)}\sum_k\; if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\left(\beta_{IJ}, \ln\frac{\omega_{Jk}\sqrt{v_I^2}}{\omega_{Ik}\sqrt{v_J^2}}\right)$$

Differential equations

- Four integral families and 59 master integrals
  - Double-real
    $$\{(k_1+k_2)^2,\ v_1\cdot k_2,\ v_1\cdot(k_1+k_2),\ \boxed{v_2}\cdot k_1,\ v_3\cdot k_1,\ v_3\cdot(k_1+k_2)\}$$
    $$\{(k_1+k_2)^2,\ v_1\cdot k_2,\ v_1\cdot(k_1+k_2),\ \boxed{v_4}\cdot k_1,\ v_3\cdot k_1,\ v_3\cdot(k_1+k_2)\}$$
  - Real-virtual
    $$\{k^2,\ l^2,\ (k+l)^2,\ v_1\cdot k,\ v_1\cdot l,\ \boxed{v_2}\cdot(k+l),\ v_3\cdot k,\ v_3\cdot l\}$$
    $$\{k^2,\ l^2,\ (k+l)^2,\ v_1\cdot k,\ v_1\cdot l,\ \boxed{v_4}\cdot(k+l),\ v_3\cdot k,\ v_3\cdot l\}$$
- $\partial_\beta \vec{g}(\epsilon,\beta,\cos\theta)=\epsilon\left(\frac{A}{\beta-1}+\frac{B}{\beta}+\frac{C}{\beta+1}+\frac{D}{\beta+1/\cos\theta}+\frac{E}{\beta-1/\cos\theta}\right)\vec{g}(\epsilon,\beta,\cos\theta)$
  - Boundary conditions(difficult parts)
  - Solution in terms of generalized polylogarithms(full analytic)

Boundary conditions

- Boundary: $\beta = \sqrt{1-\frac{4m_t^2}{M^2}} \to 0\,,\ \beta \to 1\,, \text{or}\,,\ \cdots$
  - Regular integrals, e.g.

$$g_{14}^{(1)}(\epsilon,0,y) = \frac{8\,\epsilon^2\,\Gamma(-4\epsilon)\,\omega^{1+4\epsilon}}{\pi^{2-2\epsilon}\,\Gamma(-\epsilon)^2}\int [dk_1]\,[dk_2]\,\frac{\delta[\omega - v_0\cdot(k_1+k_2)]}{(k_1+k_2)^2\;v_1\cdot k_2\;v_2\cdot k_1}$$

$$= -\,\frac{4\Gamma(-2\epsilon)\,\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)\,\Gamma(-3\epsilon)}\,{}_3F_2(-\epsilon,\,-\epsilon,\,-\epsilon;1-\epsilon,\,-3\epsilon;1)$$

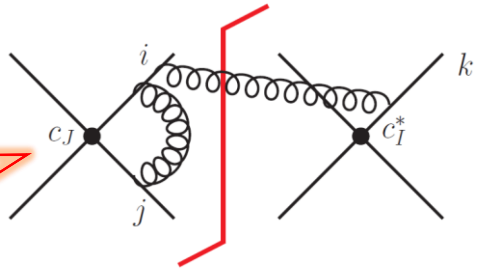
  - Singular integrals , e.g.

$$g_6^{(4)}(\epsilon,\beta,y) = \frac{ie^{-2i\pi\epsilon}\,\Gamma(-2\epsilon)\,\omega^{-1+4\epsilon}\,\beta}{2\pi^{3-2\epsilon}\,\Gamma^2(-\epsilon)\,\Gamma(2\epsilon)}\int [dk]\,d^dl\,\frac{\delta(\omega - v_0\cdot k)}{l^2\;v_3\cdot l\,[-v_4\cdot(k+l)]}\left[\frac{\omega}{-v_4\cdot(k+l)} + 2(1-4\epsilon)\right]$$

$$\approx \frac{(e^{-2i\pi\epsilon}-1)\,\boxed{\beta^{2\epsilon}}\,\Gamma(1-2\epsilon)\,\Gamma(1+\epsilon)}{4^{1-2\epsilon}\,\Gamma(1-\epsilon)}$$

Now, the full analytic NNLO bare soft function is available

Coulomb/Glauber-type singularities from virtual integrals



Validation: IR structure

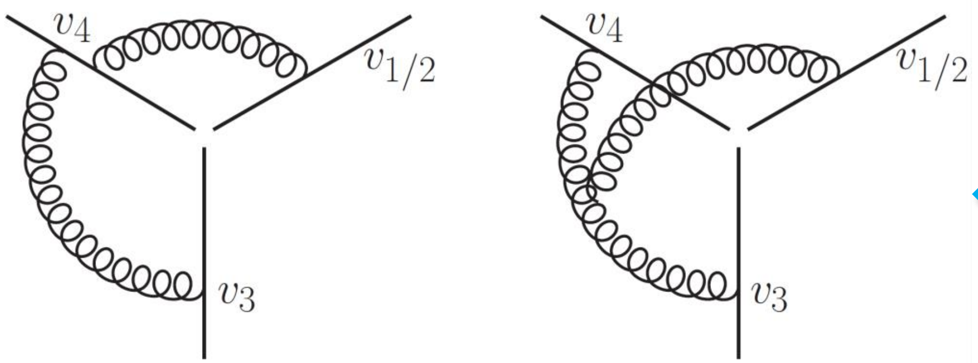
- Renormalization

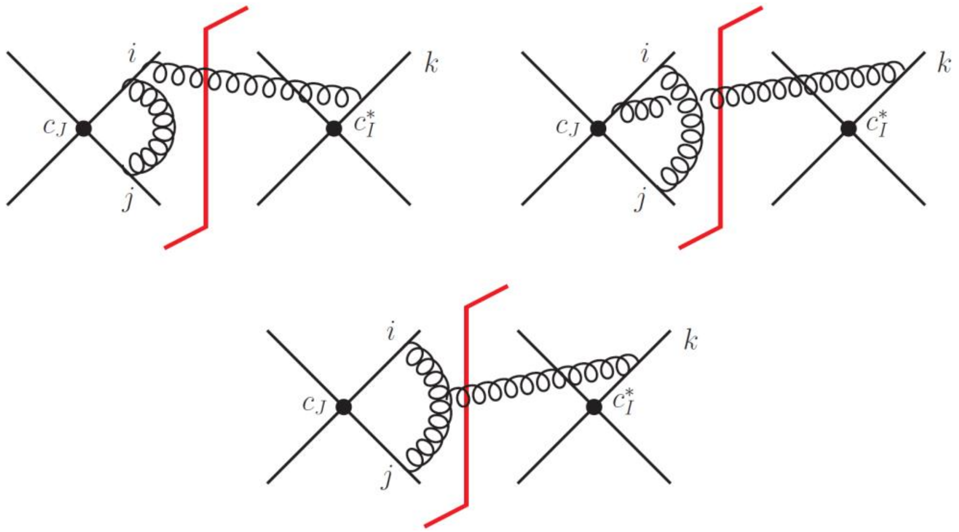
Constructed from Γ_s

$$\tilde{\boldsymbol{s}}^{(2)}(L,\mu) = \tilde{\boldsymbol{s}}^{(2)}_{\text{bare}} + \boldsymbol{Z}_s^{\dagger(2)}\tilde{\boldsymbol{s}}^{(0)} + \tilde{\boldsymbol{s}}^{(0)}\boldsymbol{Z}_s^{(2)} + \boldsymbol{Z}_s^{\dagger(1)}\tilde{\boldsymbol{s}}^{(1)}_{\text{bare}} + \tilde{\boldsymbol{s}}^{(1)}_{\text{bare}}\boldsymbol{Z}_s^{(1)} + \boldsymbol{Z}_s^{\dagger(1)}\tilde{\boldsymbol{s}}^{(0)}\boldsymbol{Z}_s^{(1)} - \frac{\beta_0}{\epsilon}\tilde{\boldsymbol{s}}^{(1)}_{\text{bare}}$$

$$\to \text{finite in } \epsilon$$

- A non-trivial IR cancellation





$$+\sum_{(I,J)}\sum_{k} if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\, f_2\!\left(\beta_{IJ}, \ln\frac{\omega_{Jk}\sqrt{v_I^2}}{\omega_{Ik}\sqrt{v_J^2}}\right)$$

Becher, Neubert: 0904.1021; Ferroglia, Neubert, Pecjak, Yang: 0907.4791, 0908,3676.

Virtual

3-parton correlations: non-trivial cross-check

Real

Validation: in threshold or boosted limit

- In threshold limit, our result is consistent with
  - color-singlet production, e.g., Drell-Yan and Higgs production; Belitsky: hep-ph/9808389
  - color-octet production. Czakon, Fiedler: 1311.2541
  - Note: singlet-octet mixing terms do NOT vanish, e.g.

Purely-imaginary and do NOT enter the NNLO cross section, NEW!

$$\tilde{\boldsymbol{s}}_{12}^{q\bar{q},(2)}\left(\ln\frac{\Lambda^2}{\mu^2},\beta\to 0,\cos\theta,\mu\right)=-2i\pi C_F C_A\left[L^2-4\left(\ln(4\beta)+1\right)L+2\ln^2(4\beta)+8\ln(4\beta)+\pi^2\right]$$

- In boosted limit, our result satisfies factorization Ferroglia, Pecjak, Yang: 1205.3663

$$\tilde{\boldsymbol{s}}_{\text{massive}}\left(\ln\frac{\Lambda^2}{\mu^2},\beta,\cos\theta,\mu\right)=\tilde{\boldsymbol{s}}_{\text{massless}}\left(\ln\frac{\Lambda^2}{\mu^2},\cos\theta,\mu\right)\tilde{s}_D^2\left(\ln\frac{m_t^2\Lambda^2}{M^2\mu^2},\mu\right)+\mathcal{O}(m_t^2/M^2)$$

Also obtain the missing 3-parton piece in Ferroglia,Pecjak,Yang:1207.4798.

Numerical impacts

- Contribution of soft function

$$\frac{d\hat{\sigma}_{ij}}{dM\, d\cos\theta_t} \propto \mathrm{Tr}\left[\boldsymbol{H}_{ij}\left(\ln\left(\frac{M^2}{\mu^2}\right), \beta, \cos\theta_t, \mu\right) \tilde{\boldsymbol{s}}_{ij}\left(\ln\left(\frac{\Lambda^2}{\mu^2}\right), \beta, \cos\theta_t, \mu\right)\right]$$

- NNLO correction to the soft function

$$\mathcal{S}_{ij}^{(n)}(\beta, \mu/\mu_{\mathrm{def}}) = \int_{-1}^{1} d\cos\theta \left(\frac{\alpha_s}{4\pi}\right)^n \mathrm{Tr}\left[\boldsymbol{H}_{ij}^{(0)}(\beta, \cos\theta)\, \tilde{\boldsymbol{s}}_{ij}^{(n)}\left(\ln\frac{\Lambda^2}{\mu^2}, \beta, \cos\theta\right)\right]$$

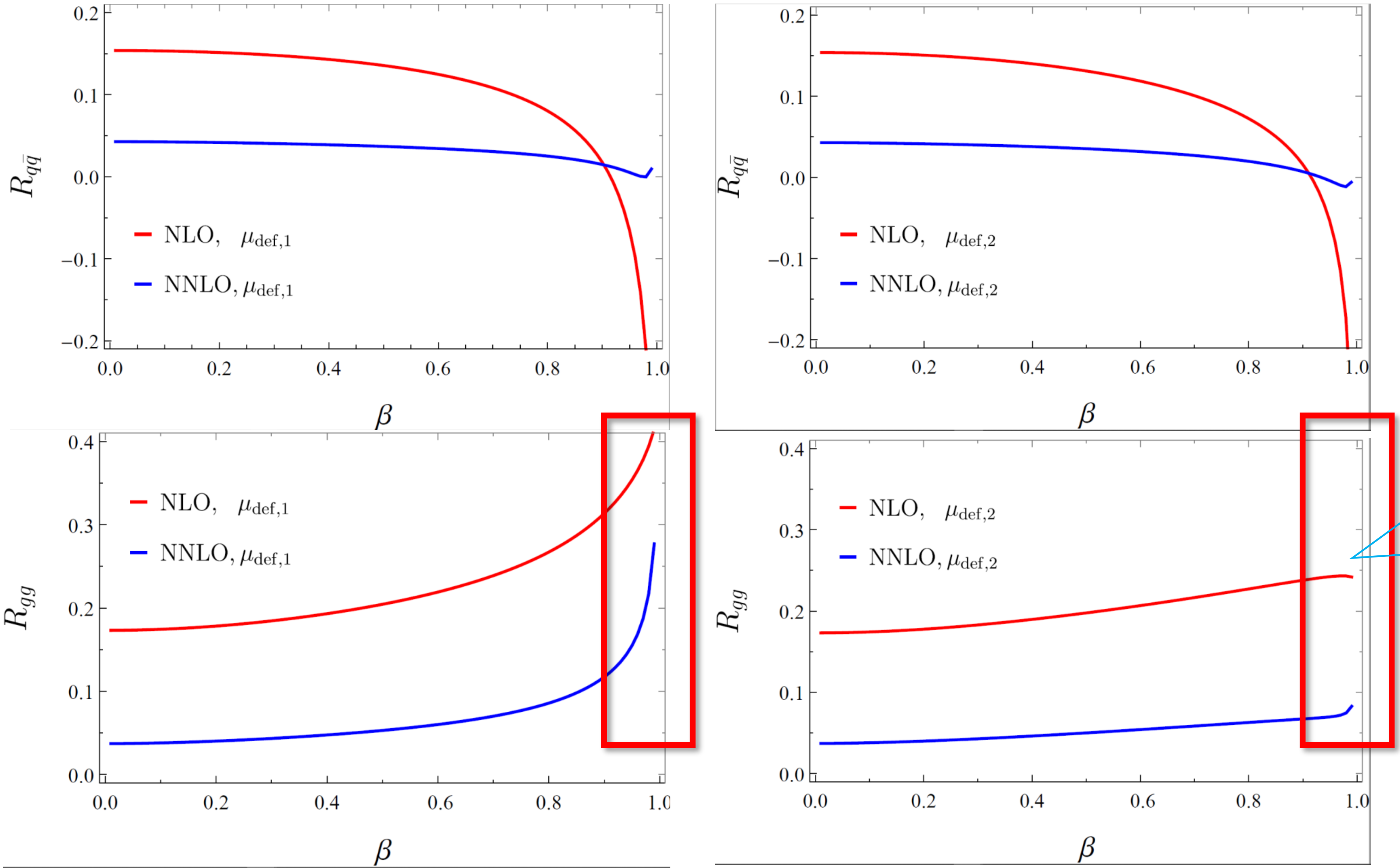
$$R_{ij}^{\mathrm{NLO}}(\beta, \mu/\mu_{\mathrm{def}}) = \frac{\mathcal{S}_{ij}^{(1)}(\beta, \mu/\mu_{\mathrm{def}})}{\mathcal{S}_{ij}^{(0)}(\beta, \mu/\mu_{\mathrm{def}})}, \qquad R_{ij}^{\mathrm{NNLO}}(\beta, \mu/\mu_{\mathrm{def}}) = \frac{\mathcal{S}_{ij}^{(2)}(\beta, \mu/\mu_{\mathrm{def}})}{\mathcal{S}_{ij}^{(0)}(\beta, \mu/\mu_{\mathrm{def}})}$$

- Two scale choices

Pecjak, Scott, Wang, Yang: 1601.07020,
Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, Yang: 1803.07623

$$\mu_{\mathrm{def},\,1} = \Lambda, \qquad \mu_{\mathrm{def},\,2} = \Lambda\sqrt{1-\beta^2\cos^2\theta_t}$$

Numerical impacts



Relative soft corrections as a function of β for two choices of the default scale $\mu_{\rm def,\,1} = \Lambda$ and $\mu_{\rm def,\,2} = \Lambda\sqrt{1-\beta^2\cos^2\theta_t}$

t channel and u channel singularities

$\mu_{\rm def,\,2}$ is a better choice, in coincidence with Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang and Yang, arXiv:1803.07623

Conclusion on NNLO soft function

- Our NNLO soft function shows mangy interesting properties and is checked in different aspects;
- It's a good verification of the cancellation of the IR singularity with non-trivial 3-parton correlation;
- $\mu_{\text{def},\,2} = \Lambda\sqrt{1-\beta^2\cos^2\theta_t}$ is indeed a better scale choice;
- Our result gives a chance to extend the theoretical precision to NNLO + NNNLL in soft limit in the future.

NNLO hard function is also needed.

Hard function-review

- Color summed QCD corrections:
  - Full analytic results at NLO
    Beenakker, Kuijf, Neerven, Smith: Phys.Rev.D **40**,54(1989)
  - Full analytic one-loop squared results at NNLO
    Korner, Merebashvili, Rogal:0802.0106
    Anastasiou, Aybat: 0809.1355
    Kniehl, Merebashvili, Korner, Rogal:0809.3980
  - Numerical two-loop results at NNLO
    Czakon: 0803.1400;
    Barnreuther, Czakon, Fiedler: 1312.6279

Please forgive me for missing your works

- Progresses on two-loop analytic calculation
  - $q\bar{q}$ channel
    Bonciani, Ferroglia, Gehrmann, Maitre, Studerus: 0806.2301
    Bonciani, Ferroglia, Gehrmann, Studerus: 0906.3671
    Becchetti, Bonciani, Casconi, Ferroglia, Lavacca, Manteuffel:1904.10834
    Vita, Gehrmann, Laporta, Mastrolia, Primo, Schubert: 1904.10964:
  - gg channel
    Bonciani, Ferroglia, Gehrmann, Manteuffel, Studerus: 1011.6661, 1309.4450
    Manteuffel, Studerus: 1306.3504
    Chen, Wang: 1903.04320

5 planner and 7 non-planner double-box integral families remain unknown

Hard function

GXW, Yang, in preparation

- One-loop contributions to the hard functions for top-quark pair production

Up to $\mathcal{O}(\epsilon^2)$ can be found in Korner, Merebashvili, Rogal: hep-ph/0412088

  - One-loop integrals for top quark pair production to arbitrary orders in the dimensional regulator ϵ
  - One-loop contributions to the unpolarized and polarized hard functions at NLO
  - NNLO unpolarized and polarized hard functions, apart from two-loop contributions.
- Polarized hard function:

$$\tilde{H}_{IJ}^{(n)}(s_t, s_{\bar{t}}) = A_{IJ}^{(n)} + B_{IJ}^{(n)} \frac{m_t}{\hat{s}^2} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} p_{2\nu} p_{3\alpha} \left(s_{t\beta} + s_{\bar{t}\beta}\right) + C_{IJ}^{(n)} \left(s_t \cdot s_{\bar{t}}\right)$$

$$+ D_{IJ}^{(n)} \left[\frac{(p_1 \cdot s_t)(p_1 \cdot s_{\bar{t}})}{\hat{s}} + \frac{(p_2 \cdot s_t)(p_2 \cdot s_{\bar{t}})}{\hat{s}} \right]$$

$$+ E_{12,IJ}^{(n)} \frac{(p_1 \cdot s_t)(p_2 \cdot s_{\bar{t}})}{\hat{s}} + E_{21,IJ}^{(n)} \frac{(p_2 \cdot s_t)(p_1 \cdot s_{\bar{t}})}{\hat{s}}$$

Can be used to study top-quark pair production and decay, e.g., Azimuthal opening angle distribution.

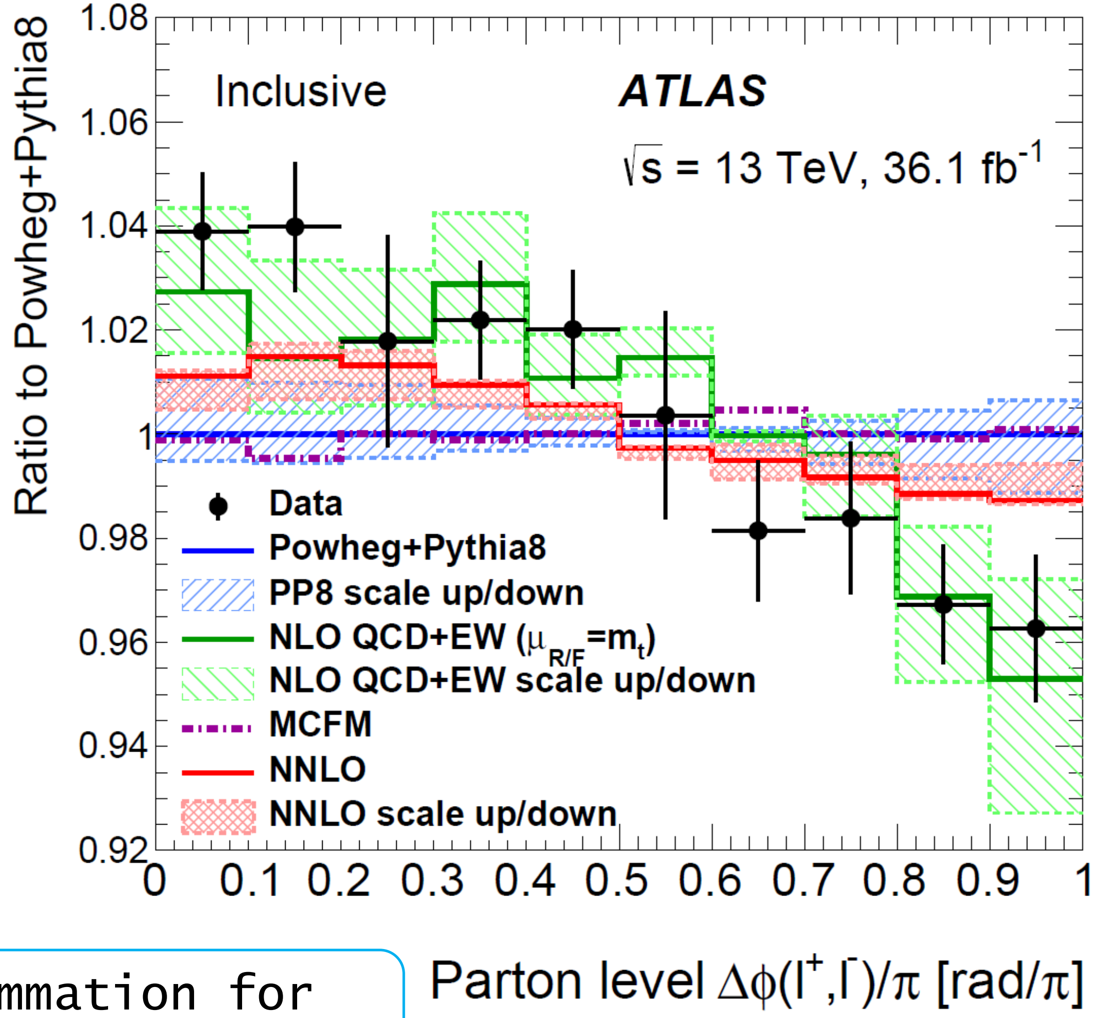
Azimuthal opening angle distribution

$$R = \frac{1}{\sigma}\left(\frac{d\sigma}{d(\Delta\phi(l^+,l^-)/\pi)}\right)$$

One can expand R with α

$$\sigma = \sigma^{(0)} + \alpha\sigma^{(1)} + \alpha^2\sigma^{(2)} + \mathcal{O}(\alpha^3)$$

- Green curve: Bernreuther, Si: 1305.2066
  - The expanded Ratio makes NLO agree with data.
  - The result has large scale uncertainties
- Red curve: Behring, Czakon, Mitov, Papanastasiou, Poncelet: 1901.05407
  - The difference are tiny between expanded and unexpanded Ratio.
- The discrepancy need to be studied further.



ATLAS: arXiv:1903.07570

transverse momentum resummation for top-quark pair production and decay

Li, **GXW**, Yan, Yang, Yuan, on progress

Summary and outlook

- Top-quark pair production is important;
- Resummation effect has significant impacts on top-quark pair production near threshold and the extraction of the top-quark mass;
- NNLO soft function is a very crucial and interesting ingredient in soft resummation;
- Ongoing:
  - Combine the resummation near threshold with soft gluon resummation;
  - Extend our framework to study the $t\bar{t}$ + jet production process
  - transverse momentum resummation for top-quark pair production and decay

Thanks

If you are interested in my researches, please feel free to contact me via
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