On the high-energy evolution of the jet quenching parameter

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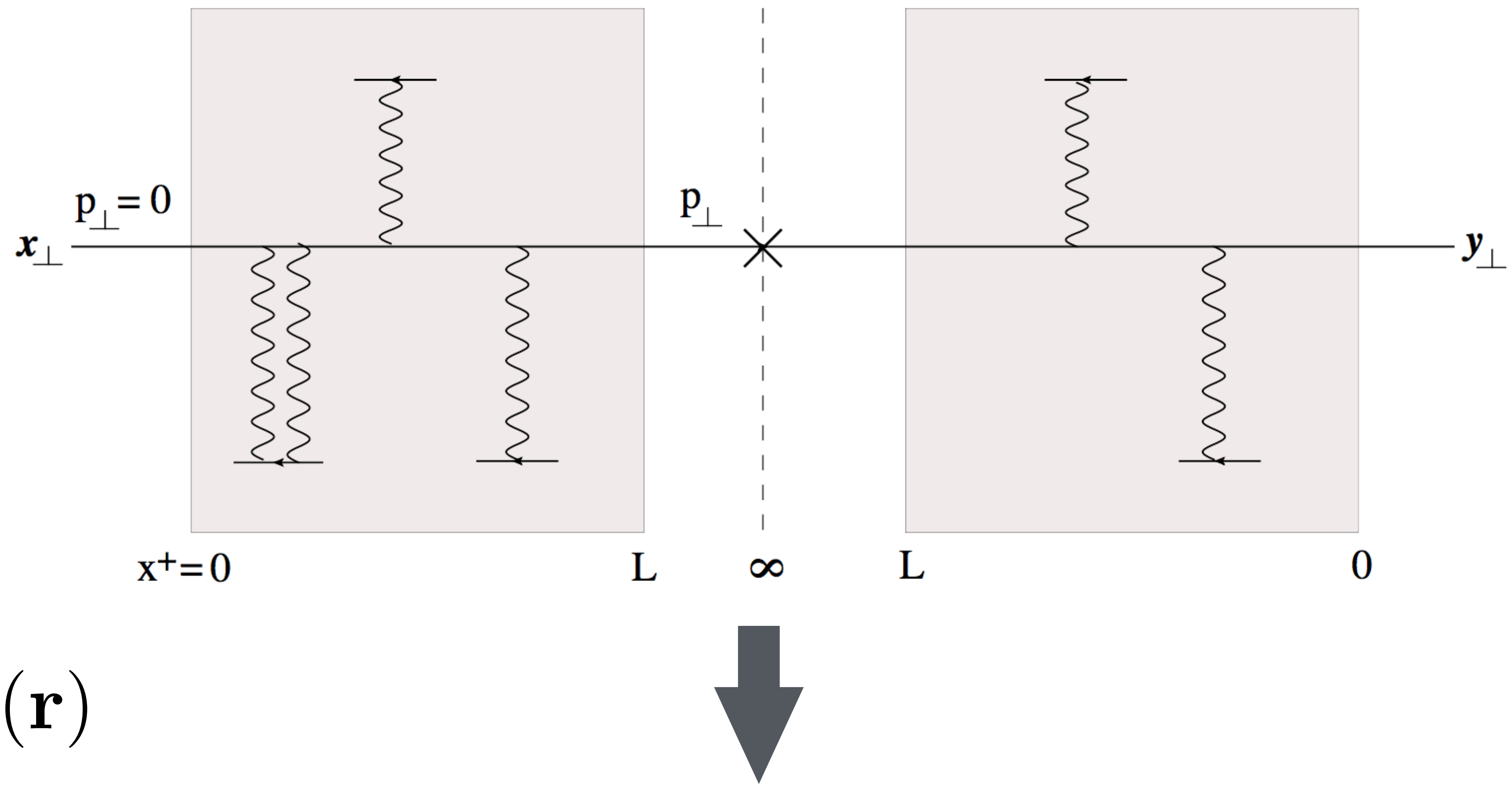
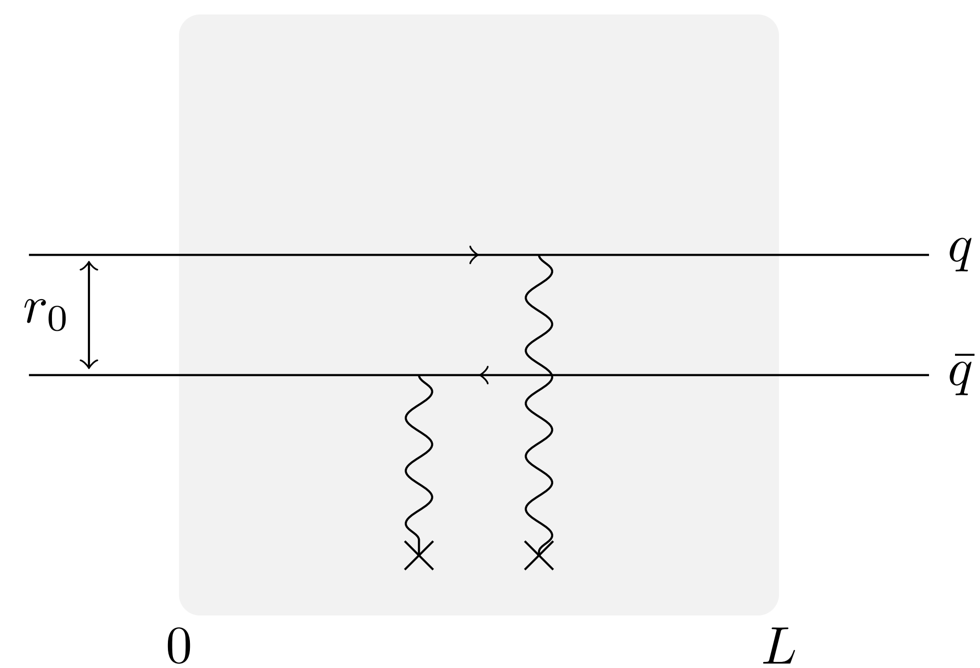
Transverse momentum broadening

Transverse-momentum distribution of a quark in a medium can be described by a 'fictitious' dipole:



$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}_\perp} = \frac{1}{(2\pi)^2}\int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}_\perp \mathbf{r}} S(\mathbf{r})$$

$$S(\mathbf{r}) = \frac{1}{N_c}\mathrm{tr}\langle V(\mathbf{r})^\dagger V(\mathbf{0})\rangle$$

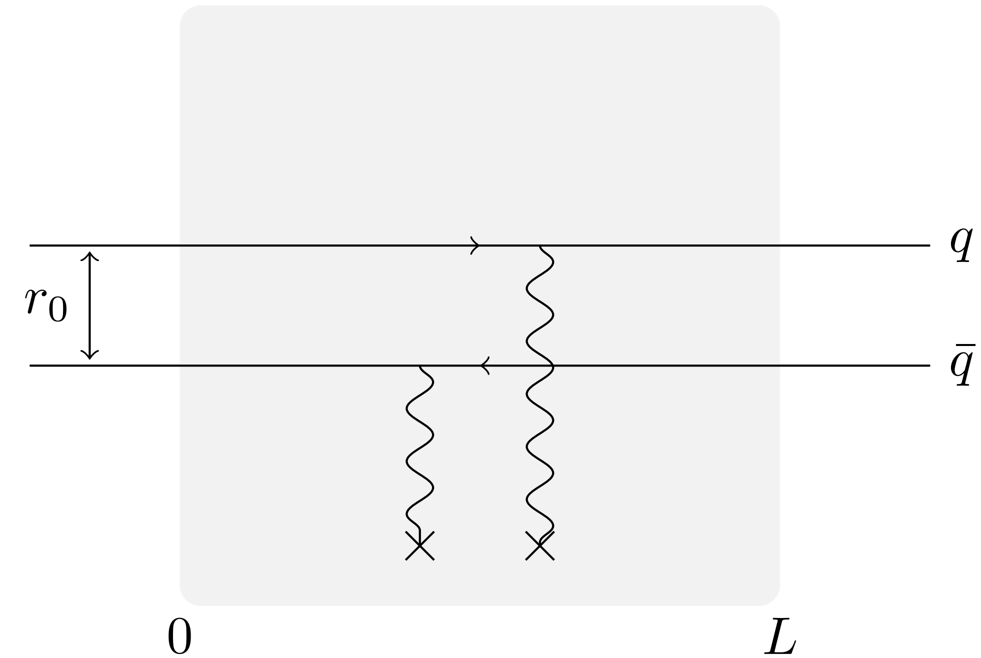


Transverse momentum broadening (**TMB**):

$$\langle \mathbf{k}_\perp^2 \rangle = \int \mathrm{d}^2\mathbf{k}_\perp \mathbf{k}_\perp^2 \frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}_\perp}$$

Transverse momentum broadening

Dipole probes the distribution of the gluon fields in the medium



Assume independent scattering centers (weakly coupled QGP)

Glauber resummation (MV model)

$$S(\mathbf{r}) \simeq e^{-\frac{\alpha_s}{4} C_F n_0 L \mathbf{r}^2 \ln \frac{1}{\mathbf{r}^2 \Lambda^2}} \simeq e^{-\frac{1}{4} Q_0^2 \mathbf{r}^2}$$

Multiple scattering important when exponent of order one

Saturation/unitarization scale: $Q_0^2\left(1/r_\perp^2\right) = \alpha_s C_F n_0 L \ln \dfrac{1}{r_\perp^2 \Lambda_{\mathrm{QCD}}^2}$

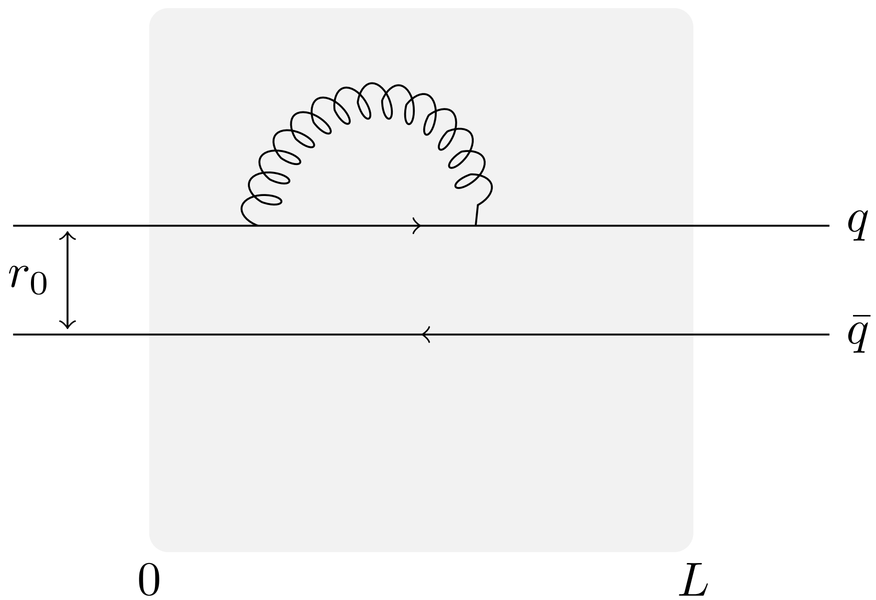
$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}_\perp} \simeq \frac{1}{\pi Q_0^2} e^{-\mathbf{k}_\perp^2/Q_0^2}$$

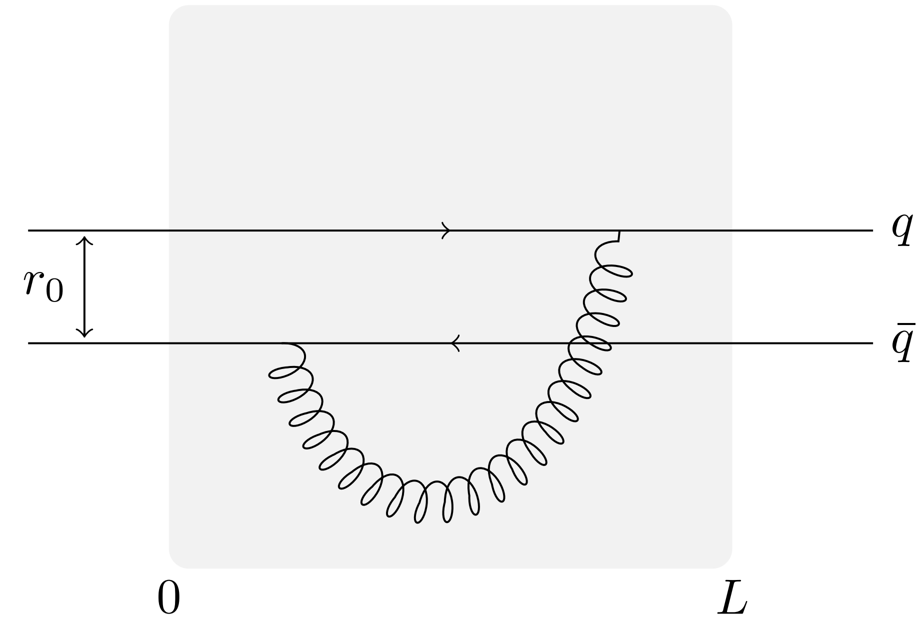
Brownian motion in transverse plane

$$\left\langle \mathbf{k}_\perp^2 \right\rangle = Q_0^2 \equiv \hat{q} L$$

Jet quenching parameter

Beyond leading order: radiative corrections





$$\mathrm{d}P = \frac{\alpha_s N_c}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

Transverse and longitudinal dynamics of the gluon are related due to the medium

$$\tau \simeq \frac{\omega}{k_\perp^2} \qquad k_{\mathrm{br}}^2 \simeq \hat{q}\tau$$

Constraints on the gluon's lifetime define phase space

$$l_0 \sim \frac{1}{T} \leq \tau \leq L$$

➡ large phase space for radiation

$$\hat{q} l_0 \leq k_\perp^2 \leq \hat{q} L$$

$$\hat{q} l_0^2 \leq \omega \leq \hat{q} L^2$$

➡ Evolution!

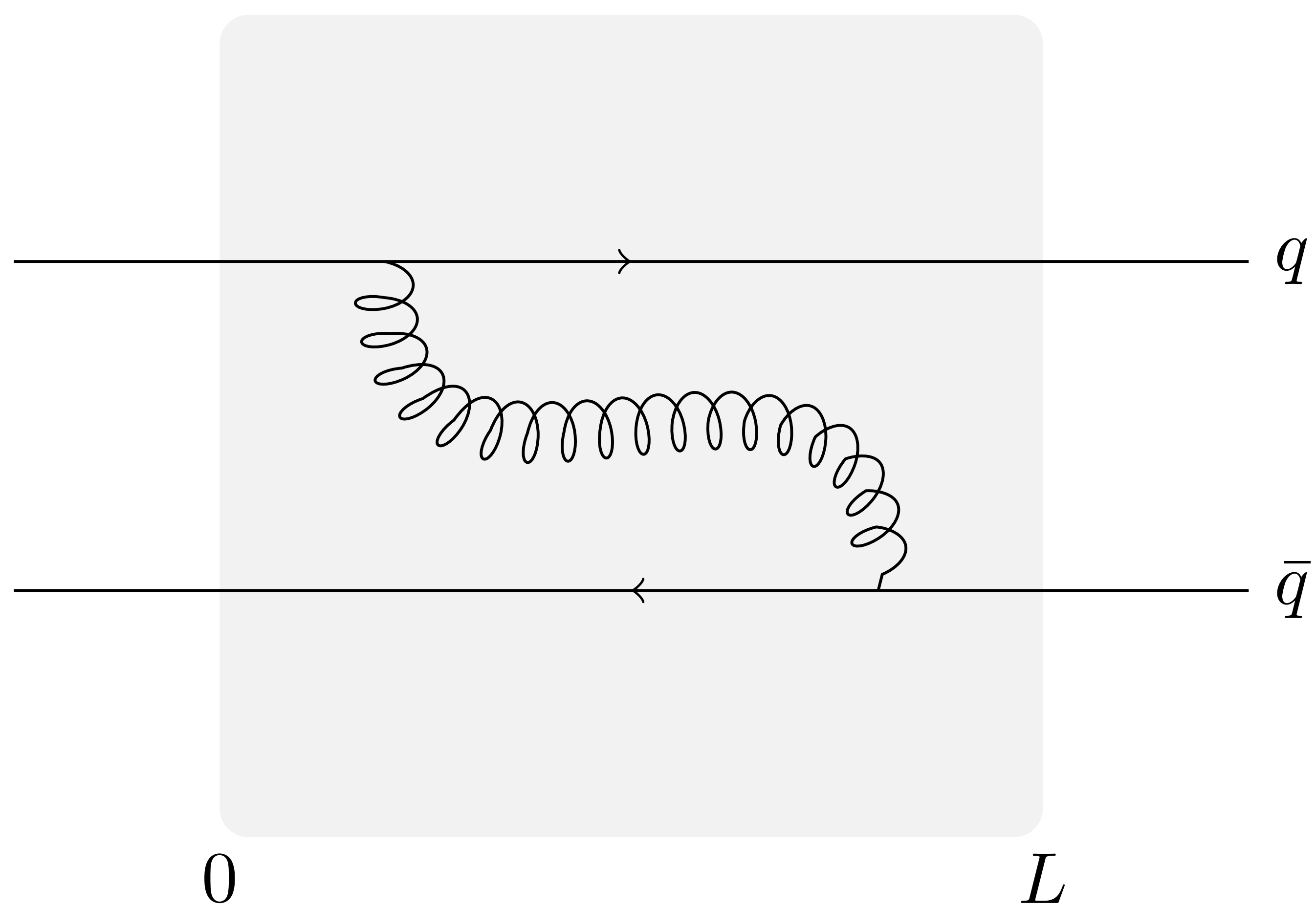
An Hamiltonian for high-energy evolution

Iancu, E., JHEP **10** (2014) 095

$$\Delta H = \frac{1}{2}\int \frac{\mathrm{d}\omega}{2\pi}\int \mathrm{d}t_2 \int \mathrm{d}t_1 \int \mathrm{d}^2\mathbf{r}_2 \int \mathrm{d}^2\mathbf{r}_1 G_{ab}^{--}\left(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1;\omega\right) J^a(t_2,\mathbf{r}_2) J^b(t_1,\mathbf{r}_1)$$

In-medium propagator

Operators generate soft gluon emission/absorption



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In-medium propagator

Operators generate soft gluon emission/absorption

$$G^{--}_{ab}\left(t_2,\mathbf{r}_2,t_1,\mathbf{r}_1;\omega\right) = \frac{1}{2\omega^3}\partial^i_{\mathbf{r}_2}\partial^i_{\mathbf{r}_1}\int \mathcal{D}\mathbf{r}(t) e^{i\frac{\omega}{2}\int_{t_1}^{t_2}\mathbf{r}^2} U^\dagger_{ab}(t_2,t_1,\mathbf{r}_\perp(t)) + \frac{i}{\omega^2}\delta_{ab}\delta(t_2-t_1)\delta^{(2)}(\mathbf{r}_2-\mathbf{r}_1)$$

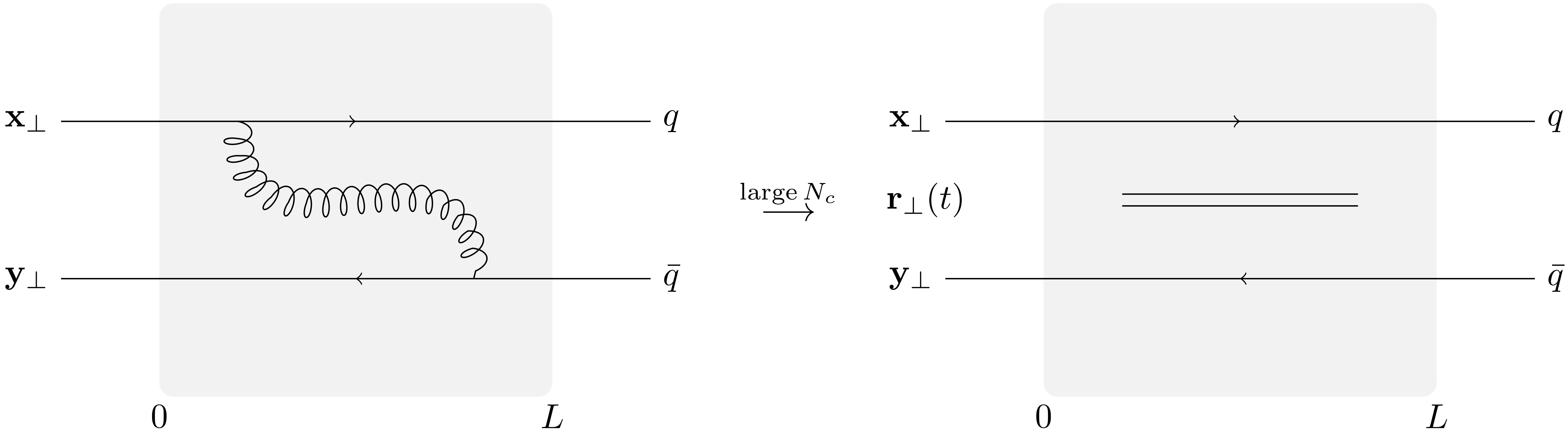
$$J^a(t,\mathbf{r}) \equiv \frac{\delta}{\delta A^-_a(t,\mathbf{r})}$$

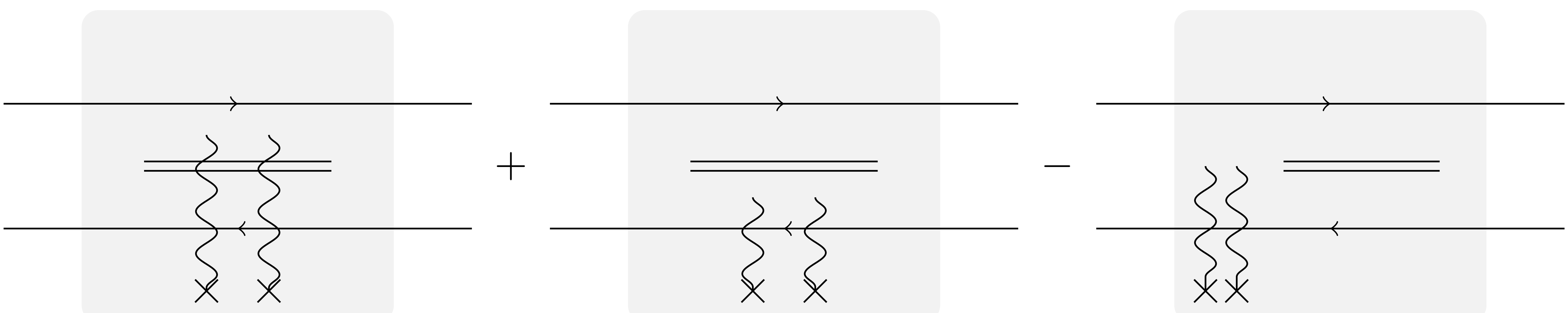
Recover B-JIMWLK in eikonal limit:

$$\Delta H = \int \frac{\mathrm{d}\omega}{\omega} \times \frac{1}{(2\pi)^3}\int_{\mathbf{r}_1\mathbf{r}_1\mathbf{z}} \mathcal{K}_{\mathbf{r}_1\mathbf{r}_1\mathbf{z}}\left(R^a_{\mathbf{r}_1}R^a_{\mathbf{r}_2} + L^a_{\mathbf{r}_1}L^a_{\mathbf{r}_2} - 2L^a_{\mathbf{r}_2}U^{\dagger ab}_{\mathbf{z}}R^b_{\mathbf{r}_1}\right)$$

Large Nc limit

In the large Nc limit, the evolution equation can be formulated in terms of dipoles = in-medium/non-eikonal analogue of BK





Large Nc limit

In the large Nc limit, the evolution equation can be formulated in terms of dipoles = in-medium/non-eikonal analogue of BK

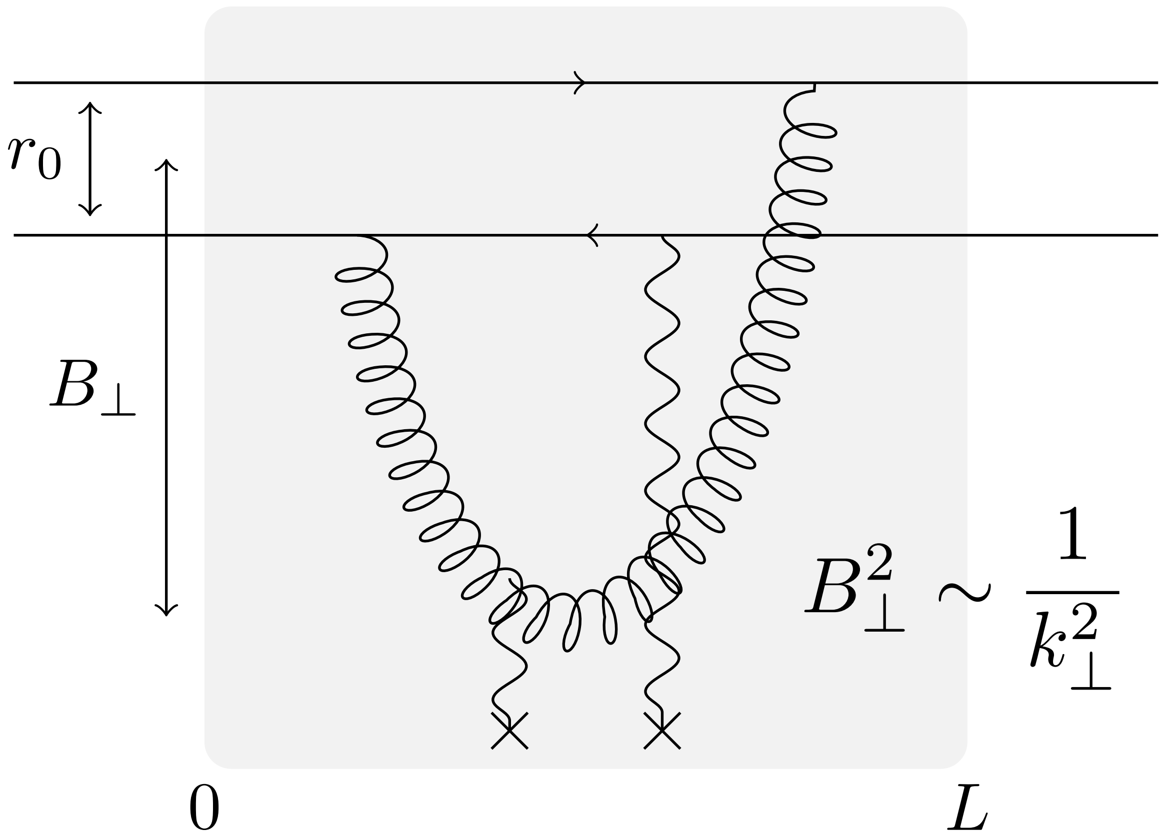
$$\Delta S_{\mathbf{xy}} = -\frac{\alpha_s N_c}{2}\int_{\omega}^{\omega_c}\frac{\mathrm{d}\omega}{\omega^3}\int_{-\infty}^{\infty}\mathrm{d}t_2\int_{-\infty}^{t_2}\mathrm{d}t_1\partial^i_{\mathbf{r}_1}\partial^i_{\mathbf{r}_2}\Bigg\{\mathcal{D}\mathbf{r}(t)e^{i\frac{\omega}{2}\int_{t_1}^{t_2}\mathrm{d}\dot{\mathbf{r}}^2}$$

$$\times S_{\infty,t_2}(\mathbf{x},\mathbf{y})\left[S_{t_2,t_1}(\mathbf{x},\mathbf{r})S_{t_2,t_1}(\mathbf{r},\mathbf{y}) - S_{t_2,t_1}(\mathbf{x},\mathbf{r})\right]S_{t_1,-\infty}(\mathbf{x},\mathbf{y})\Bigg\}\Bigg|^{\mathbf{r}_2=\mathbf{x}}_{\mathbf{r}_2=\mathbf{y}}\Bigg|^{\mathbf{r}_1=\mathbf{x}}_{\mathbf{r}_1=\mathbf{y}}$$

shockwave limit

$$\frac{\partial}{\partial\tau}S_{\mathbf{xy}} = \frac{\bar{\alpha}}{2\pi}\int_{\mathbf{z}}\mathcal{M}_{\mathbf{xyz}}\left\{S_{\mathbf{xz}}S_{\mathbf{zy}} - S_{\mathbf{xy}}\right\}$$

DLA: single scattering approximation



$$B_\perp^2 \sim \frac{1}{k_\perp^2}$$

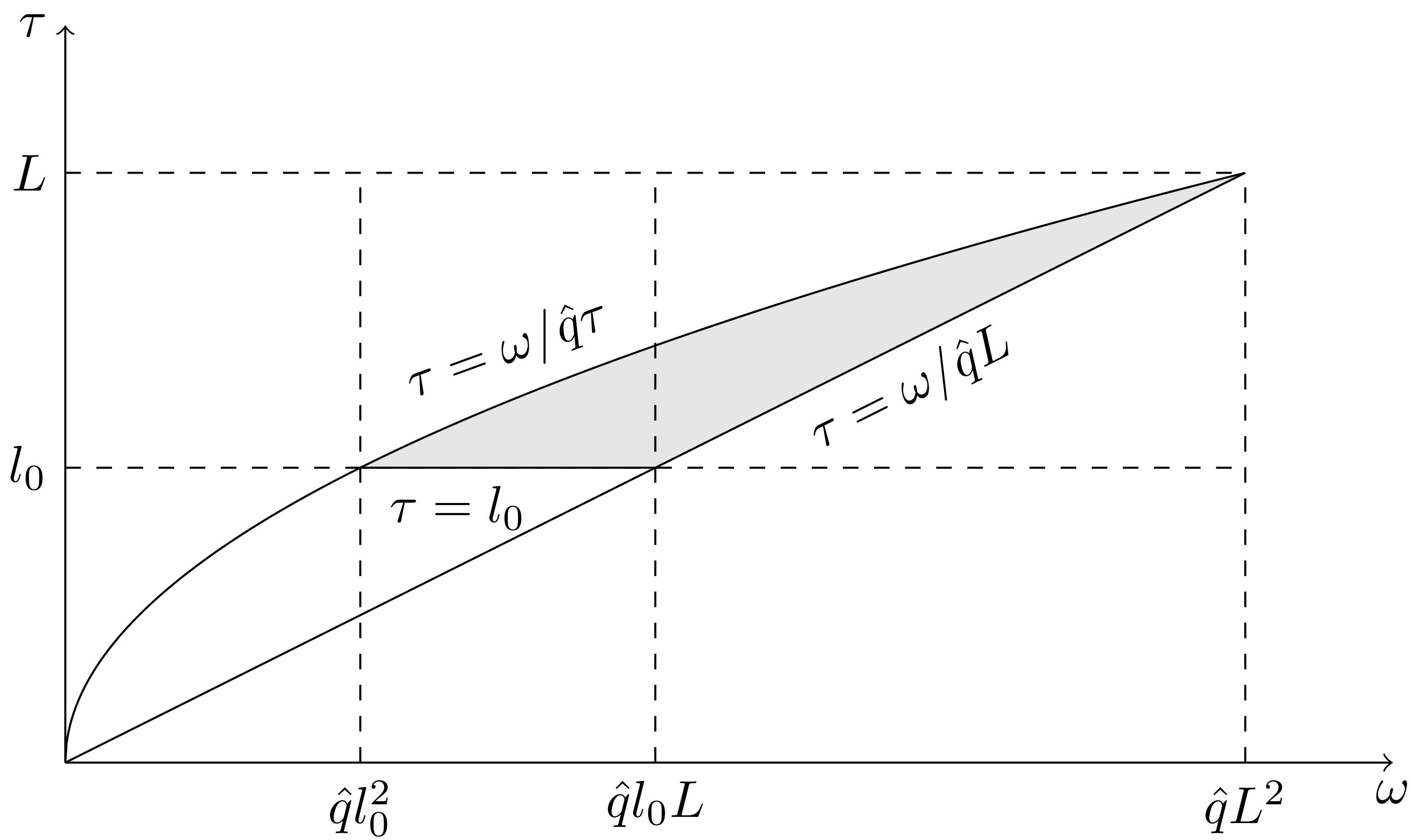
The evolution equation can be solved to double logarithmic accuracy in the approximation of a single scattering

Gluon fluctuation should be small enough for single scattering to dominate: $k_\perp^2 \geq \hat{q}\tau$

...but still large enough to be distinct from parent dipole:

$$k_\perp^2 \leq \frac{1}{r_0^2} \sim Q_0^2$$

Phase space for the DLA



More natural to describe in terms of lifetime and energy

$$l_0 \ll \tau \ll L$$

$$\hat{q}l_0^2 \ll \omega \ll \hat{q}L^2$$

remember $\tau \simeq \dfrac{\omega}{k_\perp^2}$

Single scattering region is constrained by the lines:

$$\tau = \frac{\omega}{k_{\rm br}^2} \simeq \frac{\omega}{\hat{q}\tau} \qquad \tau = \frac{\omega}{Q_0^2} = \frac{\omega}{\hat{q}L} \qquad \tau = l_0$$

Renormalization of the jet quenching parameter in DLA

In the DLA, one finds:

$$\hat{q}_L(Q_0^2) = \hat{q}^{(0)} + \bar{\alpha} \int_{l_0}^{L} \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}\tau}^{Q_0^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \hat{q}_\tau(k_\perp^2)$$

... which can be solved iteratively:

$$\hat{q}_L(Q_0^2) = \hat{q}^{(0)} \left(1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{L}{l_0} + \frac{\bar{\alpha}^2}{2!3!} \ln^4 \frac{L}{l_0} + \mathcal{O}(\bar{\alpha}^3)\right)$$

$$\hat{q}_L(Q_0^2) = \hat{q}^{(0)} \frac{1}{\sqrt{\bar{\alpha}} \ln(L/l_0)} \mathrm{I}_1 \left(2\sqrt{\bar{\alpha}} \ln \frac{L}{l_0}\right)$$

$\Downarrow$

The radiative corrections are absorbed into the renormalization of the jet quenching parameter

Implications of the DLA

From the more 'systematic' point of view of the full evolution equation, we confirm earlier results that were obtained in the spirit of BDMPS-Z

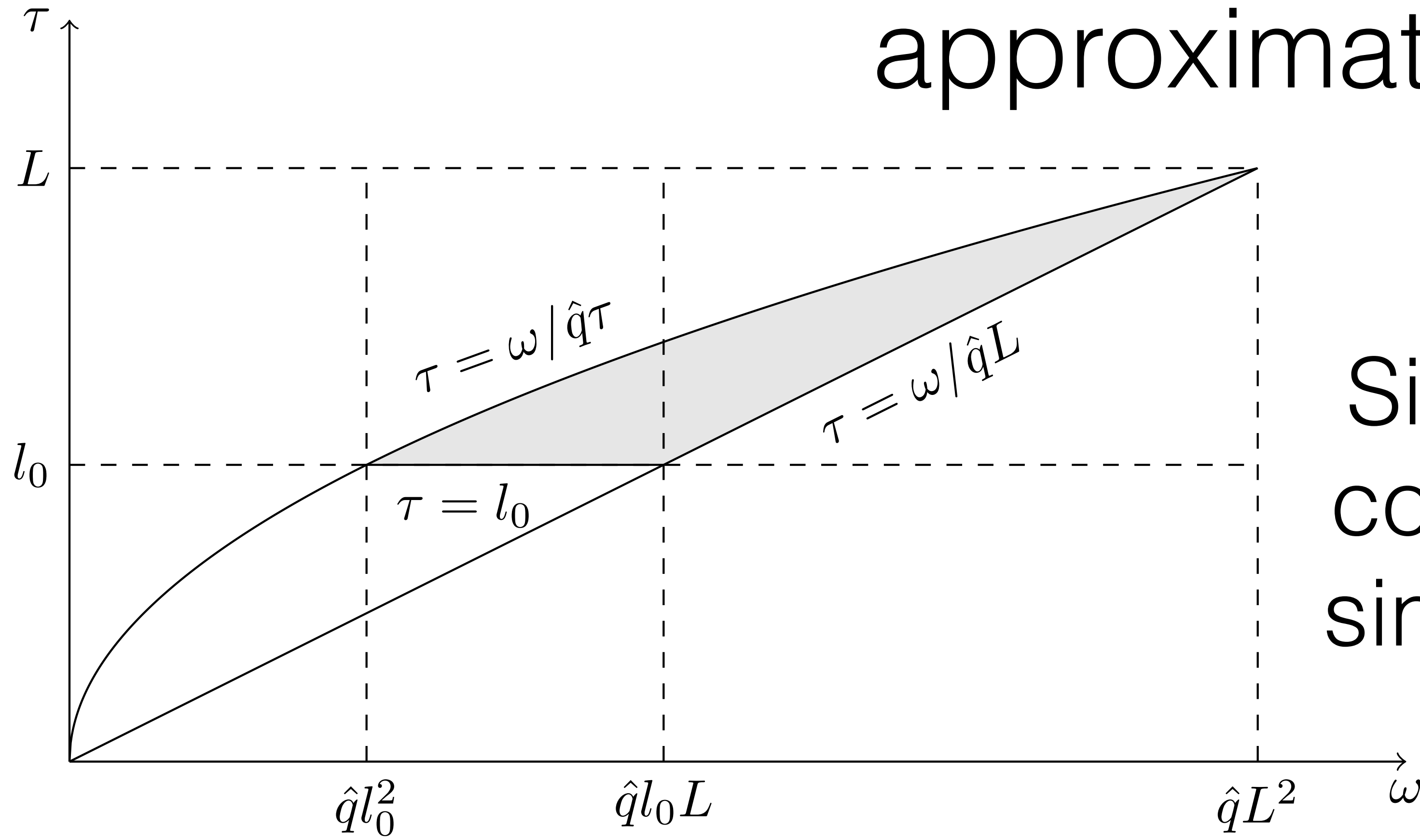
Liou, Mueller & Wu, Nucl. Phys. **A916** (2013) 102

Consequences for the initial condition of the standard BK equation: (pA collision)

$$\hat{q}^{(0)}(Q_0^2) \to \hat{q}_L(Q_0^2) \sim 3\hat{q}^{(0)}(Q_0^2)$$
$$L \sim 4\,\mathrm{fm}$$

Medium enhancement may explain a semi-hard initial value for Q_0^2

Beyond the double logarithmic approximation?



Single leading logarithmic corrections stem from both single & multiple scattering

To single logarithmic accuracy, the evolution equation remains non-linear

The first iteration of the single logarithmic correction is obtained by Mueller et al.: the full resummation is encoded in our evolution equation -> how to extract it?

Liou, Mueller & Wu, Nucl. Phys. **A916** (2013) 102

Conclusions & Outlook

A non-eikonal generalization of the JIMWLK Hamiltonian was constructed

We applied it to study the nonlinear high-energy evolution of the jet quenching parameter, for fluctuations deep inside the medium

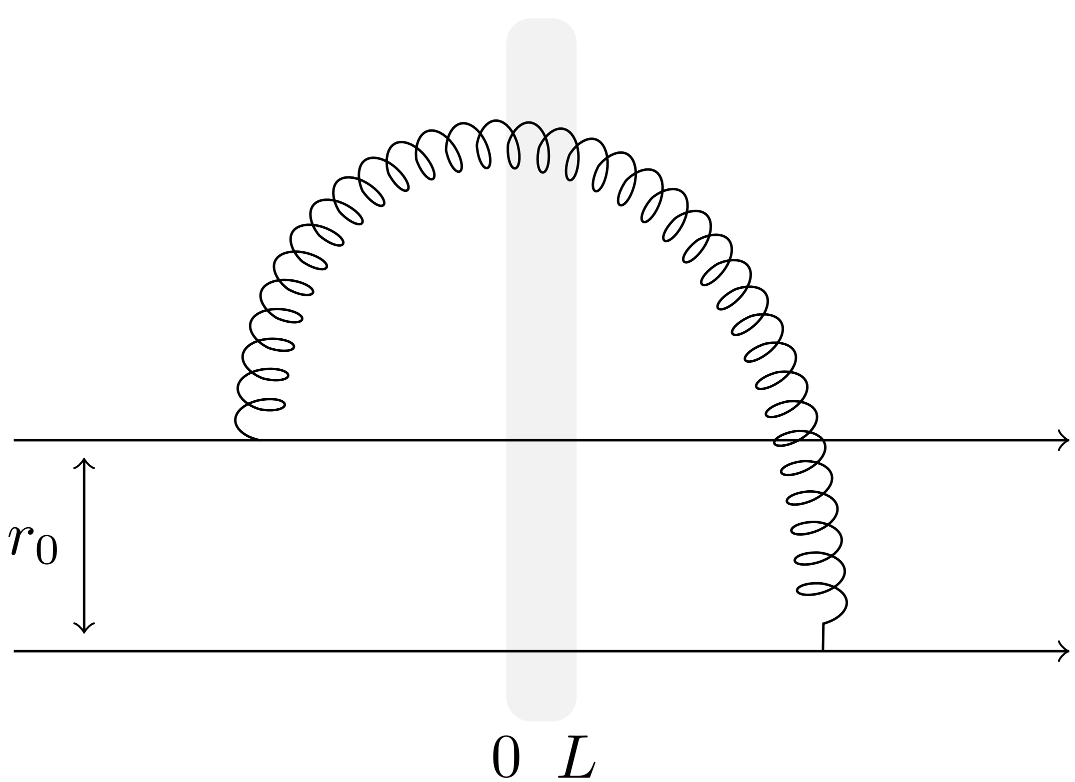
The evolution equation was solved in the double logarithmic approximation, resulting in a renormalization of the jet quenching parameter

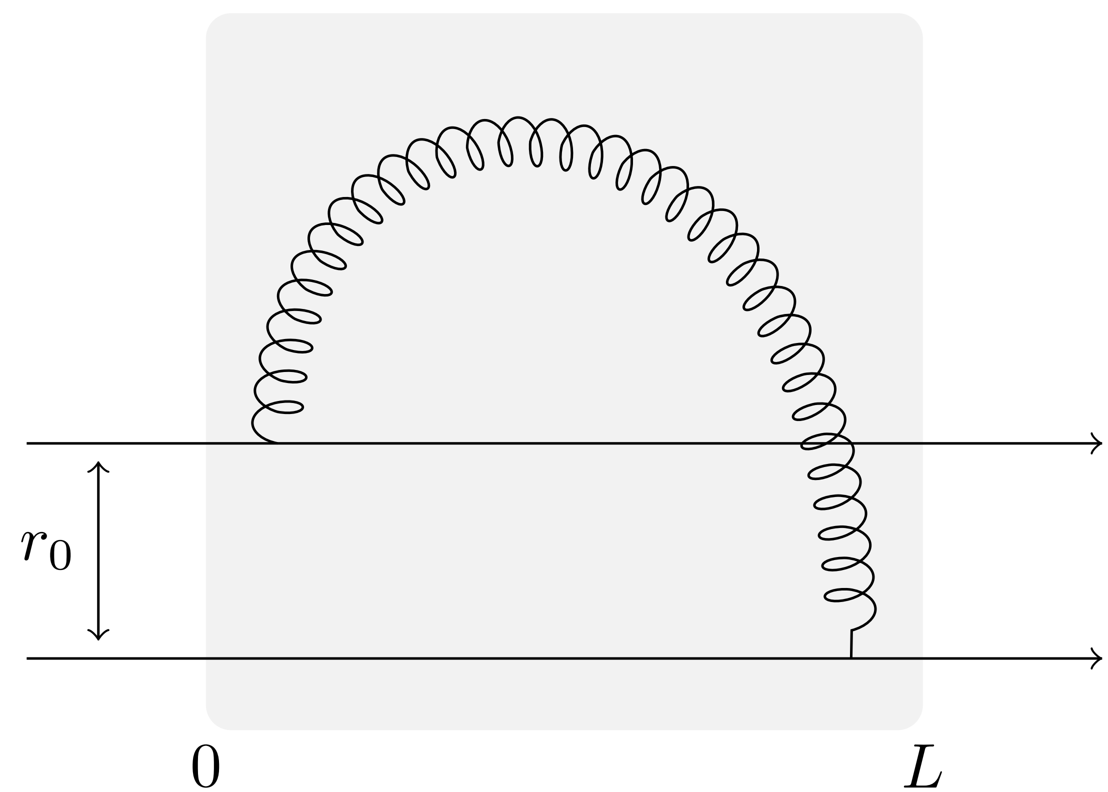
The extraction and the resummation of the single logarithmic contributions proves a much more difficult task -- work in progress

Backup: the unitarization line

In contrast to the shockwave case, the unitarization line in the medium is *strongly* dependent on the gluon's lifetime $\boldsymbol{\tau}$:

$$S\left(\omega,\tau,\mathbf{r}\right)=\exp\left\{-g^2C_F\int_0^\tau \mathrm{d}t\Gamma_\omega\left(\mathbf{r}\right)\right\}\simeq\exp\left\{-\frac{1}{4}\hat{q}(Q_s^2)\tau\mathbf{r}^2\right\}$$





$$Q_s^2(L)=\hat{q}L\sim 1/r_0^2\equiv Q_0^2 \qquad\qquad Q_s^2(\tau)=\hat{q}\tau=k_{\mathrm{br}}^2$$

jet quenching parameter

Backup: target point of view

In-medium gluon overlap function:

$$\varphi\left(\tau, \mathbf{k}\right) = \frac{4\pi^3}{N_c^2 - 1} \frac{\mathrm{d}N}{\mathrm{d}\tau \mathrm{d}^2\mathbf{b}\mathrm{d}^2\mathbf{k}} \longrightarrow f\left(k^+, \mathbf{k}\right) \simeq \frac{4\pi^3}{N_c^2 - 1} \frac{1}{\pi R_A^2} \frac{1}{L} \frac{\mathrm{d}N}{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}}$$

... at leading order: $f_0\left(k^-, \mathbf{k}_\perp\right) \simeq \dfrac{4\pi\alpha_s n_0}{k^- \mathbf{k}_\perp^2}$

$$f_0 \sim \frac{1}{\bar{\alpha}} \leftrightarrow k_\perp^2 = Q_s^2 \sim \frac{\alpha_s^2 N_c n_0}{k^-}$$

$$Q_s^2 \sim \frac{\alpha_s^2 N_c n_0}{k^-} \sim \hat{q} \frac{1}{k^-} \sim \hat{q}\tau$$

Backup: in-medium evolution of the dipole

We need to understand the behavior of the *non-eikonal* (anti)quark-gluon dipoles, that live *deeply* in the medium, with a *finite* lifetime $\tau \ll L$

$$\longrightarrow \quad S_{t_0+\tau,t_0}\left(\omega,\mathbf{r}\right) = \exp\left\{-g^2 C_F \int_{t_0}^{t_0+\tau} \mathrm{d}t \Gamma_\omega\left(\mathbf{r}\right)\right\}$$

Full high-energy evolution equation:

$$L\frac{\partial\Gamma_\omega\left(\mathbf{x},\mathbf{y}\right)}{\partial\omega} = \frac{1}{4\pi\omega^3}\int_0^L \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1 \partial^i_{\mathbf{r}_1}\partial^i_{\mathbf{r}_2}\int \mathcal{D}\mathbf{r} e^{i\frac{\omega}{2}\int_{t_1}^{t_2}\mathrm{d}t\dot{\mathbf{r}}^2}$$
$$\times\left[\exp\left(-\frac{g^2 N_c}{2}\int_{t_1}^{t_2}\mathrm{d}t\left[\Gamma_\omega\left(\mathbf{x},\mathbf{r}\left(t\right)\right)+\Gamma_\omega\left(\mathbf{r}\left(t\right),\mathbf{y}\right)-\Gamma_\omega\left(\mathbf{x},\mathbf{y}\right)\right]\right)-1\right]\Bigg|_{\mathbf{r}_2=\mathbf{y}}^{\mathbf{r}_2=\mathbf{x}}\Bigg|_{\mathbf{r}_1=\mathbf{y}}^{\mathbf{r}_1=\mathbf{x}}$$