Selected topics on QCD evolution

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arXiv:1603.0742; ZJ

Outline:

- The scale dependence of the Burkardt sum rule
- The evolution of the small x gluon TMD
- Summary

Sum rule

Longitudinal momentum conservation:

$$\sum_{q+\bar{q}+g} \int xf(x,\mu^2)dx = 1$$

The scale dependence?

$$\frac{\partial \sum_{q+\bar{q}+g} \int xf(x,\mu^2)dx}{\partial \ln \mu^2} = ?$$

DGLAP evolution equation:

$$\frac{\partial f(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_1}{x_1} \left[\frac{1+z^2}{(1-z)_+} - \frac{3}{2}\delta(1-z) \right] f(x_1,\mu^2)$$

Computing all channels:

quark ----> quark
quark ----> gluon
gluon ----> quark, antiquark
antiquark ----> antiquark
antiquark ----> gluon

One finds:

$$\frac{\partial \sum_{q+\overline{q}+g} \int xf(x,\mu^2)dx}{\partial \ln \mu^2} = 0$$

- The sum rule is stable under QCD scale evolution.

Parton transverse motion inside a nucleon

$$\Phi(x,k_T)$$

1982-1983，Collins and Soper

$$\Phi(x,k_T)=\int\frac{dy}{4\pi}\frac{d^2y_T}{(2\pi)^2}e^{-ixp^+y^-+ik_Ty_T}\langle P|\overline{\psi}(y^-,y_T)\not{n}LL^+\psi(0^-,0_T)|P\rangle$$

For an unpolarized target,

$$\int d^2k_T\Phi(x,k_T)\vec{k}_T=0$$

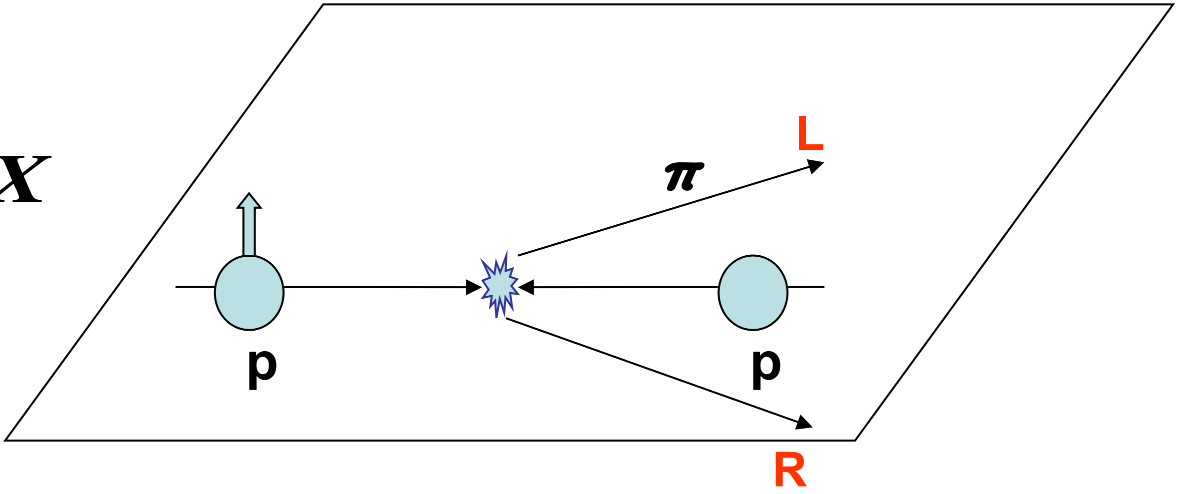
Parton distribution inside a transversely polarized target

parameterized as,

$$\Phi(x, k_T, S_T) = f(x, k_T^2) - \frac{1}{M}\varepsilon_{Tij} S_T^i k_T^j f_{1T}^{\perp}(x, k_T^2)$$

1990, D. Sivers

$$p(\uparrow) + p \longrightarrow \pi + X$$



Average transverse momentum:

$$\int dx d^2k_T \Phi(x, k_T, S_T) \vec{k}_T^{\alpha} \propto \hat{S}_T^{\alpha} \int dx d^2k_T f_{1T}^{\perp}(x, k_T^2) k_T^2 \neq 0$$

Transverse momentum conservation

Sum over all flavors

$$\sum_{q+\bar{q}+g}\int_0^1 dx\int d^2k_T f_{1T}^{\perp}(x,k_T^2)k_T^2 = 0$$

The Burkardt sum rule, **2004, Burkardt**

Alternative proof, **2015, Lorce**

The relations between the Sivers functions and twist-3 correlations,

2003, Boer, Mulders and Pijlman

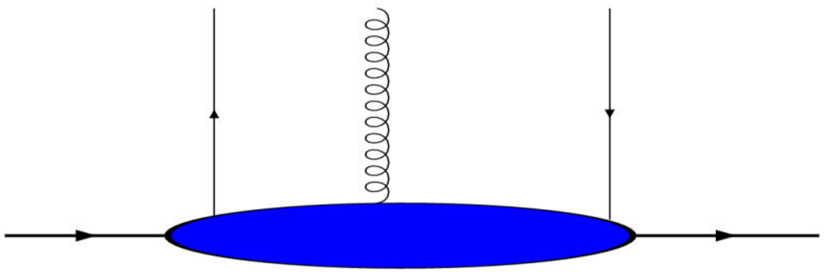
$$\int d^2k_T f_{1T}^{\perp,q}(x,k_T^2)k_T^2 \propto T_F^q(x,x) \qquad \int d^2k_T f_{1T}^{\perp,g}(x,k_T^2)k_T^2 \propto T_G^{(+)}(x,x)$$

The sum rule is re-expressed as,

$$\int_0^1 dx T_G^{(+)}(x,x) + \int_0^1 dx T_F^{q+\bar{q}}(x,x) = 0$$

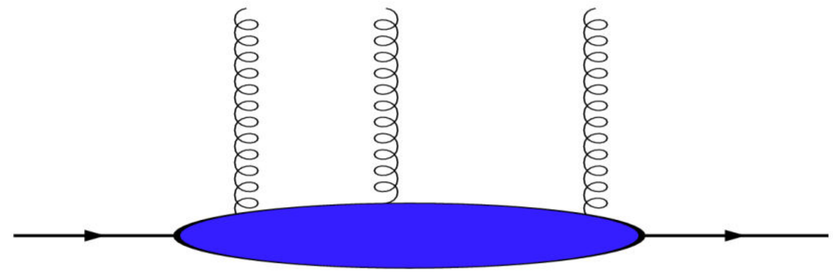
Operator definition of the twist-3 correlation functions

$$T_F(x,x,\mu^2) = \varepsilon_T^{S_T\sigma} \int \frac{dy_1^-}{2\pi} \int dy_2^- e^{ixP^+ y_1^-} \langle P | \overline{\psi}(0) \frac{\not{n}}{2} F_\sigma^{+}(y_2^-) \psi(y_1^-) | P \rangle$$



Qiu-Sterman function

1991, Qiu and Sterman



Tri-gluon function

1992, X. D. Ji; 2010, Beppu, Koike, Tanaka and Yoshida

Twist-3 functions and SSAs, see **Koike's talk**

The scale dependence of the Burkardt sum rule

$$\sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2) = \int_0^1 dx T_G^{(+)}(x,x,\mu^2) + \int_0^1 dx T_F^{q+\bar{q}}(x,x,\mu^2) = 0$$

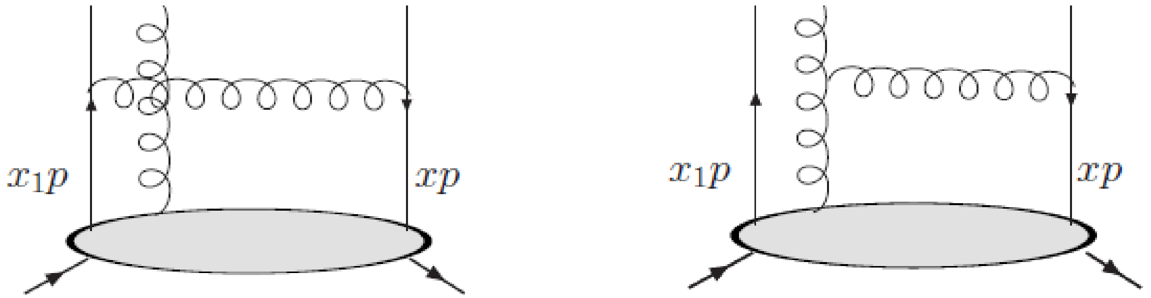
Is the sum rule stable under QCD corrections?

$$\frac{\partial \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)}{\partial \ln \mu^2} = ?$$

See also Teryaev's talk

DGLAP evolution of the twist-3 functions

Diagams contributing to the evolution kernel of the Qiu-Sterman function:



(a) (b)

$$\frac{\partial T_F^q(\xi,\xi,\mu^2)}{\partial \ln \mu^2}\bigg|_{q,\bar{q}\to q} = \frac{\alpha_s}{2\pi}\int_\xi \frac{dx}{x}\left[C_F\left\{\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right\}T_F^q(x,x)\right.$$
$$+\frac{C_A}{2}\left\{\frac{1+z}{1-z}T_F^q(\xi,x)-\frac{1+z^2}{1-z}T_F^q(x,x)-2\delta(1-z)T_F^q(x,x)\right\}$$
$$\left.-\frac{N_c}{2}\tilde{T}_F^q(\xi,x)+\frac{1}{2N_c}(1-2z)T_F^q(\xi,\xi-x)-\frac{1}{2N_c}\tilde{T}_F^q(\xi,\xi-x)\right]$$

2009, Kang and Qiu
2009, JZ, Yuan and Liang
2009, Vogelsang and Yuan
2011, Ma and Sang

2009, Braun, Manashov and Pirnay

2012, Schafer and JZ
2012, Ma and Wang
2012, Kang and Qiu
2013, Sun and Yuan

Computing all channels

quark+antiquark--->gluon

$$\left.\frac{\partial T_G^{(+)}(\xi,\xi,\mu^2)}{\partial \ln \mu^2}\right|_{q,\bar{q}\to g} = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}}\int_\xi \frac{dx}{x}\frac{C_A}{2}\left\{\frac{1+(1-z)^2}{z}\left[T_F^q(x,x)+T_F^{\bar{q}}(x,x)\right]\right.$$
$$\left. -\frac{2-z}{z}\left[T_F^q(x,x-\xi)+T_F^{\bar{q}}(x,x-\xi)\right]+\left[\tilde{T}_F^q(x-\xi,x)+\tilde{T}_F^{\bar{q}}(x-\xi,x)\right]\right\}$$

2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

gluon--->quark(or antiquark)

$$\left.\frac{\partial T_F^q(\xi,\xi,\mu^2)}{\partial \ln \mu_F^2}\right|_{g\to q} = \frac{\alpha_s}{2\pi}\int_\xi \frac{dx}{x}\frac{1}{2}\left[z^2+(1-z)^2\right]T_G^{(+)}(x,x)$$

2009, Kang and Qiu
2012, Ma and Wang

gluon--->gluon

$$\left.\frac{\partial\left[\frac{N(\xi,\xi)-N(\xi,0)}{\xi}\right]}{\partial \ln \mu_F^2}\right|_{g\to g} = \frac{\alpha_s}{2\pi}C_A\int_\xi^1 \frac{dx}{x^2}\left\{\frac{(z^2-z+1)^2}{z(1-z)_+}\left[N(x,x)-N(x,0)\right]+\frac{1+z^2}{2z(1-z)_+}N(\xi,x)-\frac{1+(1-z)^2}{2z(1-z)_+}N(x,x-\xi)\right.$$
$$\left.-\frac{z^2+(1-z)^2}{2z(1-z)_+}N(\xi,\xi-x)-\delta(1-z)\left[N(x,x)-N(x,0)\right]\right\}+\frac{\alpha_s}{2\pi}\left(C_A\frac{11}{6}-\frac{n_f}{3}\right)\left[N(\xi,\xi)-N(\xi,0)\right]$$

$$\frac{x}{2\pi}T_G^{(+)}(x,x) = -4M_N\left[N(x,x)-N(x,0)\right]$$

2009, Kang and Qiu
2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

We now agree on all other evolution equations except for the tri-gluon one.

2009, Kang and Qiu
2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

- Here we use the one derived by the third group.

$$\left.\frac{\partial\left[\frac{N(\xi,\xi)-N(\xi,0)}{\xi}\right]}{\partial\ln\mu_F^2}\right|_{g\to g} = \frac{\alpha_s}{2\pi}C_A\int_\xi^1\frac{dx}{x^2}\left\{\frac{(z^2-z+1)^2}{z(1-z)_+}[N(x,x)-N(x,0)]+\frac{1+z^2}{2z(1-z)_+}N(\xi,x)-\frac{1+(1-z)^2}{2z(1-z)_+}N(x,x-\xi)\right.$$
$$\left.-\frac{z^2+(1-z)^2}{2z(1-z)_+}N(\xi,\xi-x)-\delta(1-z)[N(x,x)-N(x,0)]\right\}+\frac{\alpha_s}{2\pi}\left(C_A\frac{11}{6}-\frac{n_f}{3}\right)[N(\xi,\xi)-N(\xi,0)]$$

Beppu et.al.'s parametrization for the tri-gluon correlation used in our calculation.

Collecting all results:

$$\frac{\partial \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)}{\partial \ln \mu^2} = -\frac{\alpha_s}{2\pi} C_A \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)$$

2015, ZJ

Solution:

$$\sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (Q^2) = e^{-\frac{C_A}{2\pi}\int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \alpha_s(\mu^2)} \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (Q_0^2)$$

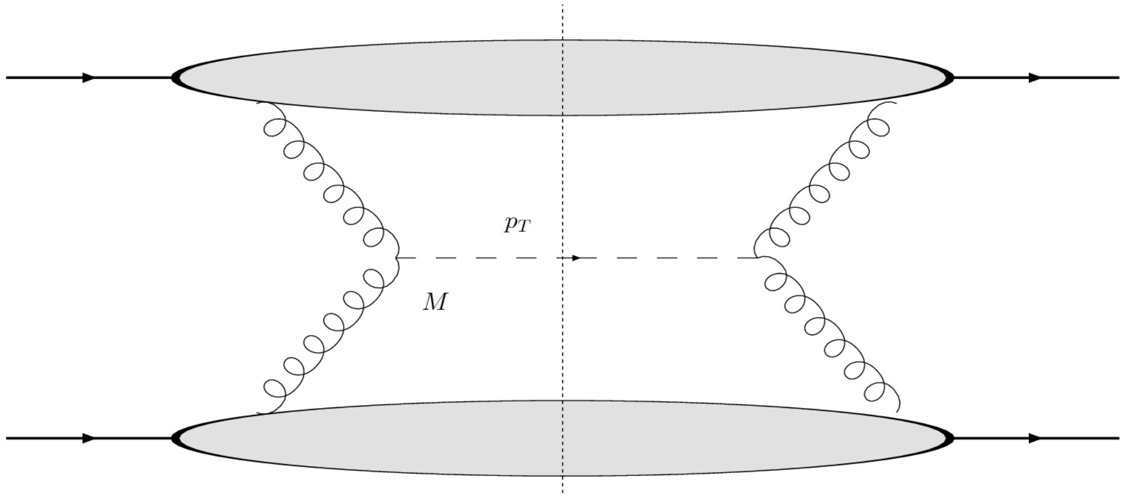
The evolution of the small x gluon TMD

See also Balitsky's talk

Extraction of small x gluon TMDs

Pisano's talk, Lansberg's talk, Yajin's talk and Mukherjee's talk

Gluon initiated Drell-Yan process



Inclusive process: Collinear factorization;

large logarithm $\ln \frac{M^2}{\mu^2}$ resummed by DGLAP equation

Semi-inclusive process:

- $M^2 >> p_T^2$, TMD factorization, $\ln \frac{M^2}{p_T^2}$ resummed by Collins-Soper equation

1982-1983, Collins and Soper

- $S >> M^2$, Kt factorization, $\ln \frac{S}{M^2}$ resummed by BFKL equation

1991, Catani, Ciafaloni and Hautmann

1991, Collins and Ellis

The overlap region

$$S >> M^2 >> p_T^2$$

Both the TMD and Kt factorization apply.

An explicit NLO cross section calculation shows that both the large logarithm appear.

$$\ln \frac{S}{M^2} \qquad \ln \frac{M^2}{p_T^2}$$

2013, Mueller, Xiao, Yuan

Such joint resummation has been also disscussed in other literatures.

2015, Balitsky and Tarasov; 2015, Kovchegov and Sievert; 2015 Marzani

Our method is closer to Mueller, Xiao and Yuan's formulation.

Our starting point

$$xG(x,l_\perp,x\zeta) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3 P^+} e^{-ixP^+y^- + il_\perp\cdot y_\perp} \langle P|F_\mu^+(y^-,y_\perp)\mathcal{L}_{\tilde{n}}^\dagger(y^-,y_\perp)\mathcal{L}_{\tilde{n}}(0,0_\perp)F^{\mu+}(0)|P\rangle$$

Gloun---->gloun splitting kernel;

$$\mathcal{P}_{gg}(z) = 2C_A \frac{(z^2 - z + 1)^2}{z(1-z)}$$

When z(or x) --->0, large logarithm $\ln\frac{1}{x}$ summed by BFKL

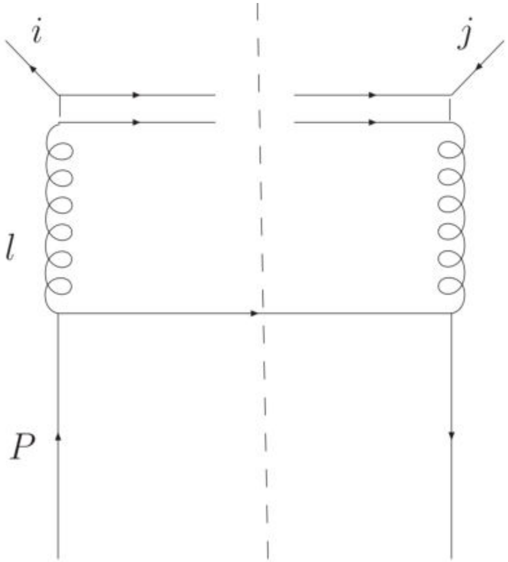
When z--->1, light cone divergence, introduce ζ to regularize

large logarithm $\ln\frac{x^2\varsigma^2}{k_T^2}$ summed by CS

$$\ln\frac{1}{x} \longrightarrow \ln\frac{S}{M^2} \qquad \ln\frac{x^2\varsigma^2}{k_T^2} \longrightarrow \ln\frac{M^2}{p_T^2}$$

The main strategy

In a simple quark model, at tree level:



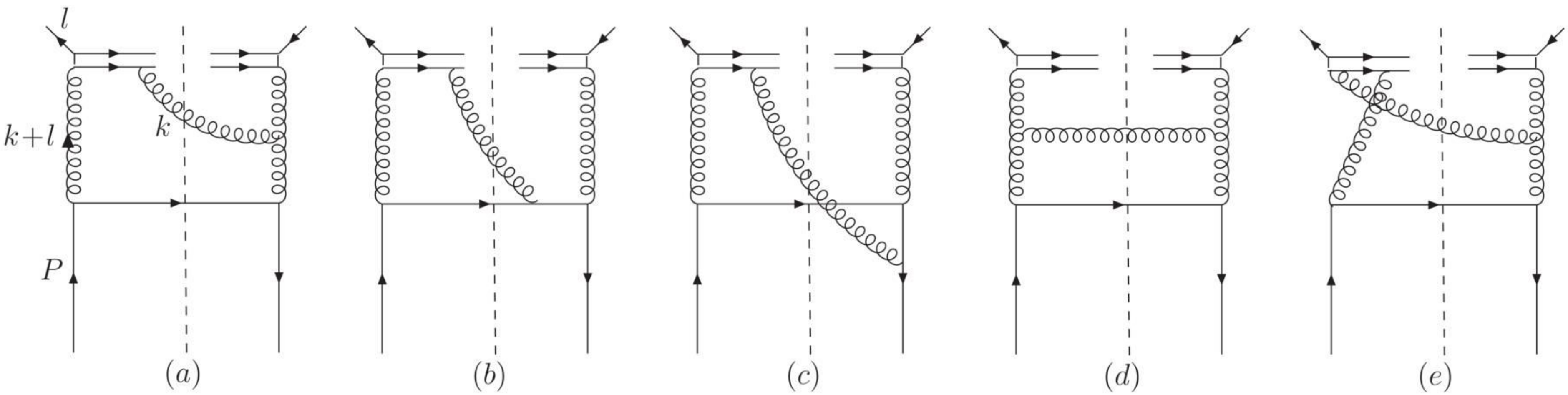
$$xG(x, l_\perp, x\zeta)_{LO}|_{x\to 0} = \frac{\alpha_s C_F}{\pi^2}\frac{1}{l_\perp^2}$$

- both large logarithms are absent at LO;

  how is it dressed by quantum corrections at NLO?

Real graphs

Sample diagrams:

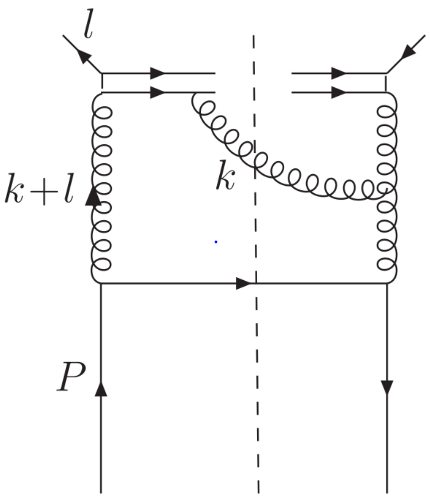


- Fig.a is the only diagram contributing to both the CS and the BFKL evolution kernel

- Calculation formulated in the Ji-Ma-Yuan scheme.

- Other regularization schemes, Collins 2011, SCET, EIS

One remark



In the infinite momentum frame $P^+ \dashrightarrow \infty$

BFKL rapidity divergence

$$\int_0^\infty \frac{dk^+}{k^+} = \int_0^{l^+} \frac{dk^+}{k^+} + \int_{l^+}^\infty \frac{dk^+}{k^+}$$

Collins-Soper rapidity divergence

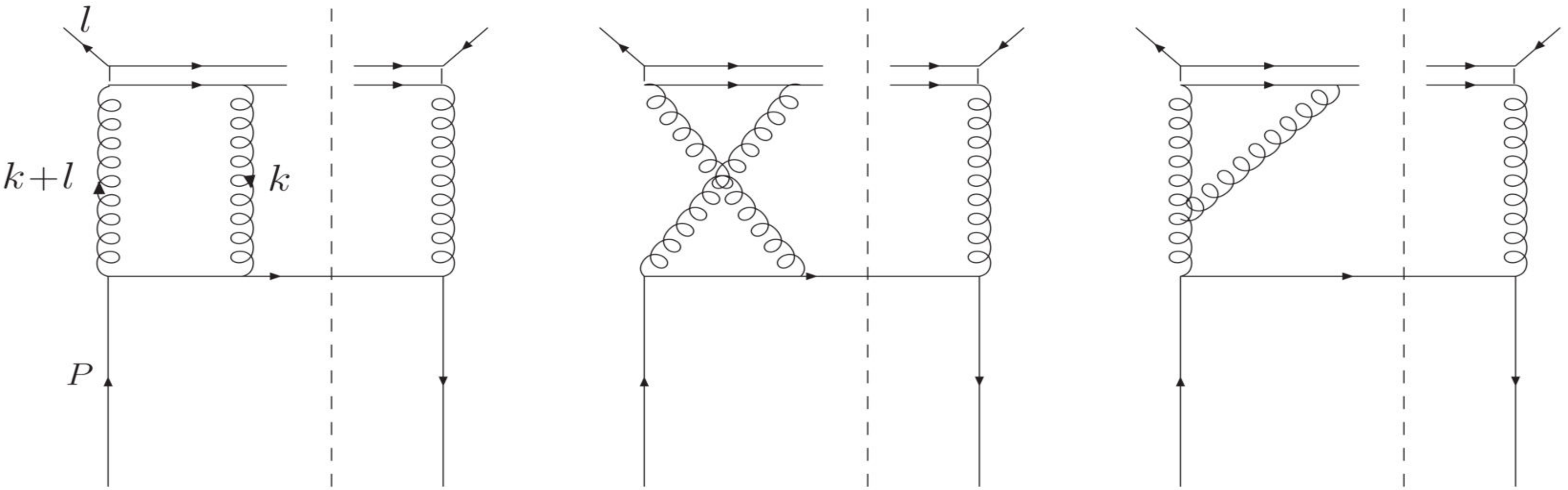
Real corrections:

$$xG(x,l_\perp,x\zeta)_{Rel}|_{x\to 0} = \mathcal{C}\int d^2k_\perp \frac{1}{(k_\perp+l_\perp)^2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}$$
$$+2\mathcal{C}\ln\frac{1}{x}\int_0 \frac{d^2k_\perp}{(k_\perp+l_\perp)^2 k_\perp^2}$$

Using $xG(x,k_\perp+l_\perp)_{LO}=\frac{\alpha_s C_F}{\pi^2}\frac{1}{(k_\perp+l_\perp)^2}$ reexpressed as,

$$xG(x,l_\perp,x\zeta)_{Rel}|_{x\to 0} = \frac{\alpha_s N_c}{\pi^2}\int d^2k_\perp \left\{\frac{\ln\frac{1}{x}}{k_\perp^2}+\frac{\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}}{2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\right\} xG_{LO}(x,k_\perp+l_\perp,x\zeta)$$

Virtual graphs



(a) (b) (c)

Fig.a & Fig.b ⟹ BFKL kernel

Fig.c ⟹ CS kernel

Final result(in the leading logarithm approximation),

$$
\begin{aligned}
xG(x,l_\perp,x\zeta)_{NLO} &= xG(x,l_\perp,x\zeta)_{LO} \\
&+ \frac{\alpha_s N_c}{\pi^2}\ln\frac{1}{x}\int\frac{d^2k_\perp}{k_\perp^2}\left[xG_{LO}(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG_{LO}(x,l_\perp,x\zeta)\right] \\
&+ \frac{\alpha_s N_c}{2\pi}\left[\ln\frac{x^2\zeta^2}{l_\perp^2}-\frac{1}{2}\ln\frac{x^2\zeta^2}{\mu^2}-\left(\ln\frac{x^2\zeta^2}{l_\perp^2}\right)^2\right]xG_{LO}(x,l_\perp,x\zeta) \\
&+ \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}xG_{LO}(x,k_\perp+l_\perp,x\zeta)
\end{aligned}
$$

2016, ZJ

- renormalization scale dependence is not yet taken into account.

The resulting gluon TMD indeed simultaneously satisfies the both

BFKL equation:

$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int \frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta) - \frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG(x,l_\perp,x\zeta)\right\}$$

CS equation:

$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial \ln\zeta} = -\frac{\alpha_s N_c}{\pi}\ln\left[\frac{x^2\zeta^2 b_\perp^2}{4}e^{2\gamma_E-\frac{1}{2}}\right]G(x,b_\perp,x\zeta)$$

slightly different from Ji-Ma-Yuan's result

Propose a new matching procedure

- At moderate/large x, matching procedure:

$$G(x,k_\perp,x\xi)\propto H(k_\perp,x/z)\otimes G(z,\mu^2)$$

Logarithm $\ln\frac{\mu^2}{Q^2}$ is important

- At small x, matching procedure:

$$xG(x,k_\perp,x\xi)\propto H(x,k_\perp,p_\perp)\otimes xG(x,p_\perp)$$

Logarithm $\ln\frac{1}{x}$ is more important

Work in progress with **Xiao and Yuan**

Summary

- Transverse momentum conservation is preserved under QCD scale evolution.
- Unified picture : TMD factorization V.S. kt factorization.

Thank you for your attention!