An Effective Field Theory for both Hard and Forward scattering

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QCD Evolution, Amsterdam
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Outline

- Modes in Soft-Collinear Effective Theory (SCET)
- Factorization and Factorization Violation

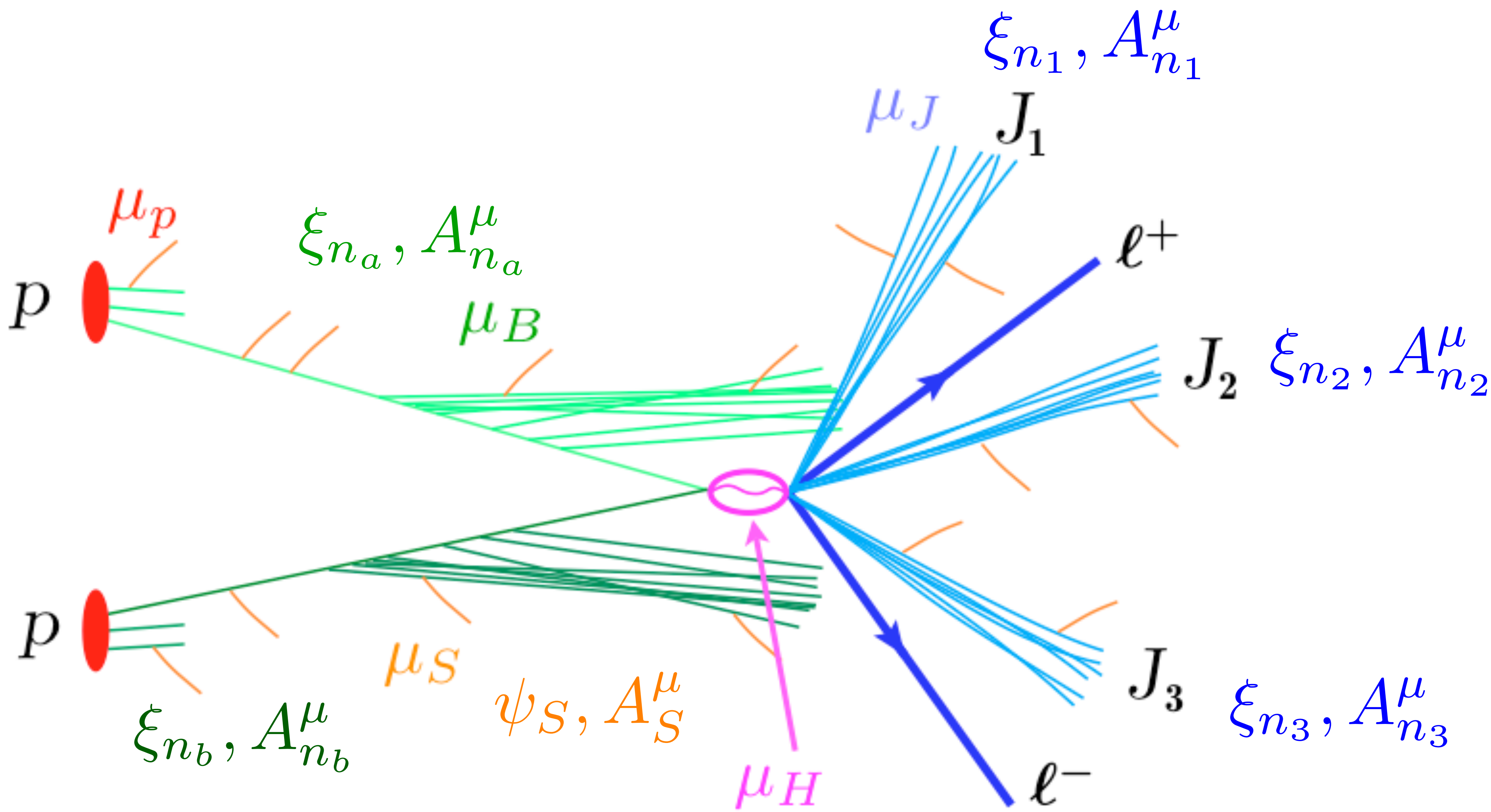
Formalism:

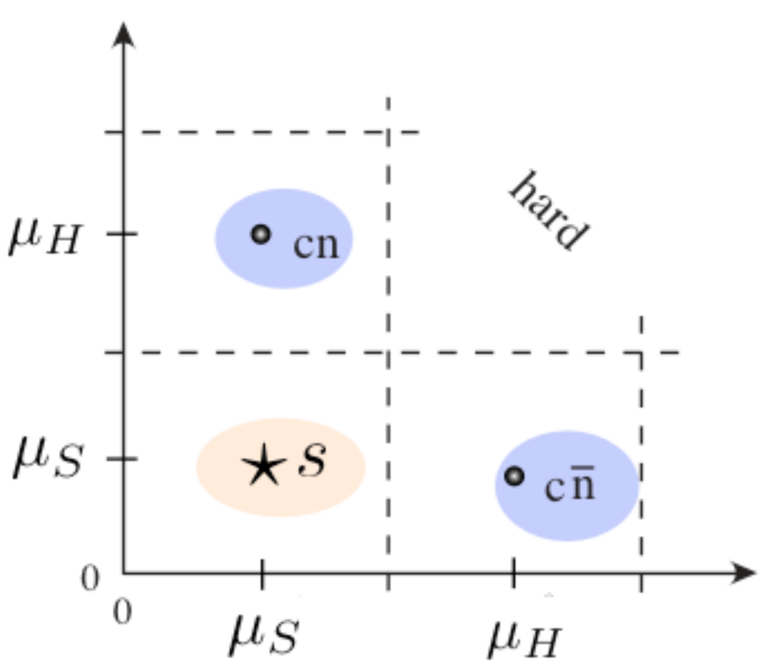
- Glauber Operators in SCET
- One loop examples

Illustrative Applications:

- Rapidity RGE & BFKL evolution
- Glauber's and Eikonalization
- Hard Matching: "The Cheshire Glauber"
- Glauber's in Hadron Scattering

SCET Fields for various Modes





- dominant contributions from particular regions of momentum space
- use subtractions rather than sharp boundaries to preserve symmetry

Relevant Modes

Infrared Structure of Amplitudes (Landau eqtns, CSS, ...)

Method of Regions (Beneke & Smirnov)

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n_a-collinear	$\xi_{n_a},\ A^\mu_{n_a}$	$(n_a\cdot p, \bar n_a \cdot p, p_{\perp a}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet a	onshell
n_b-collinear	$\xi_{n_b},\ A^\mu_{n_b}$	$(n_b\cdot p, \bar n_b \cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b	onshell
n_j-collinear	$\xi_{n_j},\ A^\mu_{n_j}$	$(n_j\cdot p, \bar n_j \cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in $\hat n_j$	onshell
soft	$\psi_S,\ A^\mu_S$	$p^\mu \sim Q(\lambda,\lambda,\lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A^\mu_{us}$	$p^\mu \sim Q(\lambda^2,\lambda^2,\lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a,\lambda^b,\lambda)$, $a+b>2$	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out these modes

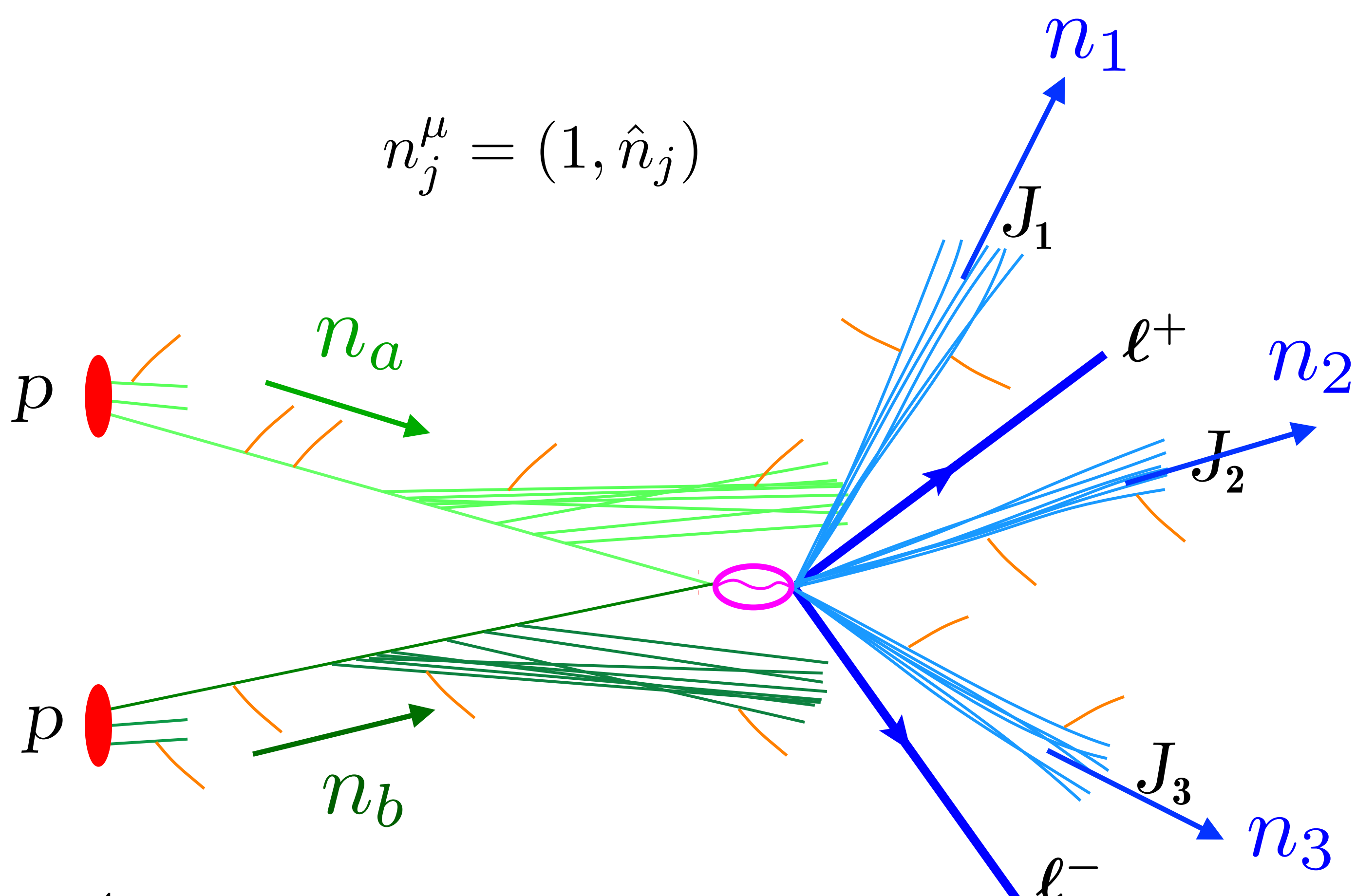
$$p^\mu = \bar n_i \cdot p\,\frac{n_i^\mu}{2} + n_i\cdot p\,\frac{\bar n_i^\mu}{2} + p_\perp^\mu$$

$$n_i^2 = 0$$

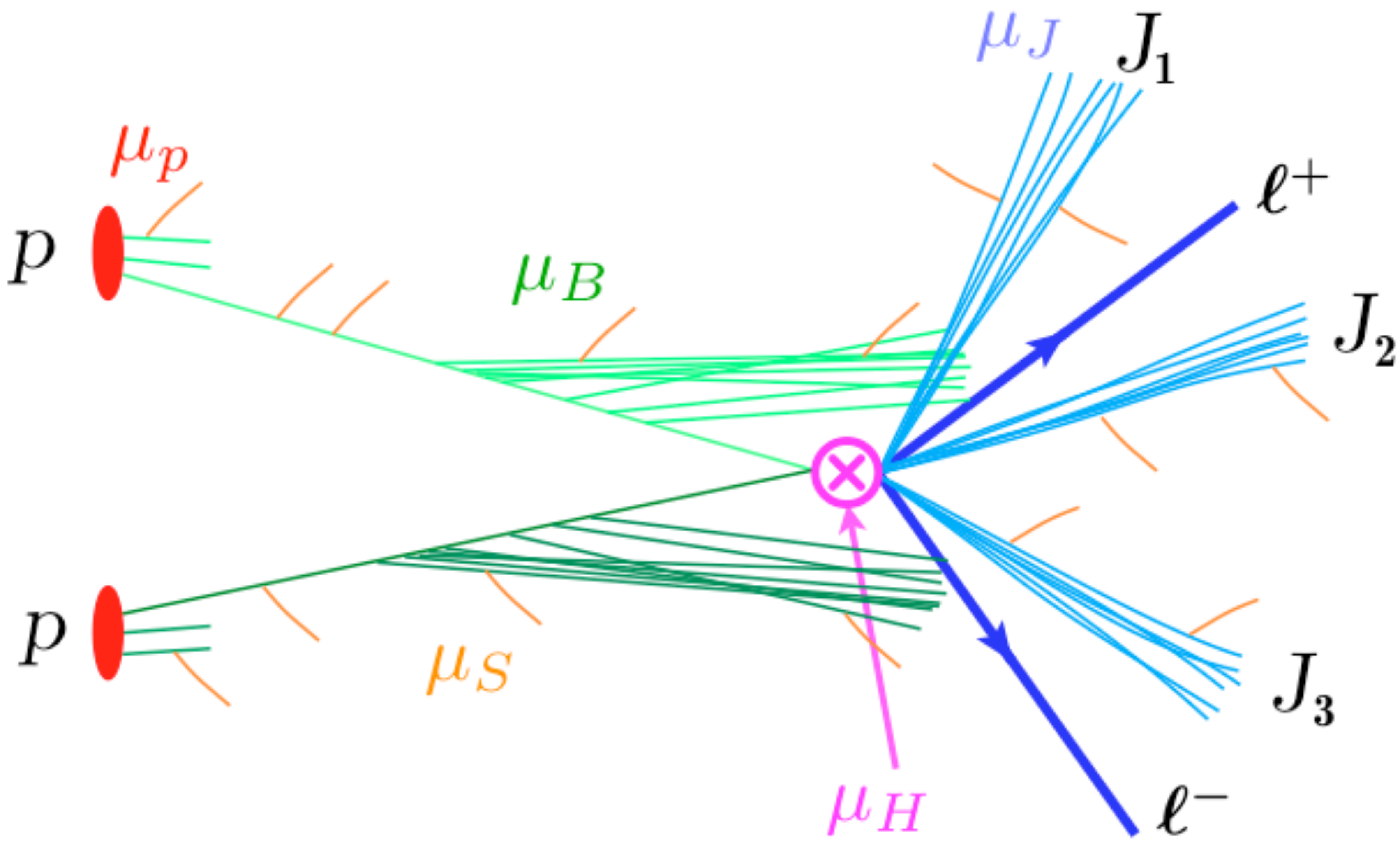
$$\bar n_i^2 = 0$$

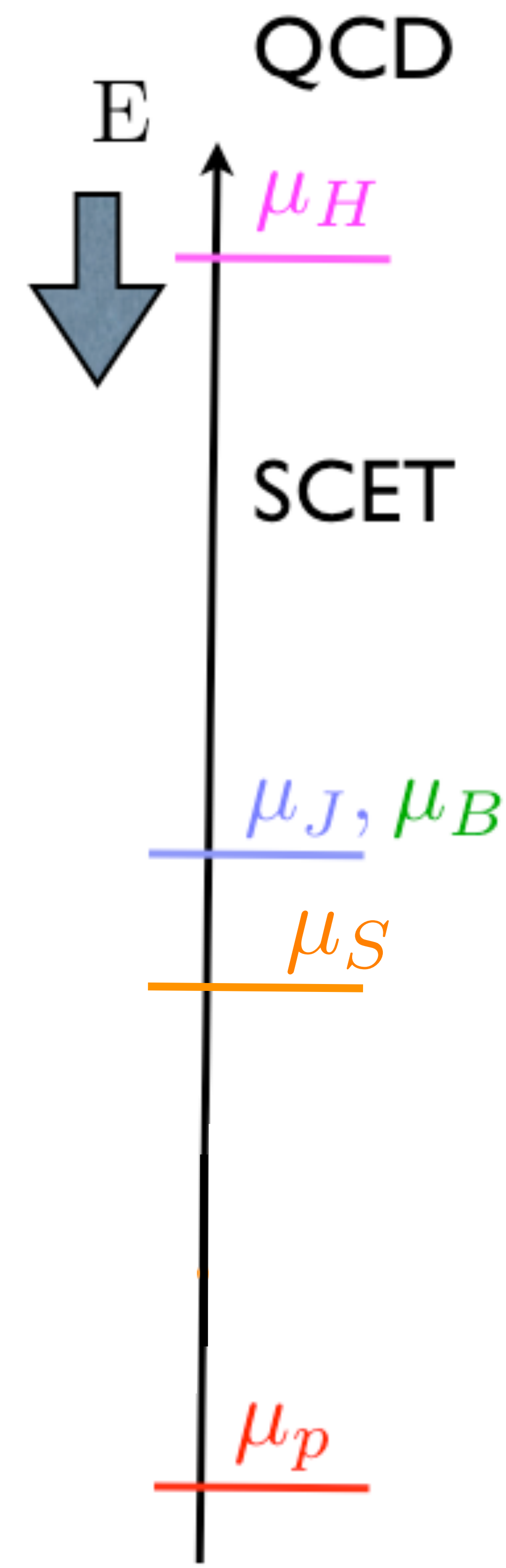
$$n_i \cdot \bar n_i = 2$$

$n_j^\mu = (1, \hat n_j)$



Hard-collinear factorization





μ_H: Wilson coefficients for SCET Hard Scattering Operators

$$C \otimes \mathcal{O}$$

Principles used for Soft-Collinear Effective Theory

- **Matching** QCD & SCET must agree at long distances
  short distance encoded by coefficients, C
- **Power Counting** for fields, states
  Carried out at level of the Lagrangian
  Power counting theorems
- **Symmetry** Gauge symmetry within sectors
  Lorentz & Reparameterization symmetries

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \Big[\mathcal{L}^{(0)}_{\mathrm{S}}(\psi_{\mathrm{S}}, A_{\mathrm{S}}) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})\Big]$$

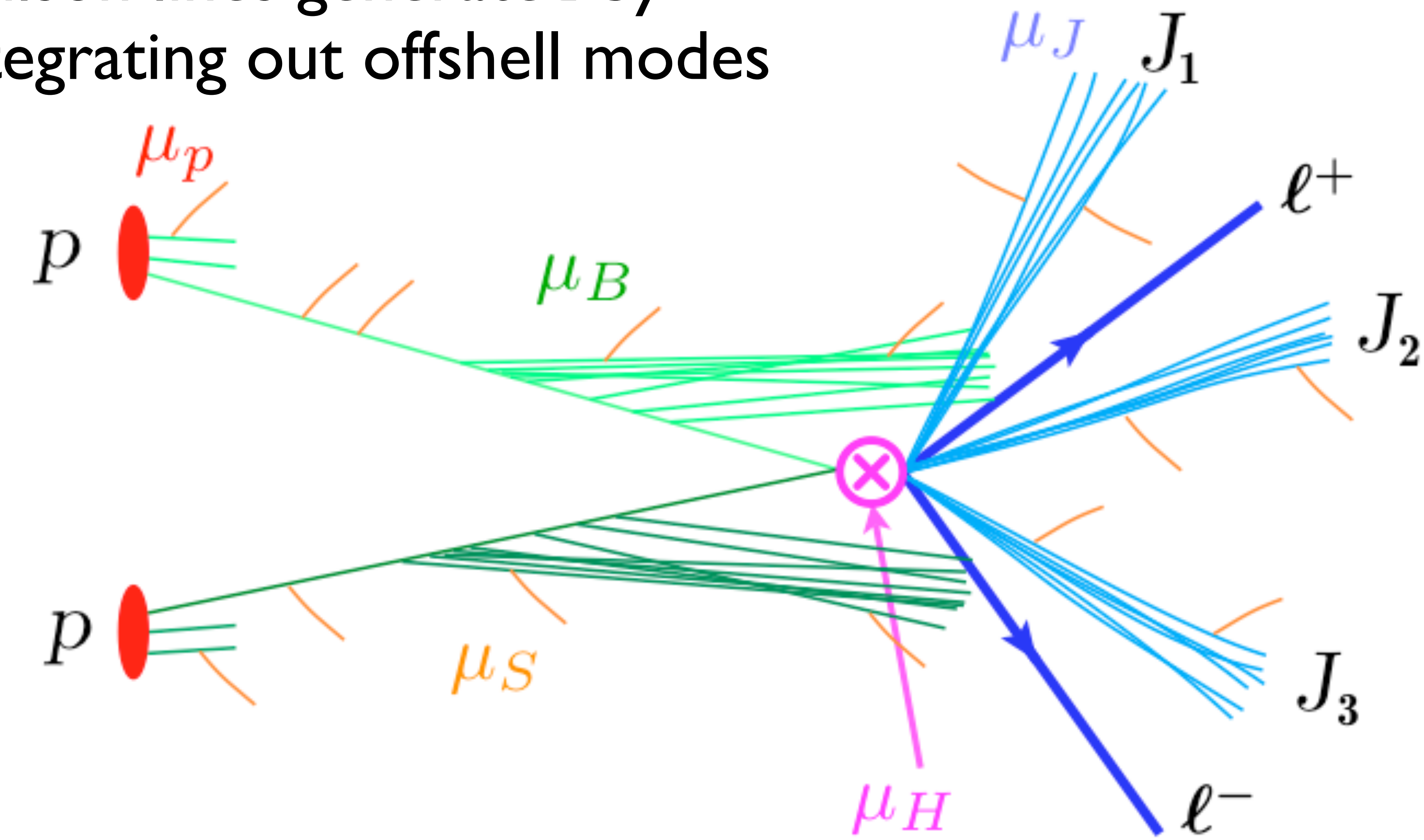
these Lagrangians factorize at leading power

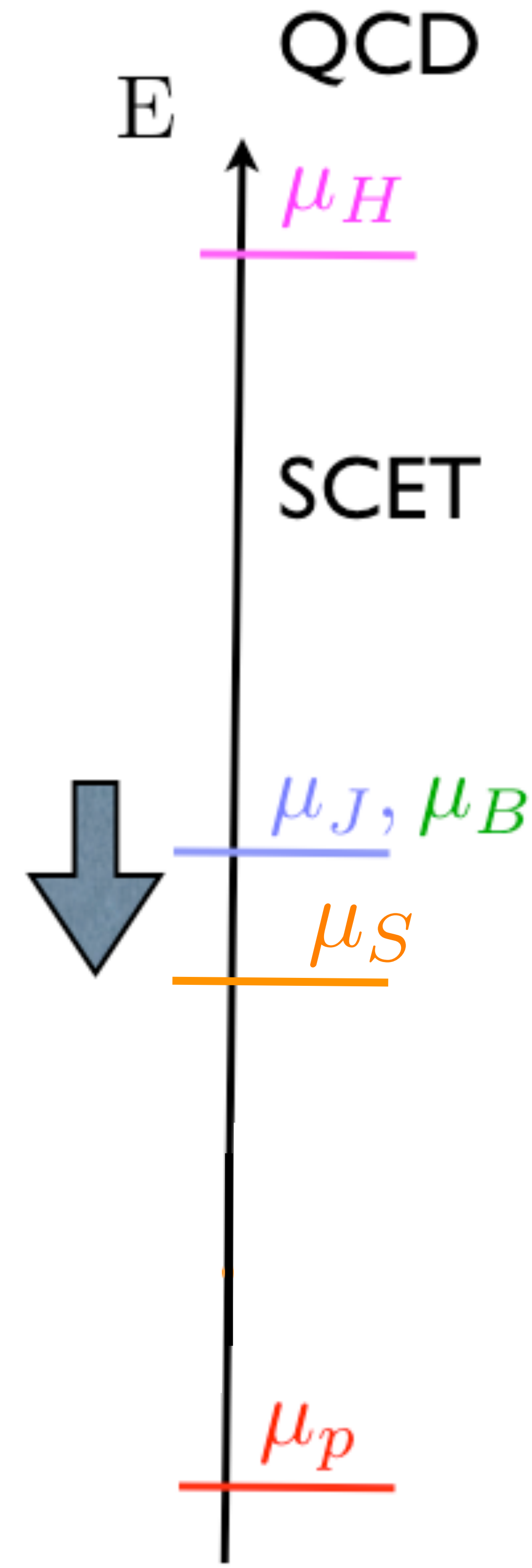
$$+\mathcal{L}^{(0)}_{G}(\psi_S, A_S, \xi_{n_i}, A_{n_i})$$

Glauber operators that can spoil factorization

Hard-collinear factorization

Wilson lines generated by integrating out offshell modes





Operators are built of building block fields:

$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

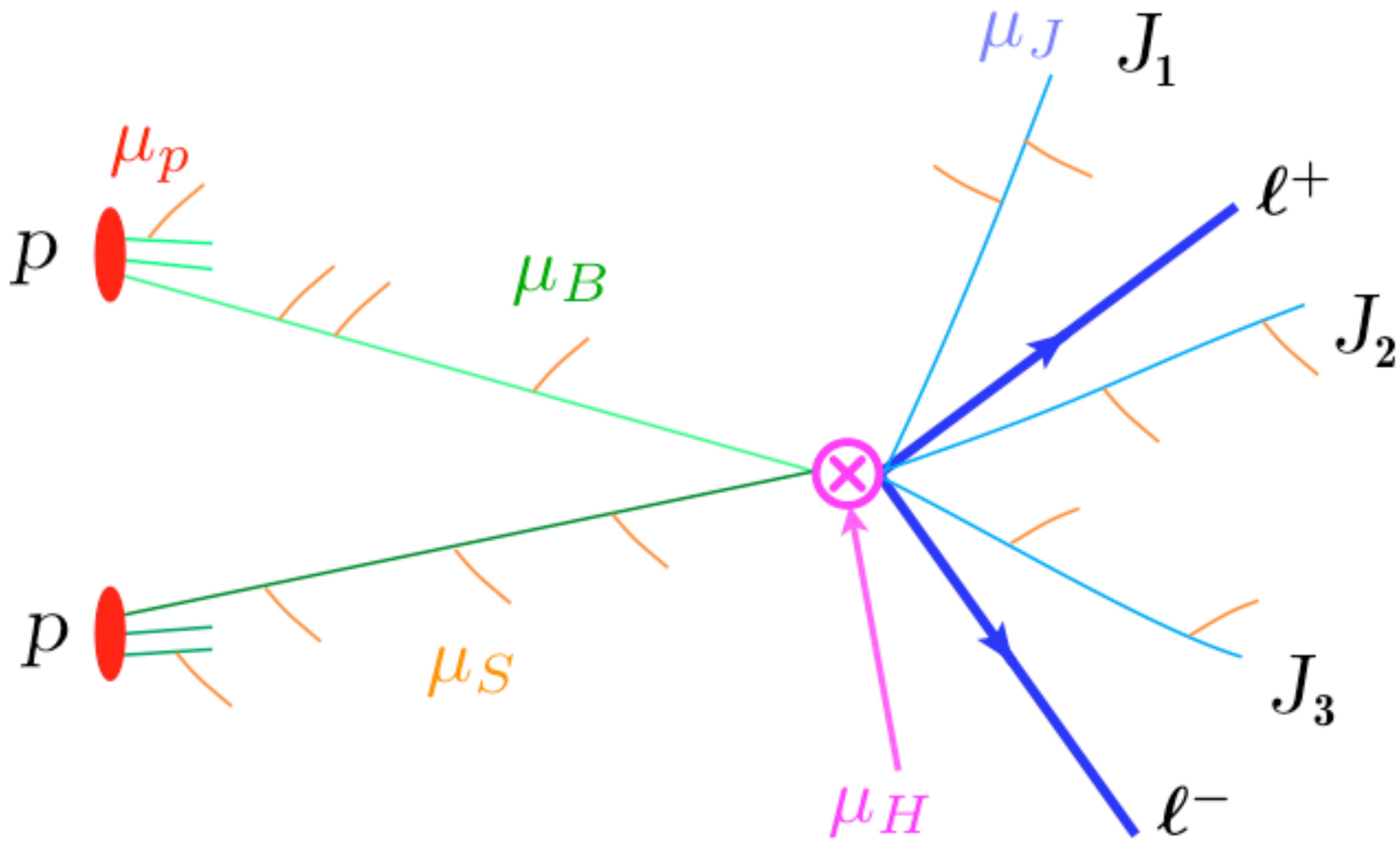
$$\chi_n = (W_n^\dagger \xi_n) \qquad \text{“quark jet”}$$

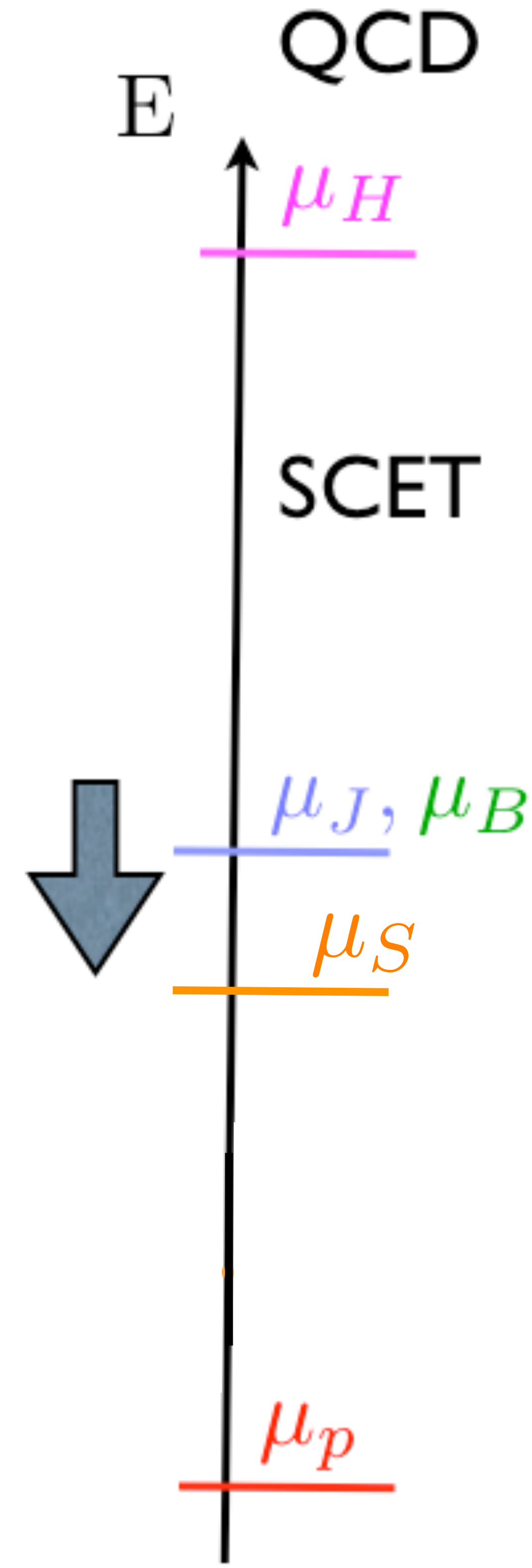
$$g\,\mathcal{B}^\mu_{n\perp} = [W_n^\dagger i D^\mu_\perp W_n] \qquad \text{“gluon jet”}$$

$$W_n = P\exp\Big(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(x+\bar{n}s)\Big)$$

Soft-collinear factorization

Exploit field redefinitions in Lagrangians



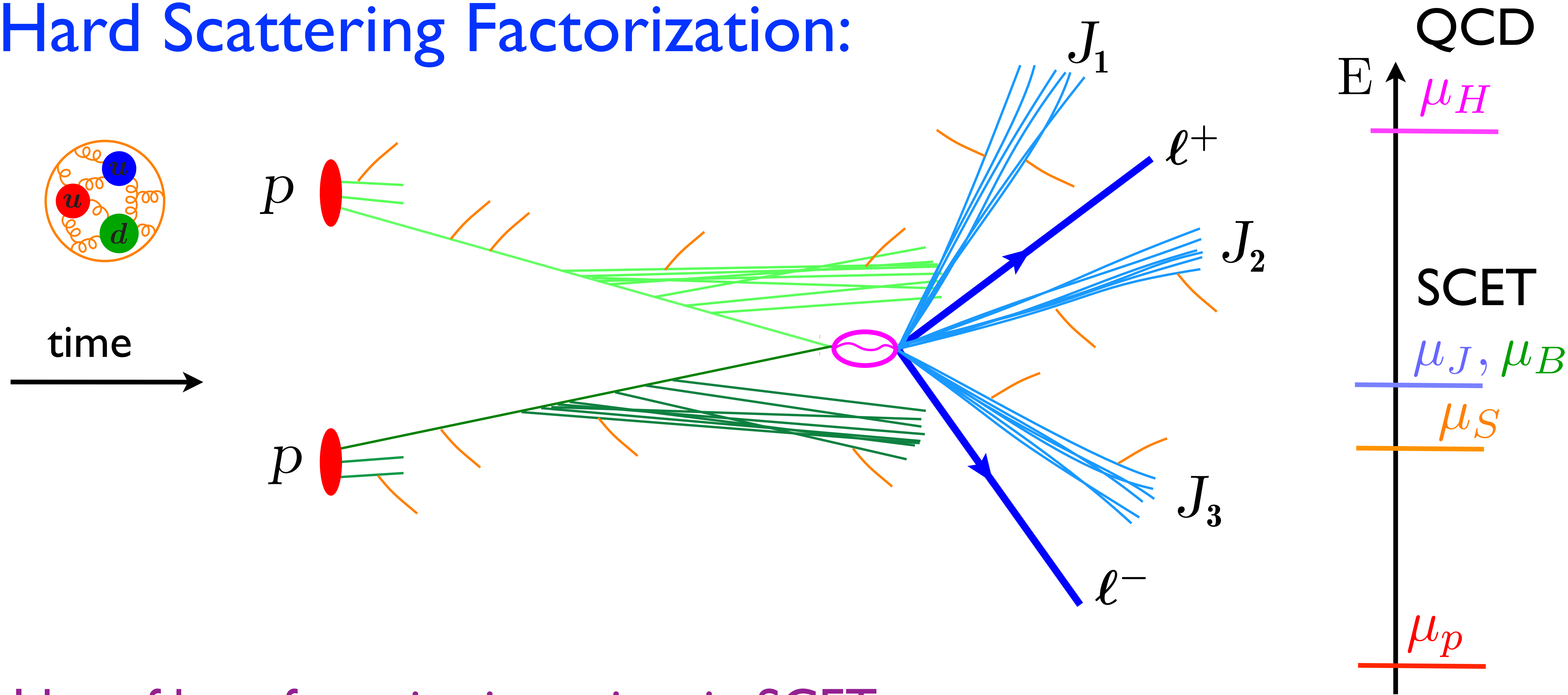


Soft radiation knows only about bulk properties of jet radiation

$$\left(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3}\right)$$

Soft Wilson Lines

Hard Scattering Factorization:



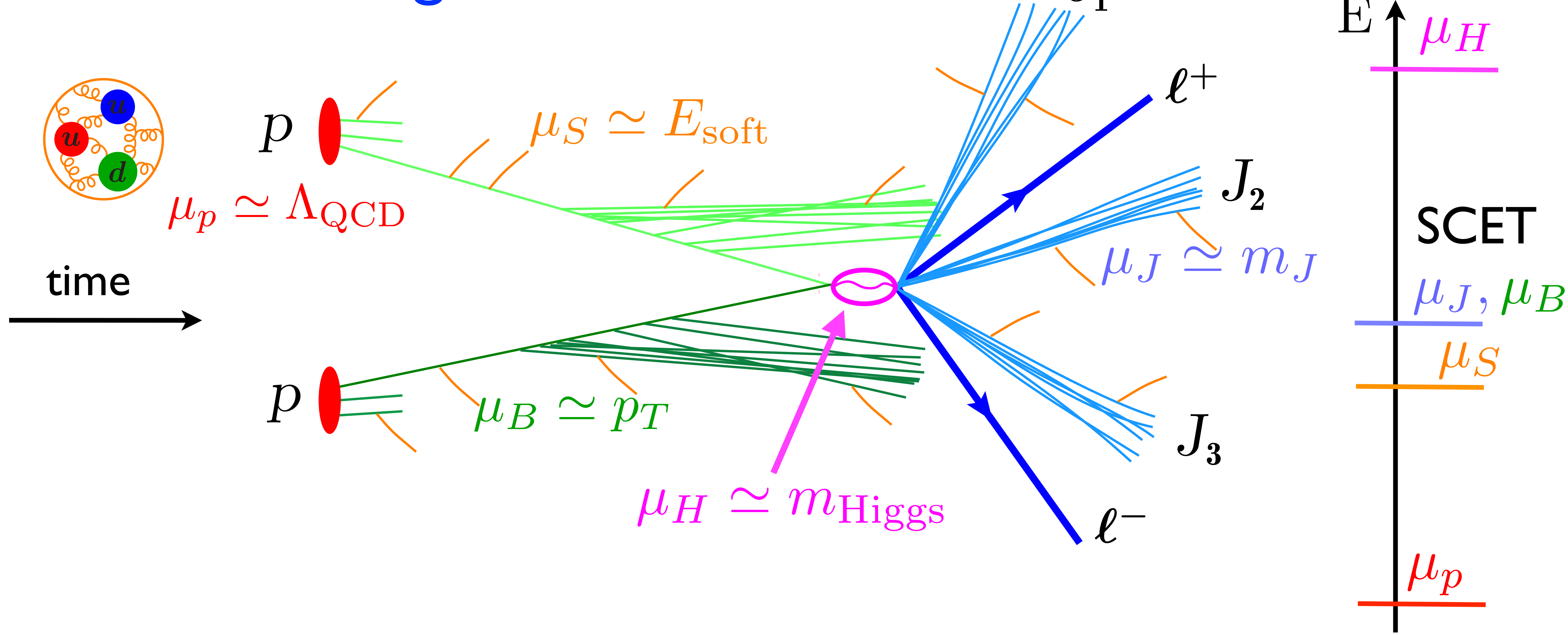
Idea of how factorization arises in SCET:

factorized Lagrangian: $\mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} = \mathcal{L}^{(0)}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i}) \quad \cancel{+\mathcal{L}^{(0)}_G}$

factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3})$

→ factorized squared matrix elements defining jet, soft, … functions

Hard Scattering Factorization:



Nonperturbative: $$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

$\mu_p \simeq \Lambda_{\rm QCD}$

hadronization

(In some cases by Operators, or is power suppressed)

eg. Perturbative: $$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

Used to Sum Logs

	μ_B	μ_H	μ_J	μ_S
Universal Functions:	beam	hard	jet	pert. soft

Examples of Factorization:

- Inclusive Higgs production $\quad pp \to \text{Higgs} + \text{anything}$

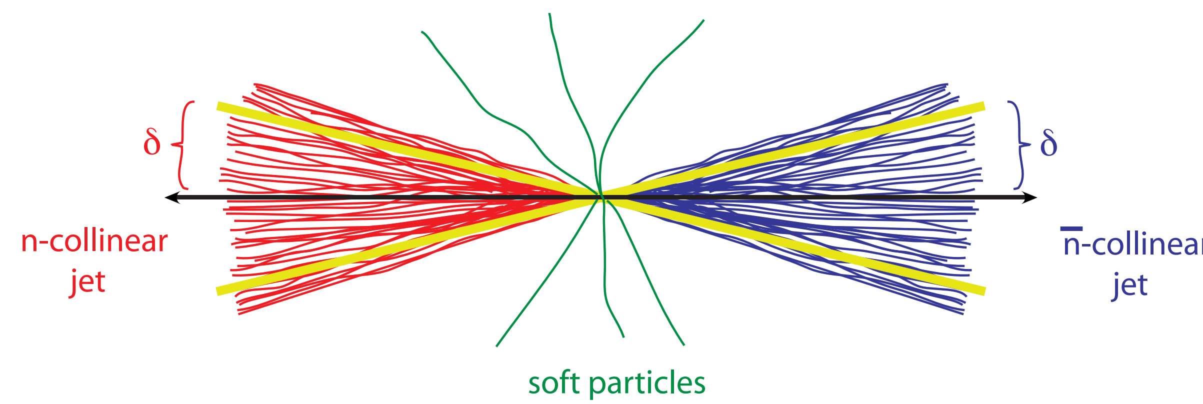
$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\; H_{ij}^{\text{incl}}\Big(\frac{m_H e^{Y}}{E_{\text{cm}}\xi_a}, \frac{m_H e^{-Y}}{E_{\text{cm}}\xi_b}, m_H, \mu\Big)$$

(Collins, Soper, Sterman)

(PDFs contribute, No Glaubers, No Softs)

- Dijet production $\quad e^+e^- \to 2 \text{ jets}$

thrust $\quad \tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\; J_T(Q^2\tau - Q\ell, \mu)\, S_T(\ell - \ell', \mu)\, F(\ell')$$

hard function

jet functions

perturbative soft function

non-perturbative soft function

(No PDFs, No Glaubers, Softs contribute)

Examples of Factorization:

- Inclusive Higgs production $\qquad pp \to \text{Higgs} + \text{anything}$

$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\, H^{\rm incl}_{ij}\Big(\frac{m_H e^{Y}}{E_{\rm cm}\xi_a}, \frac{m_H e^{-Y}}{E_{\rm cm}\xi_b}, m_H, \mu\Big)$$

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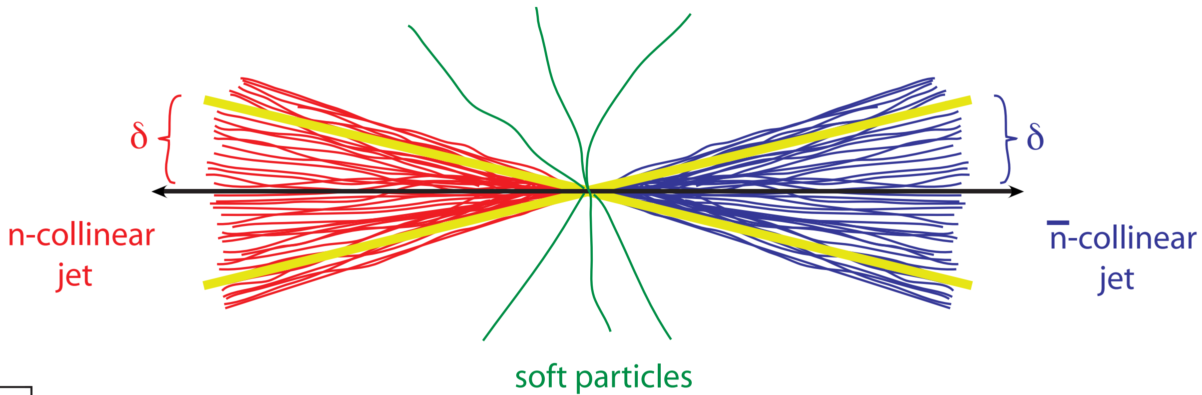
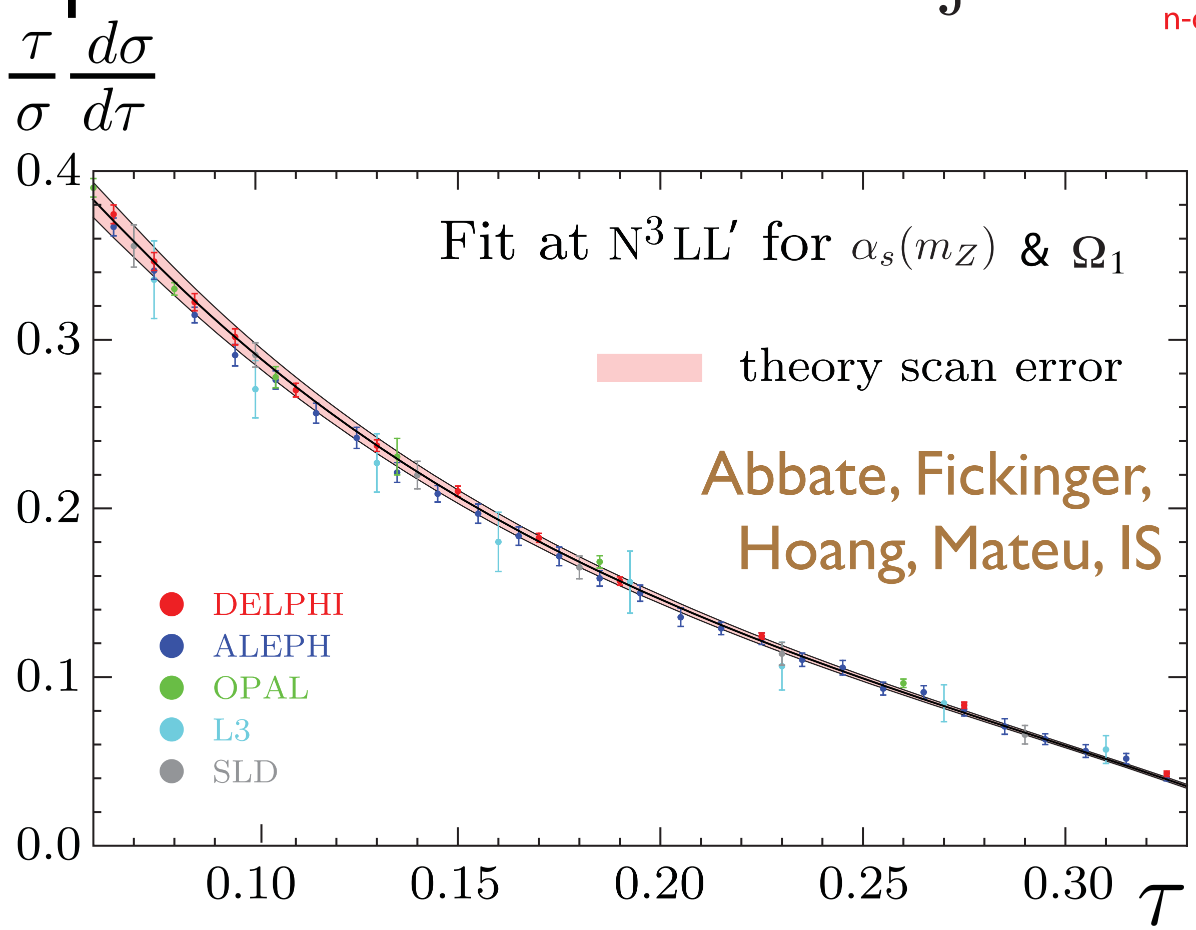


thrust



$\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$

Two parameter fit:

$$\{\alpha_s(m_Z), \Omega_1\}$$

$$\frac{\chi^2}{\rm dof} = \frac{440}{485} = 0.91$$

- Higs with a Jet Veto

(anti-kT jets, radius R)

$p_T^{\text{jet}} \le p_T^{\text{cut}} \ll m_H$

$\Lambda_{\text{QCD}} \ll p_T^{\text{cut}}$

Berger, Marcantonini, IS Tackmann, Waalewijn

Banfi, Salam, Zanderighi

Becher & Neubert

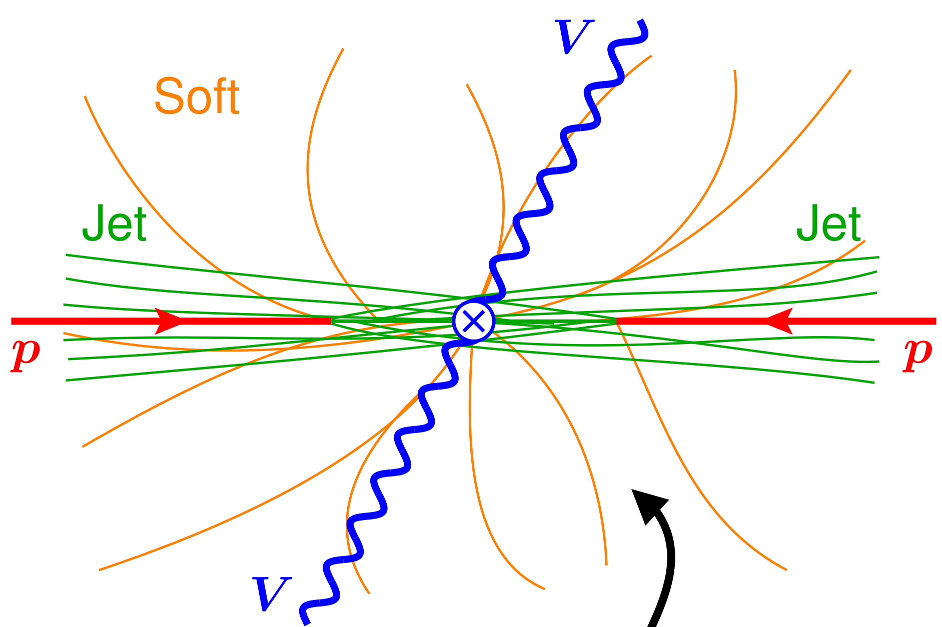
I.S., Tackmann, Walsh, Zuberi

$$\sigma_0(p_T^{\text{cut}}) = H_{gg}(m_H) \times [B_g(m_H, p_T^{\text{cut}}, R)]^2 \times S_{gg}(p_T^{\text{cut}}, R)$$

$$B_g = \mathcal{I}_{gj}(m_H, p_T^{\text{cut}}, R) \otimes f_j$$

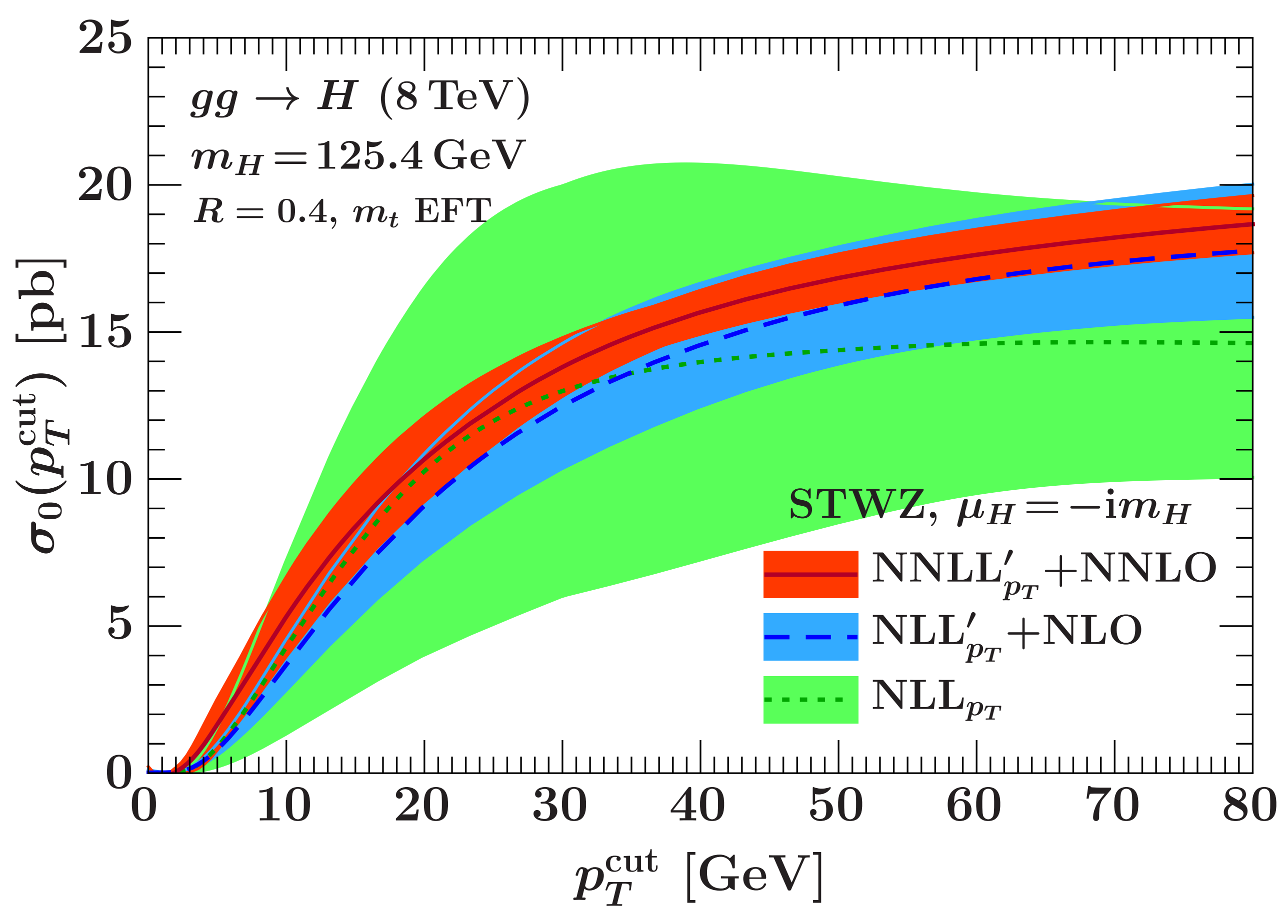
$pp \to H + $ 0-jets

(PDFs and Softs contribute, Glaubers?)



V= γ, W, or Z

veto anti-kT jets here



I.S., Tackmann, Walsh, Zuberi

Modes: $\lambda \ll 1$ large Q

can do calculations with back-to-back collinear particles, then generalize

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out these modes

Glaubers are offshell and must be integrated out (despite having $p^2 \sim \lambda^2$)

Otherwise one has problems with simultaneously having gauge invariant operators and homogeneous power counting

Modes: $\lambda \ll 1$ large Q

can do calculations with back-to-back collinear particles, then generalize

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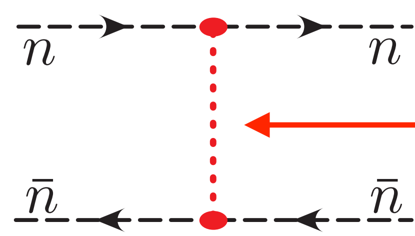
Integrate out

Need 3-types of Glauber momenta: $(+,-,\perp)$

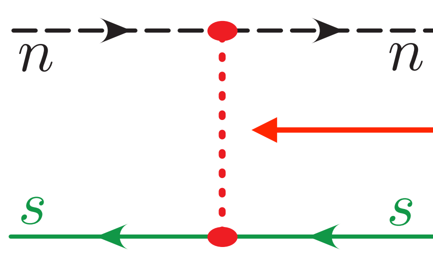
n-$\bar n$
fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$

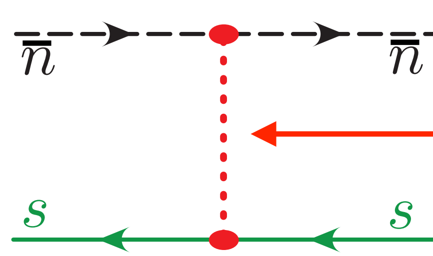
$s \gg t$

n-s
fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda, \lambda)$

$\bar n$-s
fwd. scattering



$p^\mu \sim Q(\lambda, \lambda^2, \lambda)$

(also scatter forward gluons)

Modes:

$\lambda \ll 1 \qquad \text{large } Q$

can do calculations with back-to-back collinear particles, then generalize

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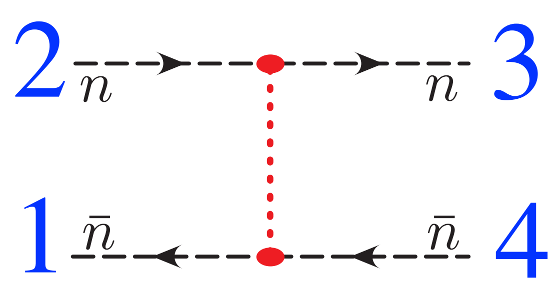
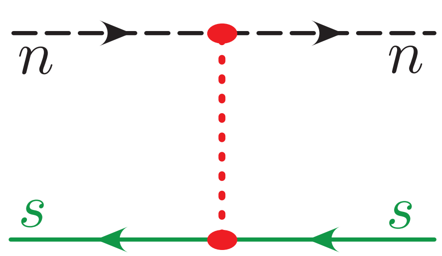


$$\bar n \cdot p_2 = \bar n \cdot p_3$$
$$n \cdot p_1 = n \cdot p_4$$

forward conditions

n-s
fwd. scattering



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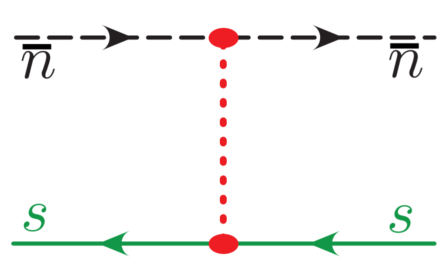
$\bar n$-s
fwd. scattering



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Modes:

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fwd. scattering

n-s
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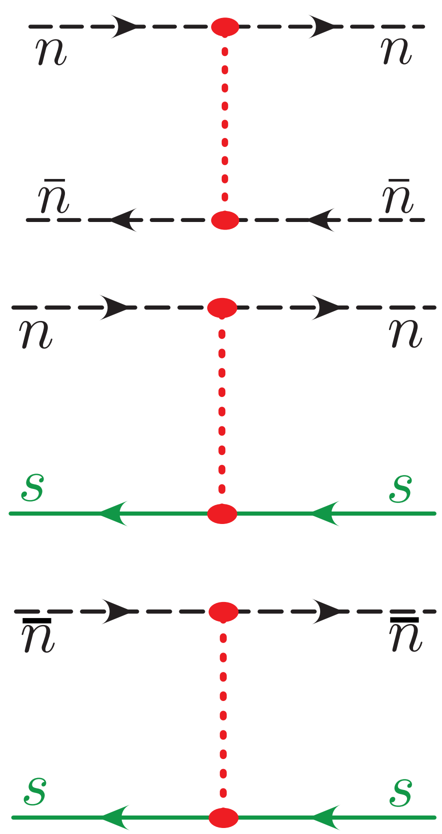
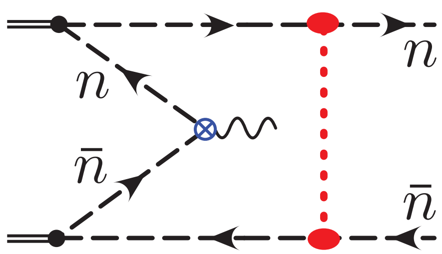
fwd. scattering



- $\dfrac{1}{k_\perp^2}$ potentials
- instantaneous in x^+, x^- (t and z)

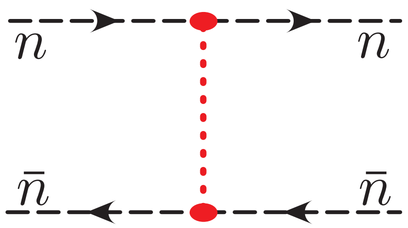
Glauber Exchange could violate factorization:



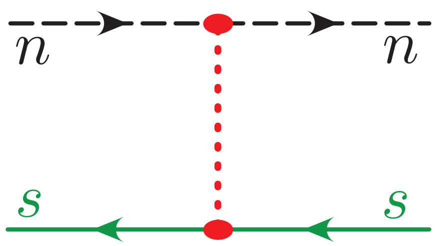
couples n-collinear, n̄-collinear, and soft modes

Glauber's dominate Forward Scattering:

n-$\bar{n}$
fwd. scattering



n-s
fwd. scattering



(small-x logs, reggeization, BFKL, BK/BJMWLK, …)

"Factorization Violation"

Phrase is used in different ways.

Factorization formula is invalid.

Reasons Factorization could fail:

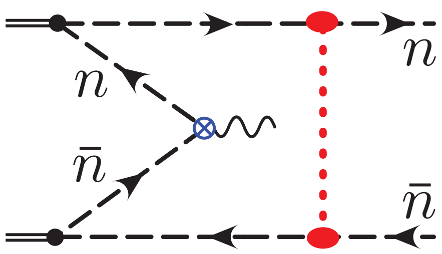
- Measurement doesn't factor: no simple factorization with universal functions. (eg. Jade algorithm)

- Divergent convolutions, not controlled by ones regulation procedures. (Requires more careful definition of functions.)

$$\int_0^1 \frac{dx}{x^2}\,\phi_\pi(x,\mu)$$

- Interactions that couple other modes and spoil factorization.

Glauber exchange



spectator-spectator cancel in proof for Drell-Yan

Glauber Related Examples of Factorization Violation

- Violation of Cross Section factorization, for example PDFs entangled

  $|pp\rangle, \text{ not } |p\rangle|p\rangle$ (Collins, Soper, Sterman; Bodwin; Bodwin, Brodsky, Lepage)

- Violation of Collinear Amplitude Factorization (Catani,de Florian,Rodrigo) (Forshaw, Seymour, Siodmok)

$$
\begin{aligned}
|\mathcal{M}^{(1)}(p_1,p_2,\dots,p_n)\rangle \;\simeq\; & \boldsymbol{Sp}^{(1)}(p_1,p_2;\widetilde{P};p_3,\dots,p_n)\;|\mathcal{M}^{(0)}(\widetilde{P},\dots,p_n)\rangle \\
+\; & \boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})\;|\mathcal{M}^{(1)}(\widetilde{P},\dots,p_n)\rangle\;.
\end{aligned}
$$

for space-like collinear limits (collinear incoming/outgoing particles)

- Violation of Regge Amplitude Factorization (Del Duca, Falcioni, Magnea, Vernazza; Glover, Duhr)

  $s \gg |t|$

$$
\mathcal{M}^{[8]}_{\rm rs}\left(\frac{s}{\mu^2},\frac{t}{\mu^2},\alpha_s\right) = 2\pi\alpha_s\, H^{(0),[8]}_{\rm rs}\; C_{\rm r}\left(\frac{t}{\mu^2},\alpha_s\right)\left[A_+\left(\frac{s}{t},\alpha_s\right) + \kappa_{\rm rs}\, A_-\left(\frac{s}{t},\alpha_s\right)\right] C_{\rm s}\left(\frac{t}{\mu^2},\alpha_s\right)
$$

$$
A_\pm\left(\frac{s}{t},\alpha_s\right) = \left(\frac{-s}{-t}\right)^{\alpha(t)} \pm \left(\frac{s}{-t}\right)^{\alpha(t)}
$$

violated at NNLL by $\dfrac{(i\pi)^2\,\alpha_s^2}{\epsilon^2}$

- Collinear Wilson Line universality fails.

  examples studied by Collins, Qiu, Mulders, Rogers, …

  $H_2, H_3 \simeq$ back-to-back

  $H_1 + H_2 \to H_3 + H_4 + X$

  pT dependent

SCET Glauber Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \overbrace{\Big[\mathcal{L}^{(0)}_{\mathrm{S}}(\psi_{\mathrm{S}}, A_{\mathrm{S}}) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})\Big]}^{\text{classic SCET}_{\text{II}}} + \mathcal{L}^{\mathrm{II}(0)}_{G}(\{\xi_{n_i}, A_{n_i}\}, \psi_{\mathrm{S}}, A_{\mathrm{S}})$$

If $\mathcal{L}^{(0)}_{G}$ does not contribute $\Longrightarrow$ can derive usual types of Factorization Formulae

(Power suppressed $\mathcal{L}^{(k\geq 1)}_{G}$ alone do not spoil factorization since they are only inserted a finite number of times.)

Goals for treating Glauber Operator in EFT:

- Hard Scattering and Forward Scattering in single framework
- Distinct Infrared Modes in Feyn. Graphs + Power Counting $\rightarrow$ derive when eikonal approximation is relevant
- $\overline{\mathrm{MS}}$ style renormalization for rapidity divergences (counterterms, renormalization group equations, ...)
- Sum Large Logs: $\ln\left(\frac{Q^2}{m^2}\right)$, $\ln(x)$
- Valid to all orders in α_s & clear path to study subleading power amplitudes with Glauber effects (subleading ops & Lagrangians)
- Factorization violating interactions may also have factorization formulae (could predict things about UE, etc.)
- Framework to (re)derive factorization theorems via $\cancel{+\mathcal{L}_G^{(0)}}$

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\rm II(0)} = \sum_{n,\bar n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar n}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

sum pairwise
on all collinears

sum on all
collinears

- Interactions with more sectors are given by T-products
- No Wilson coefficients ie. no new structures at loop level.

Uses SCET building blocks:

$$\chi_n = W_n^\dagger \xi_n \qquad \psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\left[W_n^\dagger i D_{n\perp}^\mu W_n\right] \qquad \mathcal{B}_{S\perp}^{n\mu} = \frac{1}{g}\left[S_n^\dagger i D_{S\perp}^\mu S_n\right] \qquad \widetilde{\mathcal{B}}_{S\perp}^{nAB} = -if^{ABC}\mathcal{B}_{S\perp}^{nC}$$

$$\widetilde{G}_s^{\mu\nu\,AB} = -if^{ABC} G_s^{\mu\nu\,A}$$

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar n}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

sum pairwise on all collinears

(3 rapidity sectors)

sum on all collinears

(2 rapidity sectors)

- Interactions with more sectors are given by T-products
- No Wilson coefficients ie. no new structures at loop level.

$$\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n \qquad \mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^{C} \frac{\bar n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar n}^{qB} = \overline{\chi}_{\bar n} T^B \frac{n\!\!\!/}{2} \chi_{\bar n} \qquad \mathcal{O}_{\bar n}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar n\perp\mu}^{C} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{\bar n\perp}^{D\mu}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} - \mathcal{S}_n^T \mathcal{S}_{\bar n} g \widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n} - \frac{n_\mu \bar n_\nu}{2} \mathcal{S}_n^T i g \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar n} \Big\}^{BC}$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar\psi_S^n T^B \frac{n\!\!\!/}{2} \psi_S^n \right) \qquad \mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

$$\mathcal{O}_s^{q_{\bar n} B} = 8\pi\alpha_s \left(\bar\psi_S^{\bar n} T^B \frac{\bar{n}\!\!\!/}{2} \psi_S^{\bar n} \right) \qquad \mathcal{O}_s^{g_{\bar n} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar nC} \frac{\bar n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar nD\mu} \right)$$

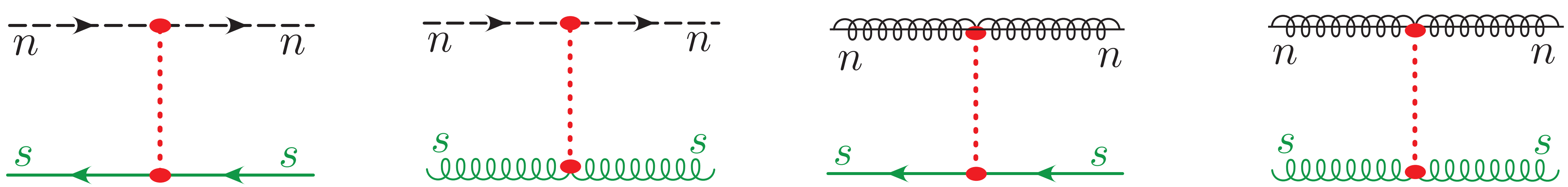
Construction: $\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

integrated out

n-s fwd. scattering $\quad s \gg t$

$$\lambda^2 = \frac{t}{s} \ll 1$$



determine $\quad \mathcal{O}(\lambda^3):\quad \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$

$\lambda^2 \quad \lambda^{-2} \quad \lambda^3$

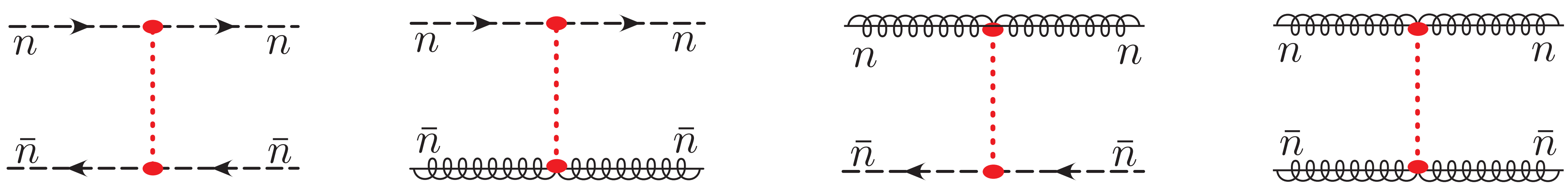
(2 rapidity sectors)

interaction between bilinear octet operators

Construction: $\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

n-$\bar{n}$ fwd. scattering $s \gg t$



actually $\mathcal{O}(\lambda^2)$:

(3 rapidity sectors)

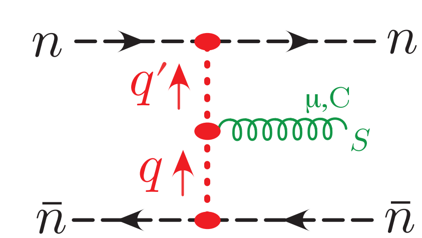
$$\sum_{i,j=q,g} \underset{\lambda^2}{\mathcal{O}_n^{iB}} \underset{\lambda^{-2}}{\frac{1}{\mathcal{P}_\perp^2}} \underset{\lambda^2}{\mathcal{O}_s^{BC}} \underset{\lambda^{-2}}{\frac{1}{\mathcal{P}_\perp^2}} \underset{\lambda^2}{\mathcal{O}_{\bar{n}}^{jC}}$$

same $\mathcal{O}_n^{iB}$

analogous $\mathcal{O}_{\bar{n}}^{jC}$

must allow for soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} + \ldots.$$



Soft $\mathcal{O}_s^{BC}$ Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu}, \qquad O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar n}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^n)(\mathcal{S}_n^T \mathcal{S}_{\bar n}) + (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n}) \cdot \mathcal{P}_\perp, \qquad O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n})(\mathcal{S}_{\bar n}^T \mathcal{S}_n) + (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^n) \cdot \mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})(\mathcal{S}_n^T \mathcal{S}_{\bar n})\mathcal{P}_\mu^\perp, \qquad O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})(\mathcal{S}_{\bar n}^T \mathcal{S}_n)\mathcal{P}_\mu^\perp,$$

$$O_7 = (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})\mathcal{S}_n^T \mathcal{S}_{\bar n}(g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n}), \qquad O_8 = (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})\mathcal{S}_{\bar n}^T \mathcal{S}_n(g\widetilde{\mathcal{B}}_{S\perp\mu}^{n}),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{\bar n}, \qquad O_{10} = \mathcal{S}_{\bar n}^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_n,$$

octet Wilson line

octet reps

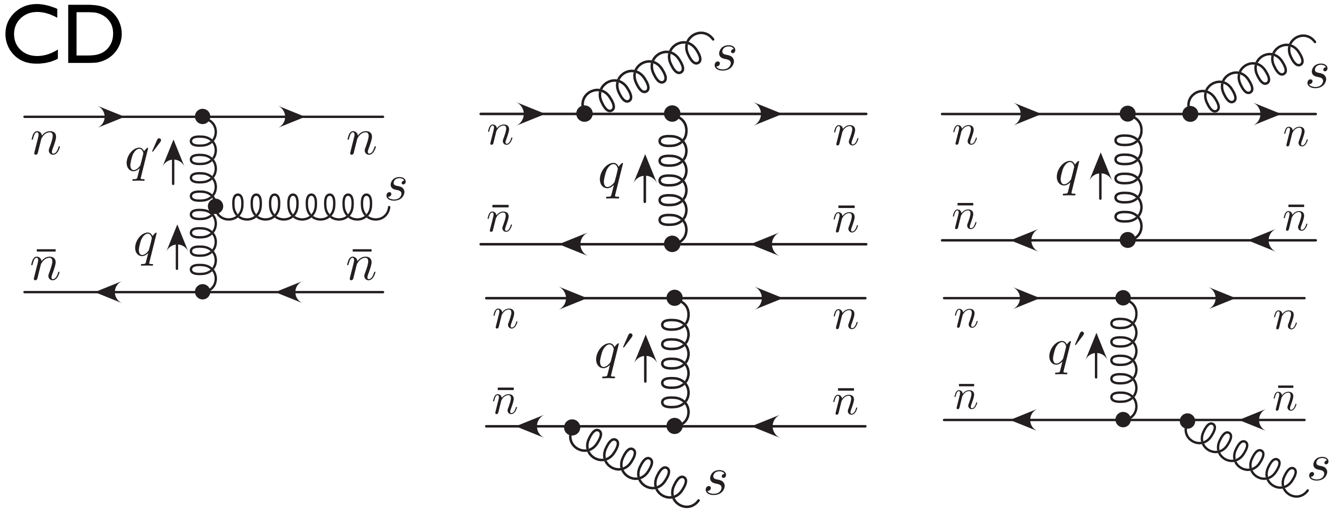
Restricted by: Hermiticity $O_i^\dagger\big|_{n\leftrightarrow\bar n} = O_i$, one $\mathcal{S}_n$, one $\mathcal{S}_{\bar n}$

operator identities: eg. $[\mathcal{P}_\perp^\mu(\mathcal{S}_n^T\mathcal{S}_{\bar n})] = -g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}(\mathcal{S}_n^T\mathcal{S}_{\bar n}) + (\mathcal{S}_n^T\mathcal{S}_{\bar n})g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}$

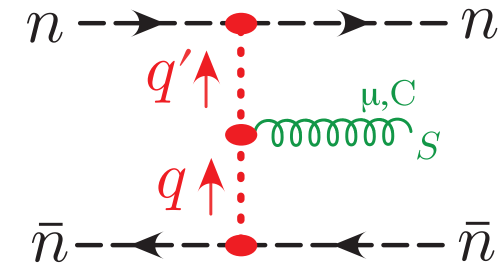
Matching with up to 2 soft gluons fixes all coefficients

One Soft Gluon:

QCD



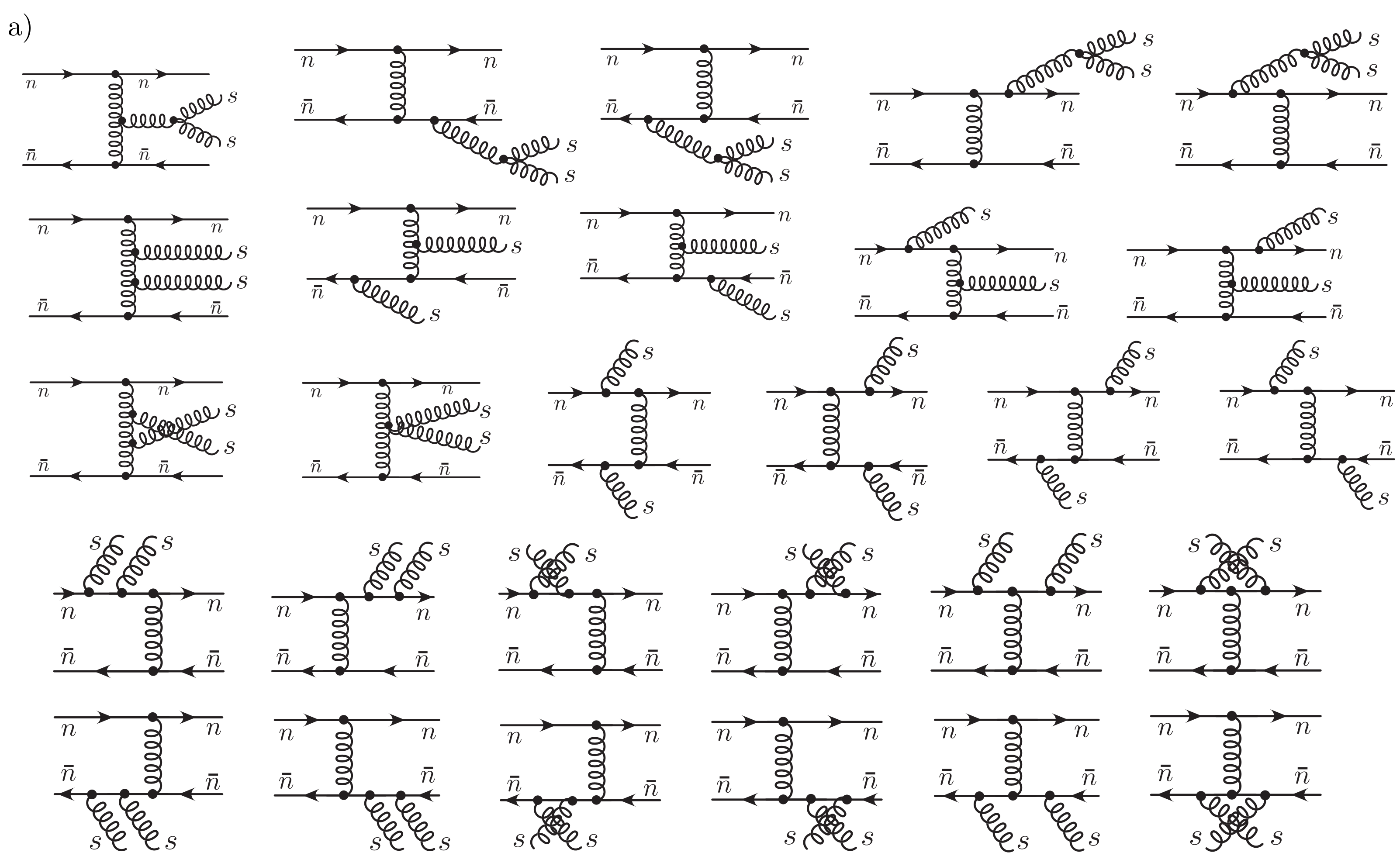
EFT



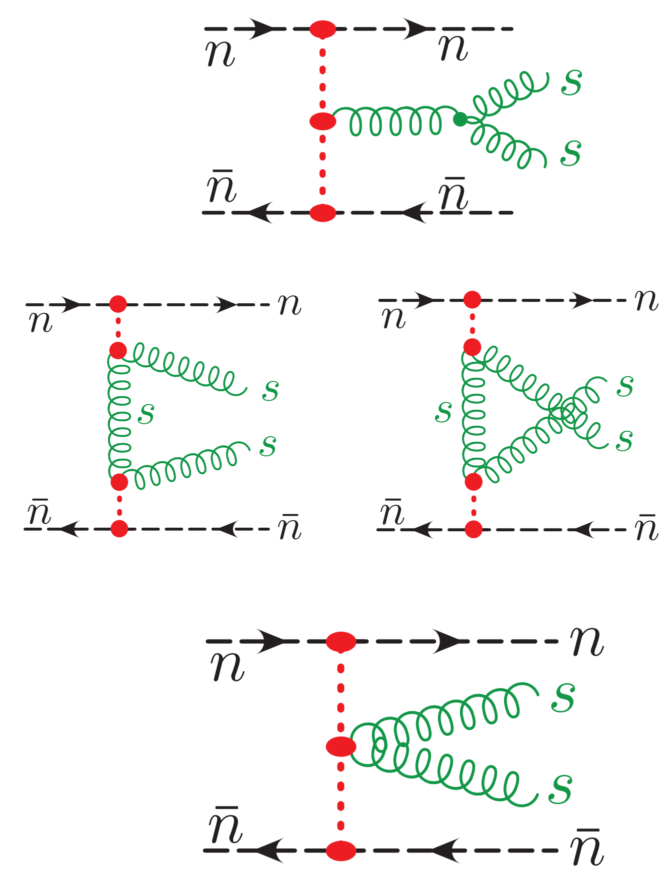
Lipatov vertex

Two Soft Gluons:

QCD



EFT



2-gluon vertex

Find:

$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0},$$

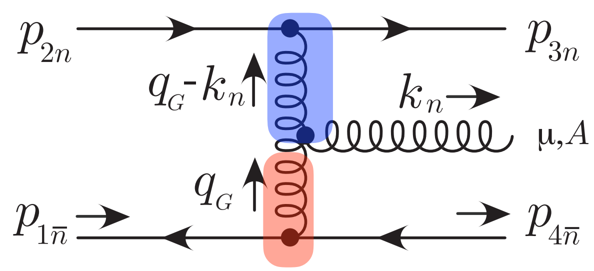
$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}$$

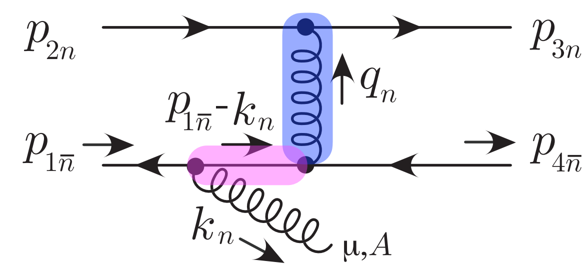
$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} - \mathcal{S}_n^T \mathcal{S}_{\bar n} g \widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n} - \frac{n_\mu \bar n_\nu}{2} \mathcal{S}_n^T i g \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar n} \Big\}^{BC} .$$

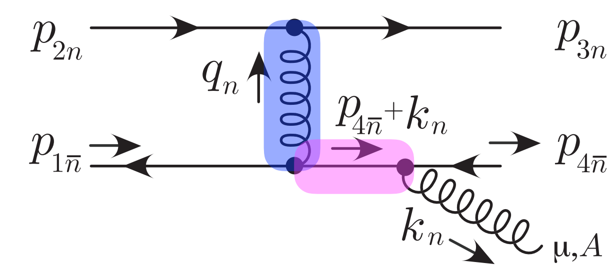
Wilson Lines in the operators are obtained from Matching:

eg. $W_n^\dagger$ in χ_n

QCD







offshell glauber

onshell collinear

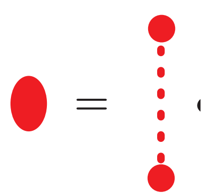
offshell hard

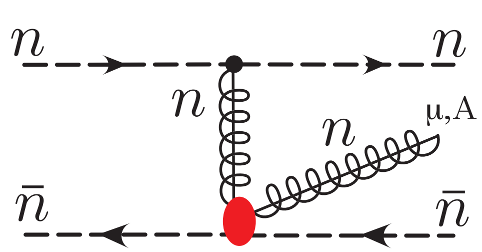
onshell collinear

offshell hard

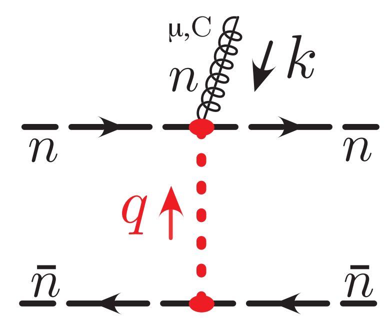
onshell collinear

SCET





has onshell collinear

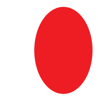
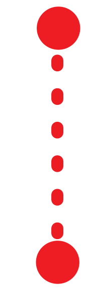


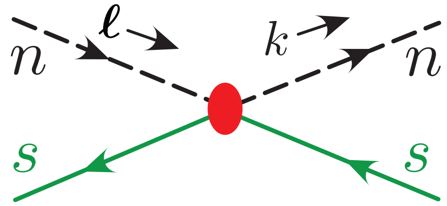
from W Wilson lines

just offshell Glauber potential

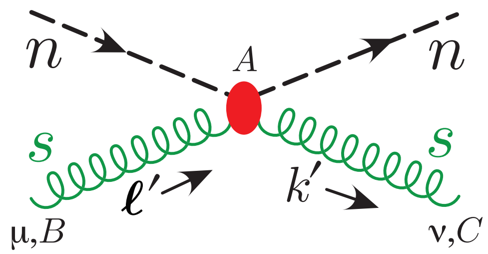
sums agree after using equations of motion

Feynman Rule examples:

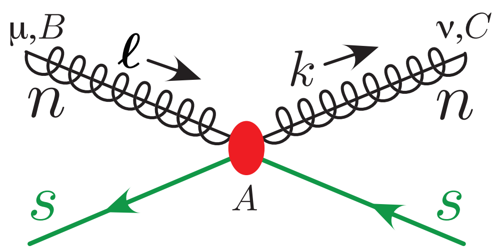
shorthand  $=$ 



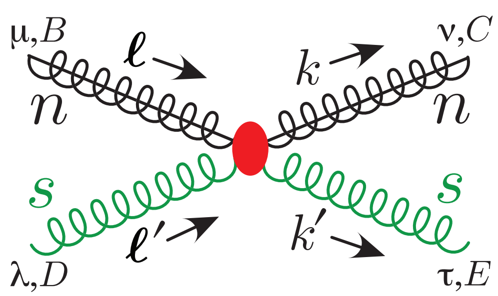
$$= \frac{-8\pi i\alpha_s}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\left[\bar{u}_n \frac{\bar{\slashed{n}}}{2} T^A u_n\right]\left[\bar{v}_s \frac{\slashed{n}}{2} \bar{T}^A v_s\right]$$



$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}'_\perp - \vec{k}'_\perp)^2}\left[\bar{u}_n \frac{\bar{\slashed{n}}}{2} T^A u_n\right]\left[n\cdot k'\, g_\perp^{\mu\nu} - n^\mu \ell'^\nu_\perp - n^\nu k'^\mu_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\mu n^\nu}{n\cdot k'}\right]$$

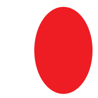
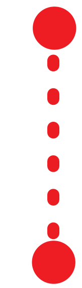


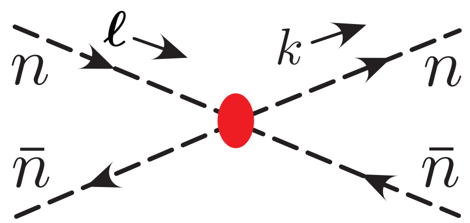
$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\left[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\right]\left[\bar{v}_s \frac{\slashed{n}}{2} \bar{T}^A v_s\right]$$



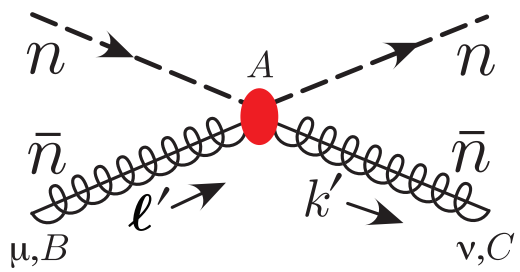
$$= \frac{8\pi i\alpha_s f^{ABC} f^{ADE}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\left[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\right]$$
$$\times\left[n\cdot k'\, g_\perp^{\lambda\tau} - n^\lambda \ell'^\tau_\perp - n^\tau k'^\lambda_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\lambda n^\tau}{n\cdot k'}\right]$$

Feynman Rule examples:

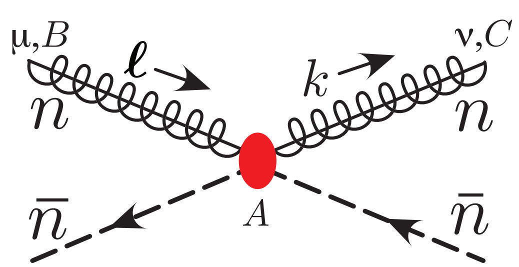
shorthand  $=$ 



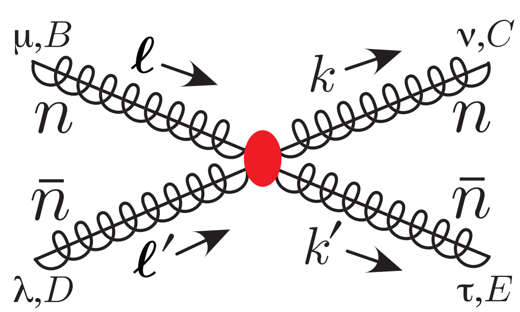
$$= \frac{-8\pi i\alpha_s}{(\vec{\ell}_\perp-\vec{k}_\perp)^2}\left[\bar{u}_n \frac{\bar{\not{n}}}{2} T^A u_n\right]\left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}}\right]$$



$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}'_\perp-\vec{k}'_\perp)^2}\left[\bar{u}_n \frac{\bar{\not{n}}}{2} T^A u_n\right]\left[n\cdot k'\, g_\perp^{\mu\nu} - n^\mu \ell'^\nu_\perp - n^\nu k'^\mu_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\mu n^\nu}{n\cdot k'}\right]$$

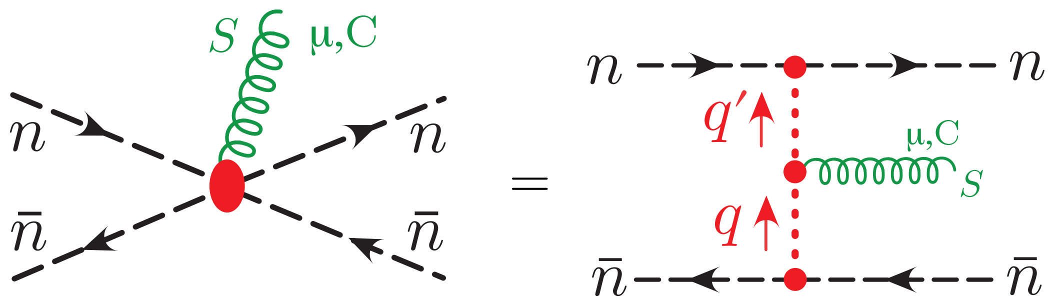


$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}_\perp-\vec{k}_\perp)^2}\left[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\right]\left[\bar{v}_{\bar{n}} \frac{\not{n}}{2} \bar{T}^A v_{\bar{n}}\right]$$



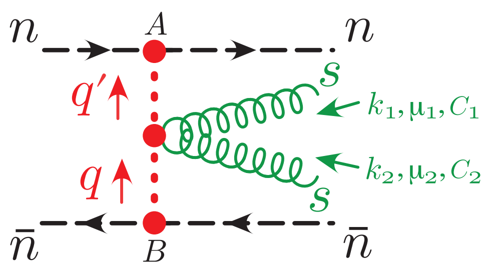
$$= \frac{8\pi i\alpha_s f^{ABC} f^{ADE}}{(\vec{\ell}_\perp-\vec{k}_\perp)^2}\left[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\right]$$
$$\times\left[n\cdot k'\, g_\perp^{\lambda\tau} - n^\lambda \ell'^\tau_\perp - n^\tau k'^\lambda_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\lambda n^\tau}{n\cdot k'}\right]$$

Feynman Rule examples:



Lipatov vertex

$$= i\Big[\bar{u}_n \frac{\bar{n}\!\!\!/}{2} T^A u_n\Big]\Big[\frac{8\pi\alpha_s}{\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2}}\, i g f^{ABC}\Big(q_\perp^\mu + q_\perp'^\mu - n\cdot q\,\frac{\bar{n}^\mu}{2} - \bar{n}\cdot q'\,\frac{n^\mu}{2} - \frac{n^\mu \vec{q}_\perp^{\,\prime 2}}{n\cdot q} - \frac{\bar{n}^\mu \vec{q}_\perp^{\,2}}{\bar{n}\cdot q'}\Big)\Big]\Big[\bar{v}_{\bar{n}} \frac{n\!\!\!/}{2} \bar{T}^B v_{\bar{n}}\Big]$$



$$= i\Big[\bar{u}_n \frac{\bar{n}\!\!\!/}{2} T^A u_n\Big]\Big[\bar{v}_{\bar{n}} \frac{n\!\!\!/}{2} \bar{T}^B v_{\bar{n}}\Big]\Big(\frac{8\pi\alpha_s}{\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2}}\Big)$$

$$\times\Big\{g^2 f^{C_1AE} f^{C_2BE}\Big[-g_\perp^{\mu_1\mu_2} - \frac{n^{\mu_1}(2q_\perp^{\mu_2}+k_{2\perp}^{\mu_2})}{n\cdot k_1} + \frac{(2q_\perp'^{\mu_1}-k_{1\perp}^{\mu_1})\bar{n}^{\mu_2}}{\bar{n}\cdot k_2} + \frac{\bar{n}^{\mu_1}n^{\mu_2}-n^{\mu_1}\bar{n}^{\mu_2}}{2}$$

$$+\frac{n^{\mu_1}\bar{n}^{\mu_2}}{n\cdot k_1\,\bar{n}\cdot k_2}\Big(\vec{q}_\perp\cdot\vec{q}_\perp^{\,\prime} + \vec{k}_{1\perp}\cdot\vec{k}_{2\perp} + \vec{k}_{1\perp}\cdot\vec{q}_\perp^{\,\prime} - \vec{k}_{2\perp}\cdot\vec{q}_\perp - \tfrac{1}{2}n\cdot k_2\,\bar{n}\cdot k_2 - \tfrac{1}{2}n\cdot k_1\,\bar{n}\cdot k_1\Big)$$

$$+n^{\mu_1}n^{\mu_2}\Big(\frac{\vec{q}_\perp^{\,\prime 2}}{n\cdot q\; n\cdot k_2} + \frac{\bar{n}\cdot k_2}{2n\cdot k_1}\Big) + \bar{n}^{\mu_1}\bar{n}^{\mu_2}\Big(\frac{-\vec{q}_\perp^{\,2}}{\bar{n}\cdot k_1\;\bar{n}\cdot q'} + \frac{n\cdot k_1}{2\bar{n}\cdot k_2}\Big)\Big]$$

$$+g^2 f^{C_2AE} f^{C_1BE}\Big[-g_\perp^{\mu_1\mu_2} + \frac{\bar{n}^{\mu_1}(2q_\perp'^{\mu_2}-k_{2\perp}^{\mu_2})}{\bar{n}\cdot k_1} - \frac{(2q_\perp^{\mu_1}+k_{1\perp}^{\mu_1})n^{\mu_2}}{n\cdot k_2} + \frac{n^{\mu_1}\bar{n}^{\mu_2}-\bar{n}^{\mu_1}n^{\mu_2}}{2}$$

$$+\frac{\bar{n}^{\mu_1}n^{\mu_2}}{n\cdot k_2\,\bar{n}\cdot k_1}\Big(\vec{q}_\perp\cdot\vec{q}_\perp^{\,\prime} + \vec{k}_{1\perp}\cdot\vec{k}_{2\perp} + \vec{k}_{2\perp}\cdot\vec{q}_\perp^{\,\prime} - \vec{k}_{1\perp}\cdot\vec{q}_\perp - \tfrac{1}{2}n\cdot k_2\,\bar{n}\cdot k_2 - \tfrac{1}{2}n\cdot k_1\,\bar{n}\cdot k_1\Big)$$

$$+n^{\mu_1}n^{\mu_2}\Big(\frac{\vec{q}_\perp^{\,\prime 2}}{n\cdot q\; n\cdot k_1} + \frac{\bar{n}\cdot k_1}{2n\cdot k_2}\Big) + \bar{n}^{\mu_1}\bar{n}^{\mu_2}\Big(\frac{-\vec{q}_\perp^{\,2}}{\bar{n}\cdot k_2\;\bar{n}\cdot q'} + \frac{n\cdot k_2}{2\bar{n}\cdot k_1}\Big)\Big]\Big\}$$

2 gluon vertex

plus analogs replacing collinear quarks by gluons

- Requires rapidity regulator for Glauber potential $|2k^z|^{-\eta}\nu^{\eta}$ and for Wilson lines

$$S_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{n\cdot\mathcal{P}}\left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]\right\} \qquad W_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar{n}\cdot\mathcal{P}}\right] \frac{w^2|\bar{n}\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\left]\right\}$$

(ala Chiu, Jain, Neill, Rothstein) $\nu\frac{\partial}{\partial\nu}w^2(\nu) = -\eta\, w^2(\nu)\,, \qquad \lim_{\eta\to 0} w(\nu) = 1$

- Zero-bin subtractions, avoid double counting IR regions

(ala Manohar & IS)

eg. 1-loop SCET$_{\mathrm{II}}$ graphs:

$$S = \tilde{S} - S^{(G)}$$

naive soft graph

glauber limit of soft graph

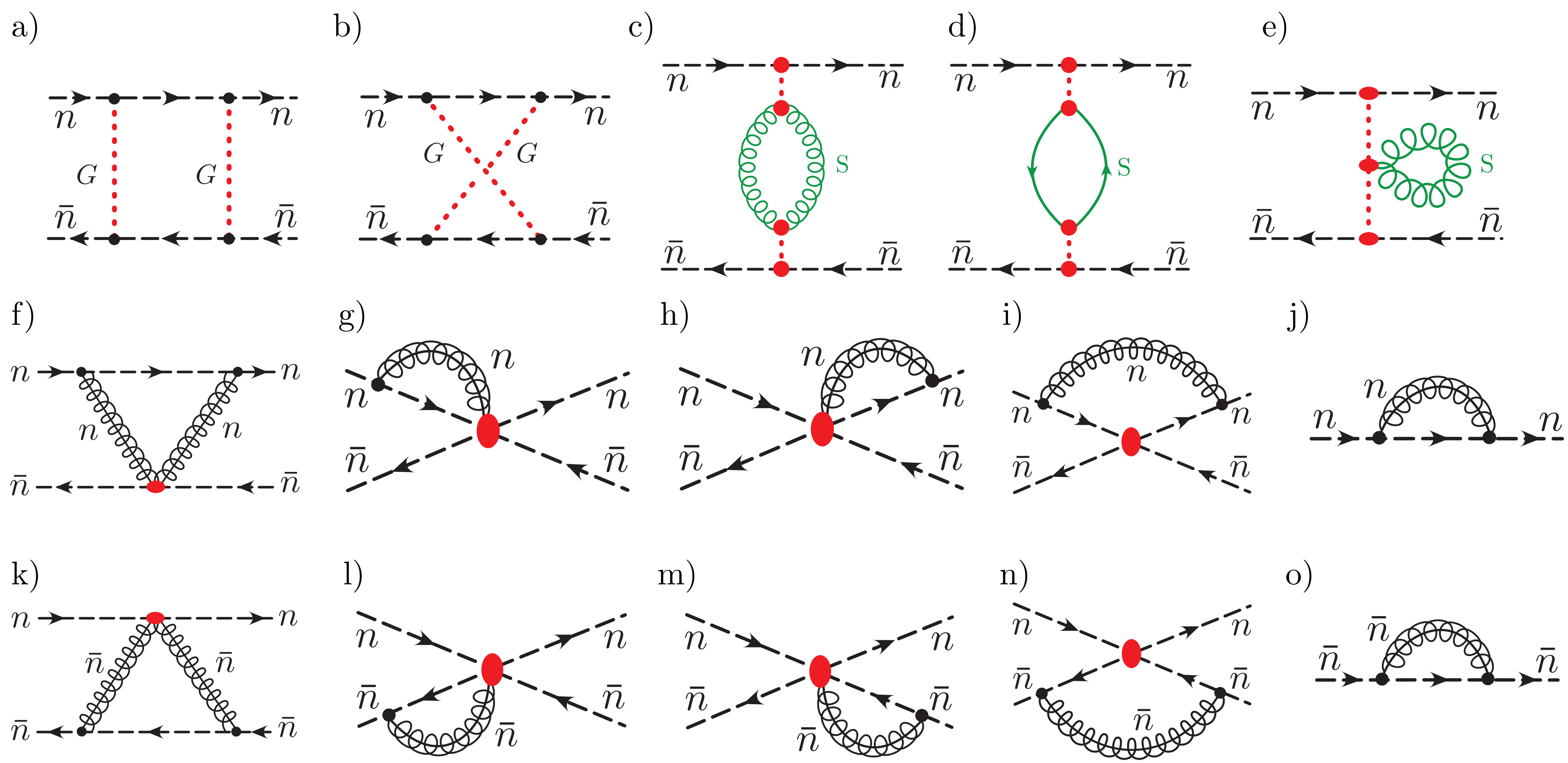
$$C_n = \tilde{C}_n - C_n^{(S)} - C_n^{(G)} + C_n^{(GS)}$$

naive collinear graph

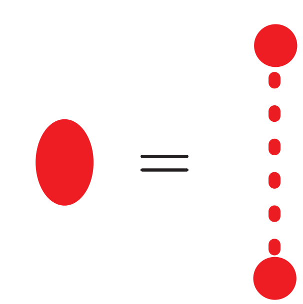
glauber limit of collinear graph

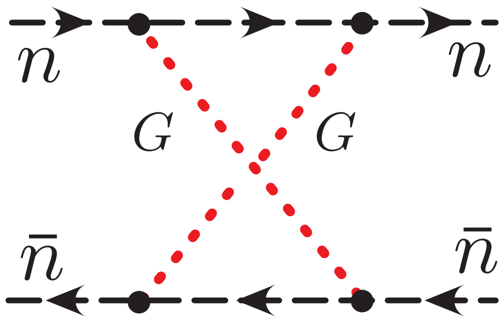
eq. One Loop $q\bar{q}$ scattering

Leading Power EFT graphs (Glauber, Soft, & Collinear Loops)



shorthand

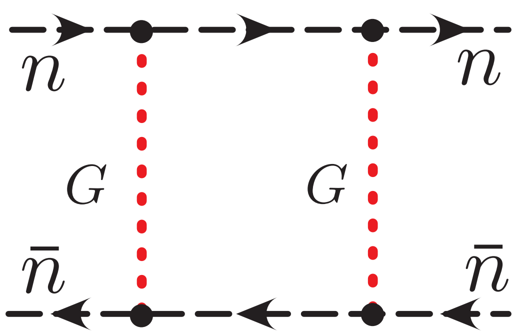




$$I_{\mathrm{Gcbox}} = \int \frac{\text{đ}^{d-2}k_\perp \; \text{đ}k^0 \; \text{đ}k^z \; |k^z|^{-2\eta} (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ - \Delta_1(k_\perp) + i0\Big)\Big(k^- - \Delta_2(k_\perp) + i0\Big)}$$

$$= 0$$

k^0 poles on same side



$$I_{\mathrm{Gbox}} = \int \frac{\text{đ}^{d-2}k_\perp \; \text{đ}k^0 \; \text{đ}k^z \; |k^z|^{-2\eta} (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ - \Delta_1(k_\perp) + i0\Big)\Big(-k^- - \Delta_2(k_\perp) + i0\Big)}$$

$$= -i \int \frac{\text{đ}^{d-2}k_\perp \text{đ}k^z \; |k^z|^{-2\eta} \; (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 (-2k^z - \Delta + i0)}$$

$$= \quad \ldots$$

$$= \left(\frac{-i}{4\pi}\right) \int \frac{\text{đ}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2} \Big[-i\pi + \mathcal{O}(\eta) \Big]$$

generic, Glauber phase

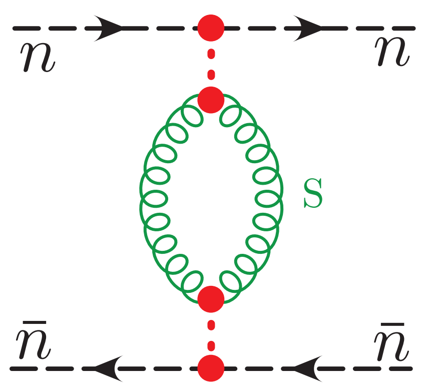
independent of Δ hence collinears behave like eikonal propagators here (but not always!)

Similar for other Glauber boxes (softs, gluons, ...).

eg. Rapidity divergences from Wilson lines

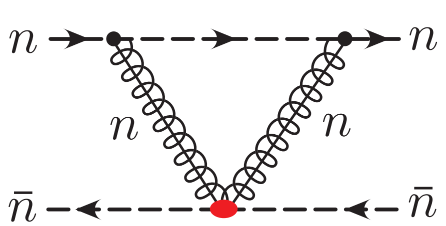
$\mu^2 \sim \nu^2 \sim -t$

rapidity divergent

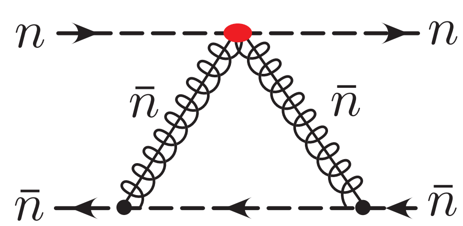


$$= -\frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t) + \frac{4}{\epsilon^2} + \frac{4}{\epsilon}\ln\Big(\frac{\mu^2}{\nu^2}\Big) + \boxed{4\ln\Big(\frac{\mu^2}{\nu^2}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)} - 2\ln^2\Big(\frac{\mu^2}{-t}\Big) + \frac{\pi^2}{3}$$
$$+ 2\Big(-\frac{11}{3\epsilon} - \frac{11}{3}\ln\frac{\mu^2}{-t} - \frac{67}{9}\Big)\Big\}$$

β -function (without ghosts)



+



(contribution to impact factors)

$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big[\Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t) - \frac{4}{\epsilon}\ln\Big(\frac{\nu}{\bar{n}\cdot p_3}\Big) - 4\ln\Big(\frac{\nu}{\bar{n}\cdot p_3}\Big)\ln\Big(\frac{\mu^2}{-t}\Big) - \frac{3}{\epsilon} - 3\ln\Big(\frac{\mu^2}{-t}\Big) - 6 + \frac{4\pi^2}{3}\Big\}$$
$$+ \Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t) - \frac{4}{\epsilon}\ln\Big(\frac{\nu}{n\cdot p_4}\Big) - 4\ln\Big(\frac{\nu}{n\cdot p_4}\Big)\ln\Big(\frac{\mu^2}{-t}\Big) - \frac{3}{\epsilon} - 3\ln\Big(\frac{\mu^2}{-t}\Big) - 6 + \frac{4\pi^2}{3}\Big\}\Big]$$
$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t) - \frac{4}{\epsilon}\ln\Big(\frac{\nu^2}{s}\Big) - \boxed{4\ln\Big(\frac{\nu^2}{s}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)} - \frac{6}{\epsilon} - 6\ln\Big(\frac{\mu^2}{-t}\Big) - 12 + \frac{8\pi^2}{3}\Big\}.$$

opposite sign for this rapidity divergence

$\mu^2 \sim -t$

$\nu^2 \sim s$

One Loop Results & Matching

$$\mathcal{S}_1^{n\bar n} = -\Big[\bar u_n T^A T^B \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \bar T^B \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_2^{n\bar n} = C_F\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,,$$

$$\mathcal{S}_3^{n\bar n} = C_A\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_4^{n\bar n} = T_F n_f\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,.$$

$$\text{Glauber Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar n}\Big[8i\pi \ln\Big(\frac{-t}{m^2}\Big)\Big]$$

m = gluon mass IR regulator

$$\text{Soft Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{-\frac{8}{\eta}h(\epsilon,\mu^2/m^2) - \frac{8}{\eta}g(\epsilon,\mu^2/t) - 4\ln\Big(\frac{-t}{\nu^2}\Big)\ln\Big(\frac{m^2}{-t}\Big) - 2\ln^2\Big(\frac{-t}{m^2}\Big) - \frac{2\pi^2}{3} + \frac{22}{3}\ln\frac{\mu^2}{-t} + \frac{134}{9}\bigg\} + \frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\Big[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big) - \frac{40}{9}\Big].$$

no $1/\epsilon$ poles

(after coupling renormalization)

$$\text{Collinear Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{\frac{8}{\eta}h\Big(\epsilon,\frac{\mu^2}{m^2}\Big) + \frac{8}{\eta}g\Big(\epsilon,\frac{\mu^2}{-t}\Big) + 4\ln\Big(\frac{\nu^2}{s}\Big)\ln\Big(\frac{-t}{m^2}\Big) + 2\ln^2\Big(\frac{m^2}{-t}\Big) + 4 + \frac{4\pi^2}{3}\bigg\} + \frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\Big[-4\ln^2\Big(\frac{m^2}{-t}\Big) - 12\ln\Big(\frac{m^2}{-t}\Big) - 14\Big]$$

$$\text{Total SCET} = \text{Total QCD}\ (s \gg t)$$

- IR divergences reproduced, <u>no hard matching</u>
- offshell lines in loop graphs are sequestered

No loop corrections to $\mathcal{L}_G^{\mathrm{II}(0)}$

Applications

To illustrate how (familiar) results
emerge in this formalism

Gluon Reggeization

$\nu \sim \sqrt{s}$

$$\mathcal{O}_n^A \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_S^{AB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^B$$

$\nu \sim \sqrt{-t}$

Consider separate rapidity renormalization of soft & collinear component operators

Either run collinear operators from $\nu \sim \sqrt{s}$ to $\nu \sim \sqrt{-t}$, or run soft operator.

$$\nu \frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$\gamma_{n\nu} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$

(IR divergent)

gives: $\left(\frac{s}{-t}\right)^{-\gamma_{n\nu}}$

virtual anom.dim. is Regge exponent for gluon

Forward Scattering & BFKL

Expand time evolution, do soft-collinear factorization term by term:

$$T\exp i\int d^4x\,\mathcal{L}_G^{\mathrm{II}(0)}(x) = \left[1 + i\int d^4y_1\,\mathcal{L}_G^{\mathrm{II}(0)}(y_1) + \frac{i^2}{2!}T\int d^4y_1\,d^4y_2\,\mathcal{L}_G^{\mathrm{II}(0)}(y_1)\mathcal{L}_G^{\mathrm{II}(0)}(y_2) + \ldots\right]$$

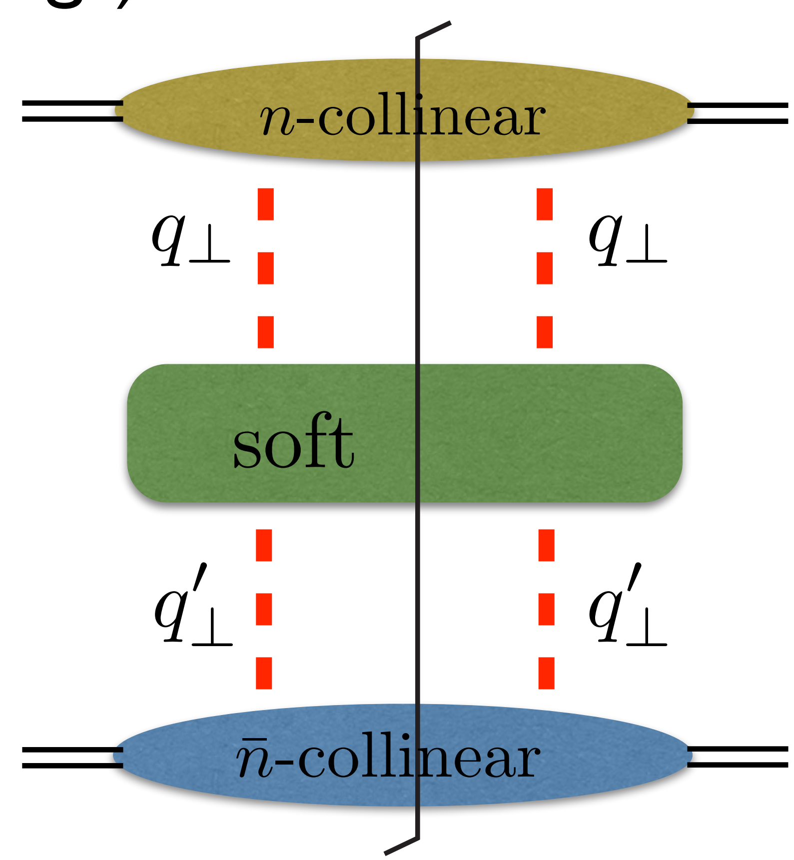
$$\sim 1 + T\sum_{k=1}^{\infty}\sum_{k'=1}^{\infty}\left[\mathcal{O}_n^{jA_i}(q_{i\perp})\right]^k\left[\mathcal{O}_{\bar{n}}^{j'B_{i'}}(q_{i'\perp})\right]^{k'}\otimes O_{s(k,k')}^{A_1\cdot A_k,B_1\cdots B_{k'}}(q_{\perp 1},\ldots,q_{\perp k'})$$

$$\equiv 1 + \sum_{k=1}^{\infty}\sum_{k'=1}^{\infty}U_{(k,k')}$$

Consider (linearized) forward scattering with one Glauber exchange, but all orders in other interactions (eg. leading logs):

$$T_{(1,1)} = \frac{1}{V_4}\sum_X \langle pp'|U_{(1,1)}^{\dagger}|X\rangle\langle X|U_{(1,1)}|pp'\rangle \quad = \ldots$$

$$= \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp,p^-)S_G(q_\perp,q'_\perp)C_{\bar{n}}(q'_\perp,p'^+)$$



after rapidity renormalization:

$$T_{(1,1)} = \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp,p^-,\nu)S_G(q_\perp,q'_\perp,\nu)C_{\bar{n}}(q'_\perp,p'^+,\nu)$$

collinear and soft functions

n-collinear function:

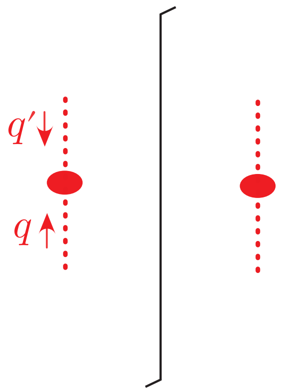
$$\frac{1}{V_1}\sum_{X_n}\Big\langle p\Big|\sum_{j=q,g}\mathcal{O}^{jA'}_{n,k'^-}(q''_\perp)(\tilde{x}'')\Big|X_n\Big\rangle\Big\langle X_n\Big|\sum_{i=q,g}\mathcal{O}^{iA}_{n,k^-}(q_\perp)(\tilde{x})\Big|p\Big\rangle$$

$$= \delta^{AA'}\, 2\delta(x^+)\delta(x''^+)\delta^2(q_\perp - q''_\perp)\, \vec{q}_\perp^{\,2}\, C_n(q_\perp, p^-, x^-, x''^-)$$

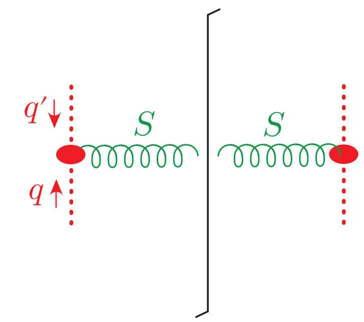
Soft function:

$$S_G(q_\perp, q'_\perp) = \frac{1}{V_2}\frac{\delta^{AA'}\delta^{BB'}}{(\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2})}\sum_X \langle 0|O^{AB}_{s(1,1)}(q_\perp, q'_\perp)|X\rangle\langle X|O^{\dagger A'B'}_{s(1,1)}(q_\perp, q'_\perp)|0\rangle$$

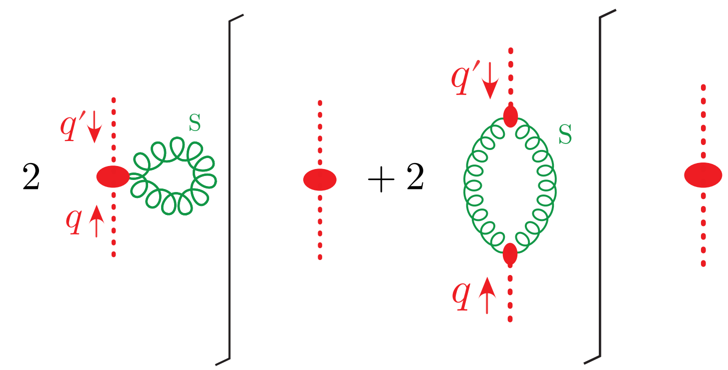
Consider rapidity renormalization for soft function that appears here:



$$= S_G^{(0)}(q_\perp, q'_\perp)$$
$$= (8\pi\alpha_s)^2 \delta^{AA} \, (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp)$$



rapidity divergent
real emission



rapidity divergent
virtual

cancel $1/\eta$ divergences

$$S_G(\vec{q}_\perp, \vec{q}'_\perp, \nu) = \int d^2k_\perp \; Z_{S_G}(q_\perp, k_\perp) \; S_G^{\rm bare}(k_\perp, q'_\perp) \qquad\qquad 0 = \nu \frac{d}{d\nu} S_G^{\rm bare}(q_\perp, q'_\perp)$$

$$\nu \frac{d}{d\nu} S_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp \; \gamma_{S_G}(q_\perp, k_\perp) \; S_G(k_\perp, q'_\perp, \nu)$$
$$= \frac{2C_A\alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} \; S_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 \, (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

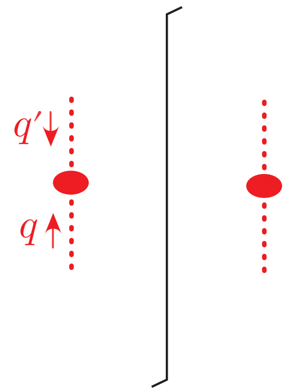
evolution given by BFKL equation

(see also work by S. Fleming)

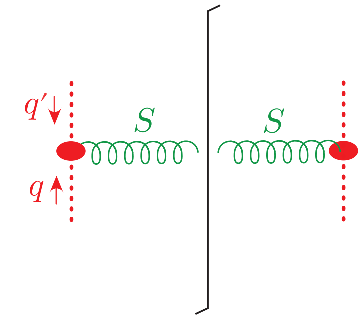
Sum usual LL: $\alpha_s^k \ln^k\left(\frac{s}{-t}\right)$ in forward cross-section

[or ln(x) in DIS]

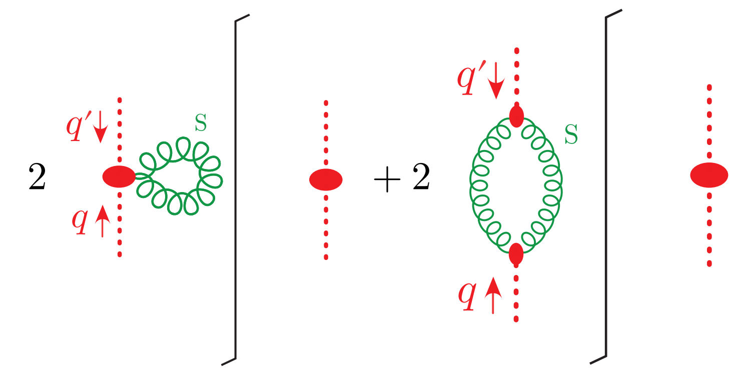
Consider rapidity renormalization for soft function that appears here:



$$= S_G^{(0)}(q_\perp, q'_\perp)$$
$$= (8\pi\alpha_s)^2 \delta^{AA} \, (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}\,'_\perp)$$



rapidity divergent
real emission



rapidity divergent
virtual

$$S_G(\vec{q}_\perp, \vec{q}\,'_\perp, \nu) = \int d^2k_\perp \, Z_{S_G}(q_\perp, k_\perp) \, S_G^{\rm bare}(k_\perp, q'_\perp) \qquad\qquad 0 = \nu \frac{d}{d\nu} S_G^{\rm bare}(q_\perp, q'_\perp)$$

$$\nu \frac{d}{d\nu} S_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp \, \gamma_{S_G}(q_\perp, k_\perp) \, S_G(k_\perp, q'_\perp, \nu)$$

$$= \frac{2C_A\alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} \, S_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

evolution given by BFKL equation

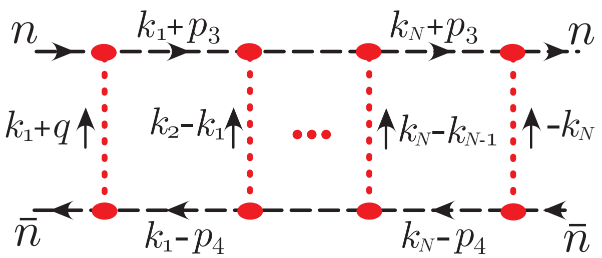
RGE consistency:

$$\nu \frac{d}{d\nu} C_n(q_\perp, p^-, \nu) = -\frac{C_A\alpha_s}{\pi^2} \int d^2k_\perp \left[\frac{C_n(k_\perp, p^-, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} \, C_n(q_\perp, p^-, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right] \qquad -\frac{1}{2}(\text{BFKL})$$

Eikonal Scattering

Rapidity regulator consistent with eikonal phase

Sum up Glauber Boxes



$$= i(-2g^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\int \frac{đk_1^z\cdots đk_N^z\,\big|2k_1^z(2k_1^z-2k_2^z)\cdots(2k_{N-1}^z-2k_N^z)2k_N^z\big|^{-\eta}\nu^{N\eta}}{2^N(-k_1^z+\Delta_1+i0)\cdots(-k_N^z+\Delta_N+i0)}$$

Fourier transform k_i^z :

$$= 2(-ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\Big(\kappa_\eta\frac{\eta}{2}\Big)^{N+1}\int_{-\infty}^{+\infty}\Big[\prod_{j=1}^{N+1} dx_j\,|x_j|^{-1+\eta}\Big]\ \theta(x_2-x_1)\theta(x_3-x_2)\cdots\theta(x_{N+1}-x_N)\exp\Big[\sum_{m=1}^{N} i\Delta_m(x_{m+1}-x_m)\Big]$$

need $x_j \to 0$

ordered collapse to equal longitudinal postion

$$= -2(ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)} I_\perp^{(N)}(q_\perp)\ \frac{1}{(N+1)!}\Big[1+\mathcal{O}(\eta)\Big]$$

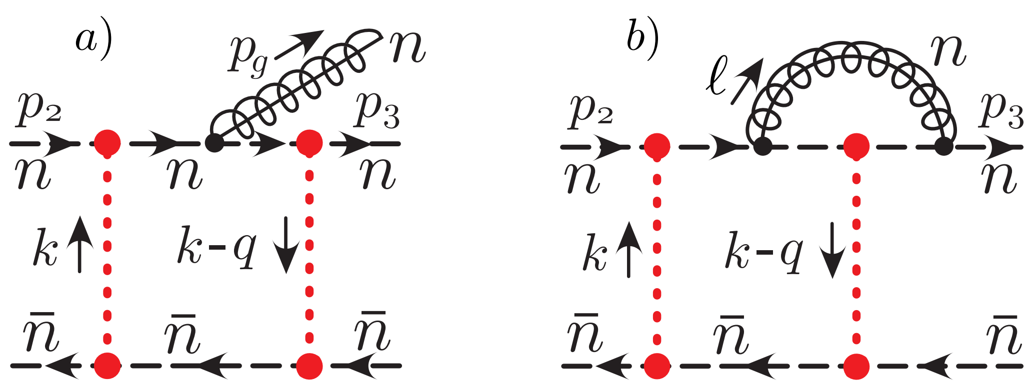
Fourier transform $q_\perp$: $\displaystyle\int đ^{d-2}q_\perp\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp}\sum_{N=0}^{\infty}\mathrm{G.Box}_N^{2\to 2}(q_\perp) = (\tilde{G}(b_\perp)-1)2\mathcal{S}^{n\bar{n}}$

gives classic eikonal scattering result: $\boxed{\tilde{G}(b_\perp) = e^{i\phi(b_\perp)}}$

$$\phi(b_\perp) = -\mathbf{T}_1^A\otimes\mathbf{T}_2^A\, g^2(\mu)\int\frac{đ^{d-2}q_\perp\,(\iota^\epsilon\mu^{2\epsilon})}{\vec{q}_\perp^{\,2}}\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp}$$

Multi-Glauber properties

- When do graphs vanish?
- When do propagators eikonalize?



an interrupted collapse gives a vanishing graph

$$(\text{pre})\int \!\!d\!\!{}^{-}\!{}^{d}k\, \frac{\left(|k_z|^{-2\eta}\,\nu^{2\eta}\right)\,\mathrm{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp-\vec{q}_\perp)^2\,[n\cdot k-\Delta_1+i0]\,[n\cdot k-\Delta_2+i0]\,[\bar{n}\cdot k+\bar{\Delta}_1'-i0]} = 0$$

or

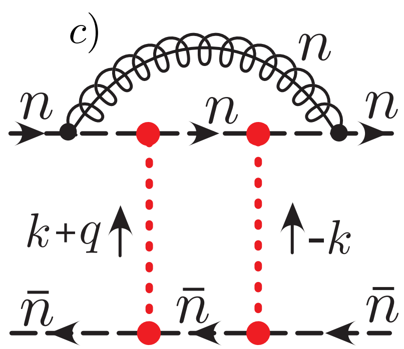
$$= -\frac{i}{4}\Big(\kappa_\eta\frac{\eta}{2}\Big)^2\int \!\!d\!\!{}^{-}\!{}^{d-2}k\int_{-\infty}^{+\infty}\! dx_1 dx_2 d\alpha\,\theta(x_1-\alpha)\theta(\alpha-x_2)|x_1x_2|^{-1+\eta}\big[1+\mathcal{O}(\eta)\big]$$
$$= \mathcal{O}(\eta)$$

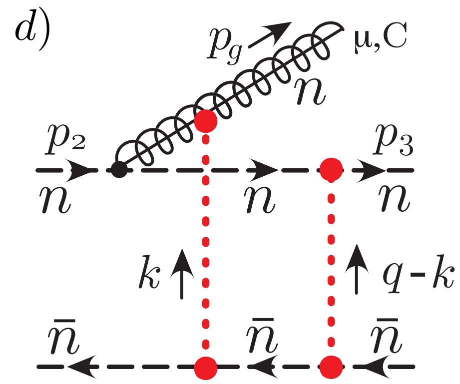
- need unimpeded exchanges:

Graphs with more than one Glauber exchange will vanish unless the exchanges can be moved towards each other unimpeded, so that they all occur at the same longitudinal position x_0 for both sources.

Multi-Glauber properties

- When do graphs vanish?
- When do propagators eikonalize?





nonzero

eikonal

$$(\text{pre})\int d\!\!{}^-{}^d k \, \frac{(|k_z|^{-2\eta}\,\nu^{2\eta})\ \mathrm{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp-\vec{q}_\perp)^2\,[n\cdot k+\Delta_1'-i0]\,[n\cdot k-\Delta_1+i0]\,[\bar{n}\cdot k+\bar{\Delta}_1'-i0]}$$

$$=\frac{(\text{pre})}{4}\int d\!\!{}^-{}^{d-2}k_\perp\,\frac{\mathrm{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp-\vec{q}_\perp)^2(\Delta_1+\Delta_1'-i0)} \qquad \neq 0$$

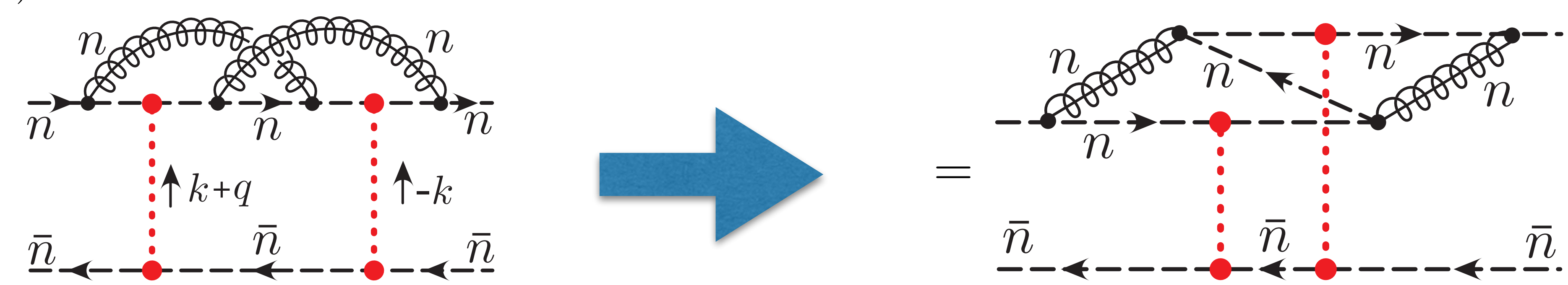
partly non-eikonal

$$\Delta_1=\frac{(\vec{k}_\perp+\vec{p}_{3\perp}-\vec{q}_\perp)^2}{\bar{n}\cdot p_3}-n\cdot p_3$$

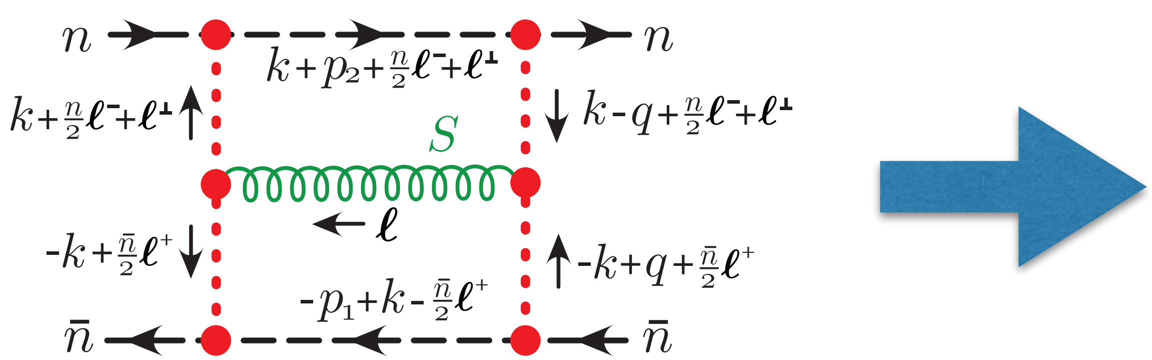
$$\Delta_1'=\frac{(\vec{k}_\perp-\vec{p}_{g\perp})^2}{\bar{n}\cdot p_g}-n\cdot p_g$$

Multi-Glauber rules

e)



nonzero



nonzero

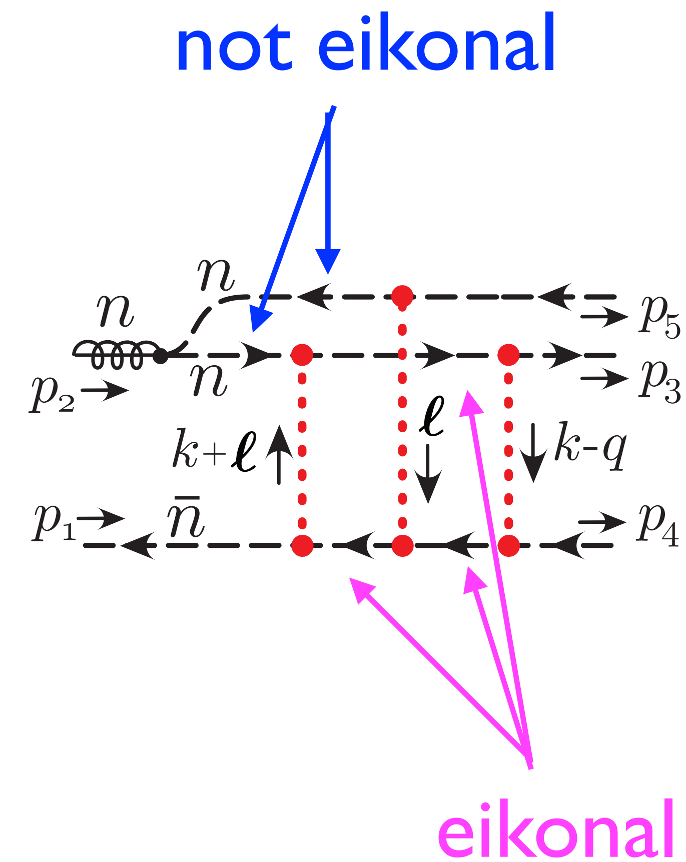
$$\ell^\mu \sim \lambda$$

$$k_\perp \sim \lambda$$

$$k^\pm \sim \lambda^2$$

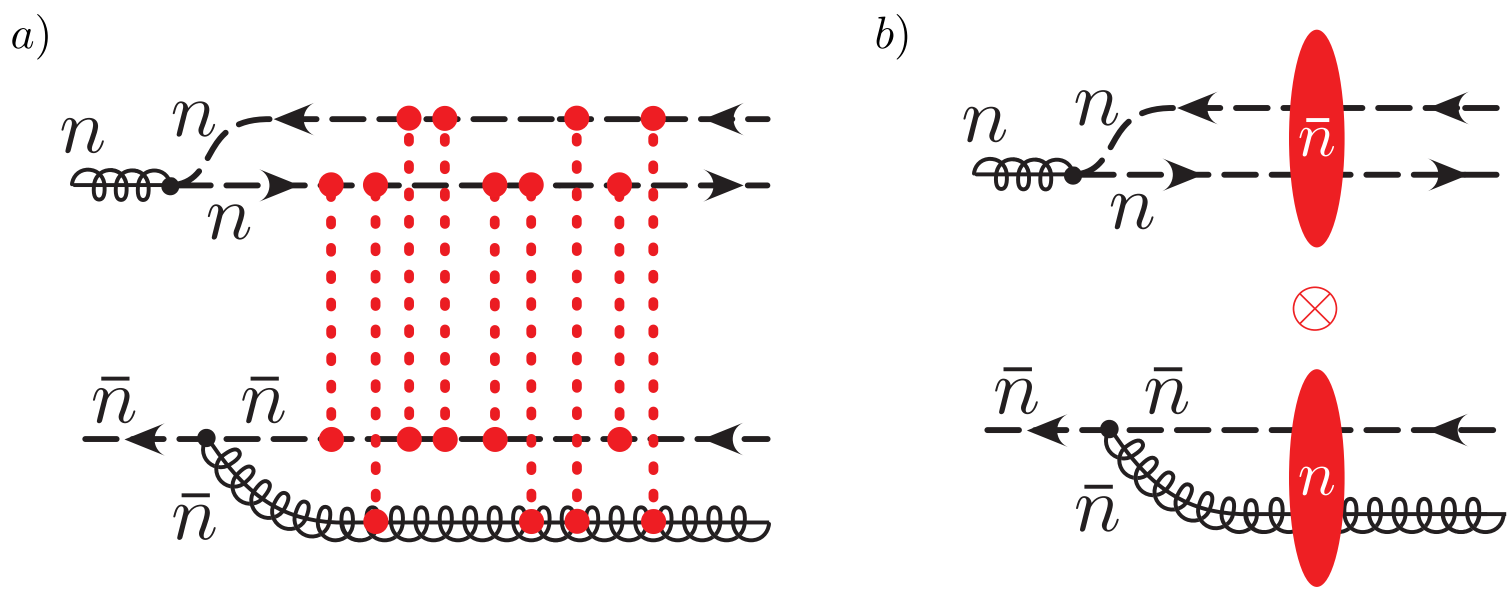
- unimpeded exchanges

Multi-Glauber rules



- unimpeded exchanges
- propagators in Glauber loops are effectively eikonal if and only if they are naively log-divergent

Together these two rules lead to the picture of multiple eikonal Wilson lines crossing a shockwave:



connection to:

Balitsky's High Energy EFT (Wilson Lines),

setup for B-JIMWLK evolution, etc.

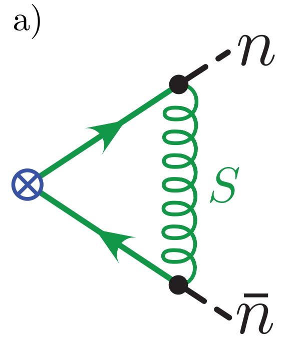
Hard Scattering $\otimes$

The Cheshire Glauber

e.g. $J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$ **Active-Active and Soft Overlap**

naive soft:

a)



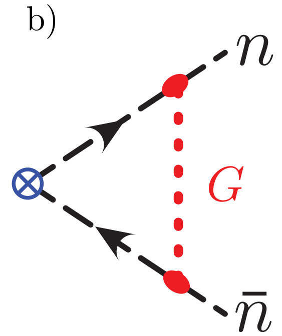
$$\tilde{S} = -2ig^2 C_F \mathcal{S}_\Gamma \int đ^d k \, \frac{(\iota^\epsilon \mu^{2\epsilon} |k_z|^{-\eta} \nu^\eta)}{[k^2 - m^2][n\cdot k + i0][\bar{n}\cdot k - i0]}$$

$$= \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \Big\{ \Big[\frac{-2h(\epsilon, \mu^2/m^2)}{\eta} + \ln\frac{\mu^2}{\nu^2}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big) + \frac{1}{\epsilon^2} - \frac{1}{2}\ln^2\frac{\mu^2}{m^2} - \frac{\pi^2}{12} \Big] + \Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big) \Big] \Big\}$$

with physical directions for soft Wilson lines in hard scattering

true soft: includes 0-bin subtraction $S = \tilde{S} - S^{(G)}$ has no $i\pi$ term

b)



Glauber:

$$G = S^{(G)} = \bar{u}_n \Gamma v_{\bar{n}} \frac{C_F \alpha_s}{2\pi} \Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big) \Big]$$

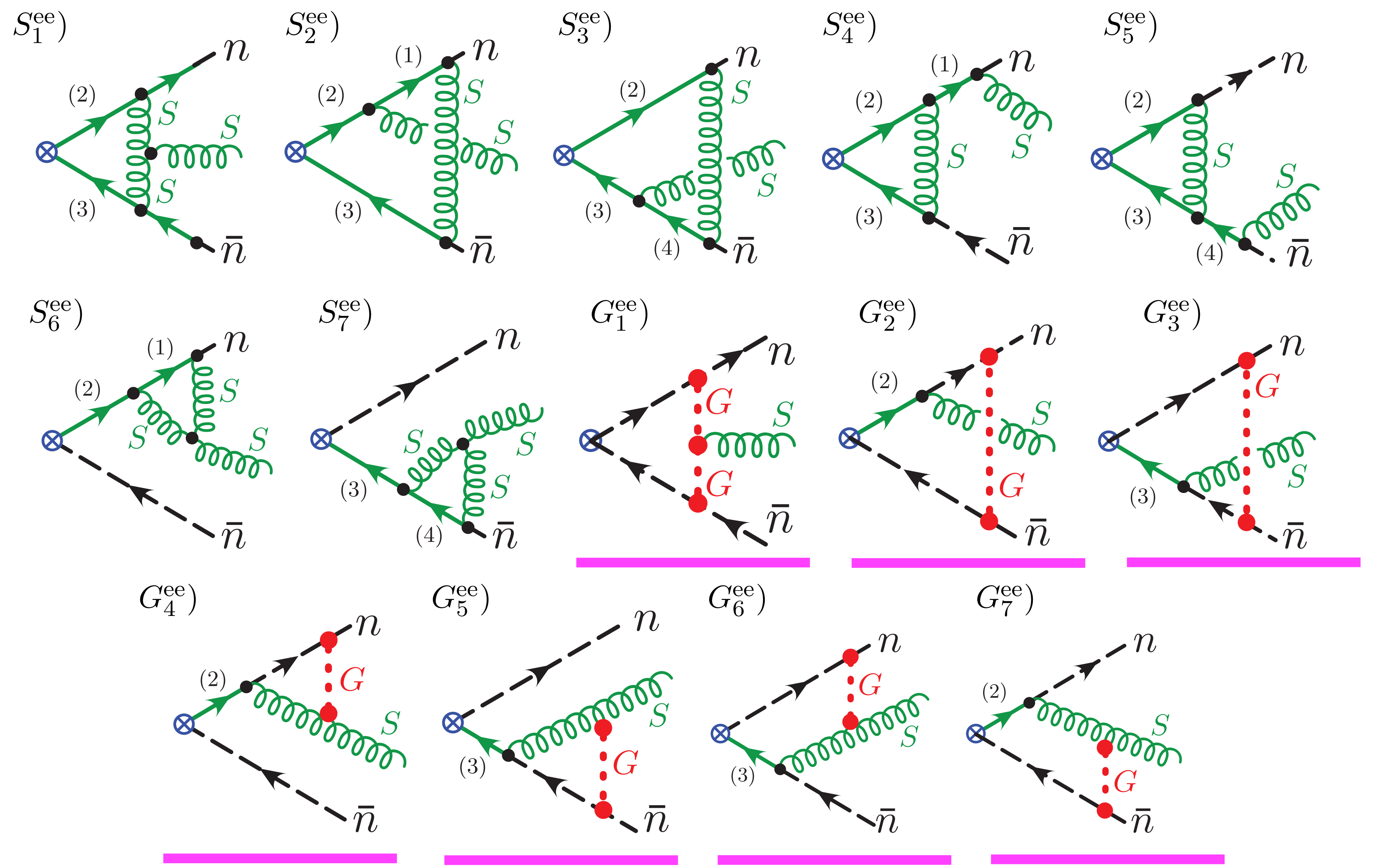
Glaubers give $(i\pi)$ terms

BUT $(\tilde{S} - S^{(G)}) + G = \tilde{S}$

- so we don't see Glauber in Hard Matching
- can absorb this Glauber into Soft Wilson lines if they have proper directions

Also true in the presence of additional emissions:

e^+e^-

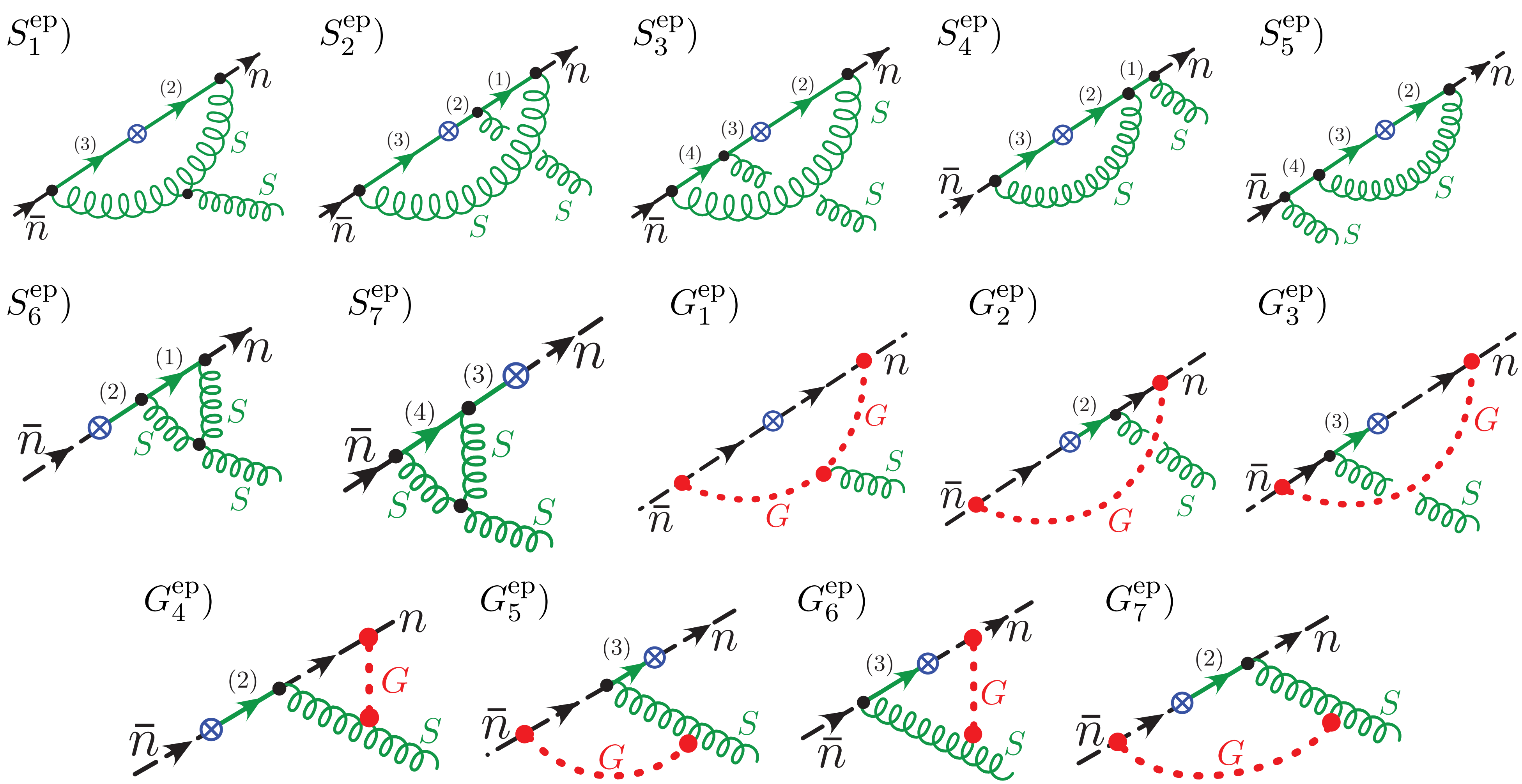


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

$e^- p$

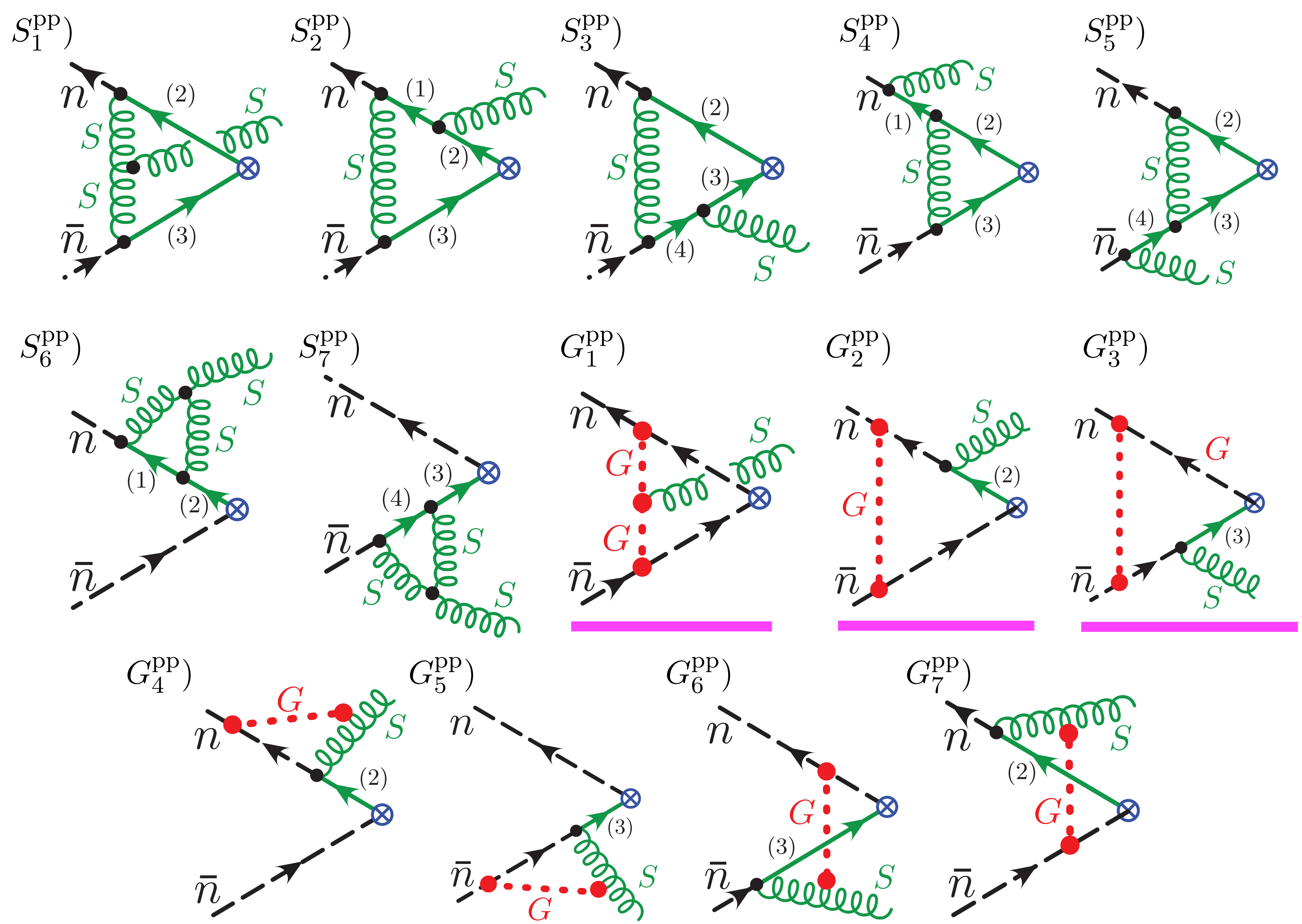


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

pp

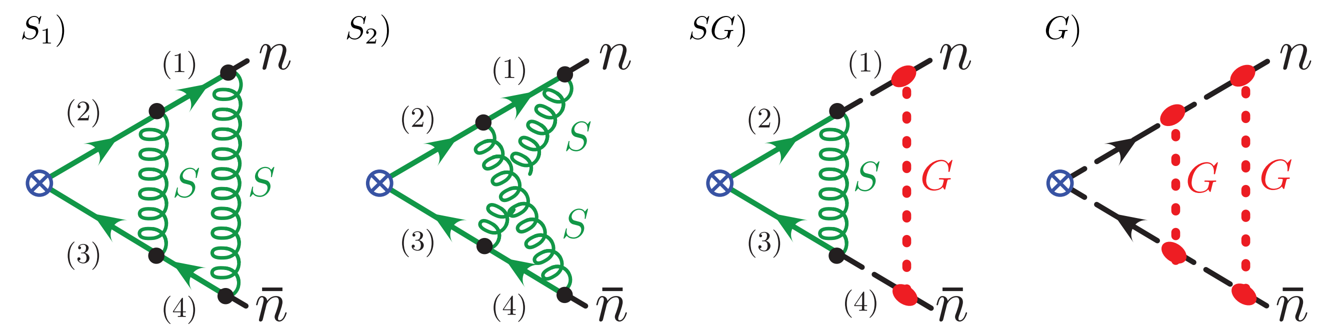


physical:

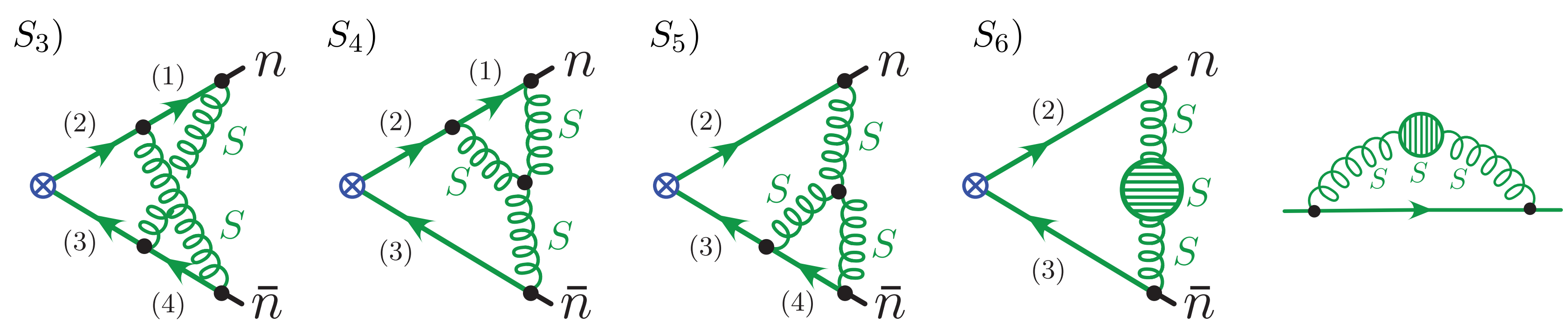
Glauber again gives all $(i\pi)$ terms here.

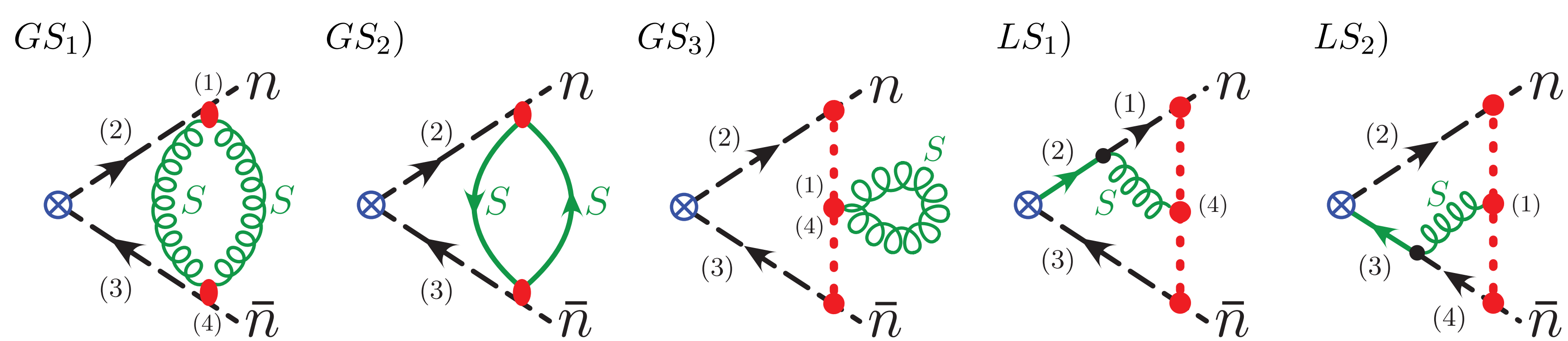
Continues at higher orders. Checked explicitly at 2-loops:

abelian



non-abelian



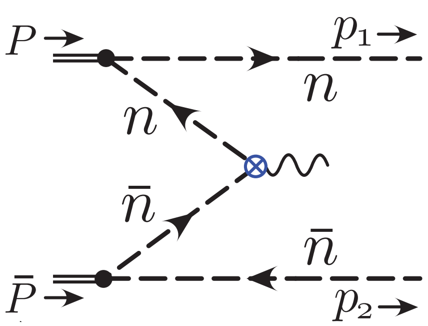


$$S_3 + S_4 + S_5 + S_6 + GS_1 + GS_2 + GS_3 + LS_1 + LS_2 = \tilde{S}_3 + \tilde{S}_4 + \tilde{S}_5 + \tilde{S}_6$$

soft with subtractions + Glauber = naive soft

Hadron Scattering

Add interpolating fields for initial state hadrons.

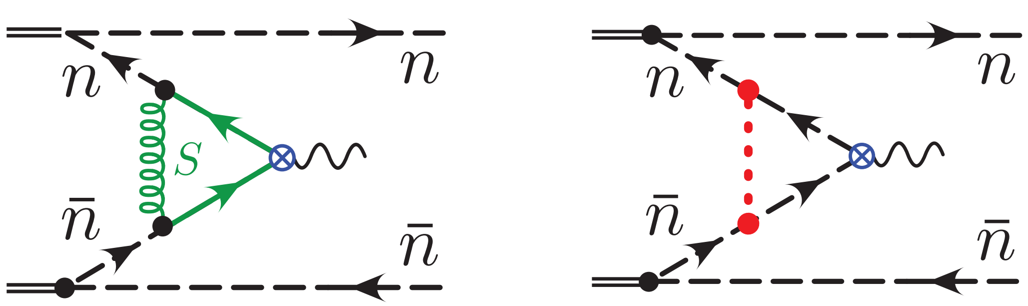


$$= \mathcal{S}^\gamma \left[\frac{1}{\vec{p}_{1\perp}^{\,2}}\frac{1}{\vec{p}_{2\perp}^{\,2}}\right]\left[\frac{\bar{n}\cdot p_1\,\bar{n}\cdot(P-p_1)}{\bar{n}\cdot P}\frac{n\cdot p_2\,n\cdot(\bar{P}-p_2)}{n\cdot\bar{P}}\right]$$
$$\equiv \mathcal{S}^\gamma\, E(p_{1\perp},p_{2\perp}),$$

$$\mathcal{S}^\gamma = \bar{u}_n\gamma_\perp^\mu v_{\bar{n}}^*$$

"an end E"

Active-Active

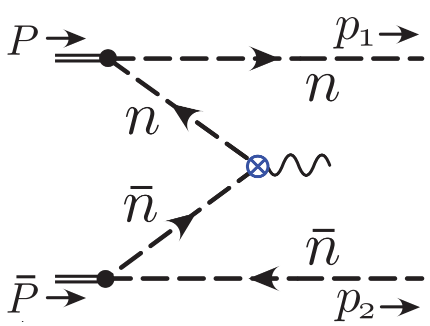


Same correspondence:

$$S = \tilde{S} - S^{(G)}$$
$$G = S^{(G)}$$
$$(\tilde{S} - S^{(G)}) + G = \tilde{S}$$

Hadron Scattering

Add interpolating fields for initial state hadrons.

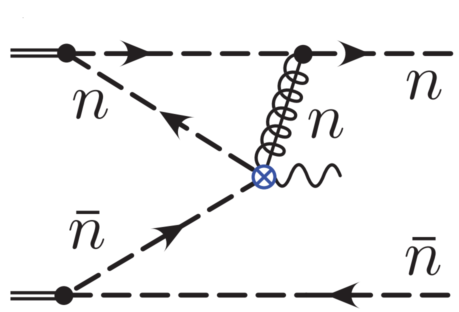


$$= \mathcal{S}^\gamma \left[\frac{1}{\vec{p}_{1\perp}^{\,2}}\frac{1}{\vec{p}_{2\perp}^{\,2}}\right]\left[\frac{\bar{n}\cdot p_1\, \bar{n}\cdot(P-p_1)}{\bar{n}\cdot P}\frac{n\cdot p_2\, n\cdot(\bar{P}-p_2)}{n\cdot\bar{P}}\right]$$
$$\equiv S^\gamma\, E(p_{1\perp}, p_{2\perp}),$$

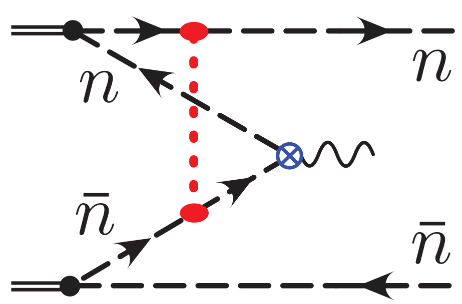
$$\mathcal{S}^\gamma = \bar{u}_n \gamma_\perp^\mu v_{\bar{n}}^*$$

"an end E"

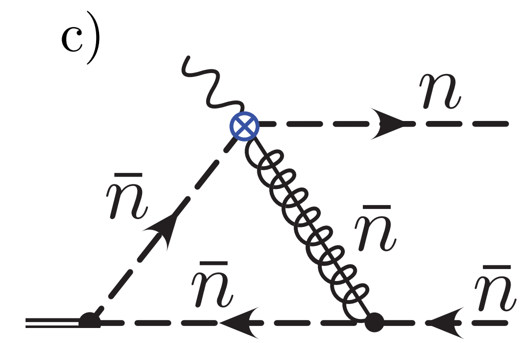
Active-Spectator

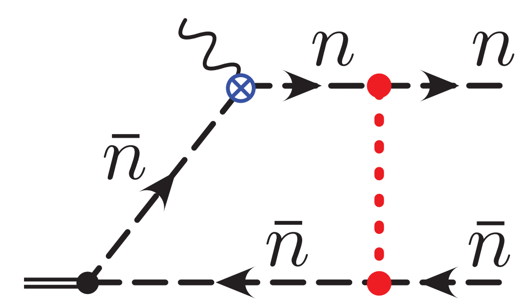


$$C_n = \tilde{C}_n - C_n^{(G)}$$



$$G = C_n^{(G)}$$

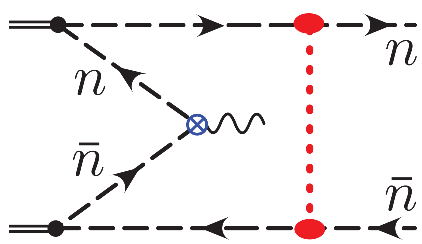




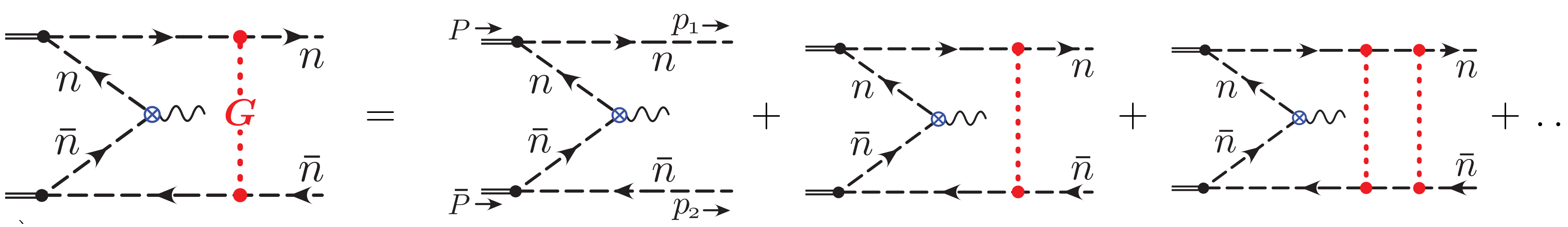
- can absorb this Glauber into the Collinear Wilson line with physical directions (note: connection to eikonalization)

$$J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$$

Spectator Scattering



no analogous soft or collinear diagrams at leading power



$$= -\mathcal{S}^{\gamma} \int \text{đ}^{d-2}k_\perp \, G(k_\perp)\, E(p_{1\perp}+k_\perp, p_{2\perp}-k_\perp)$$

$G(k_\perp) =$ Fourier Transform of $e^{i\phi}$

$$= -\mathcal{S}^{\gamma} \int \text{đ}^{d-2}k_\perp \, G(k_\perp)\, E\Big(k_\perp - \Delta p_\perp - \frac{q_\perp}{2}, \Delta p_\perp - k_\perp - \frac{q_\perp}{2}\Big)$$

$$q_\perp = -p_{1\perp} - p_{2\perp}$$

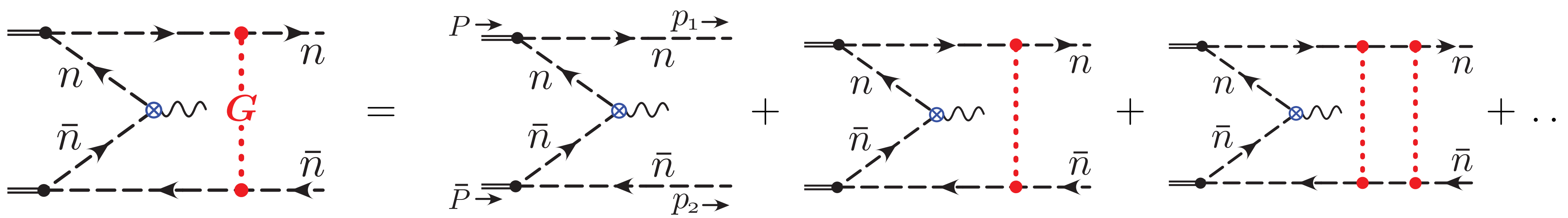
$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$\equiv -\mathcal{S}^{\gamma} \int \text{đ}^{d-2}k_\perp \, G(k_\perp)\, E'(\Delta p_\perp - k_\perp, q_\perp)$$

$$= -\mathcal{S}^{\gamma} \int \text{đ}^{d-2}k_\perp \int d^{d-2}b_\perp \, e^{-i\vec{k}_\perp\cdot\vec{b}_\perp}\, \tilde{G}(b_\perp) \int d^{d-2}b'_\perp \, e^{-i(\Delta\vec{p}_\perp - \vec{k}_\perp)\cdot\vec{b}'_\perp}\, \tilde{E}'(b'_\perp, q_\perp)$$

$$= -\mathcal{S}^{\gamma} \int d^{d-2}b_\perp \, e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp}\, \tilde{E}'(b_\perp, q_\perp)\, e^{i\phi(b_\perp)} \equiv \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

Spectator Scattering



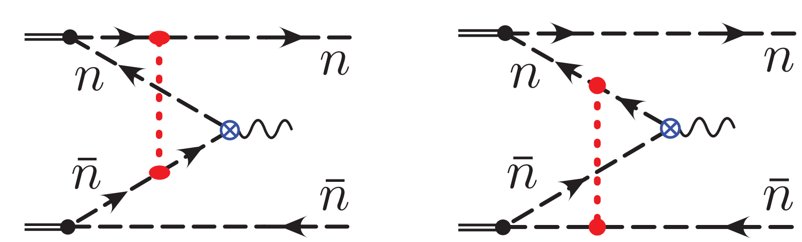
$$q_\perp = -p_{1\perp} - p_{2\perp}$$
$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$= -\mathcal{S}^\gamma \int d^{d-2} b_\perp \, e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp} \, \tilde{E}'(b_\perp, q_\perp) \, e^{i\phi(b_\perp)} \equiv \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

phase cancels IF we integrate over $\Delta p_\perp$

$$\int \!\!{d\!\!\!^-}^{d-2}\Delta p_\perp \left|\mathcal{A}_{SS}(\Delta p_\perp, q_\perp)\right|^2$$
$$= |\mathcal{S}^\gamma|^2 \int {d\!\!\!^-}^{d-2}\Delta p_\perp \int d^{d-2}b_\perp \, d^{d-2}b'_\perp \, e^{i\Delta\vec{p}_\perp\cdot(\vec{b}'_\perp-\vec{b}_\perp)} \, \tilde{E}'(b_\perp, q_\perp)\tilde{E}'^\dagger(b'_\perp, q_\perp) \, e^{i\phi(b_\perp)-i\phi(b'_\perp)}$$
$$= |\mathcal{S}^\gamma|^2 \int d^{d-2}b_\perp \left|\tilde{E}'(b_\perp, q_\perp)\right|^2$$
$$= |\mathcal{S}^\gamma|^2 \int {d\!\!\!^-}^{d-2}\Delta p_\perp \left|E'(\Delta p_\perp, q_\perp)\right|^2$$

Active-Spectator and the Collinear Overlap

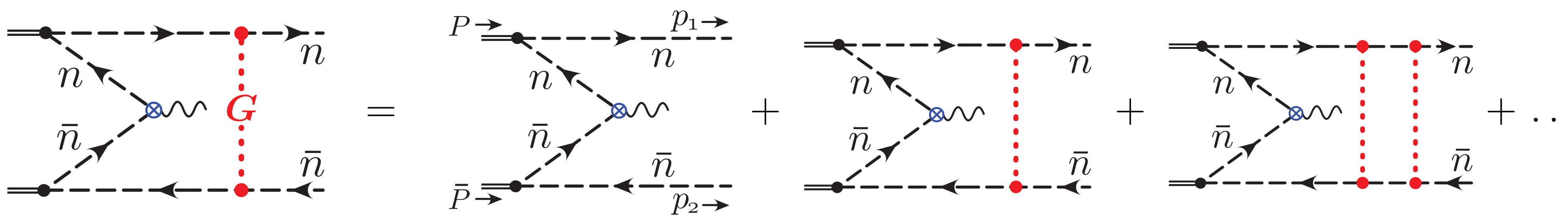


- cancel? $= \frac{1}{2} S^\gamma \int {d\!\!\!^-}^{d'} k_\perp \, G^0(k_\perp) E(p_{1\perp} + k_\perp, p_{2\perp})$

now need to integrate over $p_{i\perp}$

(importance of Wilson line directions for TMDs)

Spectator Scattering



$$q_\perp = -p_{1\perp} - p_{2\perp}$$
$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

cancel IF we integrate over $\Delta p_\perp$

Measurements (like beam thrust & transverse thrust) that disrupt this integration can cause a non-cancellation.

(Gaunt; Zeng)

Single t-scale SCET:

$\Delta p_\perp \sim \Lambda_{\rm QCD} \ll \mathcal{T}$ cancel as in inclusive DY, up to power corrections $\frac{\Lambda_{\rm QCD}}{\mathcal{T}} \ll 1$

(Aybat & Sterman)

$\Delta p_\perp \sim \mathcal{T}, \sqrt{Q\mathcal{T}}$ starts at $\mathcal{O}(\alpha_s^4)$, calculable factorization violation $(\mathcal{I}\mathcal{I}) \otimes f \otimes f$

(cf. Gaunt; Zeng)

Need multi t-scale SCET for most interesting effects

(not discussed here)

Underlying Event

- Radiation not described by primary hard scattering.
- Modeled by Multiple Particle Interactions (MPI)

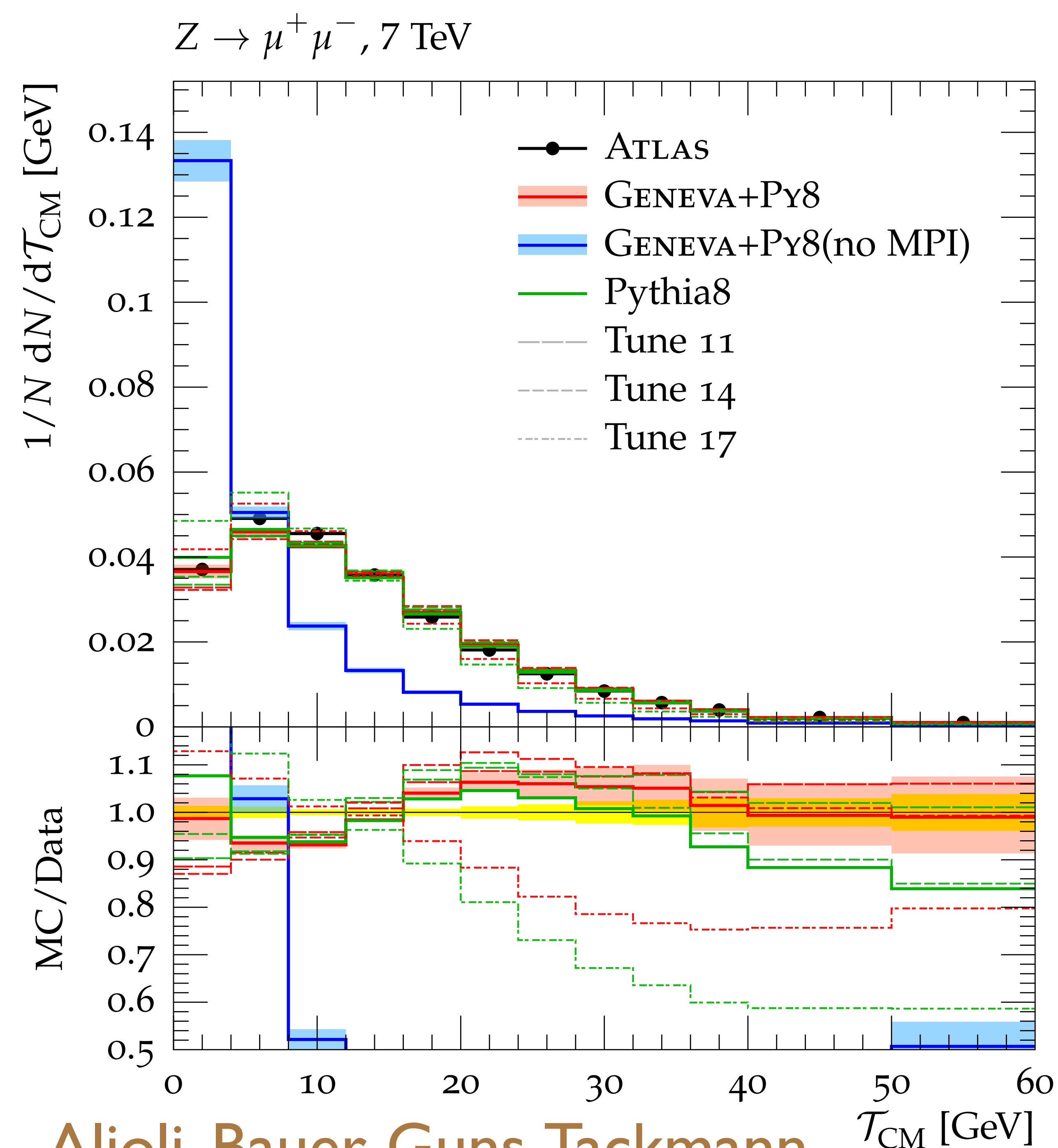
Some observables are sensitive:

beam thrust,

$$\mathcal{T}_{\mathrm{CM}} = \sum_i p_{T,i}\, e^{-|\eta_i|}$$

transverse thrust, …

Connection between fact. violation, and small-x dynamics may allow us to directly calculate these effects.



Alioli, Bauer, Guns, Tackmann, 1605.07192

Conclusion

- EFT formalism for $s \gg t$, Hard & Fwd. Scattering & Fact. Violation
- Universal Operators that can be used for many processes & problems
- Reggeization, BFKL, Shockwave picture, S-G & C-G overlaps, …

Future Directions

- Study joint DGLAP(μ) and BFKL(ν) resummation for small-x
- Study Regge & BFKL type resummation at NNLL
- Reproduce classic (CSS) proofs of factorization
- Improve theoretical description of Underlying Event
- ….

The End

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar n$-collinear	$\xi_{\bar n},\ A_{\bar n}^\mu$	$(\bar n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar n$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$\sim \lambda^\delta$ (gauge invariant)

topological factors

operator insertions

$$\delta = 6 - N^n - N^{\bar n} - N^{nS} - N^{\bar n S} + 2u$$

$$+\sum_k (k-8)V_k^{us} + \underbrace{(k-4)\big(V_k^n + V_k^{\bar n} + V_k^S\big)}_{\text{standard SCET}} + (k-3)\big(V_k^{nS} + V_k^{\bar n S}\big) + (k-2)V_k^{n\bar n}$$

need $\sim \lambda^3$ $\sim \lambda^2$

Glauber

operators at leading power