

Towards three loop evolution equation for GPDs

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based on

V. Braun, A. Manashov, Eur. Phys. J. C 73 (2013) 2544

V. Braun, A. Manashov, Phys. Lett. B 734 (2014) 137

V. Braun, A. Manashov, S. Moch, M. Strohmaier, JHEP 1603 (2016) 142

QCD Evolution, Amsterdam, 31.05.16

- **DIS:** Structure functions : scale dependence is governed by anomalous dimensions of twist-2 operators (known at three loops), Larin, Vermaseren, Moch, Vogt, et al)
- **Deeply Virtual Compton Scattering:** (Müller, Ji, Radyushkin) Generalized Parton Distributions: scale dependence \longleftrightarrow anomalous dimension matrix (nonforward kernel) (two loops) Belitsky, Müller, (2000)

Explore the road to NNLO (three-loop) evolution equations for GPDs

One loop: anomalous dimensions+conformal symmetry → full anomalous dimension matrix.
(Makeenko, 1980)

$$O_N \sim (\partial_{z_1} + \partial_{z_2})^N C_N^{3/2} \left(\frac{\partial_{z_1} - \partial_{z_2}}{\partial_{z_1} + \partial_{z_2}} \right) \bar{q}(z_1 n) \gamma_+ q(z_2 n)$$

In any realistic $d = 4$ QFT the conformal symmetry is broken, $\beta(g) \neq 0$.

D. Müller, Constraints for anomalous dimensions of local light cone operators in ϕ^3 in six-dimensions theory, Z. Phys. C 49 (1991) 293.

(Conformal Ward Identities, Conformal anomaly, Conformal scheme, etc)

Belitsky, Müller, (2000) two loop kernels in QCD.

Difference to D.Müller:

Instead of considering consequences of broken conformal symmetry in QCD we make use of exact conformal symmetry of a modified theory: Large N_f QCD in $4 - 2\epsilon$ dimensions at critical coupling:

$$\beta^{QCD}(a) = 2a[-\epsilon - \beta_0 a + \dots] \quad a_* = -4\pi\epsilon/\beta_0 + \dots \quad \beta^{QCD}(a_*) = 0$$

$$a_* = a_*(\epsilon) \qquad \longleftrightarrow \qquad \epsilon = \epsilon(a_*)$$

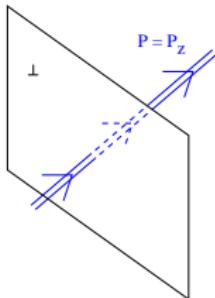
$1/N_f$ expansion, Gracey, Cuccini, Derkachov, A.M.

Premium:

- Exact symmetry \Rightarrow algebraic group-theory methods
- Answer obtained directly in $\overline{\text{MS}}$ scheme
- Light-ray operator basis, do not need to restore evolution eqs. from local operators.

$$\begin{aligned} \mathcal{O}(x; z_1, z_2) &\equiv [\bar{q}(x + z_1 n) \not{p} q(x + z_2 n)] \equiv \sum_{m,k} \frac{z_1^m z_2^k}{m!k!} [(D_+^m \bar{q})(x) \not{p} (D_+^k q)(x)] \\ &= \sum_{N,k} \Psi_{Nk}(z_1, z_2) \partial_+^k \mathcal{O}_N(x) \end{aligned}$$

Collinear Subgroup



$$\begin{aligned} p_+ &= \frac{1}{\sqrt{2}}(p_0 + p_z) \rightarrow \infty \\ p_- &= \frac{1}{\sqrt{2}}(p_0 - p_z) \rightarrow 0 \\ px &\rightarrow p_+ x_- \end{aligned}$$

- Special conformal transformation

$$x_- \rightarrow x'_- = \frac{x_-}{1 + 2ax_-}$$

- translations $x_- \rightarrow x'_- = x_- + c$

- dilatations $x_- \rightarrow x'_- = \lambda x_-$

form the so-called **collinear subgroup** $SL(2, R)$

$$\alpha \rightarrow \alpha' = \frac{a\alpha + b}{c\alpha + d}, \quad ad - bc = 1$$

$$\Phi(\alpha) \rightarrow \Phi'(\alpha) = (c\alpha + d)^{-2j} \Phi\left(\frac{a\alpha + b}{c\alpha + d}\right)$$

where $\Phi(x) \rightarrow \Phi(x_-) = \Phi(\alpha n_-)$ is the quantum field with scaling dimension ℓ and spin projection s “living” on the light-ray

Conformal spin:

$$j = (\ell + s)/2$$

Light-ray operators satisfy the RG equation

Balitsky, Braun '89

$$\left(M\partial_M + \beta(a)\partial_a + \mathbb{H}(a) \right) \mathcal{O}(z_1, z_2) = 0$$

where \mathbb{H} is an integral operator acting on the light-cone coordinates of the fields:

$$\mathbb{H}(a)\mathcal{O}(z_1, z_2) = \int_0^1 d\alpha \int_0^1 d\beta h(\alpha, \beta) \mathcal{O}(z_{12}^\alpha, z_{21}^\beta)$$

$$\begin{aligned} z_{12}^\alpha &\equiv z_1 \bar{\alpha} + z_2 \alpha \\ \bar{\alpha} &= 1 - \alpha \end{aligned}$$

$$h(\alpha, \beta) = a h^{(1)}(\alpha, \beta) + a^2 h^{(2)}(\alpha, \beta) + \dots \text{ [does not depend on } \epsilon \text{]}$$

One can show that the powers $\mathcal{O}(z_1, z_2) \mapsto (z_1 - z_2)^N$ are eigenfunctions of \mathbb{H} , and the corresponding eigenvalues are the anomalous dimensions of local operators of spin N (with $N - 1$ derivatives)

$$\gamma_N = \int d\alpha d\beta h(\alpha, \beta) (1 - \alpha - \beta)^{N-1} \quad (\text{NNLO})$$

Is it possible to restore $h(\alpha, \beta)$ from γ_N ?

It is expected that the evolution kernel \mathbb{H} commutes with the generators of $SL(2, R)$ collinear subgroup, $[S_\alpha, \mathbb{H}] = 0$, $\alpha = \pm, 0$.

- Leading order generators

$$S_+^{(0)} = z_1^2 \partial_{z_1} + z_2^2 \partial_{z_2} + 2(z_1 + z_2), \quad S_0^{(0)} = z_1 \partial_{z_1} + z_2 \partial_{z_2} + 2, \quad S_-^{(0)} = -\partial_{z_1} - \partial_{z_2}$$

if $[\mathbb{H}, S_\alpha] = 0$ **then** $h(\alpha, \beta) = h(\tau)$, $\tau = \alpha\beta/\bar{\alpha}\bar{\beta}$.

$$\gamma_N \Leftrightarrow h(\tau)$$

- Exact conformal symmetry, but the generators are modified by quantum corrections

$$S_- = S_-^{(0)},$$

$$a = \frac{\alpha_s}{4\pi}$$

$$S_0 = S_0^{(0)} - \epsilon(a_*) + \frac{1}{2} \mathbb{H}(a_*), \quad \mathbb{H}(a_*) = a_* \mathbb{H}^{(1)} + \dots$$

$$S_+ = S_+^{(0)} + (z_1 + z_2) \left(-\epsilon(a_*) + \frac{1}{2} \mathbb{H}(a_*) \right) + (z_1 - z_2) \Delta_+(a_*),$$

Expansion over conformal operators:

$$\mathcal{O}(x; z_1, z_2) = \sum_{N,k} \Psi_{Nk}(z_1, z_2) \partial_+^k \mathcal{O}_N(x),$$

$$\mathbb{H}(a_*) \Psi_{Nk} = \gamma_N \Psi_{Nk}$$

$$\Psi_{Nk}(z_1, z_2) = S_+^k (z_1 - z_2)^N$$

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$$\Psi_{Nk}(z_1, z_2) = S_+^k (z_1 - z_2)^N$$

$\mathbb{H}(a_*)$ commutes with all generators $\implies \mathbb{H}(a_*) = f(J)$

$$J(J-1) = S_+ S_- + S_0(S_0-1)$$

The lowest eigenfunctions are known: $\Psi_{N0}(z_1, z_2) = z_{12}^N$ (for any coupling).

$$J(a_*) z_{12}^N = (N + 2 - \epsilon + \gamma_N/2) z_{12}^N, \quad \mathbb{H}(a_*) z_{12}^N = \gamma_N z_{12}^N$$

$$\gamma_N = f(N + 2 - \epsilon + \gamma_N/2).$$

One can restore the function f from anomalous dimensions: $f(x) = a f_1(x) + a^2 f_2(x) + \dots$

$$\mathbb{H}(a) = a f_1(J(a)) + a^2 f_2(J(a)) + a^3 f_3(J(a)) + \dots$$

Conformal Ward Identity $\longmapsto \Delta_+$

- Expansion over conformal operators

$$\mathcal{O}^{(n)}(x; z_1, z_2) = \sum_{N,k} \Psi_{Nk}(z_1, z_2) \partial_+^k \mathcal{O}_N^{(n)}(x),$$

- Conformal symmetry at the critical coupling implies

$$\left(S_+^{(z)} - \frac{1}{2} x^2 (\bar{n} \partial_x) \right) \langle \mathcal{O}_n(0, z_1, z_2), \mathcal{O}_{\bar{n}}(x, w_1, w_2) \rangle = 0$$

$$(nx) = (\bar{n}x) = 0$$

- To find explicit expression for S_+ , consider Ward identity ($\delta_+ = K\bar{n}$)

$$\langle \delta_+ \mathcal{O}^{(n)}(z) \mathcal{O}^{(\bar{n})}(x, w) \rangle + \langle \mathcal{O}^{(n)}(z) \delta_+ \mathcal{O}^{(\bar{n})}(x, w) \rangle = \langle \delta_+ S_R \mathcal{O}^{(n)}(z) \mathcal{O}^{(\bar{n})}(x, w) \rangle$$

Then

$$\delta_+ \mathcal{O}^{(\bar{n})}(x; w_1, w_2) = -x^2 (\bar{n} \partial_x) \mathcal{O}^{(\bar{n})}(x; w_1, w_2)$$

$$\delta_+ \mathcal{O}^{(n)}(0; z_1, z_2) = 2(n\bar{n}) \left(S_+^{(0)} - \epsilon(z_1 + z_2) - \frac{a}{2} [\mathbb{H}^{(1)}, z_1 + z_2] + \dots \right) \mathcal{O}^{(n)}(0; z_1, z_2)$$

and for the last term

$$\delta_+ S^{QCD} = 4\epsilon \int d^d x (x\bar{n}) L^{QCD} + 2(d-2)\bar{n}^\mu \int d^d x \delta_{BRST}(\bar{c}^a A_\mu^a).$$

$\langle \mathcal{O}^{(n)}(0; z_1, z_2) q(x) \bar{q}(y) \rangle$ – bad object for an analysis

- Reexpand ϵL in terms of renormalized operators

A.N. Vasil'ev, The field theoretic renormalization group in critical behaviour theory and stochastic dynamics

$$2\epsilon L = -\beta(a)/a [L^{YM+gf}] + EOM + BRST$$

$$[\beta(a_*) = 0]$$

$$\Delta S_+ \langle \mathcal{O}_n(0, z), \mathcal{O}_{\bar{n}}(x, w) \rangle = KR' \left(\langle \int d^D y (\bar{n}y) L^{YM+gf}(y) \mathcal{O}_n(0, z), \mathcal{O}_{\bar{n}}(x, w) \rangle \right) \text{ SIMPLE RESIDUE IN } \epsilon$$

One loop: A. Belitsky, D. Müller, 2001, (V. Braun, A.M., (2014))

$$\Delta_+^{(1)} \mathcal{O}(z_1, z_2) = -2C_F \int_0^1 du \int_0^1 d\alpha \frac{\bar{\alpha}}{\alpha} [\mathcal{O}(z_{12}^{u\alpha}, z_2) - \mathcal{O}(z_1, z_{21}^{u\alpha})]$$

$$z_{12}^\alpha \equiv z_1 \bar{\alpha} + z_2 \alpha$$

Δ_+ at two loops

$$[\Delta_+^{(2)} \mathcal{O}](z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta [\omega(\alpha, \beta) + \omega^{\mathbb{P}}(\alpha, \beta) \mathbb{P}_{12}] [\mathcal{O}(z_{12}^\alpha, z_{21}^\beta) - \mathcal{O}(z_{12}^\beta, z_{21}^\alpha)] \\ + \int_0^1 du \int_0^1 d\alpha \varkappa(\alpha) [\mathcal{O}(z_{12}^{u\alpha}, z_2) - \mathcal{O}(z_1, z_{21}^{u\alpha})].$$

Color structures: $\beta_0 C_F$, C_F^2 and C_F/N_C : ($\tau = \alpha\beta/\bar{\alpha}\bar{\beta}$)

$$\begin{aligned} \omega_{\mathbf{P}}(\alpha, \beta) &= \frac{4}{\alpha} [\text{Li}_2(\bar{\alpha}) - \text{Li}_2(1)] + \frac{1}{\alpha} \ln^2 \bar{\alpha} - (\alpha - \beta) \ln^2 \left(\frac{\alpha}{\bar{\alpha}} \right) - \beta \ln^2 \alpha \\ &\quad + 2\alpha \left(\frac{\pi^2}{3} - \frac{15}{2} \right) - 2 \left(\alpha + \beta + \frac{1}{\bar{\alpha}} \right) \ln \alpha + (\beta - 2\bar{\alpha}) \left(1 + \frac{2}{\alpha} \right) \ln \bar{\alpha}. \\ \omega_{\mathbf{NP}}(\alpha, \beta) &= 2 \left[\left(\frac{1}{\alpha} - \alpha \right) \left[\text{Li}_2 \left(\frac{\beta}{\bar{\alpha}} \right) - \text{Li}_2(\beta) - 2 \text{Li}_2(\alpha) - \ln \alpha \ln \bar{\alpha} \right] + \frac{\alpha}{\tau} \left[\tau \ln \tau + \bar{\tau} \ln \bar{\tau} \right] \right. \\ &\quad \left. - \bar{\beta} \ln \alpha - \frac{\bar{\alpha}}{\alpha} \ln \bar{\alpha} \right], \\ \omega_{\mathbf{NP}}^{\mathbb{P}}(\alpha, \beta) &= 2 \left[\left(\bar{\alpha} - \frac{1}{\bar{\alpha}} \right) \left[\text{Li}_2 \left(\frac{\alpha}{\beta} \right) - \text{Li}_2(\alpha) - \ln \bar{\alpha} \ln \bar{\beta} \right] + \alpha \bar{\tau} \ln \bar{\tau} + \frac{\beta^2}{\bar{\beta}} \ln \bar{\alpha} \right] \end{aligned}$$

two loop kernel : $[S_+, \mathbb{H}] = 0$

• **one loop:** $[S_+^{(0)}, \mathbb{H}^{(1)}] = 0$

$$\gamma_N = 4C_F \left(S_1(N+1) + S_1(N-1) - 3/2 \right) \quad \Rightarrow \quad \mathbb{H}^{(1)} = 4C_F \left(\widehat{H} - H^+ + \frac{1}{2} \right)$$

$$\widehat{H}f(z_1, z_2) = \int_0^1 d\alpha \frac{\bar{\alpha}}{\alpha} \left[2f(z_1, z_2) - f(z_{12}^\alpha, z_2) - f(z_1, z_{21}^\alpha) \right]$$

$$H^+f(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta f(z_{12}^\alpha, z_{21}^\beta)$$

• **two loop: first step is to determine the noninvariant part**

$$[S_+^{(0)}, \mathbb{H}^{(2)}] = \left[\mathbb{H}^{(1)}, z_1 + z_2 \right] \left(\beta_0 + \frac{1}{2} \mathbb{H}^{(1)} \right) + [\mathbb{H}^{(1)}, (z_1 - z_2) \Delta_+^{(1)}]$$

$$\underline{\mathbb{H}^{(2)} = \mathbb{H}_{inv}^{(2)} + \mathbb{T}^{(1)} \left(\beta_0 + \frac{1}{2} \mathbb{H}^{(1)} \right) + [\mathbb{H}^{(1)}, \mathbb{X}^{(1)}]}$$

$$[S_+^{(0)}, \mathbb{X}^{(1)}] = (z_1 - z_2) \Delta^{(1)} \qquad \qquad [S_+^{(0)}, \mathbb{T}^{(1)}] = [\mathbb{H}^{(1)}, z_1 + z_2]$$

e.g.

$$\mathbb{X}^{(1)} f(z_1, z_2) = 2C_F \int_0^1 d\alpha \frac{\ln \alpha}{\alpha} \left[2f(z_1, z_2) - f(z_{12}^\alpha, z_2) - f(z_1, z_{21}^\alpha) \right]$$

- Restoration of \mathbb{H}_{inv} :

$$\mathbb{H}_{inv}(N) = \mathbb{H}_2(N) - T_1(N) \left(\beta_0 + \frac{1}{2} \mathbb{H}_1(N) \right).$$

$$\mathbb{H}_1(N) = 4C_F \left(S_1(N+1) + S_1(N-1) - 3/2 \right) \quad T_1(N) = 4C_F \left(2 - S_2(N+1) - S_2(N-1) \right)$$

$$\begin{aligned} \mathbb{H}_{inv}^{(2)} = & 4C_F \left\{ \beta_0 \left(\frac{13}{12} + \frac{5}{3}\hat{\mathcal{H}} - \frac{11}{3}\mathcal{H}^+ \right) + 4C_F \left(\frac{19}{48} - \frac{1}{3}\hat{\mathcal{H}} - \frac{2}{3}\mathcal{H}^+ - \frac{1}{4}(\mathcal{H}^+)^2 \right) \right. \\ & + \frac{2}{N_c} \left[\left(3\zeta(3) - \frac{\pi^2}{3} + \frac{52}{48} \right) - \frac{\pi^2 - 4}{6} (\hat{\mathcal{H}} - \mathcal{H}^+) - \mathcal{H}^+ - (\mathcal{H}^+)^2 - (\mathcal{H}^+)^3 \right. \\ & \left. \left. + \mathcal{H}^{1/\tau \ln \bar{\tau}} - \frac{1}{2} P_{12} \left(\mathcal{H}^{\ln^2 \bar{\tau}} - 2\mathcal{H}^{\tau \ln \bar{\tau}} \right) \right] \right\} \end{aligned}$$

Conclusions

- QCD evolution equations possess a "hidden" conformal symmetry in $d = 4$:
Evolution kernels commute with modified symmetry generators
- n -loop evolution kernels for twist-two operators in MS scheme can be restored from the $(n - 1)$ -loop calculation of the special conformal anomaly and n -loop anomalous dimensions
- The generator S_+ for the nonsinglet twist-2 operators is known with two loop accuracy \implies three three loop evolution kernel