A quantum information perspective on relativistic fluid dynamics and quantum fields out-of-equilibrium

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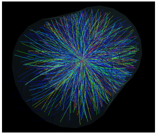
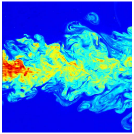
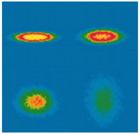
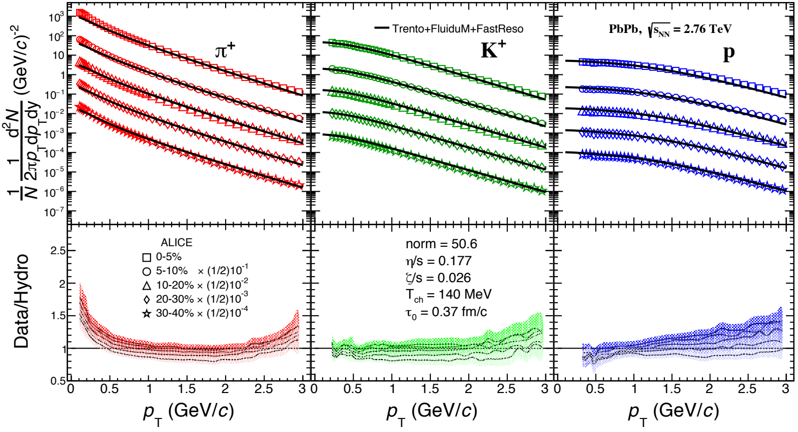
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
    - equation of state $p(T, \mu)$
    - shear viscosity $\eta(T, \mu)$
    - bulk viscosity $\zeta(T, \mu)$
    - relaxation times, ...
- *ab initio* calculation of transport properties difficult but in principle fixed by **microscopic** properties encoded in lagrangian
- standard model of high energy nuclear collisions based on relativistic dissipative fluid dynamics
- ongoing experimental and theoretical effort to understand this better

[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas & Selyuzhenkov, 1909.10485]

Identified particle transverse momentum spectra at the LHC

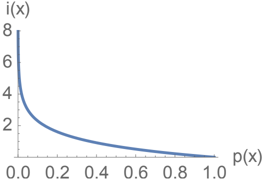


Entropy and information

[Claude Shannon (1948)]

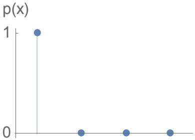
- consider a random variable x with probability distribution $p(x)$
- information content or "surprise" associated with outcome x

$$i(x) = -\ln p(x)$$

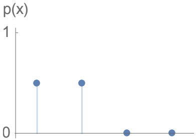


- entropy is expectation value of information content

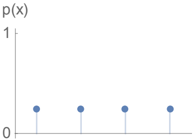
$$S = \langle i(x) \rangle = -\sum_x p(x) \ln p(x)$$



$S = 0$



$S = \ln(2)$



$S = 2\ln(2)$

Entropy at thermal equilibrium

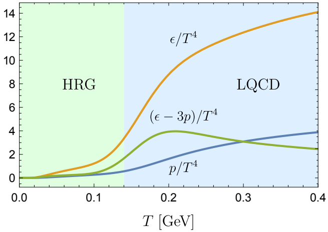
- micro canonical ensemble: **maximal entropy** S for given **conserved quantities** E, N in given volume V
- **universality** at equilibrium
- starting point for development of thermodynamics ...

$$S(E,N,V), \qquad dS = \frac{1}{T}dE - \frac{\mu}{T}dN + \frac{p}{T}dV$$

- ... grand canonical ensemble with density operator ...

$$\rho = \frac{1}{Z}e^{-\frac{1}{T}(H-\mu N)}$$

- ... Matsubara formalism for quantum fields ...



Ideal fluid dynamics

- thermal equilibrium

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p(u^\mu u^\nu + g^{\mu\nu}), \qquad N^\mu = n u^\mu, \qquad s^\mu = s u^\mu$$

- fluid velocity u^μ
- thermodynamic equation of state $p(T, \mu)$ with $dp = s dT + n d\mu$
- local thermal equilibrium approximation: $u^\mu(x)$, $T(x)$, $\mu(x)$
- neglect gradients: lowest order of a derivative expansion
- evolution of $u^\mu(x)$, $T(x)$ and $\mu(x)$ from conservation laws

$$\nabla_\mu T^{\mu\nu}(x) = 0, \qquad \nabla_\mu N^\mu(x) = 0.$$

- entropy current also conserved

$$\nabla_\mu s^\mu(x) = 0.$$

Out-of-equilibrium

- is non-equilibrium dynamics also governed by information?
- approach to equilibrium
- universality

Entropy in quantum theory

[John von Neumann (1932)]

$$S = -\mathrm{Tr}\{\rho \ln \rho\}$$

- based on the quantum density operator ρ
- for pure states $\rho = |\psi\rangle\langle\psi|$ one has $S = 0$
- for mixed states $\rho = \sum_j p_j |j\rangle\langle j|$ one has $S = -\sum_j p_j \ln p_j > 0$
- unitary time evolution conserves entropy

$$-\mathrm{Tr}\{(U\rho U^\dagger)\ln(U\rho U^\dagger)\} = -\mathrm{Tr}\{\rho \ln \rho\} \qquad \rightarrow \qquad S = \text{const.}$$

- quantum information is globally conserved

Dissipative relativistic fluid dynamics

- approximate description of quantum field dynamics
- local dissipation = local entropy production

$$\nabla_\mu s^\mu(x) > 0$$

- e. g. in Navier-Stokes approximation

$$\nabla_\mu s^\mu = \frac{1}{T}\left[2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta(\nabla_\rho u^\rho)^2\right] \geq 0$$

- crucial difference to quantum field theory: entropy not conserved

What is an entropy current?

- *can not* be density of global von-Neumann entropy for closed system

$$\int_\Sigma d\Sigma_\mu \; s^\mu(x) \neq -\mathrm{Tr}\,\{\rho \ln \rho\}$$

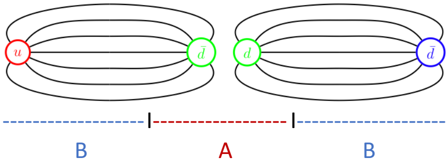
- kinetic theory for weakly coupled (quasi-) particles [Boltzmann (1890)]

$$s^\mu(x) = -\int \frac{d^3p}{p^0}\,\{p^\mu f(x,p)\ln f(x,p)\}$$

- molecular chaos: keep only single particle distribution $f(x,p)$
- how to go beyond weak coupling / quasiparticles?
- aim: local notion of entropy in QFT

Entropy and entanglement

- consider a split of a quantum system into two $A + B$



- reduced density operator for system A

$$\rho_A = \mathsf{Tr}_B\{\rho\}$$

- entropy associated with subsystem A

$$S_A = -\mathsf{Tr}_A\{\rho_A \ln \rho_A\}$$

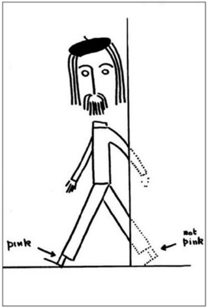
- pure **product** state $\rho = \rho_A \otimes \rho_B$ leads to $S_A = 0$
- pure **entangled** state $\rho \neq \rho_A \otimes \rho_B$ leads to $S_A > 0$
- S_A is called **entanglement entropy**

Why is entanglement interesting?

- *Can quantum-mechanical description of physical reality be considered complete?* [Einstein, Podolsky, Rosen (1935), Bohm (1951)]

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right) \\
&= \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle_A |\leftarrow\rangle_B - |\leftarrow\rangle_A |\rightarrow\rangle_B\right)
\end{aligned}
$$

- *Bertlemann's socks and the nature of reality* [Bell (1980)]



Bell's inequalities and Bell tests

[John Stewart Bell (1966)]

- most popular version [Clauser, Horne, Shimony, Holt (1969)]

$$S = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2$$

holds for local hidden variable theories

- expectation value of product of two observables

$$E(a,b) = \langle A(a)B(b) \rangle$$

with possible values $A = \pm 1$, $B = \pm 1$.

- depending on measurement settings a, a' and b, b' respectively
- quantum mechanical bound is $S \leq 2\sqrt{2}$
- experimental values $2 < S \leq 2\sqrt{2}$ rule out local hidden variables
- one measurement setting but at different times [Leggett, Garg (1985)]

Entanglement in high energy (QCD) physics

[. . ., Elze (1996), Kovner, Lublinsky (2015), Kharzeev & Levin (2017), Berges, Floerchinger & Venugopalan (2017), Shuryak & Zahed (2017), Kovner, Lublinsky, Serino (2018), Baker & Kharzeev (2018), Tu, Kharzeev & Ullrich (2019), Armesto, Dominguez, Kovner, Lublinsky, Skokov (2019), . . .]

- entanglement of *quantum fields* instead of *particles*
- entanglement on sub-nucleonic scales
- entanglement in non-Abelian gauge theory / color / confinement
- discussions in mathematical physics [e. g. Witten (2018)]
- connections to black holes and holography [Ryu & Takayanagi (2006)]
- thermalization in closed quantum systems

Classical statistics

- consider system of two random variables x and y
- joint probability $p(x, y)$, joint entropy

$$S = -\sum_{x,y} p(x, y) \ln p(x, y)$$

- reduced or marginal probability $p(x) = \sum_y p(x, y)$
- reduced or marginal entropy

$$S_x = -\sum_{x} p(x) \ln p(x)$$

- one can prove: **joint entropy is greater than** or equal to **reduced entropy**

$$S \geq S_x$$

- **globally pure** state $S = 0$ is also **locally pure** $S_x = 0$

Quantum statistics

- consider system with two subsystems A and B
- combined state ρ , combined or full entropy

$$S = -\mathsf{Tr}\{\rho \ln \rho\}$$

- reduced density matrix $\rho_A = \mathsf{Tr}_B\{\rho\}$
- reduced or entanglement entropy

$$S_A = -\mathsf{Tr}_A\{\rho_A \ln \rho_A\}$$

- for quantum systems **entanglement makes a difference**

$$S \ngeq S_A$$

- **coherent information** $I_{B\rangle A} = S_A - S$ can be **positive**!
- **globally pure** state $S = 0$ can be **locally mixed** $S_A > 0$

Entanglement entropy in quantum field theory

- entanglement entropy of region A is a local notion of entropy

$$S_A = -\mathrm{tr}_A \{\rho_A \ln \rho_A\} \qquad\qquad \rho_A = \mathrm{tr}_B \{\rho\}$$

- however, it is infinite already in vacuum state

$$S_A = \frac{\text{const}}{\epsilon^{d-2}} \int_{\partial A} d^{d-2}\sigma \sqrt{h} \, + \, \text{subleading divergences} \, + \, \text{finite}$$

- UV divergence proportional to entangling surface
- quantum fields are very strongly entangled already in vacuum
- Theorem [Reeh & Schlieder (1961)]: local operators in region A can create all particle states

Relative entropy

- **relative entropy** of two density matrices

$$S(\rho|\sigma) = \mathsf{tr}\left\{\rho\left(\ln\rho - \ln\sigma\right)\right\}$$

- measures how well state ρ can be distinguished from a model σ
- Gibbs inequality: $S(\rho|\sigma) \geq 0$
- $S(\rho|\sigma) = 0$ if and only if $\rho = \sigma$
- quantum generalization of Kullback-Leibler divergence

Relative entanglement entropy

- consider now reduced density matrices

$$\rho_A = \mathsf{Tr}_B\{\rho\}, \qquad \sigma_A = \mathsf{Tr}_B\{\sigma\}$$

- define **relative entanglement entropy**

$$S_A(\rho|\sigma) = \mathsf{Tr}\left\{\rho_A\left(\ln \rho_A - \ln \sigma_A\right)\right\}$$

- measures how well ρ is represented by σ locally in region A
- UV divergences cancel: contains real physics information
- well defined in quantum field theory [Araki (1977)]
  [see also works by Casini, Myers, Lashkari, Witten, Liu, ...]

An approximate local description

- consider non-equilibrium situation with
  - true density matrix ρ
  - local equilibrium approximation

$$\sigma = \frac{1}{Z} e^{-\int d\Sigma_\mu \{\beta_\nu(x) T^{\mu\nu} + \alpha(x) N^\mu\}}$$

- reduced density matrices $\rho_A = \text{Tr}_B\{\rho\}$ and $\sigma_A = \text{Tr}_B\{\sigma\}$
- σ is very good model for ρ in region A when

$$S_A = \text{Tr}_A\{\rho_A(\ln \rho_A - \ln \sigma_A)\} \to 0$$

- does *not* imply that globally $\rho = \sigma$

Monotonicity of relative entropy

- monotonicity of relative entropy

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) \leq S(\rho|\sigma)$$

  with $\mathcal{N}$ completely positive, trace-preserving map

- $\mathcal{N}$ unitary evolution

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) = S(\rho|\sigma)$$

- $\mathcal{N}$ open system evolution with generation of entanglement to environment

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) < S(\rho|\sigma)$$

Local form of second law

- for small volume $A \to 0$ (hypothesis)

$$S_A(\rho|\sigma) = \int_A d\Sigma_\mu s^\mu(\rho|\sigma)$$

- local form of second law of thermodynamics

$$\nabla_\mu s^\mu(\rho|\sigma) \leq 0$$

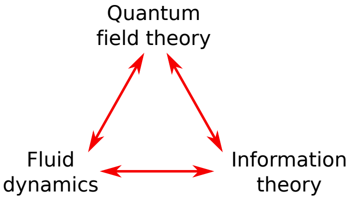
- **relative entanglement entropy between ρ and any state, in particular thermal state σ is non-increasing**

Quantum field dynamics

- new hypothesis

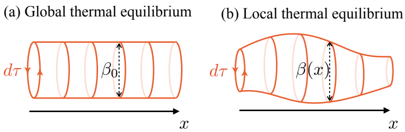
local dissipation = *quantum entanglement generation*

- quantum information is spread
- locally, quantum state approaches mixed state form
- full loss of *local* quantum information = *local* thermalization



Local equilibrium & partition function

[Floerchinger, JHEP 1609, 099 (2016)]



- local equilibrium with $T(x)$ and $u^\mu(x)$

$$\beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

- represent partition function as functional integral with periodicity

$$\phi(x^\mu - i\beta^\mu(x)) = \pm\phi(x^\mu)$$

- partition function $Z[J]$, Schwinger functional $W[J]$ in Euclidean

$$Z[J] = e^{W_E[J]} = \int D\phi \, e^{-S_E[\phi] + \int_x J\phi}$$

One-particle irreducible or quantum effective action

- in Euclidean domain $\Gamma[\phi]$ defined by Legendre transform

$$\Gamma_E[\Phi] = \int_x J_a(x)\Phi_a(x) - W_E[J]$$

with expectation values

$$\Phi_a(x) = \frac{1}{\sqrt{g}(x)} \frac{\delta}{\delta J_a(x)} W_E[J]$$

- **Euclidean** field equation

$$\frac{\delta}{\delta \Phi_a(x)} \Gamma_E[\Phi] = \sqrt{g}(x)\, J_a(x)$$

resembles classical equation of motion for $J = 0$

- need **analytic continuation** to obtain a viable equation of motion

Entropy production

[Floerchinger, JHEP 1609, 099 (2016)]

- variational principle with effective dissipation from analytic continuation
- analysis of general covariance leads to entropy current and local entropy production

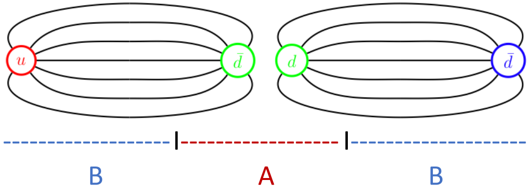
$$\nabla_\mu s^\mu = \frac{1}{\sqrt{g}} \frac{\delta \Gamma_D}{\delta \Phi_a}\Big|_{\text{ret}} \beta^\lambda \partial_\lambda \Phi_a + \beta_\mu \nabla_\nu \left(-\frac{2}{\sqrt{g}} \frac{\delta \Gamma_D}{\delta g_{\mu\nu}}\Big|_{\text{ret}} \right)$$

- can likely be understood as entanglement generation

Thermalization beyond collisions

- quantum fields can be locally thermal without collisions
- horizons: black holes, de-Sitter space
- space-time dynamics of entanglement

Entanglement, QCD strings and thermalization



- hadronization in Lund string model (e. g. PYTHIA)
- reduced density matrix for region A

$$\rho_A = \text{Tr}_B\{\rho\}$$

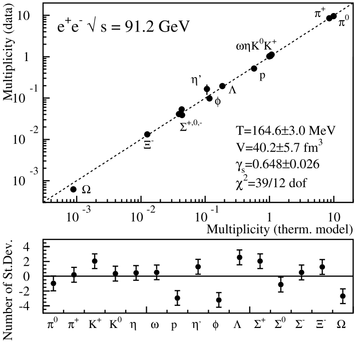
- **entanglement entropy**

$$S_A = -\text{Tr}_A\{\rho_A \ln \rho_A\}$$

- could this lead to thermal-like effects?

The thermal model puzzle

- elementary particle collision experiments such as $e^+ \, e^-$ collisions show some thermal-like features
- particle multiplicities well described by thermal model



[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by collisions unlikely
- more thermal-like features difficult to understand in PYTHIA [Fischer, Sjöstrand (2017)]
- alternative explanations needed

Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2} \mathrm{tr}\, \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu}$$

- fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling g has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed
  ['t Hooft (1974)]

Schwinger model

- QED in 1+1 dimension

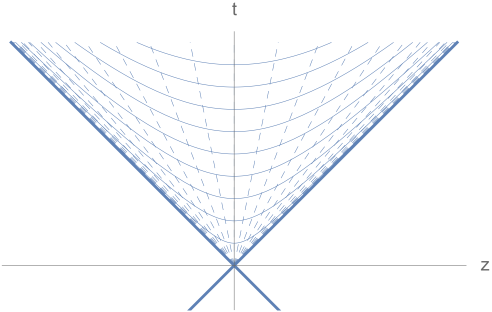
$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly
  [Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 - \frac{m\, q\, e^\gamma}{2\pi^{3/2}} \cos\left(2\sqrt{\pi}\phi + \theta\right) \Big\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- scalar mass related to U(1) charge by $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model $m = 0$ leads to free bosonic theory

Expanding string solution 1



- external quark-anti-quark pair on trajectories $z = \pm t$
- coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

Expanding string solution 2

- Schwinger boson field depends only on τ

$$\bar{\phi} = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau}\partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_{\mathsf{e}}$ for $\tau \to 0_+$

$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} \qquad (\tau \to 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} J_0(M\tau)$$

Gaussian states

- theories with quadratic action often have Gaussian density matrix
- fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

  and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- if ρ is Gaussian, also reduced density matrix ρ_A is Gaussian

Entanglement entropy for Gaussian state

- entanglement entropy of Gaussian state in region A
  [Berges, Floerchinger, Venugopalan, JHEP 1804 (2018) 145]

$$S_A = \frac{1}{2}\mathsf{Tr}_A\left\{D\ln(D^2)\right\}$$

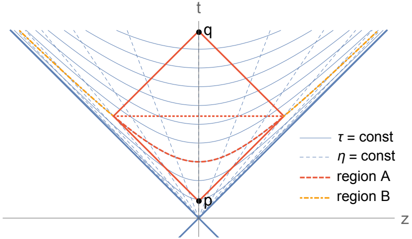
- operator trace over region A only
- matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}$$

- involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- expectation value $\bar{\phi}$ does not appear explicitly
- coherent states and vacuum have equal entanglement entropy S_A

Local density matrix and temperature in expanding string

[Berges, Floerchinger, Venugopalan, *Thermal excitation spectrum from entanglement in an expanding quantum string*, PLB778, 442 (2018)]



- Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- **local density matrix thermal at early times as result of entanglement**

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- Hawking-Unruh temperature in Rindler space $T(x) = \frac{\hbar c}{2\pi x}$

Physics picture

- coherent state at early time contains entangled pairs of quasi-particles with opposite wave numbers
- on finite rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ in- and out-flux of quasi-particles with thermal distribution via boundaries
- technically limits $\Delta\eta \to \infty$ and $M\tau \to 0$ do not commute
  - $\Delta\eta \to \infty$ for any finite $M\tau$ gives pure state
  - $M\tau \to 0$ for any finite $\Delta\eta$ gives thermal state with $T = 1/(2\pi\tau)$

Testing the picture

- explicit calculations in non-equilibrium QFT
- experimental tests with high-energy collisions
- explicit calculations in holography
- explicit calculations in small dimensions with tensor networks
- quantum simulations with universal quantum computers
- quantum simulation with ultracold atoms

Entropic uncertainty relations

Heisenberg / Robertson uncertainty relation [Robertson (1929)]

$$\sigma(X)\sigma(Z) \geq \frac{1}{2}|\langle\psi|[X,Z]|\psi\rangle|$$

Entropic uncertainty relations [Maassen & Uffink (1988), Frank & Lieb (2012)]

$$H(X) + H(Z) \geq \ln\frac{1}{c} + S(\rho)$$

- Shannon information entropy for measurement outcome

$$H(X) = -\sum_x p(x)\ln p(x)$$

- von-Neumann entropy

$$S(\rho) = -\mathrm{Tr}\{\rho\ln\rho\}$$

- maximal overlap between basis states

$$c = \max_{x,z} |\langle x|z\rangle|^2$$

Entanglement and entropic uncertainty relations

[Berta et al. (2010)]

- side information from entanglement with system B

$$H(X_A|X_B) + H(Z_A|Z_B) \geq \ln \frac{1}{c} + S(A|B)$$

- use measurement on B to infer outcome on A
- quantum conditional entropy can be negative for positive coherent information

$$S(A|B) = S(\rho) - S(\rho_B) = -I_{A\rangle B}$$

- experiments with cold atoms [with M. Gärttner and M. Oberthaler]
- towards test of *local dissipation = quantum entanglement generation*
- towards test of entanglement in horizon physics
- more applications in nuclear and high energy physics to be explored

Conclusions

- new perspectives on relativistic fluids from quantum information theory
- relative entanglement entropy useful to describe local thermalization
- quantum field theoretic description of relativistic fluid dynamics with two density matrices
  - true density matrix ρ evolves unitary
  - fluid model σ agrees locally but evolves non-unitary
- local thermalization without collisions possible
- excitations in expanding QCD strings locally thermal at early times