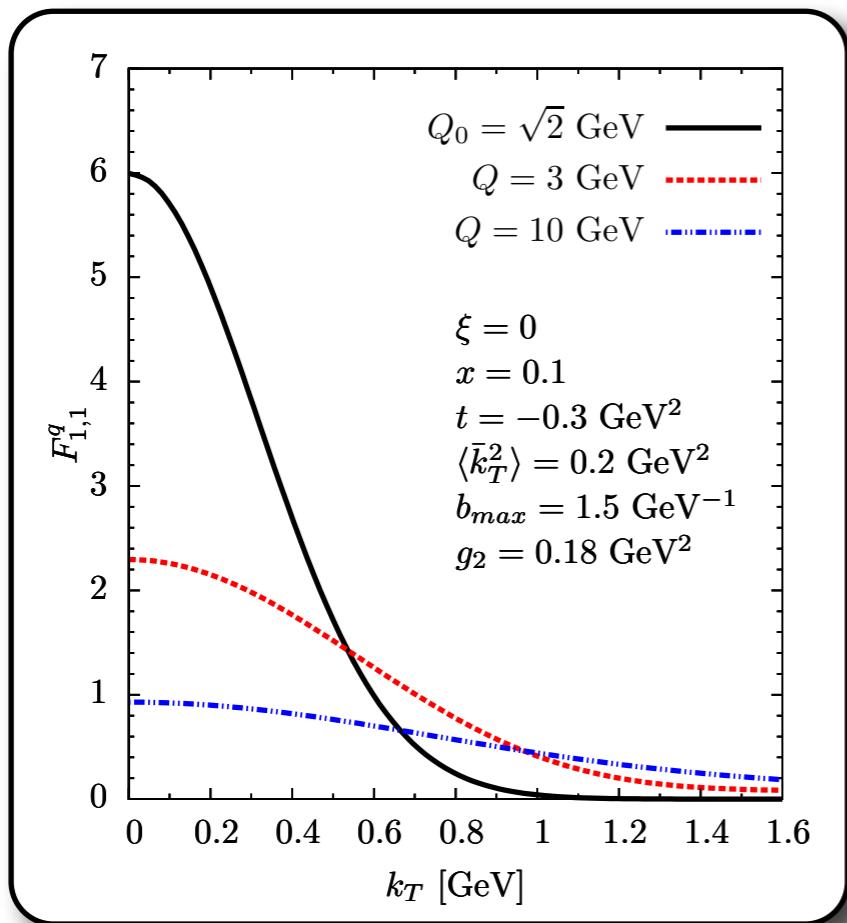


Definition and evolution of GTMDs

[MGE, Idilbi, Kanazawa, Lorcé, Metz, Pasquini, Schlegel 1602.06953, PLB]
and
[work in progress 1607.XXXX]



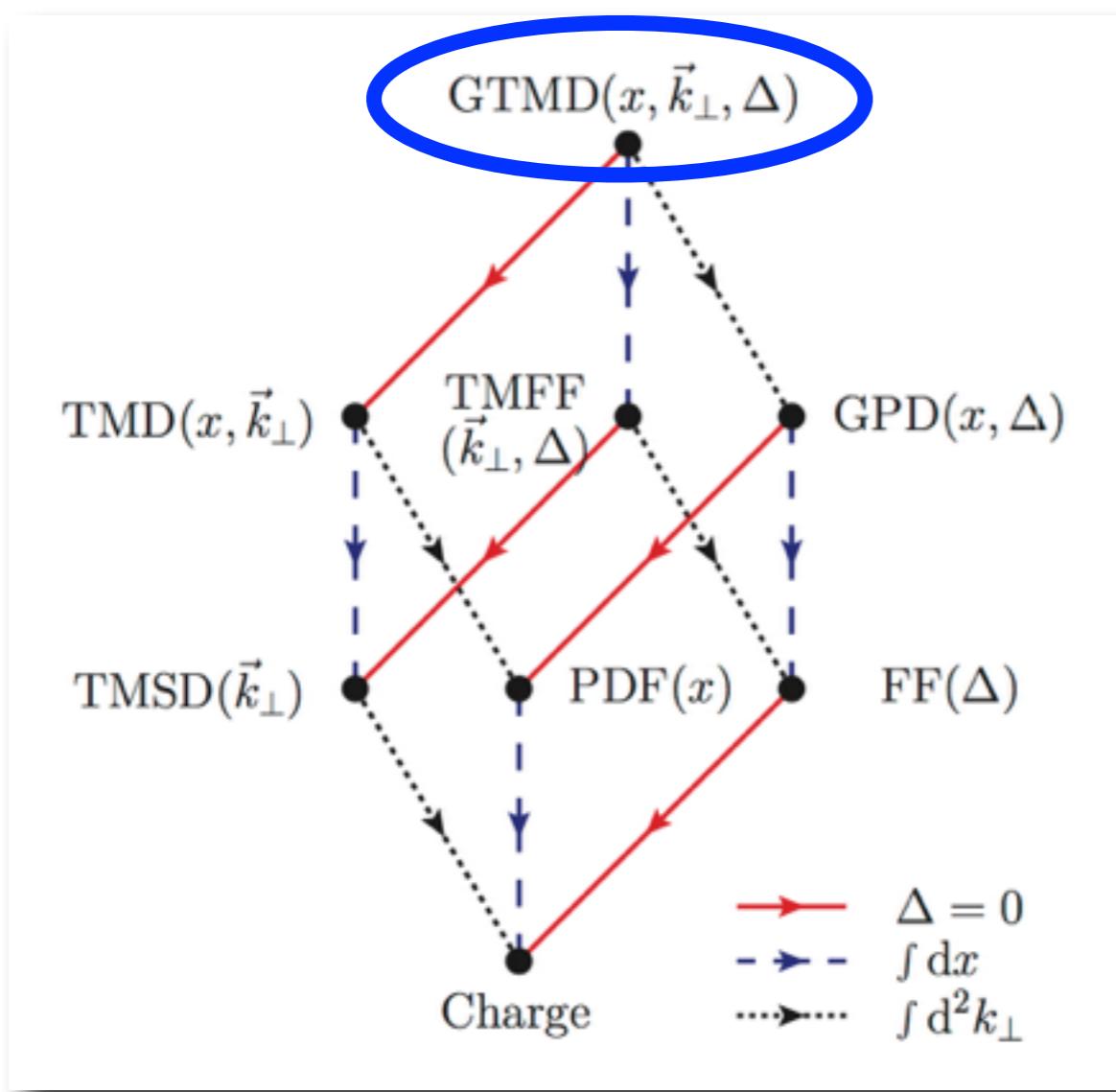
Miguel G. Echevarría



QCD Evolution 2016
May 30 - June 3
Nikhef (Amsterdam)

Motivation

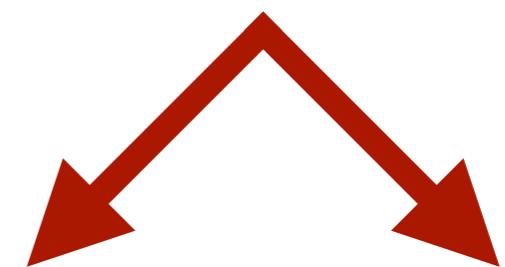
- GTMDs are the most fundamental hadronic objects (“mother functions”)
- Naive GTMDs have the same issues as naive TMDs: spurious rapidity divergences
- No clear connection with experiment yet
- Anyway we want a definition supported by pQCD (no need of factorization th.)



Not only a theoretical motivation!
(although that's enough)

2 phenomenological implications

Evolution



Collinear
expansion

[Lorce, Pasquini, Vanderhaeghen | 102.4704]

Outline

1. Naive GTMDs at NLO

- Definitions, kinematics, etc
- NLO results: problems...

2. Proper definition of GTMDs

- Soft factor
- NLO results: solution!

3. Evolution of GTMDs

- Evolution kernel at N^3LL'
- Illustration: F_{11}

4. Collinear expansion of GTMDs

- F_{11} onto H at NLO

Preliminary

5. Conclusions & Outlook

Naive GTMDs: definition

$$\phi_{\lambda\lambda'}^{[\Gamma],q} = \langle p', \lambda' | \bar{q}(-z/2) \mathcal{W}_n(-z/2) \frac{\Gamma}{2} \mathcal{W}_n^\dagger(z/2) q(z/2) | p, \lambda \rangle \Big|_{z^+=0}$$

$$\mathcal{W}_{n;\alpha\beta}(z) = \left\{ \mathcal{P} \exp \left[-ig \int_0^\infty ds \bar{n} \cdot A(z + s\bar{n}) \right] \right\}_{\alpha\beta}$$

DIS kinematics

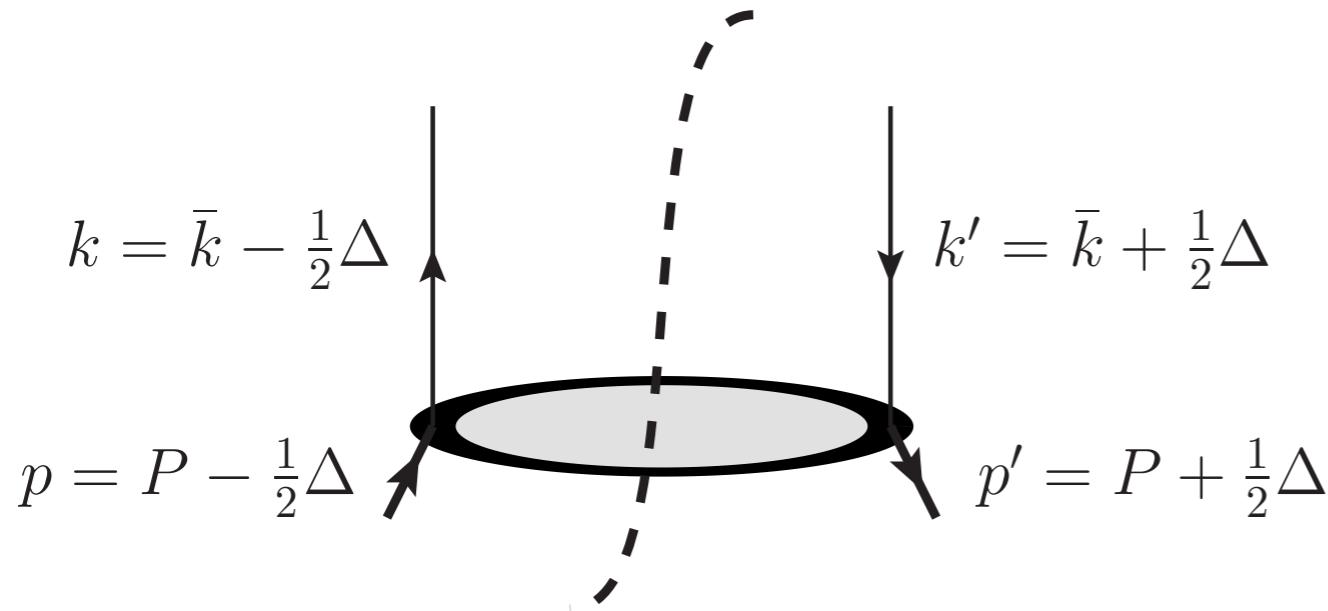
- To make it gauge invariant one also needs transverse gauge links
- This reduces to the naive/unsubtracted TMDPDF in the forward limit ($p=p'$)

$$W_{\lambda\lambda'}^{[\Gamma],q}(x, \xi, \bar{k}_T^2, \Delta_T^2, \bar{\boldsymbol{k}}_\perp \cdot \Delta_\perp) \Big|_{naive} = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{+i(\frac{1}{2}z^- \bar{k}^+ - \boldsymbol{z}_\perp \cdot \bar{\boldsymbol{k}}_\perp)} \phi_{\lambda\lambda'}^{[\Gamma],q}(0, z^-, \boldsymbol{z}_\perp)$$

Naive GTMDs: kinematics

$$\phi_{\lambda\lambda'}^{[\Gamma],q} = \langle p', \lambda' | \bar{q}(-z/2) \mathcal{W}_n(-z/2) \frac{\Gamma}{2} \mathcal{W}_n^\dagger(z/2) q(z/2) | p, \lambda \rangle \Big|_{z^+=0}$$

- We choose the *symmetric frame*



$$P^\mu = (P^+, P^-, \mathbf{0}_\perp)$$
$$\Delta^\mu = (-2\xi P^+, 2\xi P^-, \Delta_\perp)$$
$$P^- = \frac{\Delta_T^2 + 4M^2}{4(1-\xi^2)P^+}$$

- We will calculate the GTMD for an unpolarized quark (in the DGLAP region $|\xi| \leq x$)
- However this story applies as well to all (un)polarized quark/gluon GTMDs

Naive GTMDs: properties

$$W_{\Lambda'\Lambda}^{[\gamma^+]} = \frac{1}{2m} \bar{u}(p', \Lambda') \left[F_{11} + \frac{i\sigma^{j+} k_\perp^j}{P^+} F_{12} + \frac{i\sigma^{j+} \Delta_\perp^j}{P^+} F_{13} + \frac{i\sigma^{jl} k_\perp^j \Delta_\perp^l}{m^2} F_{14} \right] u(p, \Lambda)$$

$$W_{\Lambda'\Lambda}^{[\gamma^+ \gamma_5]} = \frac{1}{2m} \bar{u}(p', \Lambda') \left[-\frac{i\epsilon_T^{jl} k_\perp^j \Delta_\perp^l}{m^2} G_{11} + \frac{i\sigma^{j+} \gamma_5 k_\perp^j}{P^+} G_{12} + \frac{i\sigma^{j+} \gamma_5 \Delta_\perp^j}{P^+} G_{13} + i\sigma^{+-} \gamma_5 G_{14} \right] u(p, \Lambda)$$

$$W_{\Lambda'\Lambda}^{[i\sigma^{j+} \gamma_5]} = \frac{1}{2m} \bar{u}(p', \Lambda') \left[-\frac{i\epsilon_T^{jl} k_\perp^l}{m} H_{11} - \frac{i\epsilon_T^{jl} \Delta_\perp^l}{m} H_{12} + \frac{m i\sigma^{j+} \gamma_5}{P^+} H_{13} + \frac{k_\perp^j i\sigma^{l+} \gamma_5 k_\perp^l}{m P^+} H_{14} \right. \\ \left. + \frac{\Delta_\perp^j i\sigma^{l+} \gamma_5 k_\perp^l}{m P^+} H_{15} + \frac{\Delta_\perp^j i\sigma^{l+} \gamma_5 \Delta_\perp^l}{m P^+} H_{16} + \frac{k_\perp^j i\sigma^{+-} \gamma_5}{m} H_{17} + \frac{\Delta_\perp^j i\sigma^{+-} \gamma_5}{m} H_{18} \right] u(p, \Lambda)$$

[Meissner, Metz, Schlegel 0906.5323]

$$W_{\lambda\lambda'}^{[\Gamma],q}(P, \bar{k}, \Delta, n; \eta) = \left[W_{\lambda'\lambda}^{[\gamma^0 \Gamma^\dagger \gamma^0],q}(P, \bar{k}, -\Delta, n; \eta) \right]^* \quad \text{hermiticity}$$

$$= W_{\lambda_P \lambda'_P}^{[\gamma^0 \Gamma \gamma^0],q}(P_P, \bar{k}_P, \Delta_P, n_P; \eta) \quad \text{parity}$$

$$= \left[W_{\lambda_T \lambda'_T}^{[(i\gamma_5 C)\Gamma^*(i\gamma_5 C)],q}(P_T, \bar{k}_T, \Delta_T, n_T; -\eta) \right]^* \quad \text{time-reversal}$$

$$\eta = \pm 1 \begin{cases} \nearrow DIS \\ \searrow DY \end{cases}$$

[Meissner, Metz, Schlegel 0906.5323]
 [Lorcé, Pasquini 1307.4497]
 [Lorcé, Pasquini 1512.06744]

Naive GTMDs in p QCD

- We use the following regulator:

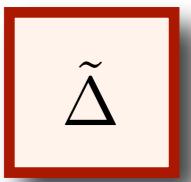
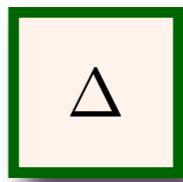
$$\frac{i(\not{p} + \not{k})}{(p+k)^2 + i\tilde{\Delta}} \rightarrow \frac{1}{k^- + i\delta}, \quad \delta = \frac{\tilde{\Delta}}{p^+}$$

+

Dimensional regularization for UV

[MGE, Idilbi, Scimemi 1111.4996]

- It consists just in keeping the “*epsilons*” of the propagators finite
- We send them to zero unless they regulate some divergence
- Not confuse *transfer* with *regulator*!



- A convenient upgrade of the regulator for higher order calculations: ➔ Alexey's talk

[MGE, Scimemi, Vladimirov '16]

Physics is regulator IN-dependent!!

Naive GTMD at LO

- We decompose the GTMD in four independent structures:

$$W_{\lambda\lambda'}^{[\gamma^+],q} = F_{1,1}^q \Gamma_1 + F_{1,2}^q \Gamma_2 + F_{1,3}^q \Gamma_3 + F_{1,4}^q \Gamma_4$$

[Meissner, Metz, Schlegel 0906.5323]

$$\Gamma_1 = \frac{1}{2M} \bar{u}(p', \lambda') u(p, \lambda),$$

$$\Gamma_3 = \frac{1}{2M} \frac{\Delta_\perp^i}{P^+} \bar{u}(p', \lambda') i\sigma^{i+} u(p, \lambda),$$

$$\Gamma_2 = \frac{1}{2M} \frac{\bar{k}_\perp^i}{P^+} \bar{u}(p', \lambda') i\sigma^{i+} u(p, \lambda),$$

$$\Gamma_4 = \frac{1}{2M} \frac{\bar{k}_\perp^i \Delta_\perp^j}{M^2} \bar{u}(p', \lambda') i\sigma^{ij} u(p, \lambda)$$

Gordon identities

$$\bar{u}' \gamma^\mu u = \bar{u}' \left[\frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} \right] u$$

$$0 = \bar{u}' \left[\frac{\Delta^\mu}{2M} + \frac{i\sigma^{\mu\nu} P_\nu}{M} \right] u$$

$$\bar{u}' \gamma^+ u = 2P^+ (1 - \xi^2) \Gamma_1 + P^+ \Gamma_3$$

$$\bar{u}' \gamma^- u = \left(\frac{-\Delta_T^2 + 4M^2}{2P^+} \right) \Gamma_1 - \left(\frac{\Delta_T^2 + 4M^2}{4(1 - \xi^2)P^+} \right) \Gamma_3$$

$$\bar{k}_T^i \bar{u}' \gamma_\perp^i u = \xi \bar{k}_T^i \Delta_T^i \Gamma_1 + \xi \left(\frac{\Delta_T^2 + 4M^2}{2(1 - \xi^2)} \right) \Gamma_2 - M^2 \Gamma_4$$

Naive GTMD at LO

- At LO we get:

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\text{LO}} = \frac{1}{2P^+} \delta(1-x) \delta^{(2)}(\bar{\mathbf{k}}_\perp) \bar{u}(p', \lambda') \gamma^+ u(p, \lambda)$$

- This result trivially reduces to the unpolarized TMDPDF in the forward limit and after taking the average over spins:

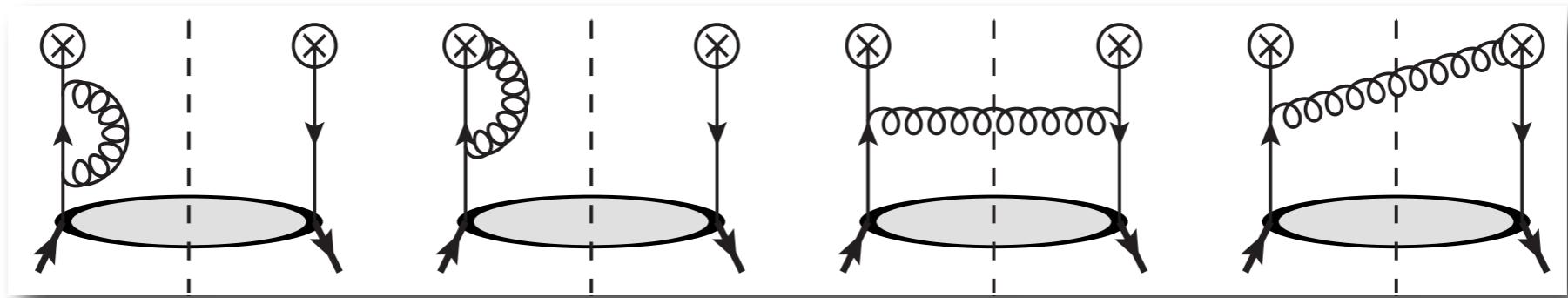
$$f_1^q \Big|_{\text{LO}} = \frac{1}{2p^+} \delta(1-x) \delta^{(2)}(\bar{\mathbf{k}}_\perp) \frac{1}{2} \sum_{\lambda} \bar{u}(p, \lambda) \gamma^+ u(p, \lambda) = \delta(1-x) \delta^{(2)}(\bar{\mathbf{k}}_\perp)$$

- We can decompose it:

$$\bar{u}' \gamma^+ u = 2P^+ (1 - \xi^2) \Gamma_1 + P^+ \Gamma_3$$

$$\begin{aligned} F_{1,1}^q &= (1 - \xi^2) \delta(1-x) \delta^{(2)}(\bar{\mathbf{k}}_\perp) + \mathcal{O}(\alpha_s) \\ F_{1,2}^q &= \mathcal{O}(\alpha_s) \\ F_{1,3}^q &= \frac{1}{2} \delta(1-x) \delta^{(2)}(\bar{\mathbf{k}}_\perp) + \mathcal{O}(\alpha_s) \\ F_{1,4}^q &= \mathcal{O}(\alpha_s) \end{aligned}$$

Naive GTMD at NLO: virtual part



$$\begin{aligned}
 W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\phi \text{ virtual}} &= \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[\frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} + \frac{3}{2\varepsilon_{\text{UV}}} \right. \\
 &\quad \left. - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - 2 \ln \frac{\tilde{\Delta}}{\mu^2} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1+\xi)} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1-\xi)} + \frac{7}{4} + \frac{5}{12} \pi^2 + i\pi \ln \frac{1-\xi}{1+\xi} \right]
 \end{aligned}$$

- **Spurious rapidity divergences** are present: no anomalous dimension!!
- The logarithm in the mixed UV-rapidity term is fixed: p^+ and p'^+ come from adding left and right virtual diagrams, and we have a single logarithm.
- Double logs will be cancelled by combining this result with real-gluon emission piece
- **Imaginary** piece allowed since GTMDs are complex. Consistent with hermiticity.

$$p^+ = P^+(1 - \xi)$$

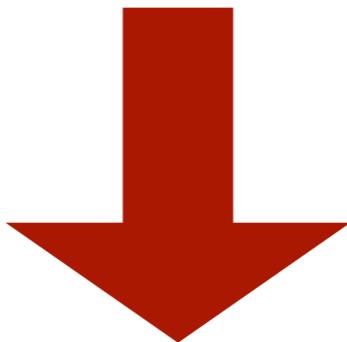
$$p'^+ = P^+(1 + \xi)$$

Naive GTMD at NLO: virtual part

- This result reduces to the TMDPDF (in DIS kinematics) in the forward limit:

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\phi \text{ virtual}} = \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[\frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} + \frac{3}{2\varepsilon_{\text{UV}}} \right. \\ \left. - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - 2 \ln \frac{\tilde{\Delta}}{\mu^2} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1+\xi)} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1-\xi)} + \frac{7}{4} + \frac{5}{12} \pi^2 + i\pi \ln \frac{1-\xi}{1+\xi} \right]$$

$p=p'$
Spin average



$$\delta = \frac{\tilde{\Delta}}{Q}, \quad \xi = 0$$

$$f_1^q \Big|_{\text{virtual}}^{\text{naive}} = \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[\frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\tilde{\Delta}}{Q^2} + \frac{3}{2\varepsilon_{\text{UV}}} \right. \\ \left. - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - 2 \ln \frac{\tilde{\Delta}}{\mu^2} \ln \frac{\tilde{\Delta}}{Q^2} - \ln^2 \frac{\tilde{\Delta}}{Q^2} + \frac{7}{4} + \frac{5}{12} \pi^2 \right]$$

[MGE, Idilbi, Scimemi 1111.4996]

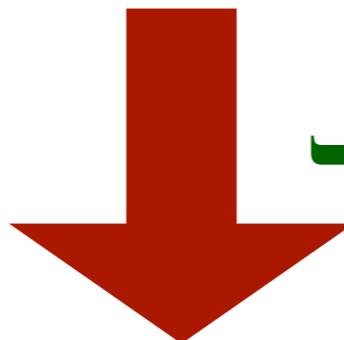
GTMDS: proper definition

- By the inclusion of the soft function we properly define the GTMDs:

$$\phi_{\lambda\lambda'}^{[\Gamma],q} = \langle p', \lambda' | \bar{q}(-z/2) \mathcal{W}_n(-z/2) \frac{\Gamma}{2} \mathcal{W}_n^\dagger(z/2) q(z/2) | p, \lambda \rangle \Big|_{z^+=0}$$

Naive definition

➡ Ahmad's talk



$$W_{\lambda\lambda'}^{[\Gamma],q} = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(\frac{1}{2}z^- \bar{k}^+ - \mathbf{z}_\perp \cdot \bar{\mathbf{k}}_\perp)} \phi_{\lambda\lambda'}^{[\Gamma],q}(0, z^-, \mathbf{z}_\perp) \sqrt{S(z_T)}$$

New proper definition

- This applies as well to **gluon** GTMDs

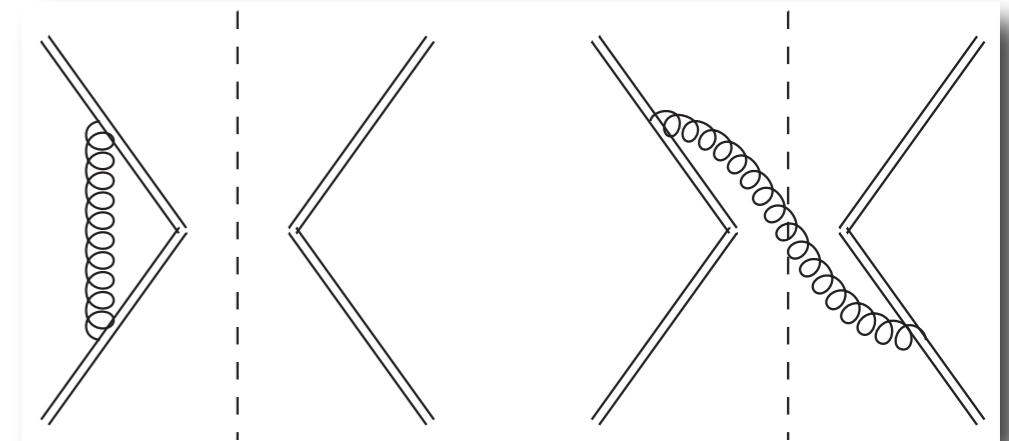
GTMDS: proper definition

- The soft function is defined as:

$$S(z_T) = \frac{\text{Tr}_c}{N_c} \langle 0 | \mathcal{S}_n^\dagger \left(-\frac{z}{2} \right) \mathcal{S}_{\bar{n}} \left(-\frac{z}{2} \right) \mathcal{S}_{\bar{n}}^\dagger \left(\frac{z}{2} \right) \mathcal{S}_n \left(\frac{z}{2} \right) | 0 \rangle \Big|_{z^\pm=0}$$

$$\mathcal{S}_{n;\alpha\beta}(z) = \left\{ \mathcal{P} \exp \left[ig \int_{-\infty}^0 ds \ n \cdot A(z + sn) \right] \right\}_{\alpha\beta}$$

$$\mathcal{S}_{\bar{n};\alpha\beta}(z) = \left\{ \mathcal{P} \exp \left[-ig \int_0^\infty ds \ \bar{n} \cdot A(z + s\bar{n}) \right] \right\}_{\alpha\beta}$$



DIS kinematics

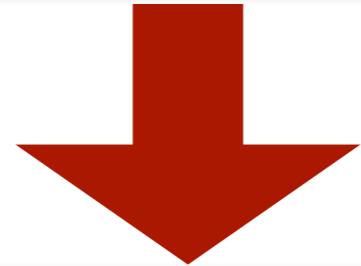
$$\tilde{S}(b_T; \mu; \delta) = 1 + \frac{\alpha_s C_A}{2\pi} \left[-\frac{2}{\varepsilon_{\text{UV}}^2} + \frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta^2}{\mu^2} + L_T^2 + 2L_T \ln \frac{\delta^2}{\mu^2} + \frac{\pi^2}{6} \right]$$

- Exactly the same soft function as for quark TMDs

GTMD at NLO: results (virtual part)

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\phi \text{ virtual}} = \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[\frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} + \frac{3}{2\varepsilon_{\text{UV}}} \right. \\ \left. - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - 2 \ln \frac{\tilde{\Delta}}{\mu^2} \ln \frac{\delta}{P^+ \sqrt{1-\xi^2}} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1+\xi)} - \frac{1}{2} \ln^2 \frac{\delta}{P^+(1-\xi)} + \frac{7}{4} + \frac{5}{12} \pi^2 + i\pi \ln \frac{1-\xi}{1+\xi} \right]$$

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{S \text{ virtual}} = \frac{1}{2} \left\{ \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[-\frac{2}{\varepsilon_{\text{UV}}^2} + \frac{2}{\varepsilon_{\text{UV}}} \ln \frac{\delta^2}{\mu^2} - \ln^2 \frac{\delta^2}{\mu^2} + \frac{\pi^2}{2} \right] \right\}$$



$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\text{virtual}} = W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\phi \text{ virtual}} - W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{S \text{ virtual}}$$

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\text{virtual}} = \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \delta(1-x) \delta^{(2)}(\bar{k}_\perp) \frac{\alpha_s C_F}{2\pi} \left[\frac{1}{\varepsilon_{\text{UV}}^2} + \frac{1}{\varepsilon_{\text{UV}}} \left(\frac{3}{2} + \ln \frac{\mu^2}{Q^2(1-\xi^2)} \right) \right. \\ \left. - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - \frac{1}{2} \ln^2 \frac{\tilde{\Delta}^2}{\mu^2 Q^2} + \ln^2 \frac{\tilde{\Delta}^2}{\mu^2} + \ln \frac{\tilde{\Delta}^2}{\mu^2 Q^2} \ln(1-\xi^2) - \frac{1}{2} \ln^2(1+\xi) - \frac{1}{2} \ln^2(1-\xi) + \frac{7}{4} + \frac{\pi^2}{6} + i\pi \ln \frac{1-\xi}{1+\xi} \right]$$

- Evolution is ok now!

GTMD at NLO: results (real part)

- Following the same procedure now for real gluon emission diagrams:

$$W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\text{real}} = W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\phi \text{ real}} - W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{S \text{ real}}$$

$$\begin{aligned} W_{\lambda\lambda'}^{[\gamma^+],q} \Big|_{\text{real}} &= \frac{1}{2P^+} \bar{u}(p', \lambda') \gamma^+ u(p, \lambda) \frac{\alpha_s C_F}{2\pi^2} \left[\frac{N_1}{D_+ D_-} - \delta(1-x) \frac{1}{\bar{k}_T^2} \ln \frac{\bar{k}_T^2}{Q^2} \right] \\ &\quad + P^+ \bar{u}(p', \lambda') \gamma^- u(p, \lambda) \frac{\alpha_s C_F}{2\pi^2} \left[\frac{N_2}{D_+ D_-} \right] \\ &\quad + \bar{k}_T^i \bar{u}(p', \lambda') \gamma_T^i u(p, \lambda) \frac{\alpha_s C_F}{2\pi^2} \left[\frac{N_3}{D_+ D_-} \right] \end{aligned}$$

- This is calculated in the DGLAP region $|\xi| \leq x$
- Deltas are dropped here, since k_T is finite.
- Deltas should be kept to properly regularize remaining $k_T=0$ divergences
- Reduces to the real part of TMDPDF in the forward limit (DIS kinematics)

GTMD at NLO: results (complete)

- Combining virtual and real and decomposing the result into the basic structures:

$$\begin{aligned}
F_{1,1}^q(|\xi| < x) = & (1 - \xi^2) \delta(1 - x) \delta^{(2)}(\bar{k}_\perp) \left\{ 1 + \frac{\alpha_s C_F}{2\pi} \left[\frac{1}{\varepsilon_{\text{UV}}^2} + \frac{1}{\varepsilon_{\text{UV}}} \left(\frac{3}{2} + \ln \frac{\mu^2}{Q^2(1 - \xi^2)} \right) \right. \right. \\
& - \frac{3}{2} \ln \frac{\tilde{\Delta}}{\mu^2} - \frac{1}{2} \ln^2 \frac{\tilde{\Delta}^2}{\mu^2 Q^2} + \ln^2 \frac{\tilde{\Delta}}{\mu^2} + \ln \frac{\tilde{\Delta}^2}{\mu^2 Q^2} \ln(1 - \xi^2) - \frac{1}{2} \ln^2(1 + \xi) - \frac{1}{2} \ln^2(1 - \xi) + \frac{7}{4} + \frac{\pi^2}{6} + i\pi \ln \frac{1 - \xi}{1 + \xi} \left. \right] \right\} \\
& + \frac{\alpha_s C_F}{2\pi^2} \left[\frac{(1 - \xi^2)N_1 - \frac{\Delta_T^2}{2}N_2 + \xi \bar{k}_\perp \cdot \Delta_\perp N_3}{D_+ D_-} - (1 - \xi^2) \delta(1 - x) \frac{1}{\bar{k}_T^2} \ln \frac{\bar{k}_T^2}{Q^2} \right]
\end{aligned}$$

$$\begin{aligned}
N_1 &= \frac{1}{(1 - \xi^2)(1 - x)_+} \left[((1 + x^2) - 2\xi^2) \bar{k}_T^2 + 2\xi(1 - x) \bar{k}_\perp \cdot \Delta_\perp - (1 - x)^2 \frac{\Delta_T^2}{2} \right], \\
N_2 &= -\frac{(1 - x)^2}{2(1 - \xi^2)} (1 + x) , \quad N_3 = -\frac{1 - x}{1 - \xi^2} (1 + x) , \quad D_\pm = \left(\bar{k}_\perp \pm \frac{1 - x}{1 \mp \xi} \frac{\Delta_\perp}{2} \right)^2
\end{aligned}$$

- Double logs are cancelled if deltas are properly kept in real part
- So all rapidity divergences cancel and the remaining delta dependence is taken care of by confinement

QCD evolution of GTMDs at N^3LL'

- *The evolution of the GTMDs w.r.t the renormalization scale is given by:*

$$\frac{d}{d\ln\mu} \ln \tilde{W}^j(b_T; \mu, Q^2) = \gamma_W^j \left(\alpha_s(\mu), \ln \frac{Q^2(1 - \xi^2)}{\mu^2} \right)$$

$$\gamma_W^j \left(\alpha_s(\mu), \ln \frac{Q^2(1 - \xi^2)}{\mu^2} \right) = -\Gamma_{\text{cusp}}^j(\alpha_s(\mu)) \ln \frac{Q^2(1 - \xi^2)}{\mu^2} - \gamma^j(\alpha_s(\mu))$$

Known at 3-loops!!

[Moch, Vermaseren, Vogt 0507039, 0403192]

- *The evolution of the GTMDs w.r.t the rapidity scale is given by:*

$$\frac{d}{d\ln Q^2} \ln \tilde{W}^j(b_T; \mu, Q^2) = -D^j(b_T; \mu)$$

[Li, Zhu 1604.01404]

$$\frac{dD^j}{d\ln\mu} = \Gamma_{\text{cusp}}^j(\alpha_s(\mu))$$

Known at NLO. Recently at NNLO

Indirect: [Becher, Neubert 1007.4005]
Direct: [MGE, Scimemi, Vladimirov 1511.05590]

QCD evolution of GTMDs at N^3LL'

- Combining both *renormalization* and *rapidity scales* evolutions:

$$\tilde{W}^j(b_T; \mu, Q^2) = R^j(\xi, b_T; \mu, Q^2, \mu_0, Q_0^2) \tilde{W}^j(b_T; \mu_0, Q_0^2)$$

$$R^j(\xi, b_T; \mu, Q^2, \mu_0, Q_0^2) = \left(\frac{Q^2}{Q_0^2} \right)^{-D^j(b_T; \mu_0)} \exp \left[\int_{\mu_0}^{\mu} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_W^j \left(\alpha_s(\hat{\mu}), \ln \frac{Q^2(1 - \xi^2)}{\hat{\mu}^2} \right) \right]$$

- D^j is the same as for quark/gluon TMDs (i.e. also the same non-perturbative piece!!)
- Evolution is **universal** among all (un)polarized quark/gluon GTMDs
- Their evolution is **identical** to the one of the TMDs
- Since evolution comes from virtual diagrams, it applies for all ξ

Example: evolution of $F_{I,I}$

- We illustrate the evolution with $F_{I,I}$ by taking a simple model:

$$F_{1,1}^q(x, \xi = 0, \bar{k}_T^2, \Delta_T^2, \bar{\mathbf{k}}_\perp \cdot \Delta_\perp; Q_0) = H^q(x, \xi = 0, t = -\Delta_T^2; Q_0) \frac{e^{-\bar{k}_T^2/\langle \bar{k}_T^2 \rangle}}{\pi \langle \bar{k}_T^2 \rangle}$$

- GPD is matched onto PDF:

$$H^q(x, \xi = 0, t; Q_0) = f_1^q(x; Q_0) \exp [\lambda t]$$

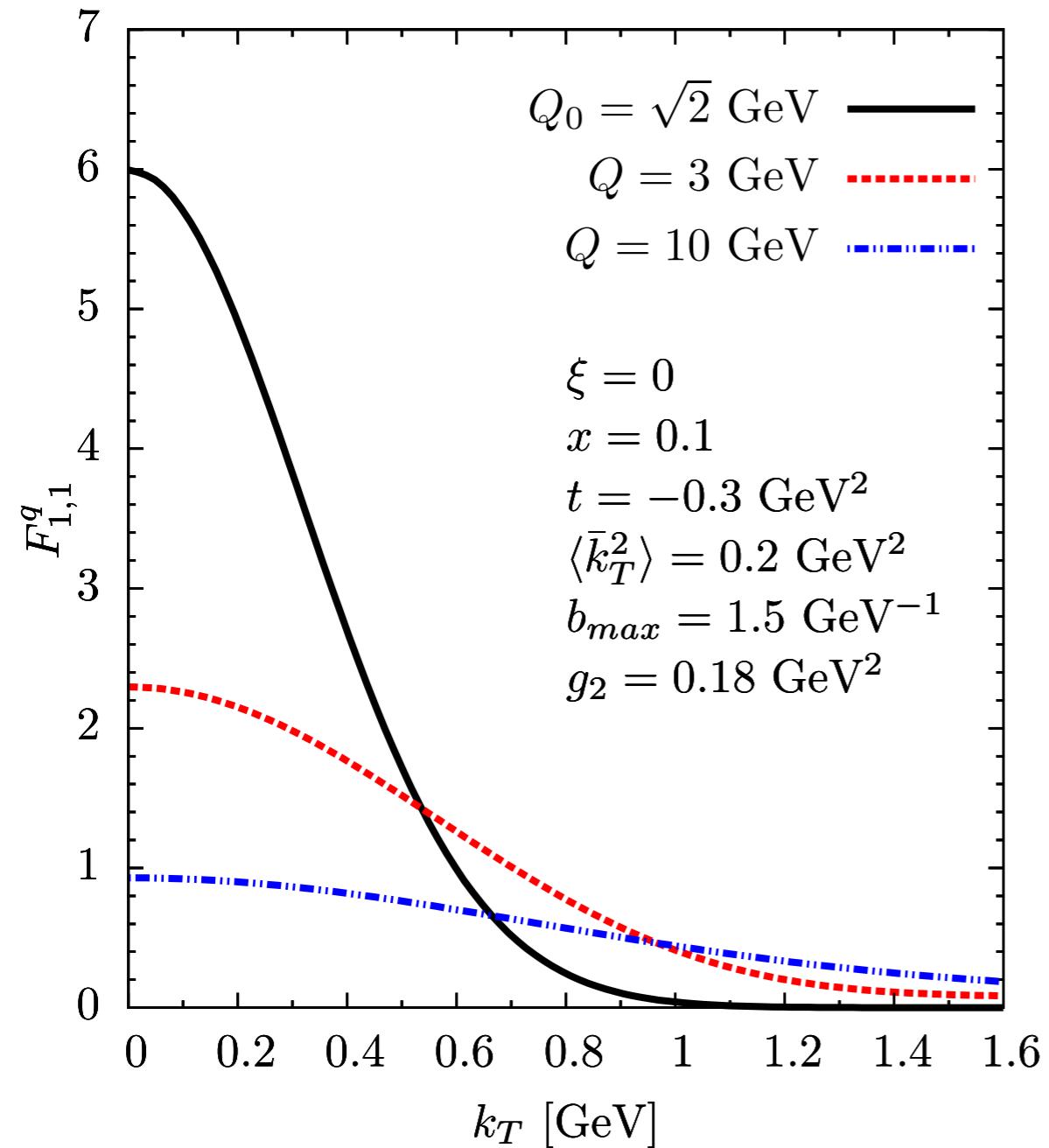
$$D^q(b_T; Q_0) = D^q(b_T^*; \mu_b) + \int_{\mu_b}^{Q_0} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{\text{cusp}}^q + \frac{1}{4} g_2 b_T^2$$

$$b_T^* = b_T \left[1 + \left(\frac{b_T}{b_{\max}} \right)^2 \right]^{-\frac{1}{2}}$$

$$\mu_b = 2e^{-\gamma_E} / b_T^*$$

- b^* prescription
- Non-perturbative input the same as for TMDs

Example: evolution of $F_{1,I}$



- NNLL evolution kernel
- Non-perturbative parameters from [Konychev-Nadolsky 0506225]
- Distribution flattens and widens, as for TMDs
- Evolution is **UNIVERSAL** among all (un)polarized quark/gluon GTMDs, and we **already know** the ingredients for **N³LL'** resummation for **both quark and gluon GTMDs**

GTMDs: Operator Product Expansion

Preliminary

- The relation between GTMDs and GPDs is through an OPE:

$$\tilde{W}_{i/H}^{\text{ren}}(x, \xi, b_T^2, \Delta_T^2, b_T \cdot \Delta_T; \mu, Q^2) = \sum_j C^{i/j}(x, \xi, b_T^2, b_T \cdot \Delta_T, \Delta_T^2; \mu, Q^2) \otimes w_{j/H}^{\text{ren}}(x, \xi, -\Delta_T^2; \mu)$$

GTMD

GPDs

- We computed the coefficient for F_{11} in the quark channel at NLO:

$$F_{1,1}^{q,\text{ren}}(x, \xi = 0, b_T^2, \Delta_T^2, b_T \cdot \Delta_T; \mu, Q^2) = \int_x^1 \frac{dy}{y} C^{q/q}(y, b_T^2, b_T \cdot \Delta_T, \Delta_T^2; \mu, Q^2) H^{q,\text{ren}}(\frac{x}{y}, 0, -\Delta_T^2)$$

GTMDS: Operator Product Expansion

Preliminary

- Need to calculate H with the delta regulator (tricky...)
- Need the Fourier transform of F_{II} (even trickier...)

$$\begin{aligned} C^{q/q}(x, b_T^2, b_T \cdot \Delta_T, \Delta_T^2, \mu, \Lambda) = & \delta(1-x) + \frac{\alpha_s C_F}{2\pi} \left[\chi(x, b_T^2, \Delta_T^2, b_T \cdot \Delta_T, \mu) \right. \\ & + (1-x)[1 + \ln(1-x)] - 2x \left(\cos \left(\frac{1-x}{2} b_T \cdot \Delta_T \right) - 1 \right) \left(\frac{\ln(1-x)}{1-x} \right)_+ \\ & - \frac{2x}{(1-x)_+} \cos \left(\frac{1-x}{2} b_T \cdot \Delta_T \right) L_T - \frac{\pi x}{(1-x)_+} \sin \left(\frac{1-x}{2} b_T \cdot \Delta_T \right) \\ & \left. + \delta(1-x) \left(-\frac{1}{2} L_T^2 - L_T \ln \left(\frac{Q^2}{\mu^2} \right) + \frac{\pi^2}{12} \right) \right] \end{aligned}$$

Regular function

- No divergences in the coefficient!
- It reduces to the TMDPDF coefficient in the forward limit

Conclusions & Outlook

- ★ Naive definition of GTMDs does not work (like TMDs, double TMDs,...)
- ★ Combination with a Soft Function cancels all *spurious rapidity divergences*
- ★ Only when properly defined it makes sense to talk about its evolution and **collinear expansion**, and thus connect it eventually to experimental data through factorization theorems (whenever we have them!)
- ★ Evolution kernel is universal among all (un)polarized quark/gluon GTMDs
- ★ We currently know the perturbative ingredients for the kernel at (almost) N^3LL

- ✿ Extension of OPE to gluon GTMDs
- ✿ Relations between GTMDs, GPDs and OAM, effect of evolution,...
- ✿ Measurement of GTMDs?...

