

Sub-gauge Conditions for the Gluon Propagator Singularities in Light-Cone Gauge

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June 2, 2016

- Motivation
- Obtain gluon propagator in temporal gauge in functional method
 - Sub-gauge condition for temporal gauge.
 - Prescription for $\frac{1}{k_0}$ pole.
- Obtain gluon propagator in light-cone gauge in functional method.
 - Introduce sub-gauge condition to regulate $\frac{1}{k^+}$ poles.
 - Consider concrete example to show how to use PV prescription.

Naive light-cone gauge propagator

η^μ and $\tilde{\eta}^\mu$ are light-cone vector

$$\eta \cdot x = x^+ \quad \tilde{\eta} \cdot x = x^-$$

- light-cone gauge: $\eta \cdot A = A^+ = 0$
- Naive light-cone propagator

$$D^{\mu\nu}(x, y) \equiv \langle 0 | T A^\mu(x) A^\nu(y) | 0 \rangle$$

$$= \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+} \right]$$

Known light-cone pole prescription

θ -function sub-gauges

The classical field of point (color) charge moving along the $x^- = 0$ light cone is proportional to

- $A_\perp^\mu \sim \theta(-x^-)$

$$D_1^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ - i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ + i\epsilon} \right]$$

- $A_\perp^\mu \sim \theta(x^-)$

$$D_2^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ + i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ - i\epsilon} \right]$$

Slavnov and Frolov (1987); Kovchegov (1997); Belitsky, Ji, Yuan (2003).

Known light-cone pole prescription

Principal value (PV) sub-gauge

$$D_{PV}^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \left(k^\mu \eta^\nu + k^\nu \eta^\mu \right) \text{PV} \left\{ \frac{1}{k^+} \right\} \right]$$

$$\text{PV} \left\{ \frac{1}{k^+} \right\} \equiv \frac{1}{2} \left(\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon} \right)$$

Known light-cone pole prescription

Mandelstam–Leibbrandt (ML) prescription

$$D_{ML}^{\mu\nu}(x, y) = \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+ + i\epsilon k^-} \right]$$

S. Mandelstam (1983); G. Leibbrandt (1984).

- There are several possible choices to regulate light-cone (or temporal) gauge. Why don't we just use one of them?
- If we choose one of the available prescriptions for light-cone pole, how do we know whether quantization of our theory really allows that particular prescription we chose? We might get wrong result for the calculation at hand.
- In calculation at high-energy, **PV prescription simplify the calculation** reducing a lot the number of diagrams to be calculated.
- Moreover, In processes like two very energetic quarks off a large nucleus, quark lines reduce to Wilson line only if one uses PV prescription for light-cone poles.

- Suppose we choose PV prescription for light-cone gauge propagator poles, how do we actually use it when we have multiple light-cone poles? Do we use the same ϵ 's or they have to be different?
- Ambiguous case where the use of PV is not clear

$$\int \frac{d^2 k_\perp dk^+}{(2\pi)^3} \frac{d^2 l_\perp dl^+}{(2\pi)^3} e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-) + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{b}_{2\perp}) + i\vec{l}_\perp \cdot (\vec{b}_{2\perp} - \vec{b}_{1\perp})}$$
$$\times \frac{ig^3 f^{abc} t^a(t^b)_2(t^c)_1}{k_\perp^2 l_\perp^2 (\vec{k}_\perp - \vec{l}_\perp)^2} \left[\frac{\vec{l}_\perp \cdot (\vec{k}_\perp - \vec{l}_\perp) k_\perp^\mu (k^+ - 2l^+)}{k^+ l^+ (k^+ - l^+)} \right]$$

- If we choose PV prescription as an *ad hoc choice*, then for each pole we have to use different $i\epsilon$'s and the result will depend on the order we send the $i\epsilon$'s to zero.

Propagator in *Naive* Temporal Gauge

The propagator in the *naive* temporal gauge is obtained by setting $A_0 = 0$ in the Lagrangian

$$\langle 0 | e^{-iH(t_f - t_i)} | 0 \rangle$$

$$= \int \mathcal{D}\mathbf{A}^i \mathcal{D}\mathbf{A}^f \Psi_0(\mathbf{A}^i) \Psi_0^*(\mathbf{A}^f) \int_{\mathbf{A}^i, \mathbf{A}^f} \mathcal{D}\mathbf{A} e^{iS_0(A_0=0)} \Delta(\mathbf{A})$$

$$\Psi_0(\mathbf{A}) = e^{\frac{1}{2} \int d^3x \mathbf{A}_k W^{kj} \mathbf{A}_j}, \quad W^{ij} = \frac{\vec{\partial}^2 \delta^{ij} - \partial^i \partial^j}{\sqrt{-\vec{\partial}^2}}$$

Propagator in *Naive Temporal Gauge*

$$\square^{ik} = -\partial^2 \delta^{ik} - \partial^i \partial^k$$

$$D^{ij}(x, y) = \int d^4 k \frac{e^{-ik \cdot (x-y)}}{k^2 + i\epsilon} \left(\delta^{ij} - \frac{k^i k^j}{k_0^2} \right)$$

- This propagator does not satisfy Gauss law. Indeed sub-gauge conditions have been ignored.
- What is the correct prescription for the pole at $\frac{1}{k_0}$?

Including the sub-gauge condition

- Sub-gauge condition: $\partial_i A_i(\mathbf{x}, t_0) = 0$,
- t_0 is a generic point between t_i and t_f .

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$$\langle 0 | e^{-iH(t_f-t_i)} | 0 \rangle = \int \mathcal{D}\mathbf{A}^i \mathcal{D}\mathbf{A}^f \Psi_0(\mathbf{A}^i) \Psi_0^*(\mathbf{A}^f) \int_{\mathbf{A}^i, \mathbf{A}^f} \mathcal{D}\mathbf{A} e^{iS_0(A_0=0)} \Delta(\mathbf{A}) \delta(\partial_i A_i)$$

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$$\begin{aligned} &= \lim_{\beta \rightarrow 0} \int \mathcal{D}\mathbf{A}^i \mathcal{D}\mathbf{A}^f \exp \left\{ \frac{1}{2} \int d^3x \mathbf{A}_k^f (i\partial_{t_f} \delta^{kj} + W^{kj}) \mathbf{A}_j^f \right\} \\ &\quad \times \exp \left\{ -\frac{1}{2} \int d^3x \mathbf{A}_k^i (i\partial_{t_i} \delta^{kj} - W^{kj}) \mathbf{A}_j^i \right\} \\ &\quad \times \int_{\mathbf{A}^i, \mathbf{A}^f} \mathcal{D}\mathbf{A} \exp \left\{ \frac{i}{2} \int_{t_i}^{t_f} dt \int d^3x \mathbf{A}_i(x) \tilde{\square}^{ij} \mathbf{A}_j(x) \right\} \Delta(\mathbf{A}) \end{aligned}$$

where

$$\tilde{\square}^{ij} = -\partial^2 \delta^{ij} - \partial^i \partial^j - \frac{1}{\beta} \partial^i \partial^j \delta(x_0 - t_0)$$

Including the sub-gauge condition

$$\tilde{\square}^{ik} \equiv \square^{ik} - \frac{1}{\beta} \partial^i \partial^k \delta(x_0 - t_0)$$

$$\begin{aligned}\tilde{D}_{kj}(x, y; t_0, \beta) = & \int d^4 k \frac{e^{-ik \cdot (x-y)}}{k^2} \left(\delta^{ij} - \frac{k^i k^j}{k_0^2} \right) \\ & - \frac{1}{2} \int d^3 k e^{i\vec{k} \cdot (\vec{x}-\vec{y})} \frac{k^i k^j}{|\vec{k}|^2} \left(|x_0 - y_0| - |x_0 - t_0| - |y_0 - t_0| - \frac{2\beta}{|\vec{k}|^2} \right)\end{aligned}$$

- $\tilde{D}_{kj}(x, y; t_0, \beta)$ satisfies Gauss law: $\langle D_\mu F^{\mu i} \rangle = -J^i$.
- Boundary conditions will set the prescriptions for poles at $k^2 = 0$ and at $k_0 = 0$.

Propagator in functional method

In functional method the propagator is obtained by functional derivatives with respect to the sources

$$\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle = - \left[\frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta J^\nu(y)} e^{-\frac{1}{2} \int d^4x' d^4y' J^\alpha(x') D_{\alpha\beta}(x', y') J^\beta(y')} \right] \Big|_{J=0}$$

- To this end one has to properly fix the gauge and to know the condition at the boundaries.

Add the source \mathbf{J}_k and make the shift

$$\mathbf{A}_k \rightarrow \mathbf{A}_k - \int d^4y \left(\tilde{\square}_{xy}^{-1} \right)_{kj} \mathbf{J}_j$$

We get

$$\begin{aligned} & \int \mathcal{D}\mathbf{A}^i \mathcal{D}\mathbf{A}^f \Psi_0(\mathbf{A}^i) \Psi_0^*(\mathbf{A}^f) \int_{\mathbf{A}^i, \mathbf{A}^f} \mathcal{D}\mathbf{A} e^{iS_0(A_0=0) + i \int d^4x \mathbf{J}_k(x) \mathbf{A}(x)} \Delta(\mathbf{A}) \delta(\partial_i A_i) \\ &= \lim_{\xi \rightarrow 0} \int \mathcal{D}\mathbf{A}^i \mathcal{D}\mathbf{A}^f e^{\frac{1}{2} \int d^3x \mathbf{A}_k^f (i\partial_{tf} \delta^{kj} + W^{kj}) \mathbf{A}_j^f} e^{-\frac{1}{2} \int d^3x \mathbf{A}_k^i (i\partial_{ti} \delta^{kj} - W^{kj}) \mathbf{A}_j^i} \\ &\quad \times \int_{\mathbf{A}^i, \mathbf{A}^f} \mathcal{D}\mathbf{A} e^{\frac{i}{2} \int d^4x \mathbf{A}_i(x) \tilde{\square}^{ij} \mathbf{A}_j(x)} \Delta(\mathbf{A}) e^{\frac{i}{2} \int d^4x d^4y \mathbf{J}_i(x) \left(\tilde{\square}_{xy}^{-1} \right) \mathbf{J}_j(y)} \\ &= \lim_{\xi \rightarrow 0} Z_0[0] \Delta(\mathbf{A}) e^{\frac{i}{2} \int d^4x d^4y \mathbf{J}_i(x) \left(\tilde{\square}_{xy}^{-1} \right) \mathbf{J}_j(y)} \end{aligned}$$

Condition at the boundaries set the prescription

only if the linear terms in the sources cancel out

$$\frac{1}{2} \int d^4y d^3x \mathbf{A}_k^{fi}(\mathbf{x}) (i\partial_{x_0} \delta^{kj} + W^{kj}) [\tilde{\square}_{xy}^{-1}]_{ji} J_i(y) = 0$$

$$-\frac{1}{2} \int d^4y d^3x \mathbf{A}_k^{in}(\mathbf{x}) (i\partial_{t_i} \delta^{kj} - W^{kj}) [\tilde{\square}_{xy}^{-1}]_{ji} J_i(y) = 0$$

Linear terms in J_k cancel only if

$$\begin{aligned}\tilde{\square}_{xy}^{-1} = \tilde{D}_{kj}(x, y; t_0, \beta) &= \int d^4k \frac{e^{-ik \cdot (x-y)}}{k^2 + i\epsilon} \left(\delta^{ij} - \text{PV} \frac{k^i k^j}{k_0^2} \right) \\ &+ \frac{1}{2} \int d^3k e^{i\vec{k} \cdot (\vec{x}-\vec{y})} \frac{k^i k^j}{|\vec{k}|^2} \left(|x_0 - t_0| + |y_0 - t_0| + \frac{2\beta}{|\vec{k}|^2} \right)\end{aligned}$$

Now this propagator satisfies Gauss law and has proper prescription for all the poles.

Naive light-cone propagator

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$$\eta \cdot x = x^+ \quad \tilde{\eta} \cdot x = x^-$$

- light-cone gauge: $\eta \cdot A = A^+ = 0$
- Naive light-cone propagator

$$D^{\mu\nu}(x, y) \equiv \langle 0 | T A^\mu(x) A^\nu(y) | 0 \rangle$$

$$= \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+} \right]$$

- Sub-gauge conditions provide prescription for the light-cone pole $\frac{1}{k^+}$.
- Get sub-gauge conditions within functional integration method.

Functional integration method

$$\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle = - \left[\frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta J^\nu(y)} \left(\frac{Z[J]}{Z[0]} \right) \right] \Big|_{J=0}$$

- Gauge condition $A^+ = 0$
- Sub-gauge condition $\vec{\partial}_\perp \cdot \vec{A}_\perp(x) \delta(x^- - \sigma) = 0$

Functional integration method

$$Z[J] = \lim_{\xi_1, \xi_2 \rightarrow 0} \int \mathcal{D}A_i \mathcal{D}A_f \Psi_0(A_i) \Psi_0^*(A_f)$$
$$\times \int \mathcal{D}A_\mu \exp \left\{ i \int_{x_i^+}^{x_f^+} dx^+ \int dx^- d^2 x_\perp \left[\mathcal{L}_0(A) + \mathcal{L}_{fix}(A) + J_\mu A^\mu \right] \right\}$$
$$\begin{aligned} & A(x_i^+, x^-, \vec{x}_\perp) = A_i \\ & A(x_f^+, x^-, \vec{x}_\perp) = A_f \end{aligned}$$

with

$$\mathcal{L}_0(A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{2} (\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2} (\partial_\mu A_\nu)(\partial^\nu A^\mu)$$

and the gauge and sub-gauge fixing terms

$$\mathcal{L}_{fix}(A) = -\frac{1}{2\xi_1} A_\mu \eta^\mu \eta^\nu A_\nu - \frac{1}{2\xi_2} \left(\vec{\partial}_\perp \cdot \vec{A}_\perp \right)^2 \delta(x^- - \sigma)$$

Light-cone vacuum wave-function

- Vacuum wave function in temporal gauge

$$\Psi_0(A) = \exp \left\{ -\frac{1}{2} \int d^3x A^i \sqrt{-\vec{\nabla}^2} \left[\delta^{ij} - \frac{\partial^i \partial^j}{\vec{\nabla}^2} \right] A^j \right\}$$

Perform an infinite boost $\Rightarrow A^0 \rightarrow A^+ = 0$

- Vacuum wave function in light-cone gauge

$$\Psi_0(A) = \exp \left\{ \frac{1}{2} \int dx^- d^2x_\perp A^\mu \sqrt{-(\partial^+)^2} A_\mu \right\}$$

Making the shift

The goal is to get the propagator from functional derivative with respect to the source

$$\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle = - \left[\frac{\delta}{\delta J^\mu(x)} \frac{\delta}{\delta J^\nu(y)} e^{-\frac{1}{2} \int d^4x' d^4y' J^\alpha(x') D_{\alpha\beta}(x',y') J^\beta(y')} \right] \Big|_{J=0}$$

- Make shift $A^\mu \rightarrow A^\mu + a^\mu$
- We need to find the right a^μ such that the generating functional is in Gaussian form.

Making the shift

$$\begin{aligned} Z[J] = & \lim_{\xi_1, \xi_2 \rightarrow 0} \int \mathcal{D}A_i \mathcal{D}A_f \Psi_0(A_i) \Psi_0^*(A_f) \Psi_0(a_i) \Psi_0^*(a_f) \\ & \times \exp \left\{ \int dx^- d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2} a_{i\mu} + A_f^\mu \sqrt{-(\partial^+)^2} a_{f\mu} \right) \right\} \\ & \times \int \mathcal{D}A_\mu \exp \left\{ i \int_{x_i^+}^{x_f^+} dx^+ \int dx^- d^2x_\perp \left[\mathcal{L}_0(A) + \mathcal{L}_{fix}(A) + \mathcal{L}_0(a) + \mathcal{L}_{fix}(a) \right. \right. \\ & \left. \left. A(x_i^+, x^-, \vec{x}_\perp) = A_i \right. \right. \\ & \left. \left. A(x_f^+, x^-, \vec{x}_\perp) = A_f \right. \right. \\ & \quad + J^\mu A_\mu + J^\mu a_\mu + -(\partial_\mu A_\nu) (\partial^\mu a^\nu) + (\partial_\mu A_\nu) (\partial^\nu a^\mu) \\ & \quad \left. \left. - \frac{1}{\xi_1} A_\mu \eta^\mu \eta^\nu a_\nu - \frac{1}{\xi_2} (\vec{\partial}_\perp \cdot \vec{A}_\perp) (\vec{\partial}_\perp \cdot \vec{a}_\perp) \delta(x^- - \sigma) \right] \right\} \end{aligned}$$

Green function

Perform integration by parts and eliminate linear terms in A^μ we get

$$a^\mu(x) = i \int d^4y D^{\mu\nu}(x, y) J_\nu(y)$$

where $D^{\mu\nu}(x, y)$ is the Green function found from

$$\left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu - \frac{1}{\xi_1} \eta^\mu \eta^\nu + \frac{1}{\xi_2} \partial_\perp^\mu \partial_\perp^\nu \delta(x^- - \sigma) \right] D_{\nu\rho}(x, y) = i \delta_\rho^\mu \delta^{(4)}(x - y)$$

And to get the generating functional in Gaussian form with respect to the sources j^μ we have to satisfy

$$\begin{aligned} & \int dx^- d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2} a_{i\mu} + A_f^\mu \sqrt{-(\partial^+)^2} a_{f\mu} \right) \\ & - i \int d\sigma_\mu \left[A_\nu(\partial^\mu a^\nu) - A_\nu(\partial^\nu a^\mu) \right] = 0 \end{aligned}$$

Looking for the Green function $D_{\mu\nu}(x, y)$

For any $x^- \neq \sigma$ the general solution for

$$\left[\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu - \frac{1}{\xi_1} \eta^\mu \eta^\nu + \frac{1}{\xi_2} \partial_\perp^\mu \partial_\perp^\nu \delta(x^- - \sigma) \right] D_{\nu\rho}(x, y) = i \delta_\rho^\mu \delta^{(4)}(x - y)$$

is

$$D^{\mu\nu}(x, y)|_{x^- \neq \sigma} = \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+} \right]$$

For $x^- = \sigma$ we get condition to fix prescription for the poles $k^2 = 0$ and $k^+ = 0$

$$\partial_\mu^\perp \partial_\rho^\perp D^{\rho\nu}(x, y)|_{x^- = \sigma} = 0$$

prescription for $\frac{1}{k^2}$ pole

The condition

$$\int dx^- d^2x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2} a_{i\mu} + A_f^\mu \sqrt{-(\partial^+)^2} a_{f\mu} \right) - i \int d\sigma_\mu \left[A_\nu (\partial^\mu a^\nu) - A_\nu (\partial^\nu a^\mu) \right] = 0$$

Set the prescription for $\frac{1}{k^2} \rightarrow \frac{1}{k^2 + i\epsilon}$

Prescription for $\frac{1}{k^+}$ pole

- It turns out that condition $\partial_\mu^\perp \partial_\rho^\perp D^{\rho\nu}(x, y)|_{x^- = \sigma} = 0$ is satisfied only for $\sigma = +\infty$ or $\sigma = -\infty$
- Light-cone gauge gluon propagator for the sub-gauge condition
 $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$

$$D_1^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ - i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ + i\epsilon} \right];$$

- Light-cone gauge gluon propagator for the sub-gauge condition
 $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$

$$D_2^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ + i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ - i\epsilon} \right].$$

- Gauss law is satisfied: $\langle D_\mu F^{\mu+} \rangle = -J^+$

Obtaining the sub-gauge condition for PV prescription

We do not know the sub-gauge (boundary condition) for PV. Start with

$$D_{PV}^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - (k^\mu \eta^\nu + k^\nu \eta^\mu) \text{PV} \left\{ \frac{1}{k^+} \right\} \right]$$

$$a_{PV}^\mu = i \int d^4 y D_{PV}^{\mu\nu}(x, y) J_\nu(y)$$

condition to be satisfied is

$$\begin{aligned} & \int dx^- d^2 x_\perp \left(A_i^\mu \sqrt{-(\partial^+)^2} a_{i\mu}^{PV} + A_f^\mu \sqrt{-(\partial^+)^2} a_{f\mu}^{PV} \right) \\ & - i \int d\sigma_\mu \left[A_\nu (\partial^\mu a_{PV}^\nu) - A_\nu (\partial^\nu a_{PV}^\mu) \right] = 0 \end{aligned}$$

we get condition

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$$

- θ -function sub-gauges

$$\partial_\mu^\perp D_1^{\mu\nu}(x, y) \Big|_{x^- = +\infty} = 0$$

$$\partial_\mu^\perp D_2^{\mu\nu}(x, y) \Big|_{x^- = -\infty} = 0$$

- PV sub-gauge

$$\partial_\mu^\perp D_{PV}^{\mu\nu}(x, y) \Big|_{x^- = +\infty} + \partial_\mu^\perp D_{PV}^{\mu\nu}(x, y) \Big|_{x^- = -\infty} = 0$$

- Weaker sub-gauge condition: it exists

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$$

- Stronger sub-gauge condition: it does not exist in non-abelian gauge theory

$$\vec{A}_\perp(x^- = +\infty) + \vec{A}_\perp(x^- = -\infty) = 0$$

Mandelstam-Leibbrandt prescription

Repeat same strategy used for PV prescription get the following sub-gauge conditions

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$$

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$$

check is not satisfied

$$\partial_\mu^\perp D_{ML}^{\mu\nu}(x, y)|_{x^- = +\infty} = -\frac{1}{2\pi} \eta^\nu \delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp) \frac{1}{x^+ - y^+ - i\epsilon} \neq 0$$

$$\partial_\mu^\perp D_{ML}^{\mu\nu}(x, y)|_{x^- = -\infty} = -\frac{1}{2\pi} \eta^\nu \delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp) \frac{1}{x^+ - y^+ + i\epsilon} \neq 0$$

- We could not justify Mandelstam-Leibbrandt prescription through functional method.

Construction of the gauge-rotation matrix

Classical gluon field of two ultrarelativistic quarks on two parallel light-cones in covariant (Feynman) $\partial_\mu A^\mu = 0$ is

$$A_{cov}^{a+}(x^-, \vec{x}_\perp) = \frac{g}{2\pi} (t^a)_1 \delta(x^- - b_1^-) \ln \left(|\vec{x}_\perp - \vec{b}_{1\perp}| \Lambda \right) \\ + \frac{g}{2\pi} (t^a)_2 \delta(x^- - b_2^-) \ln \left(|\vec{x}_\perp - \vec{b}_{2\perp}| \Lambda \right)$$

$(b_1^-, \vec{b}_{1\perp})$ and $(b_2^-, \vec{b}_{2\perp})$ determine the quarks' light-cone trajectories,

Construction of the gauge-rotation matrix

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$(b_1^-, \vec{b}_{1\perp})$ and $(b_2^-, \vec{b}_{2\perp})$ determine the quarks' light-cone trajectories,

$$A_\mu^{LC} = S A_\mu^{cov} S^{-1} - \frac{i}{g} (\partial_\mu S) S^{-1}$$

$$A_{LC}^+ = 0 \quad \Rightarrow \quad \partial^+ S = -i g S A_{cov}^+$$

Construction of the gauge-rotation matrix

Solution of $\partial^+ S = -ig S A_{cov}^+$ can be constructed perturbatively.

- Stronger sub-gauge condition:

$$\vec{A}_\perp(x^- = +\infty) + \vec{A}_\perp(x^- = -\infty) = 0$$

Solution exists only in the abelian case (LO)

- Weaker sub-gauge condition:

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$$

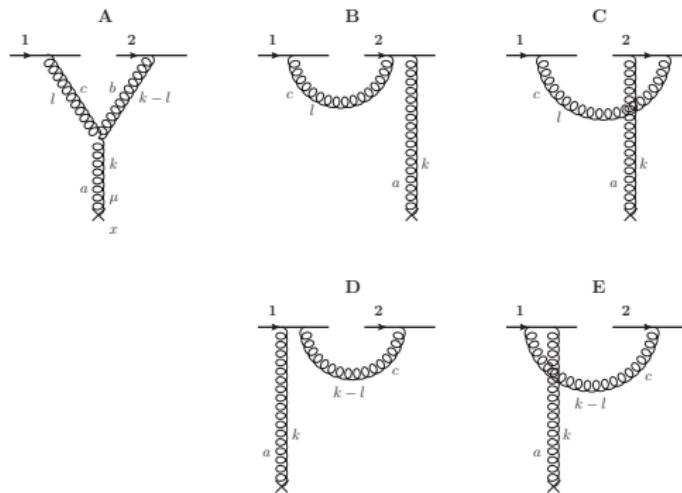
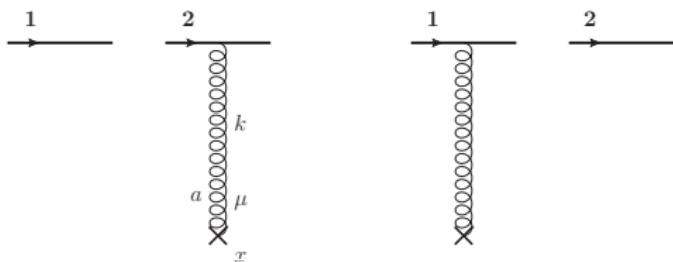
Solution exists also in non-abelian gauge theory

Classical field in the light-cone gauge

The classical field in the light-cone gauge and **weaker sub-gauge** condition is (only g^3 order)

$$\begin{aligned} \vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) \Big|_{g^3} = & \\ & - \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \text{Sign}(b_2^- - b_1^-) [\text{Sign}(x^- - b_2^-) + \text{Sign}(x^- - b_1^-)] \\ & \times \left[\frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln \left(|\vec{x}_\perp - \vec{b}_{2\perp}| \Lambda \right) + \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln \left(|\vec{x}_\perp - \vec{b}_{1\perp}| \Lambda \right) \right] \\ & + \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right] \ln \left(|\vec{b}_{1\perp} - \vec{b}_{2\perp}| \Lambda \right) \\ & - \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \text{Sign}(x^- - b_1^-) \text{Sign}(x^- - b_2^-) \\ & \times \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln \left(|\vec{x}_\perp - \vec{b}_{1\perp}| \Lambda \right) - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln \left(|\vec{x}_\perp - \vec{b}_{2\perp}| \Lambda \right) \right] + \mathcal{O}(g^5) \end{aligned}$$

Diagrammatic construction of the classical field



Diagrammatic construction of the classical field

$$\begin{aligned} \vec{A}_{\perp}^{LC}(x^-, \vec{x}_{\perp}) \Big|_{g^3} &= \int \frac{d^2 k_{\perp} dk^+}{(2\pi)^3} \frac{d^2 l_{\perp} dl^+}{(2\pi)^3} e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-) + i\vec{k}_{\perp} \cdot (\vec{x}_{\perp} - \vec{b}_{2\perp}) + i\vec{l}_{\perp} \cdot (\vec{b}_{2\perp} - \vec{b}_{1\perp})} \\ &\times \frac{ig^3 f^{abc} t^a(t^b)_2(t^c)_1}{k_{\perp}^2 l_{\perp}^2 (\vec{k}_{\perp} - \vec{l}_{\perp})^2} \left[\frac{-k_{\perp}^2 l_{\perp}^{\mu} + \vec{k}_{\perp} \cdot \vec{l}_{\perp} k_{\perp}^{\mu}}{l^+ (k^+ - l^+)} + \frac{\vec{l}_{\perp} \cdot (\vec{k}_{\perp} - \vec{l}_{\perp}) k_{\perp}^{\mu} (k^+ - 2l^+)}{k^+ l^+ (k^+ - l^+)} \right. \\ &\quad \left. + \frac{(\vec{k}_{\perp} - \vec{l}_{\perp})^2 k_{\perp}^{\mu}}{k^+ l^+} - \frac{l_{\perp}^2 k_{\perp}^{\mu}}{k^+ (k^+ - l^+)} \right] \end{aligned}$$

- All poles in $+$ components are regulated with PV.

Diagrammatic construction of the classical field

$$\begin{aligned} \vec{A}_{\perp}^{LC}(x^-, \vec{x}_{\perp}) \Big|_{g^3} &= \int \frac{d^2 k_{\perp} dk^+}{(2\pi)^3} \frac{d^2 l_{\perp} dl^+}{(2\pi)^3} e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-) + i\vec{k}_{\perp} \cdot (\vec{x}_{\perp} - \vec{b}_{2\perp}) + i\vec{l}_{\perp} \cdot (\vec{b}_{2\perp} - \vec{b}_{1\perp})} \\ &\times \frac{i g^3 f^{abc} t^a(t^b)_2(t^c)_1}{k_{\perp}^2 l_{\perp}^2 (\vec{k}_{\perp} - \vec{l}_{\perp})^2} \left[\frac{-k_{\perp}^2 l_{\perp}^{\mu} + \vec{k}_{\perp} \cdot \vec{l}_{\perp} k_{\perp}^{\mu}}{l^+ (k^+ - l^+)} + \frac{\vec{l}_{\perp} \cdot (\vec{k}_{\perp} - \vec{l}_{\perp}) k_{\perp}^{\mu} (k^+ - 2l^+)}{k^+ l^+ (k^+ - l^+)} \right. \\ &\quad \left. + \frac{(\vec{k}_{\perp} - \vec{l}_{\perp})^2 k_{\perp}^{\mu}}{k^+ l^+} - \frac{l_{\perp}^2 k_{\perp}^{\mu}}{k^+ (k^+ - l^+)} \right] \end{aligned}$$

- All poles in $+$ components are regulated with PV.
- The term in red contains pinched poles. If one uses PV with different $i\epsilon$'s for each pole then the result depends on the order one sends to zero the $i\epsilon$'s.
- Since we obtained PV prescription directly from the generating functional, all poles are regulated with the same $i\epsilon$'s and the result is unambiguous.

Classical field in the light-cone gauge

$$\begin{aligned} \vec{A}_\perp^{LC}(x^-, \vec{x}_\perp) \Big|_{g^3} = & - \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \text{Sign}(b_2^- - b_1^-) [\text{Sign}(x^- - b_2^-) + \text{Sign}(x^- - b_1^-)] \\ & \times \left[\frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln(|\vec{x}_\perp - \vec{b}_{2\perp}| \Lambda) + \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln(|\vec{x}_\perp - \vec{b}_{1\perp}| \Lambda) \right] \\ & + \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right] \ln(|\vec{b}_{1\perp} - \vec{b}_{2\perp}| \Lambda) \\ & - \frac{i}{8} \frac{g^3}{(2\pi)^2} [t^a(t^a)_1, t^b(t^b)_2] \text{Sign}(x^- - b_1^-) \text{Sign}(x^- - b_2^-) \\ & \times \left[\frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \ln(|\vec{x}_\perp - \vec{b}_{1\perp}| \Lambda) - \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \ln(|\vec{x}_\perp - \vec{b}_{2\perp}| \Lambda) \right] + \mathcal{O}(g^5) \end{aligned}$$

- Using PV-prescription with the same-i ϵ 's is equivalent to using the weak sub-gauge condition $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$ in the diagrammatic calculation of the classical field.

Conclusions

- Propagator in temporal gauge is obtained in Functional method.
- Prescriptions of the light-cone pole in light-cone gauge Gluon propagator is derived from functional method using sub-gauge conditions.
- Suitable sub-gauge condition for the PV prescription is the *Weak* condition

$$\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) + \vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$$

- The weak sub-gauge condition is confirmed from direct construction of the gauge transformation matrix which relate classical gluon field in covariant gauge to light-cone gauge.
- And it is consistent from direct diagrammatic calculation of the classical gluon field in light-cone gauge with PV prescription for light-cone poles.
- We could not reproduce Mandelstam–Leibbrandt prescription through functional integration.