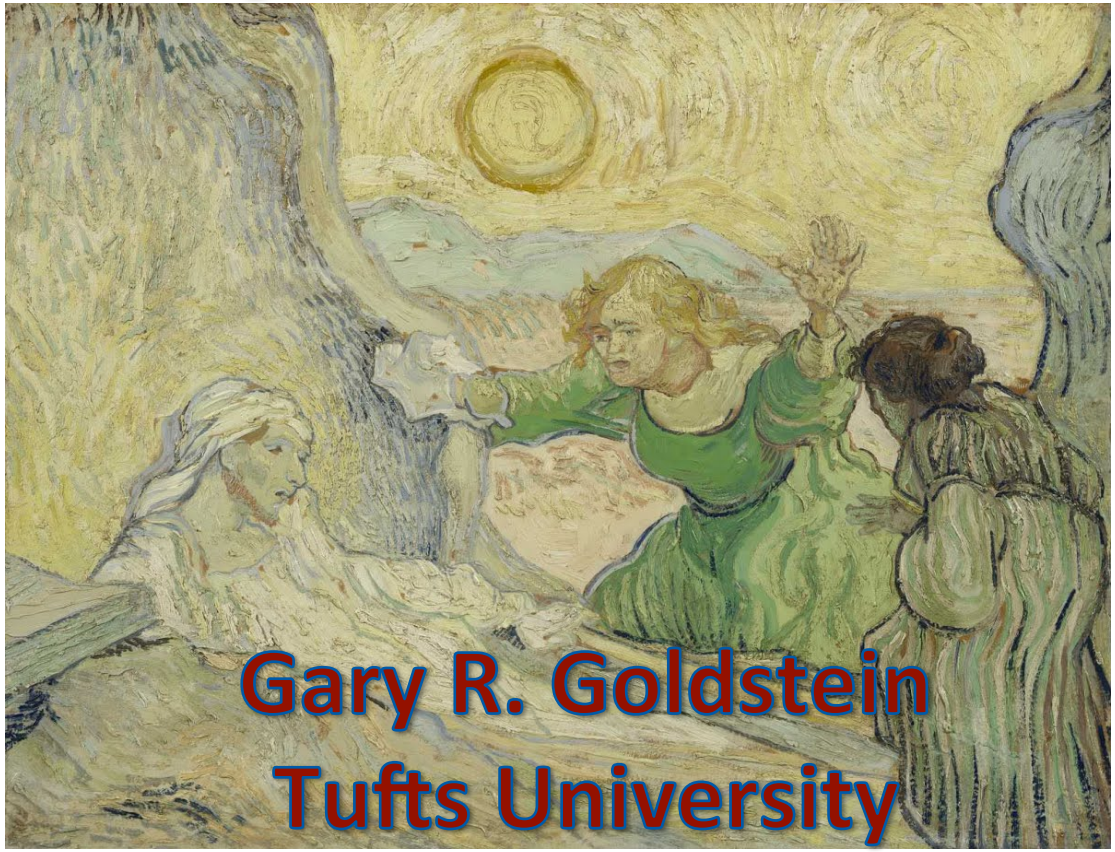
Generalized Parton Distributions in Deeply Virtual Lepton Scattering Processes



Gary R. Goldstein
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Van Gogh
Raising of Lazarus

QCD Evolution 2016

Abstract

- Spin and transverse momentum dependent Generalized Parton QCD-Distributions (GPDs) exist at the interface between the non-perturbative regime of QCD hadron structure and observable quantities. The QCD-Distributions appear as linear superpositions and convolutions within helicity amplitudes for parton-nucleon scattering processes, which, in turn, occur in amplitudes for leptoproduction processes. We have developed a "flexible model" of quark and gluon GPDs that incorporates diquark and other spectators, Regge behavior and evolution. Chiral even GPDs determine deeply virtual Compton scattering amplitudes and are compared with cross section and polarization data. The chiral odd GPDs can be generated from these via parity relations. Those chiral odd GPDs, including "transversity", lead to predictions for pseudoscalar leptoproduction. We will present relations between crucial quark-nucleon or gluon-nucleon GPDs and the rich array of angular QCD-Distributions in Deeply Virtual Scattering processes.

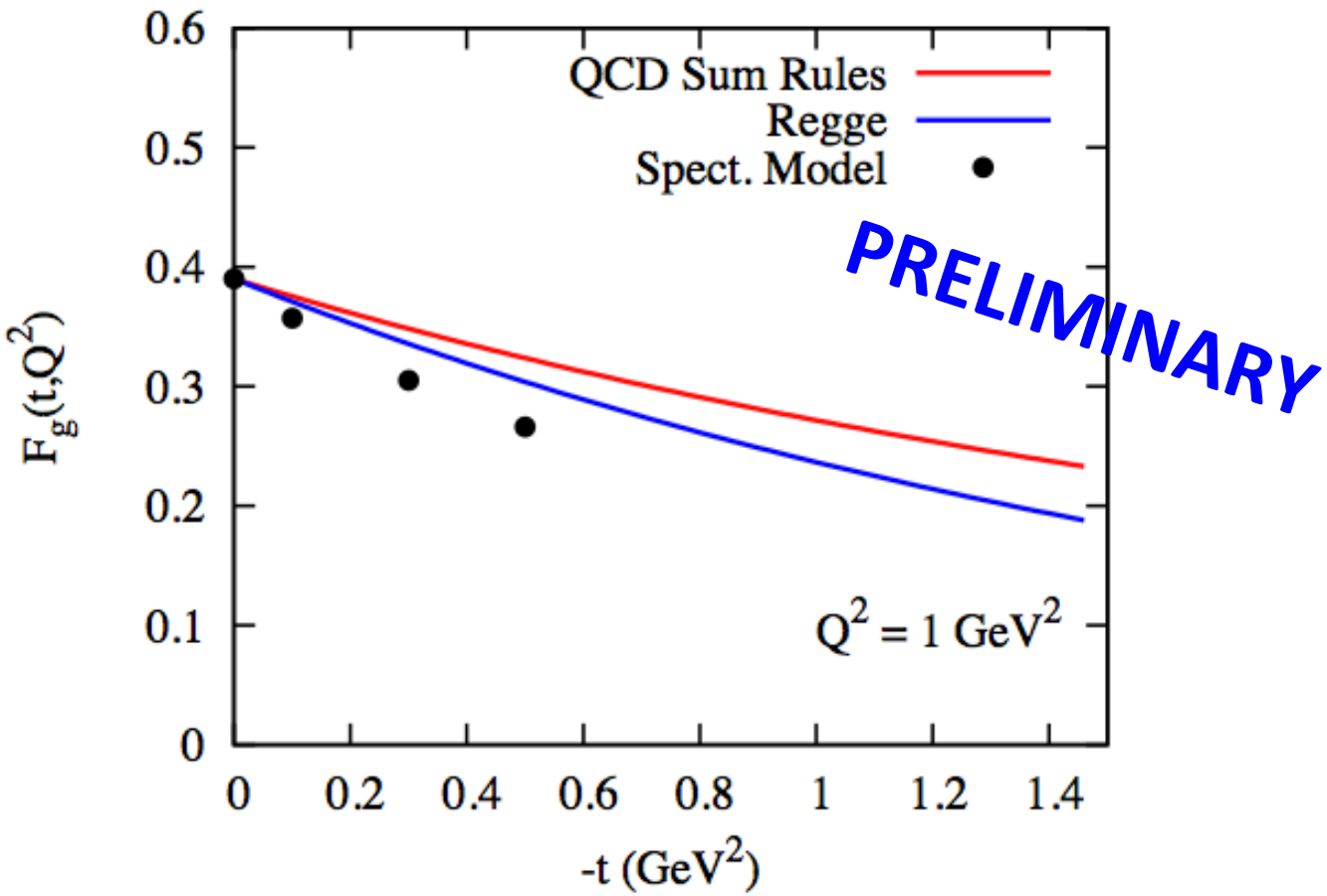
Collaborators: S. Liuti, O. Gonzalez Hernandez, A. Rajan, J. Poage

OUTLINE

- GPDs, Models to guide– Reggeized spectator "flexible"
- Valence quarks: Chiral Even
- Valence quarks: Chiral Odd
- Gluons & sea quarks - Reggeized spectator "flexible"
- Helicity amplitudes
- Observable quantities

Preview: Gluons

Spectator t-dependence w/o Regge small x behavior: hybrid Regge-Spectator model combines



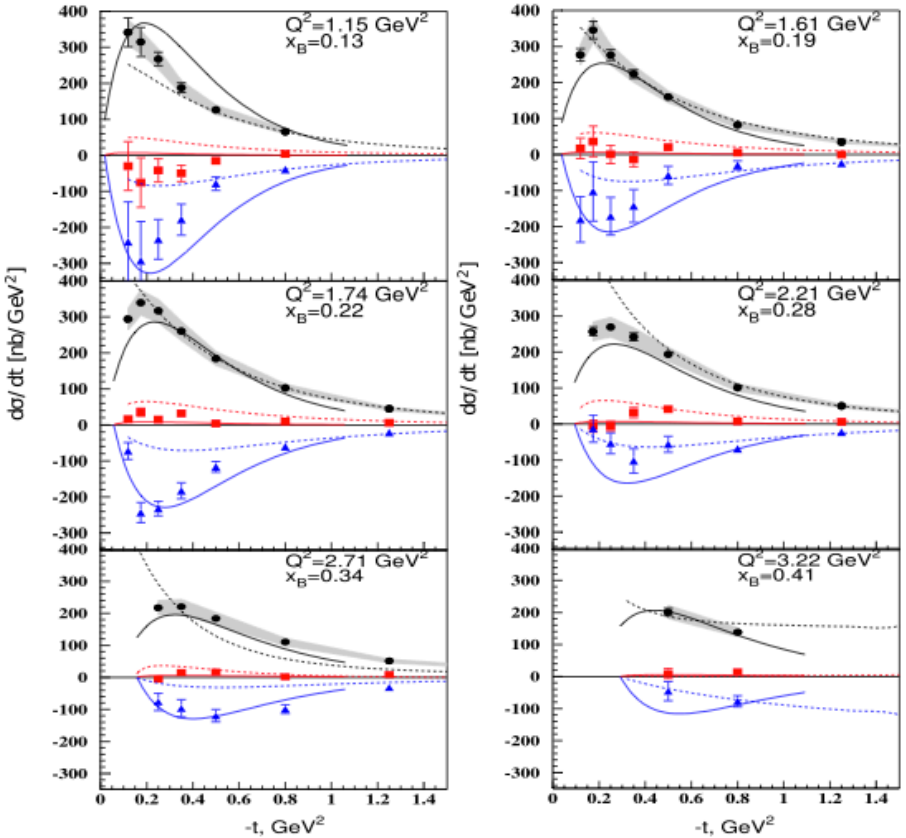
Compare Gluon form factor via QCD sum rules

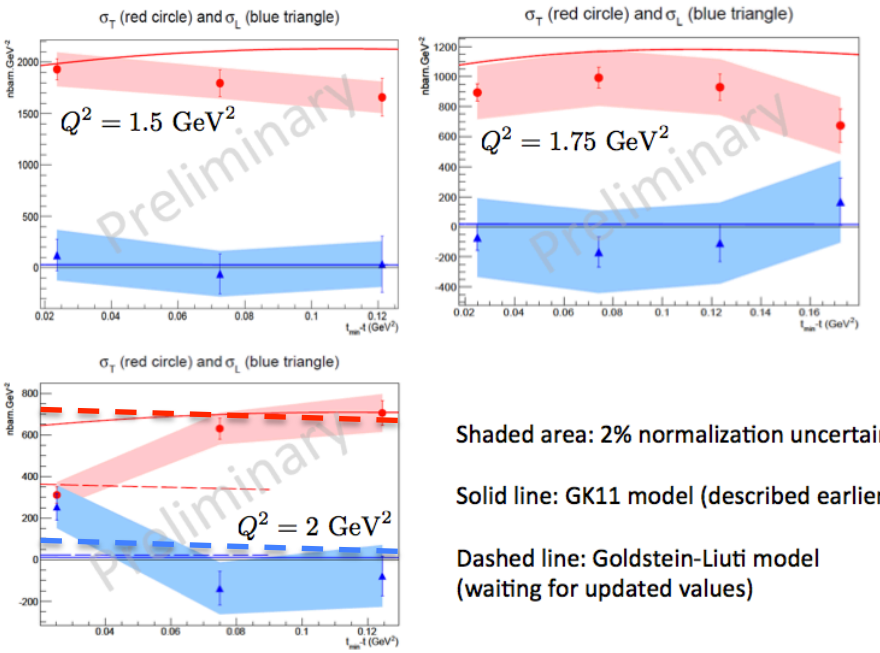
Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)

Preview: Why consider chiral-odd GPDs? Why go beyond leading twist?

$\pi 0$ electroproduction **data dictate** necessity of transverse photons

CLAS; Hall A separated cross sections ; Asymmetries distinguish models





Shaded area: 2% normalization uncertainty

Solid line: GK11 model (described earlier)

Dashed line: Goldstein-Liuti model (waiting for updated values)

courtesy F. Sabatie & M. DeFurne
Hall A @ CIPANP

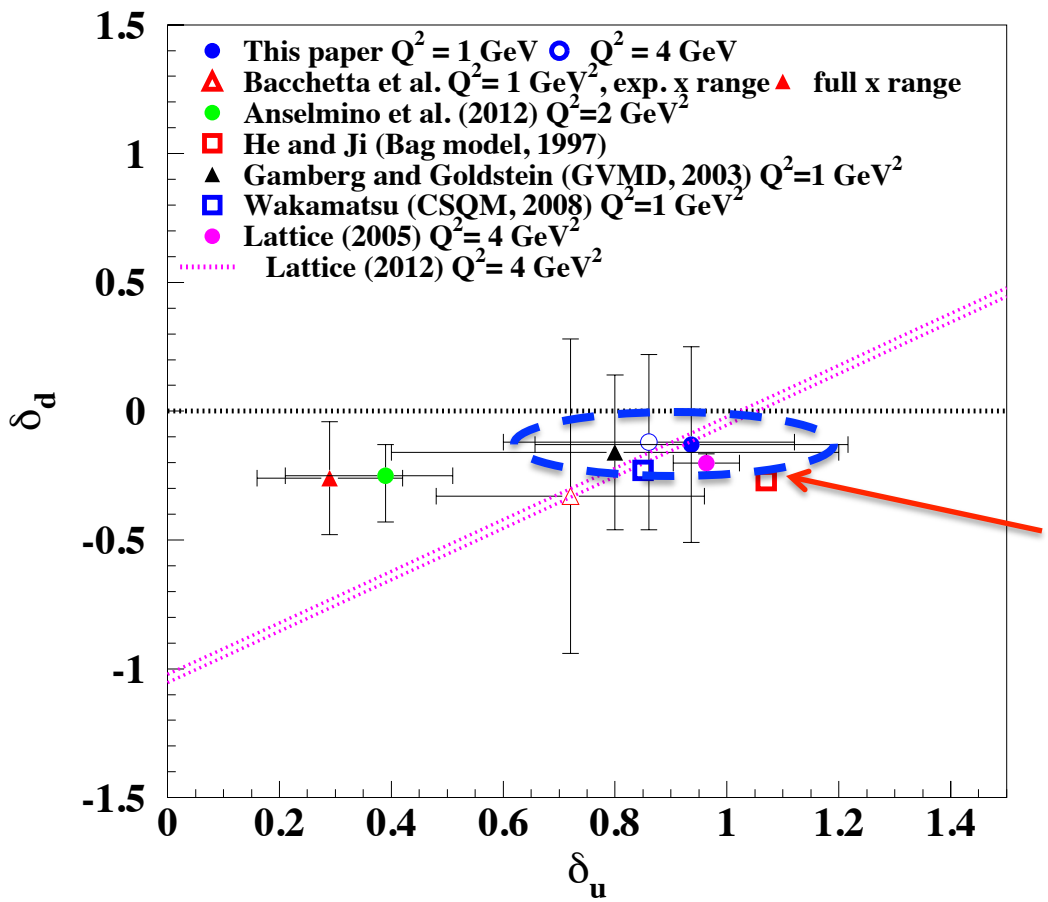
Dashed curve: GGL . . Solid curve: G&K

CLAS: Bedlinskiy, et al., PRL 109, 112001 (2012)

G. R. Goldstein, J. O. Gonzalez Hernandez, and S. Liuti, Phys. Rev. D 84, 034007 (2011).

S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011).

We look at Chiral odd GPDs - why? → $H_T(x,\xi,t)$ → $h_1(x)$ **Transversity** → **tensor charges** δ_q to get complete picture of spin decomposition



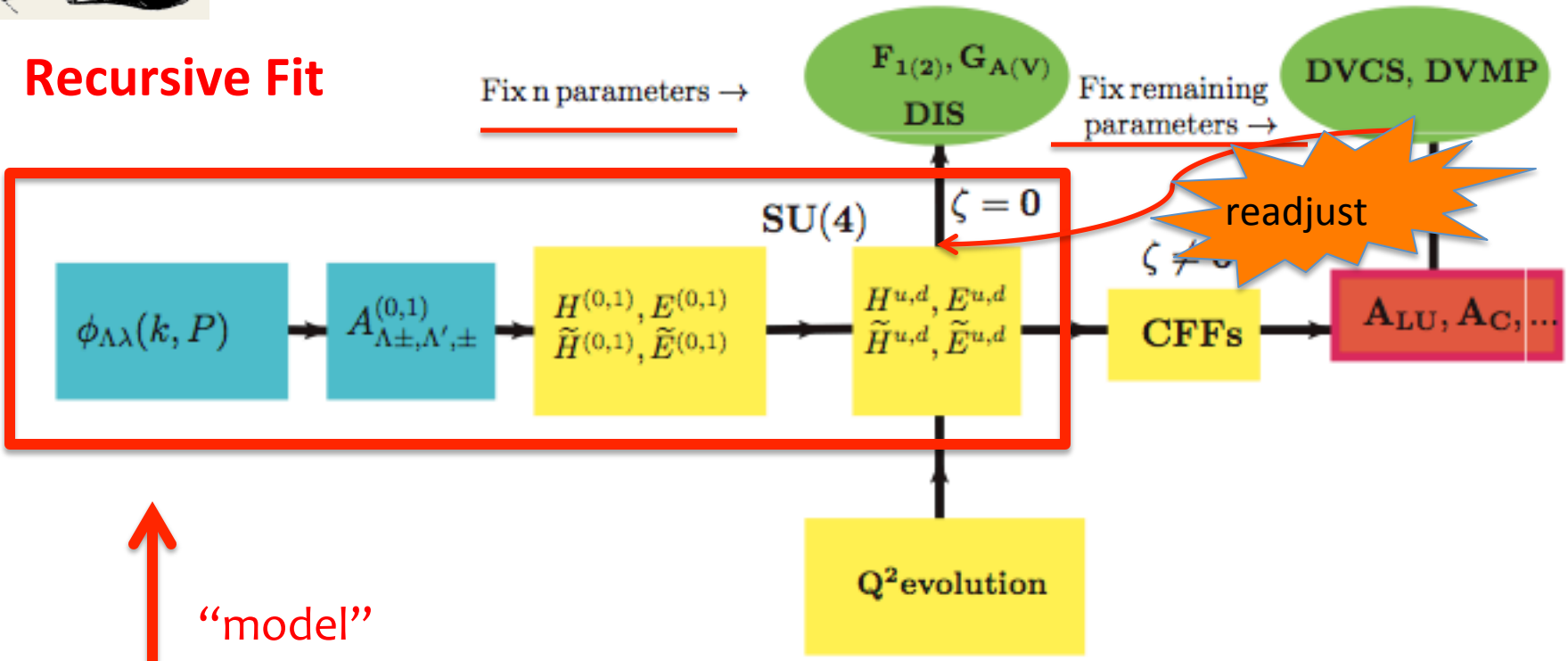
GG, Gonzalez, Liuti, PRD91, 114013 (2015))

Gluon distributions

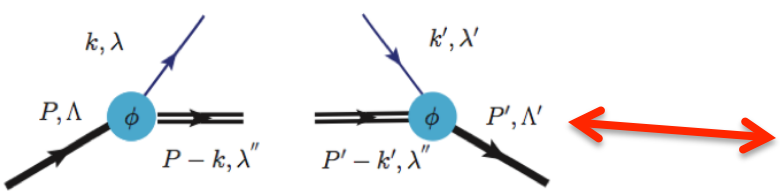
- What to expect for H_g , $E_g(x,\xi,t)$, . . . ?
- Begin with normalization
- $H_g(x,0,0) \rightarrow xG(x)|_{Q^2}$ unpolarized
- Parametrize via "spectator" model by pdf's
- Follow procedure for valence quark chiral even GPDs & then chiral odd
- Review procedure:

O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv:1206.1876

Recursive Fit



"model"



$$\phi_{\Lambda,\lambda}(k,P) = \Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2 - m^2}$$

$$\phi^*_{\Lambda'\lambda'}(k',P') = \Gamma(k')\frac{\overline{U}(P',\Lambda')u(k',\lambda')}{k'^2 - m^2},$$

GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\boxed{H^q}\gamma^+ + \boxed{E^q}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\boxed{\tilde H^q}\gamma^+\gamma_5 + \boxed{\tilde E^q}\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule
talks by Mueller, Kumericki, Guidal.
Liuti, et al. → "flexible parameterization"

$$\left\langle J_q^x\right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,i\sigma^{+i}\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\boxed{H_T^q}\,i\sigma^{+i} + \boxed{\tilde H_T^q}\,\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right.$$
$$\left. + \boxed{E_T^q}\,\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \boxed{\tilde E_T^q}\,\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]u(p,\lambda$$

Chiral odd GPDs
-> transversity
How to measure and/or parameterize them?

Normalizing quark GPDs – Chiral even

Form factor, Forward limit

$$\int_0^1 H_q(x,\xi,t)\,dx = F_1^q(t), \quad H_q(x,0,0) = q(x)$$

Integrates to charge

$$\int_0^1 E_q(x,\xi,t)\,dx = F_2^q(t)$$

→Anomalous magnetic moments

$$\int_0^1 \tilde{H}_q(x,\xi,t)\,dx = g_A^q(t), \quad \tilde{H}_q(x,0,0) = \Delta q(x) = q_{\Rightarrow}^{\rightarrow}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

Integrates to axial charge

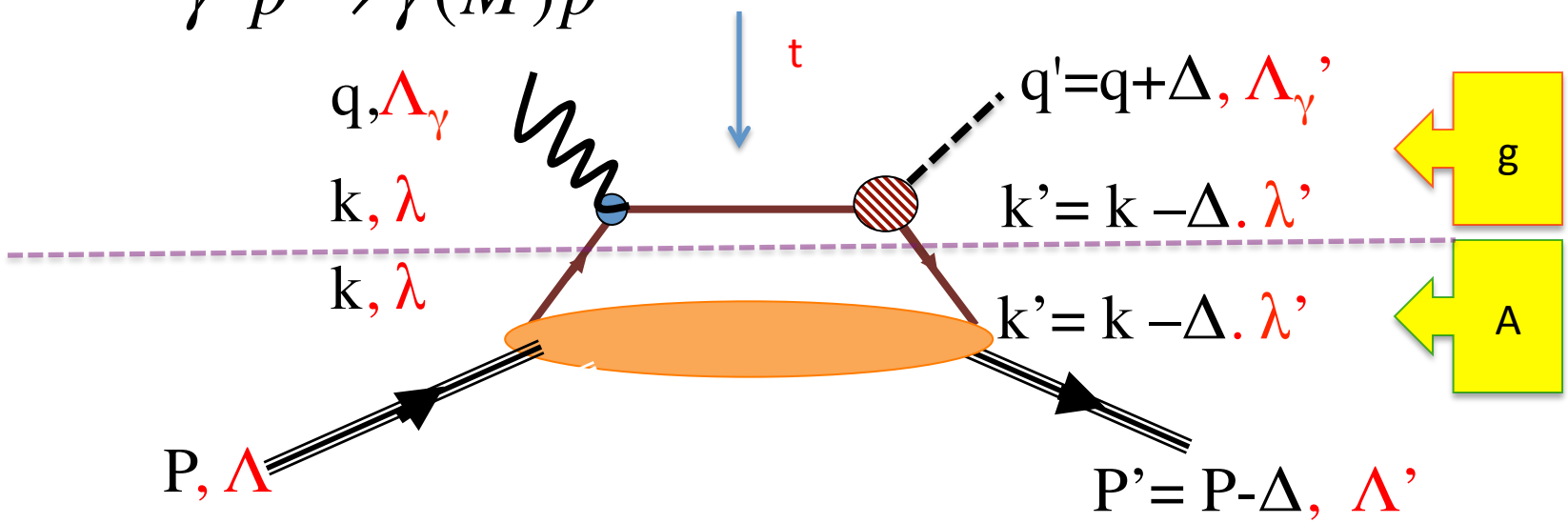
$$\int_0^1 \tilde{E}_q(x,\xi,t)\,dx = g_P^q(t)$$

The Model – Reggeized Diquarks

The Model – first for Chiral Even – Reggeized Diquark Spectator

Factorization in exclusive processes (DVCS, DVMP…)

$$\gamma^* p \to \gamma(M) p'$$



Convolution of "hard part" with quark-proton **Helicity** amplitudes

$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$

λ= +(-) λ' chiral even (odd)

see Ahmad, GG, Liuti, PRD79, 054014, (2009)
for first chiral odd GPD parameterization
Gonzalez, GG, Liuti PRD84, 034007 (2011) chiral even GPD

Recursive fit

GRG, Gonzalez Hernandez, Liuti, PRD84 (2011)

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,\ldots)$$

to DVCS, DVMP

$$H_q(x,\xi,t;Q_o^2) = N_q\, x^{-\left[\alpha_q+\alpha'_q(1-x)^p t\right]} G^{a_1 a_2 a_3 \cdots}(x,\xi,t)$$

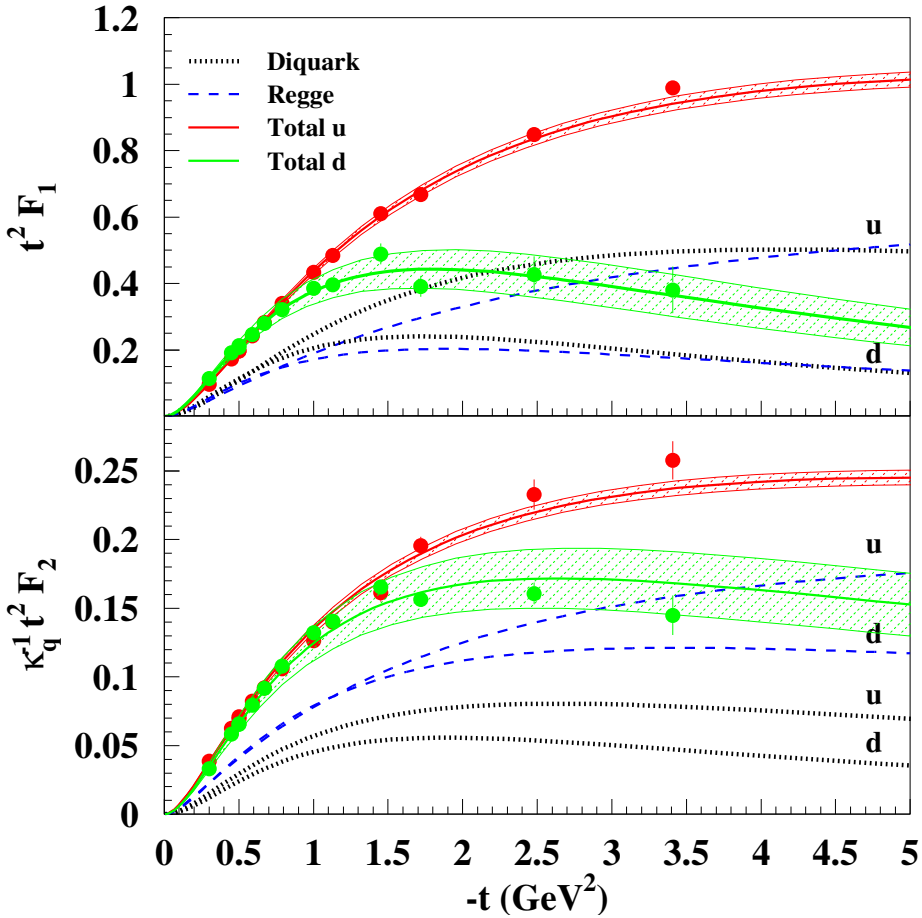
$$a_1 = m_q,\ a_2 = M_X^q,\ a_3 = M_\Lambda^q, \ldots$$

pdf's
Form Factors
dσ / dΩ
Asymmetries

"Flexible" parameterization based on the Reggeized quark-diquark model.

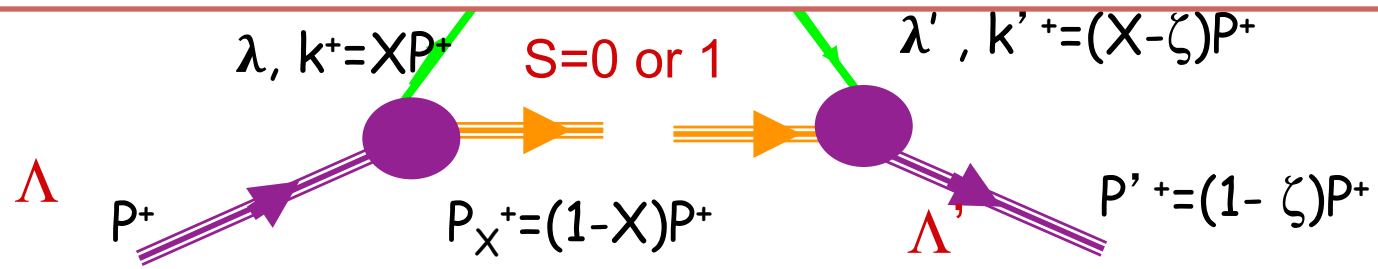
Sea quarks and gluon parametrization, work in progress

EM Form Factors (t dependence)



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)
data: G.D. Cates, et al. PRL106,252003 (2011).

Procedure to construct Chiral Odd GPDs & observables
Spectator diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\lambda;\Lambda'\lambda'=\lambda}$

↓

$A_{\Lambda\lambda;\Lambda'\lambda}$ → chiral even GPDs + Evolution

↓

g ⊗ A → exclusive process helicity amps

↓

pdf's, FF's, dσ/dΩ & Asymmetries: parameters & predictions

↓

vertex parity$\rightarrow A_{\Lambda\lambda;-\Lambda'-\lambda'}$ → chiral odd GPDs→pdf's, …

Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at ξ=0)

$$H^{M^q_\Lambda}_{M^q_X,m_q} = \mathcal{N}_q \int \frac{d^2k_\perp}{1-x} \frac{[(m_q+Mx)(m_q+Mx)+\mathbf{k}_\perp\cdot\tilde{\mathbf{k}}_\perp]}{\left[\mathcal{M}_q^2(x)-k_\perp^2/(1-x)\right]^2\left[\mathcal{M}_q^2(x)-\tilde{k}_\perp^2/(1-x)\right]^2},$$

$$E^{M^q_\Lambda}_{M^q_X,m_q} = \mathcal{N}_q \int \frac{d^2k_\perp}{1-x} \frac{-2M/\Delta_\perp^2[(m_q+Mx)\tilde{\mathbf{k}}_\perp\cdot\mathbf{\Delta}_\perp-(m_q+Mx)\mathbf{k}_\perp\cdot\mathbf{\Delta}_\perp]}{\left[\mathcal{M}_q^2(x)-k_\perp^2/(1-x)\right]^2\left[\mathcal{M}_q^2(x)-\tilde{k}_\perp^2/(1-x)\right]^2},$$

Diquark mass "spectrum"
as in Brodsky, Close & Gunion
Phys. Rev. D8, 3678 (1973)

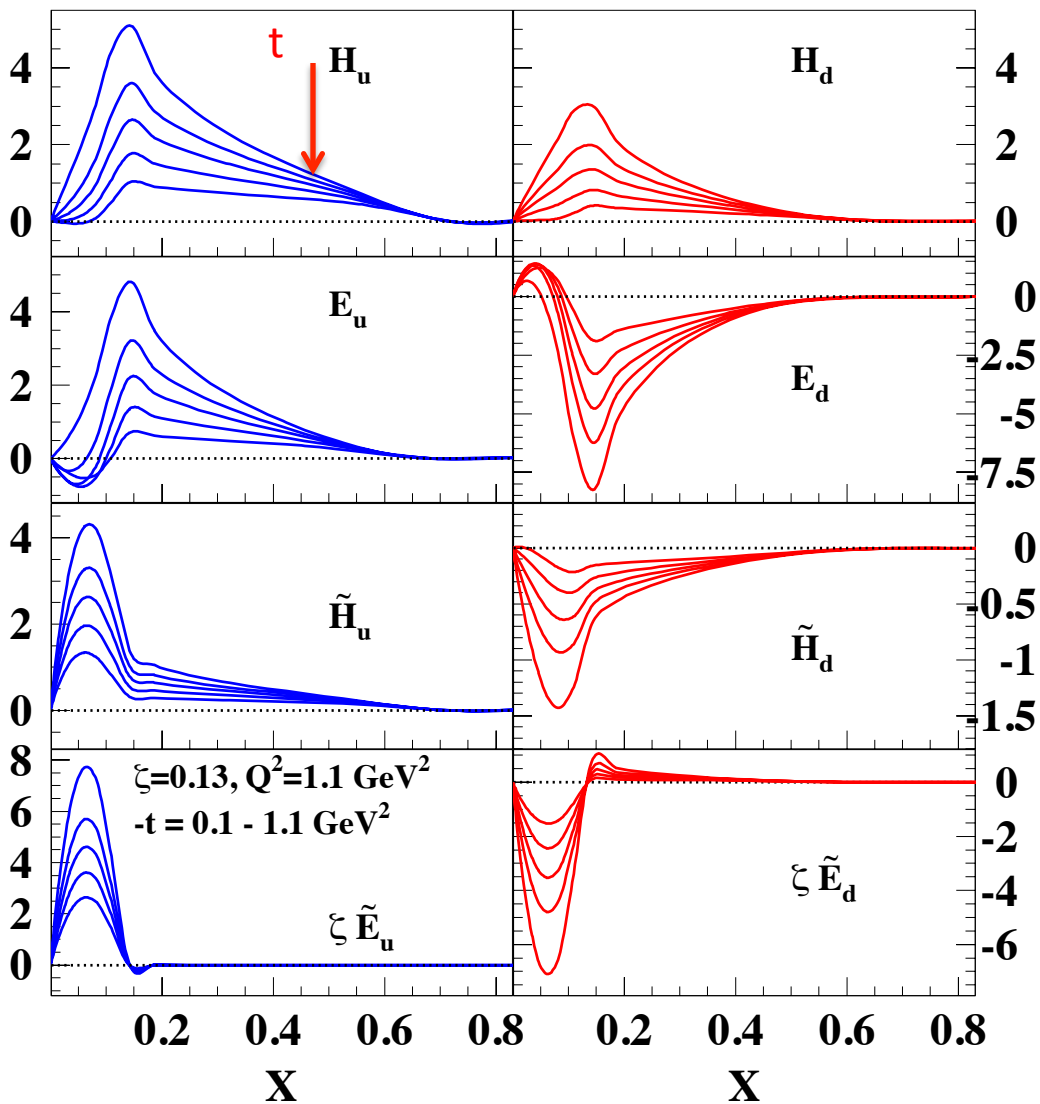
$$F_T^q(X,\zeta,t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q,M^q_\Lambda)}(X,\zeta,t;M_X).$$

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \to \infty \\ \delta(M_X^2-\overline{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

$$F_T^q(X,\zeta,t) \approx \mathcal{N}_q X^{-\alpha_q+\alpha_q'(X)t} F_T^{(m_q,M^q_\Lambda)}(X,\zeta,t;\overline{M}_X) = R_{p_q}^{\alpha_q,\alpha_q'}(X,\zeta,t)\, G^{M_\Lambda}_{M_X,m}(X,\zeta,t)$$

R✖Dq

Chiral even GPDs



From GPDs with evolution to Compton Form Factors

↓

CFFs to helicity amps

↓

helicity amps to observables

↓

<-> parameters

GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[H^q\gamma^+ + E^q\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\tilde H^q\gamma^+\gamma_5 + \tilde E^q\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule
talks by Mueller, et al.
Liuti, et al. → "flexible parameterization"

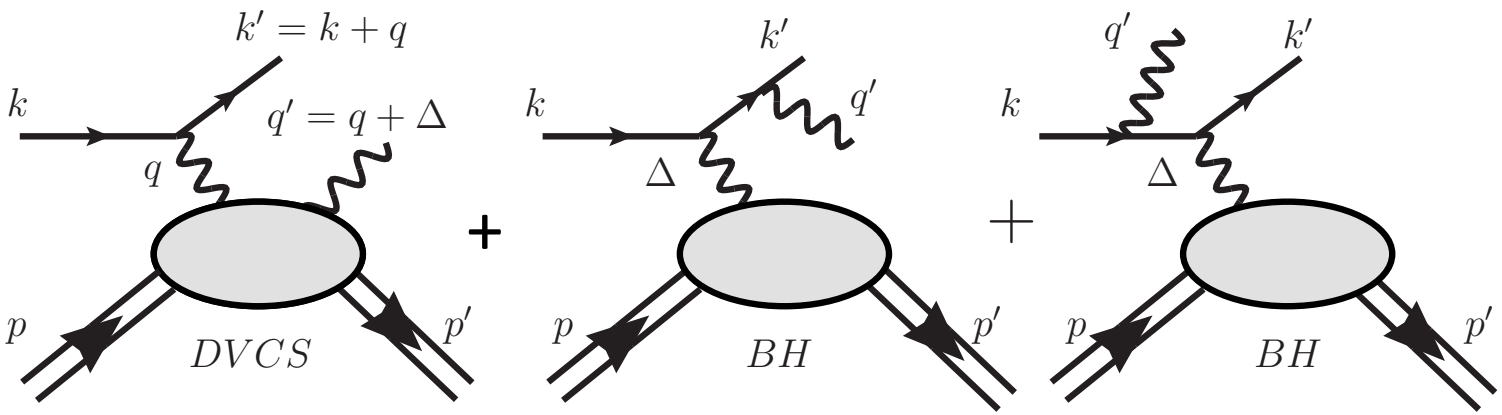
$$\left\langle J_q^x\right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,i\sigma^{+i}\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde H_T^q\,\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right.$$
$$\left. + E_T^q\,\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \tilde E_T^q\,\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]u(p,\lambda)$$

Chiral odd GPDs
-> transversity
Measure and/or parameterize them?

How to extract from measurements?

DVCS: paradigmatic GPDs



Some results – GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

GOLDSTEIN, HERNANDEZ, AND LIUTI

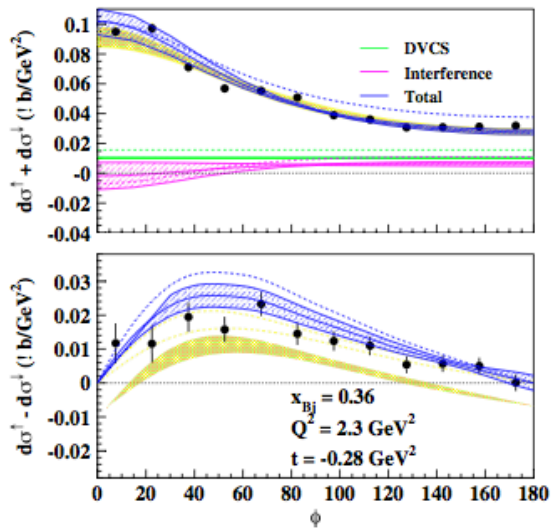


FIG. 16 (color online). Hall A data [49] for the "sum" (upper panel) and "difference" (lower panel) of the two electron beam polarizations. Shown are curves, including the contribution of the ζ-dependent factor from Eq. (34) (solid lines) and neglecting it (dashed lines). All terms (DVCS, Interference, and Total) are shown for the sum graph. The wide yellow bands in both panels represent the error of the data fit. The green band in the asymmetry graph is the theoretical error from our parametrization.

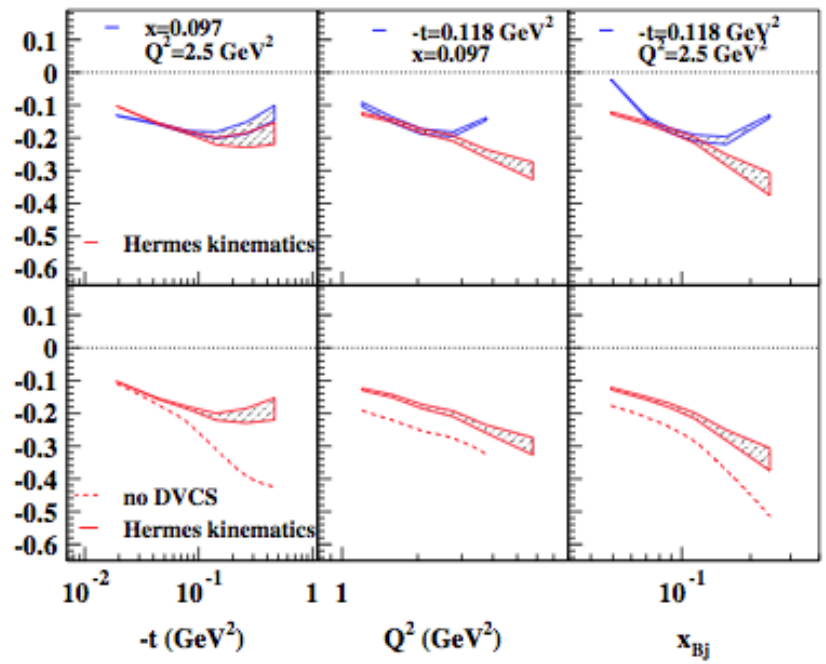


FIG. 18 (color online). Calculations at Hermes kinematics [52,53,56]. Shown is $A_{LU}(90°)$ vs $-t$, Q^2, and x_{BJ}, respectively, calculated at each kinematical bin provided by Hermes [56] (curve denoted as "Hermes kinematics") and at the nominal average values presented in each panel. It is interesting to notice that, due to the correlation between x_{Bj} and Q^2 in the data, different features arise when using the average bin values. In the lower panels, we also show the effect of disregarding the DVCS term in the denominator (dashed curves).

Some more results – GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

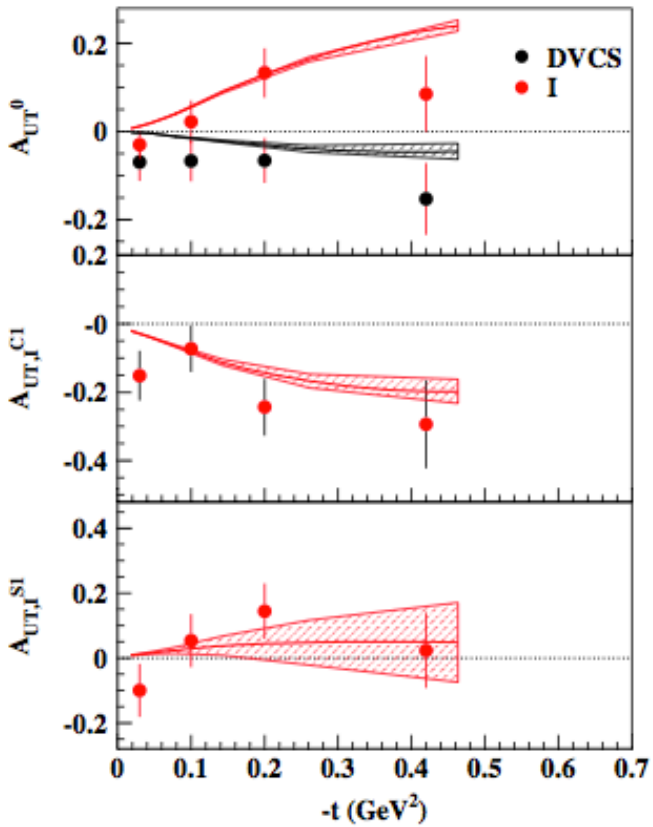
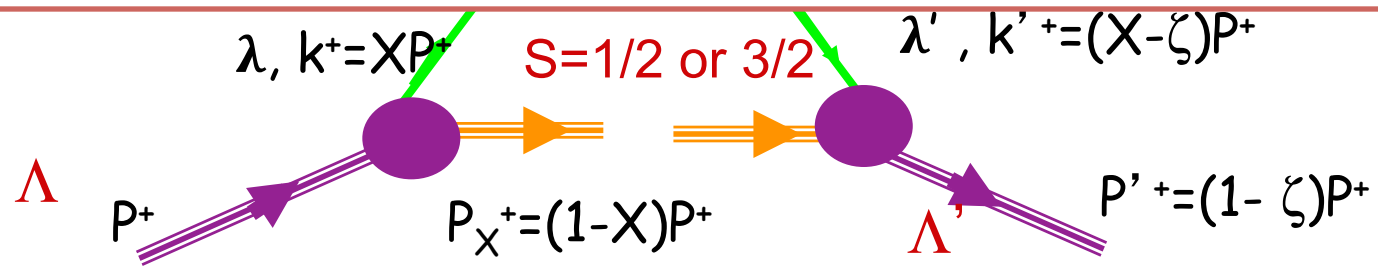


FIG. 21 (color online). Coefficients of the beam charge asymmetry, A_{UT}, extracted from experiment [52,53]. The upper panel shows the terms E and F from Eqs. (83) and (84), respectively; the middle panel shows G, and the lower panel H, both in Eq. (84). The curves are predictions obtained extending our quantitative fit of Jefferson Lab data to the Hermes set of observables.

Procedure to construct Gluon GPDs & observables
Spectator color octet "nucleon" model & Reggeization



Product of baryon l.c.w.f.'s $\rightarrow A_{\Lambda\lambda;\Lambda'\lambda'=\lambda}$

↓

$A_{\Lambda\lambda;\Lambda'\lambda} \rightarrow$ gluon GPDs

↓

$g \otimes A \rightarrow$ exclusive process helicity amps

↓

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

↓

vertex parity $\rightarrow A_{\Lambda\lambda;-\Lambda'-\lambda'} \rightarrow$ chiral odd GPDs $\rightarrow$ pdf's, …

The Model Extension– Sea Quarks & Gluons

$$\frac{1}{\bar{P}^+}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0}=$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)$$

$$\frac{-i}{\bar{P}^+}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0}=$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + E^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda)$$

- Gluon & Sea quark distributions
  - generalize Regge-spectator model

- N→ g + "color octet N" spectator
- N→ anti-u + color 3 "tetraquark"uuud

- How to normalize?
- Let $H_g(x,\xi,t)_Q^2 \rightarrow H_g(x,0,0)_Q^2 = xG(x)_Q^2$
- Sea quark distributions $H_{anti-u}(x,0,0)$ . . .

- Use pdf's to fix x dependence

- Small x ~ Pomeron

Gluon-spectator light-front variable vertices

$$\mathcal{G}_{\Lambda_X;\Lambda_g=x,\Lambda}(X,\vec{k}_T^2) \simeq \Gamma(k)\,\{\delta_{\Lambda_X,-\Lambda}(\Lambda)\frac{((1-X)M-M_X)}{\sqrt{(1-X)}} + \delta_{\Lambda_X,\Lambda}\frac{k_x-i\Lambda k_y}{\sqrt{(1-X)}}\}$$

$$\mathcal{G}_{\Lambda_X;\Lambda_g=y,\Lambda}(X,\vec{k}_T^2) \simeq \Gamma(k)\,\{\delta_{\Lambda_X,-\Lambda}(i)\frac{((1-X)M-M_X)}{\sqrt{(1-X)}} + \delta_{\Lambda_X,\Lambda}\frac{i\Lambda k_x+k_y}{\sqrt{(1-X)}}\}$$

$$\mathcal{G}_{\Lambda_X;\Lambda_g=z,\Lambda}(X,\vec{k}_T^2) \simeq \Gamma(k)\,\delta_{\Lambda_X,\Lambda}(-\frac{(1-X)}{\sqrt{1-X}}(\sqrt{2}P^+)+O(\frac{1}{P^+}))$$

LC helicity and Ordinary helicity give identical results at this order of P^+. Similarly for the outgoing gluon

$$\mathcal{G}^*_{\Lambda_X;\Lambda_g=x,\Lambda'}(X,\vec{k}_T'^2) \simeq \Gamma(k')\,\{\delta_{\Lambda',-\Lambda_X}(\Lambda')\frac{(1-X)M-(1-\zeta)M_X}{\sqrt{(1-\zeta)(1-X)}} + \delta_{\Lambda',\Lambda_X}\frac{(1-X)(\Delta_x-i(\Lambda')\Delta_y)+(1-\zeta)(k_x+i(\Lambda')k_y)}{\sqrt{(1-\zeta)(1-X)}}\}$$

$$\mathcal{G}^*_{\Lambda_X;\Lambda_g=y,\Lambda'}(X,\vec{k}_T'^2) \simeq \Gamma(k')\,\{\delta_{\Lambda',-\Lambda_X}(-i)\frac{(1-X)M-(1-\zeta)M_X}{\sqrt{(1-\zeta)(1-X)}} + \delta_{\Lambda',\Lambda_X}\frac{(1-X)(i(\Lambda')\Delta_x+\Delta_y)-(1-\zeta)(i(\Lambda')k_x-k_y)}{\sqrt{(1-\zeta)(1-X)}}\}$$

$$\mathcal{G}^*_{\Lambda_X;\Lambda_g=z,\Lambda'}(X,\vec{k}_T'^2) \simeq \Gamma(k')\,\delta_{\Lambda',\Lambda_X}(-\frac{(1-\zeta)(1-X)}{\sqrt{(1-\zeta)(1-X)}}(\sqrt{2}P^+)+\mathcal{O}(\frac{1}{P^+}))$$

Gluon-spectator light-front variable GPDs

$$H_g = \int d^2k_\perp \mathcal{N} \frac{1}{(1-X)^2}$$

$$\frac{[X(X-\zeta)((1-X)M - M_X)((\frac{1-X}{1-\zeta})M - M_X) + (1-\zeta)(1+\frac{(1-X)^2}{1-\zeta}))\vec{k_\perp}\cdot\vec{k_\perp}']}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}$$

$$+\frac{\frac{\zeta^2}{4}}{(1-\zeta)}E_g$$

$$E_g = \int d^2k_\perp \mathcal{N} \frac{-2M(1-\zeta)}{(1-X)} \frac{1}{(1-\zeta/2)}$$

$$\frac{[X((1-X)M - M_X)\frac{\vec{k_\perp}\cdot\vec{\Delta_\perp}}{\Delta_\perp^2} - (X-\zeta)((\frac{1-X}{1-\zeta})M - M_X)\frac{\vec{k_\perp}\cdot\vec{\Delta_\perp}}{\Delta_\perp^2}]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}$$

$$\tilde{H}_g = \int d^2k_\perp \mathcal{N} \frac{1}{(1-X)^2}$$

$$\frac{[X(X-\zeta)((1-X)M - M_X)((\frac{1-X}{1-\zeta})M - M_X) + (1-\zeta)(1-\frac{(1-X)^2}{1-\zeta}))\vec{k_\perp}\cdot\vec{k_\perp}']}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}$$

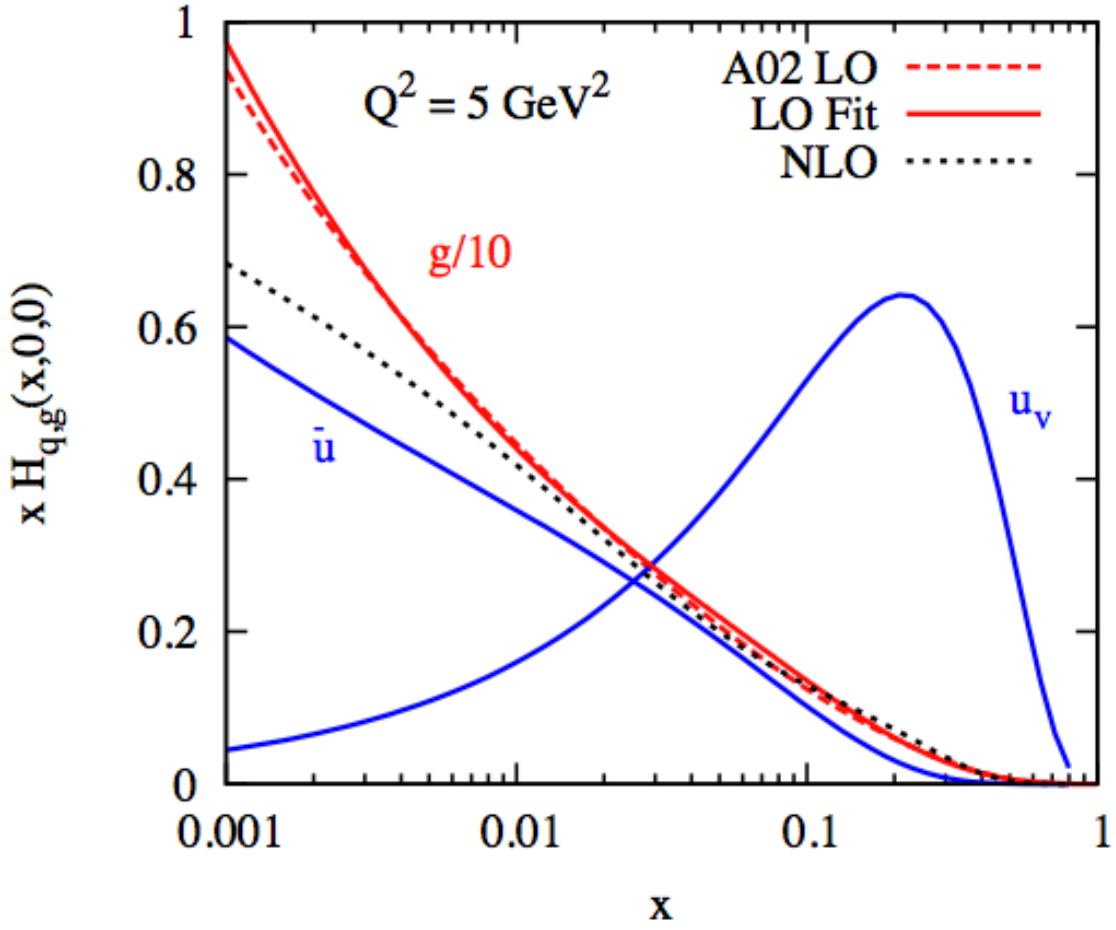
$$+\frac{\frac{\zeta^2}{4}}{(1-\zeta)}\tilde{E}_g$$

$$\tilde{E}_g = \int d^2k_\perp \mathcal{N} \frac{2}{\zeta} \frac{(-2M)(1-\zeta)}{(1-X)}$$

$$\frac{[X((1-X)M - M_X)\frac{\vec{k_\perp}\cdot\vec{\Delta_\perp}}{\Delta_\perp^2} + (X-\zeta)((\frac{1-X}{1-\zeta})M - M_X)\frac{\vec{k_\perp}\cdot\vec{\Delta_\perp}}{\Delta_\perp^2}]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}$$

Fitting gluon pdf's

c.f. Alekhin, .. etc.



pdf's fix x dependence

Gluon

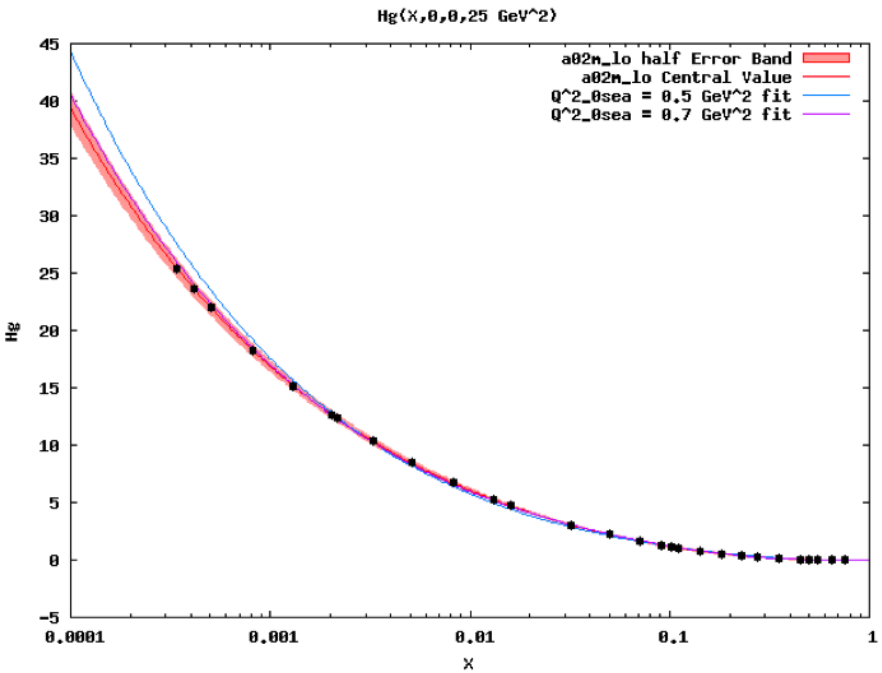


Figure 6: The plot above shows the distribution H_g(X,0,0,25 GeV^2) for the two fits. Alekhin's distribution Xg(X) used in the fit procedure is included with an error band of one half of the error for the set a02m_lo. The X points used in the fit procedure are indicated by black dots.

anti-u

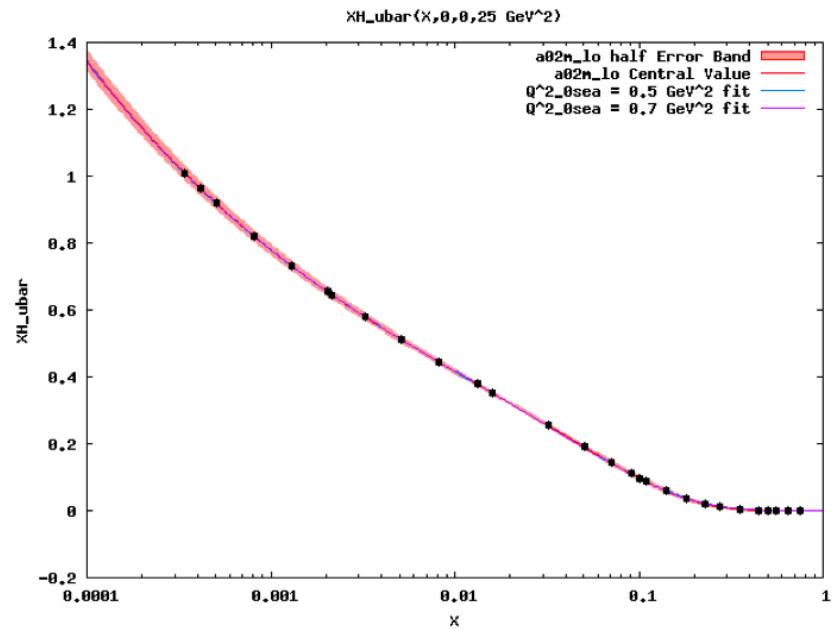


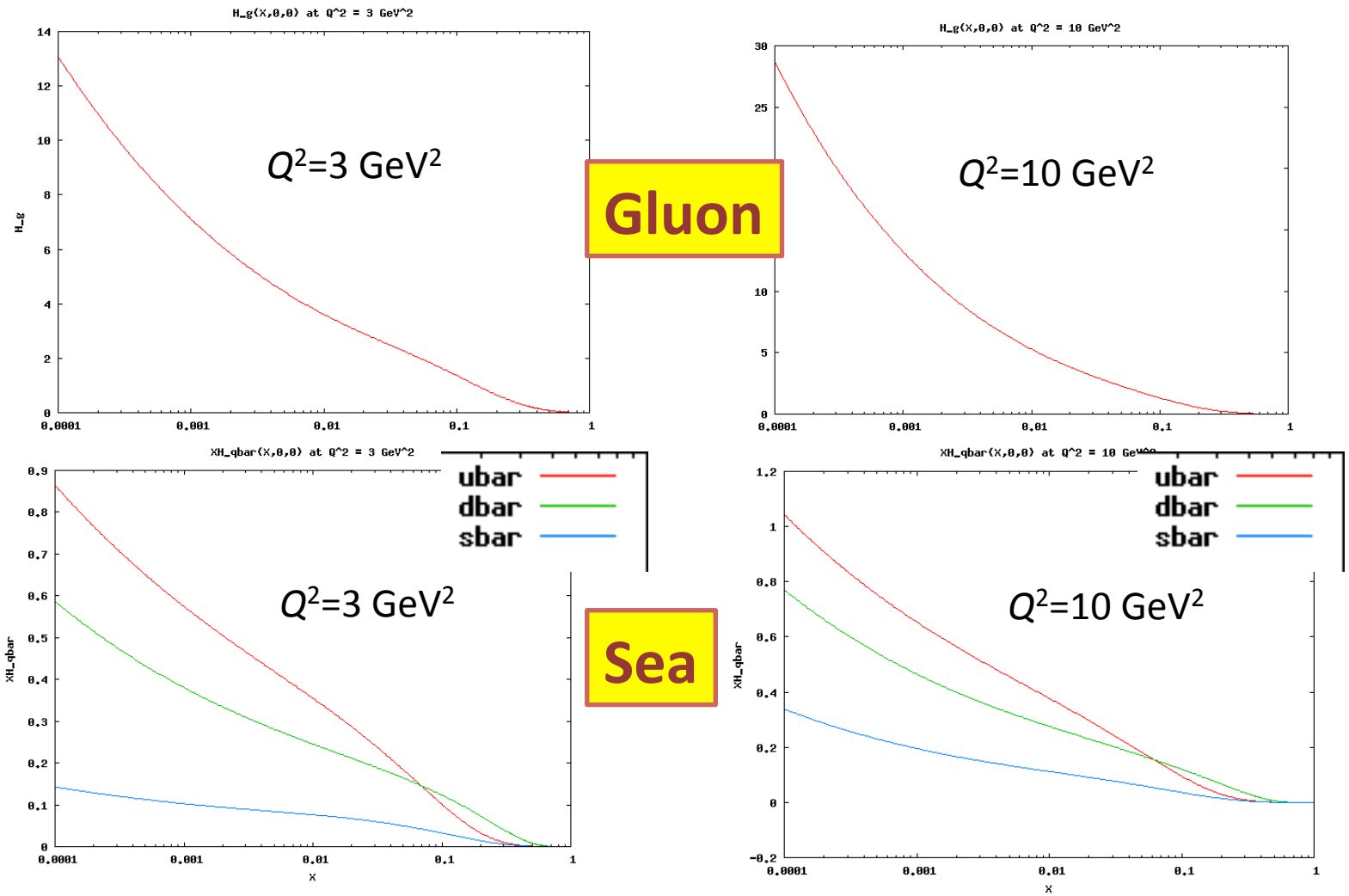
Figure 3: The plot above shows the distribution $XH_{\bar{u}}$(X,0,0,25 GeV^2) for the two fits. Alekhin's distribution $X\bar{u}$(X) used in the fit procedure is included with an error band of one half of the error for the set a02m_lo. The X points used in the fit procedure are indicated by black dots.

Single Q^2 value shown --- fit known pdf's all Q^2

from J. Poage

Gluon & sea distributions

J. Poage



- Gluon & Sea quark distributions
  - generalize spectator model
- N→ g + "color octet N" spectator
- N→ anti-u + "tetraquark"uuud color3

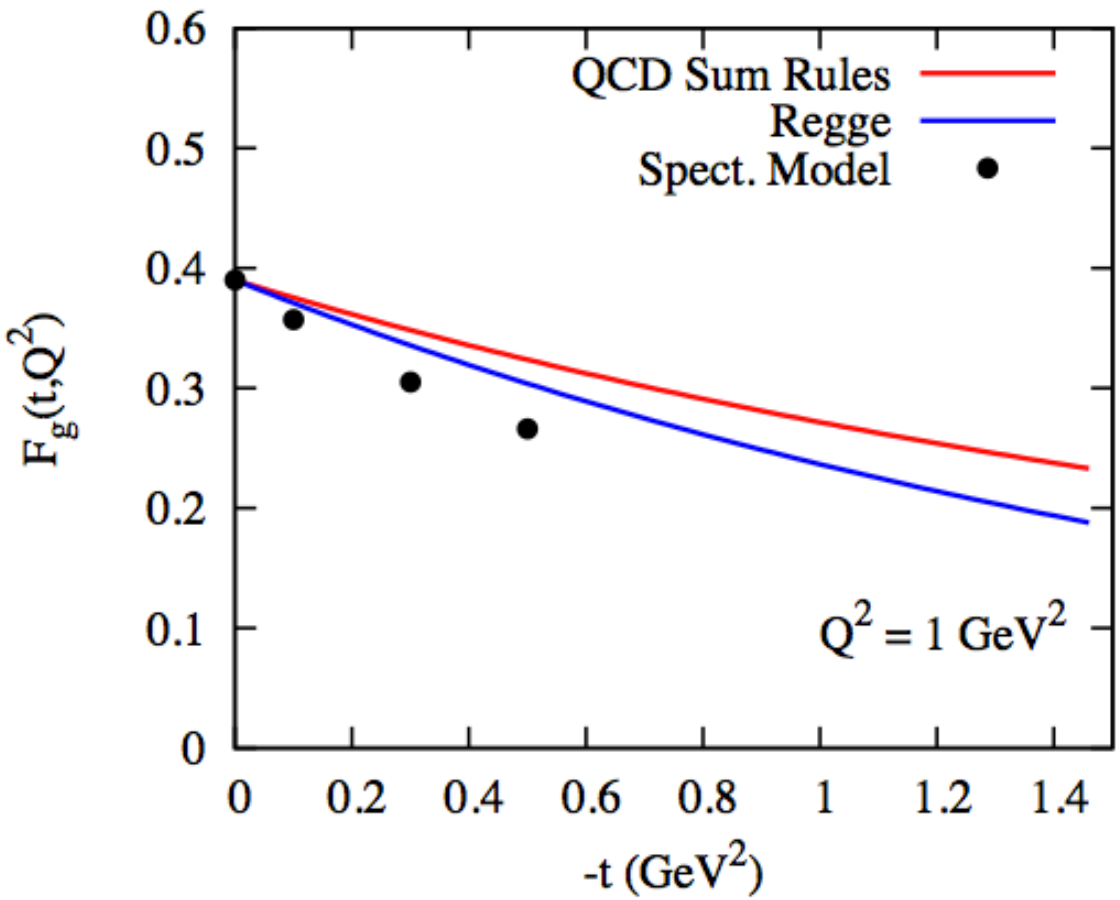
- How to normalize?
- Let $H_g(x,\xi,t)_Q^2 \rightarrow H_g(x,0,0)_Q^2 = xG(x)_Q^2$

- Use pdf's to fix x dependence

- How to fix t-dependence? For valence quarks used EM form factors:
  $\int dx\, H_q(x,0,t) = F_1^q(t)$, etc. . . .

Compare Gluon form factor via QCD sum rules

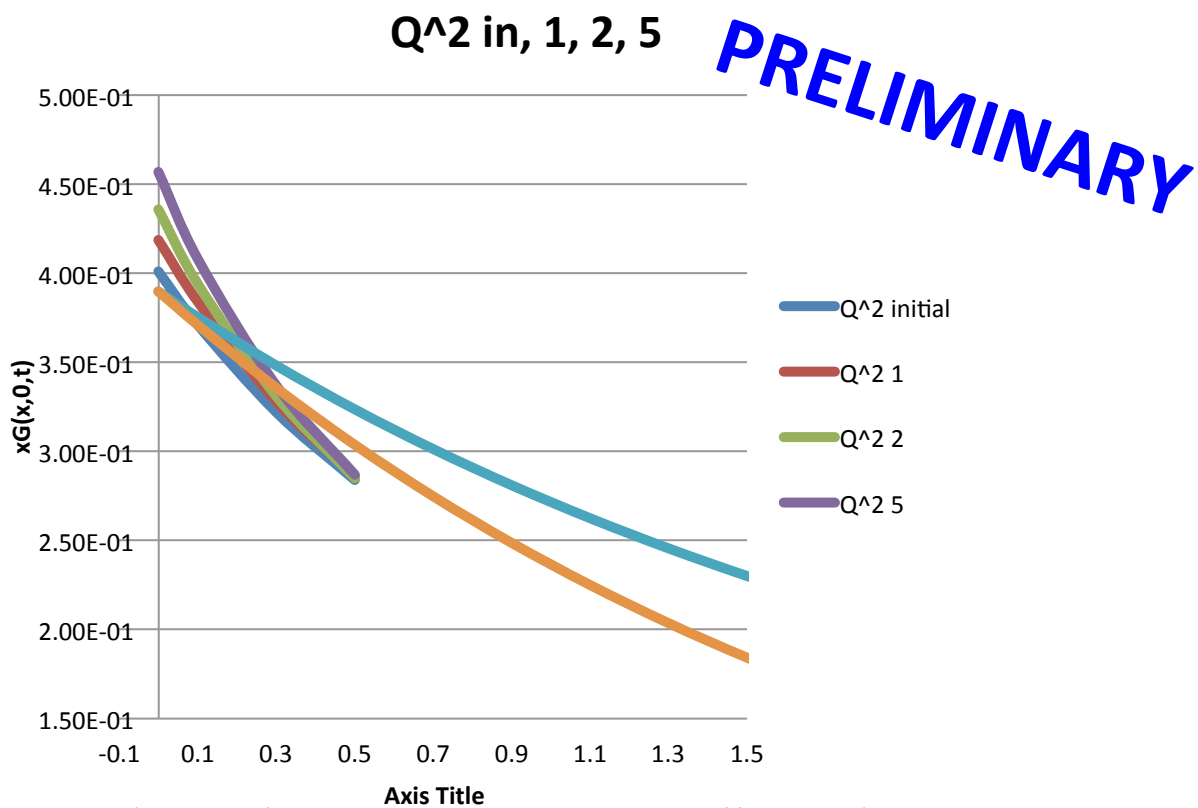
Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)



Spectator t-dependence w/o Regge small x behavior:
hybrid Regge-Spectator model combines

Compare Gluon form factor via QCD sum rules

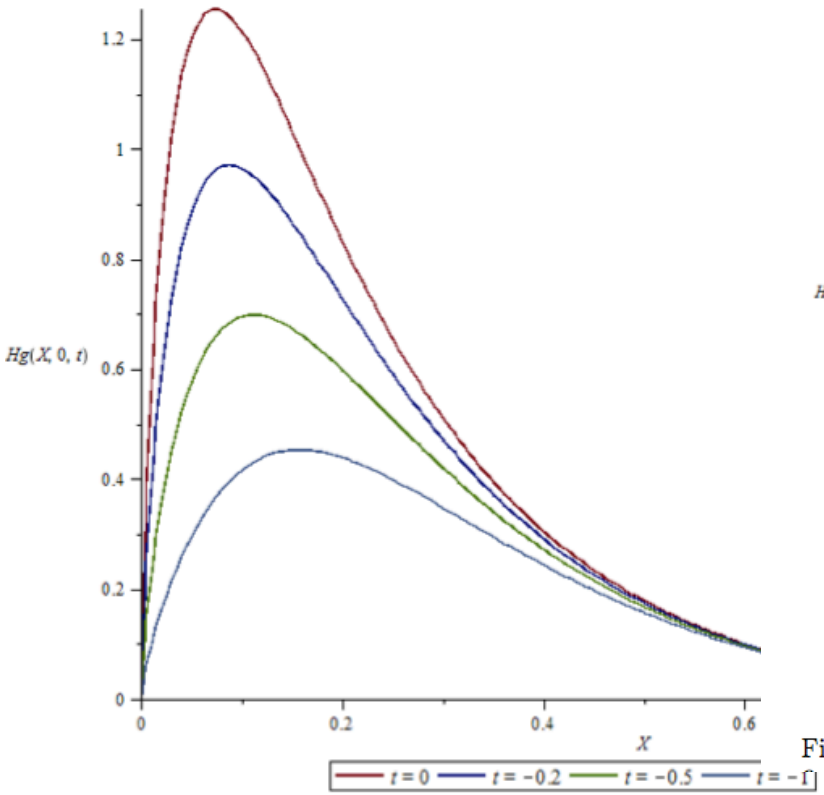
Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)



Spectator t-dependence w/o Regge small x behavior: hybrid Regge-Spectator model combines (connecting the dots)

Preliminary: x and t dependence of $H_g(x, 0, t)$ for input scale

J. Poage



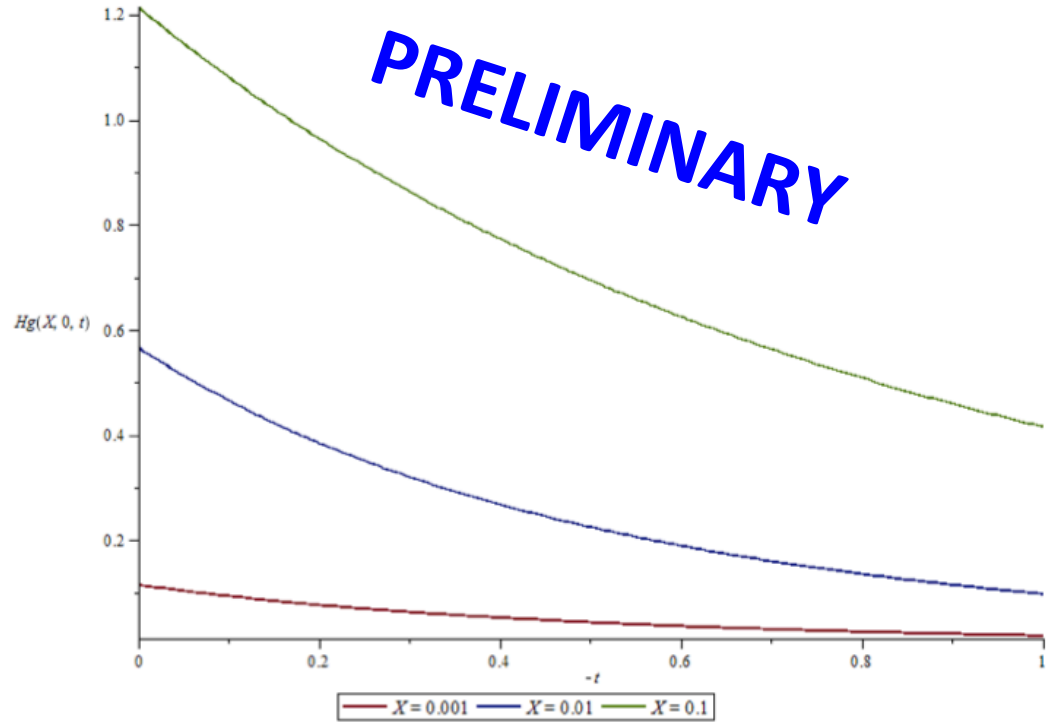


Figure 10: The plot above displays the distribution $H_g(X,0,t)$ as a function of t, for several

'igure 9: The plot above displays the distribution $H_g(X,0,t)$ for a range of t values.

Chiral odd quark GPDs

One question is: how do we normalize chiral-odd GPDs?

The only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_1(x)$$

Transversity

Form Factors

Integrates to tensor charge δ_q

$$\int H_T^q(x,\xi,t)\,dx = \delta q(t)$$

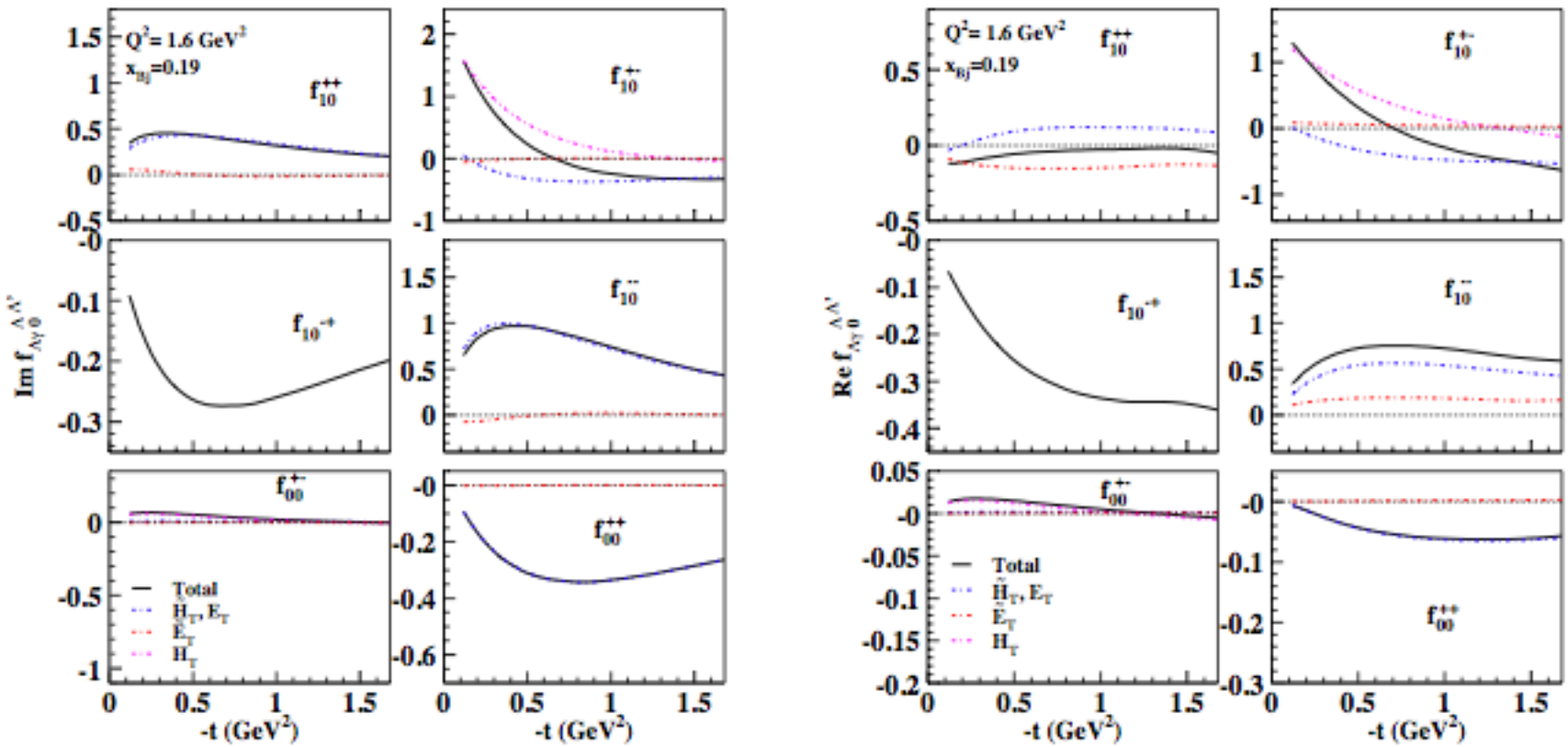
$$\int \bar{E}_T^q(x,\xi,t)\,dx = \int \left(2\tilde{H}_T^q + E_T^q\right)dx = \kappa_T^q(t)$$

"transverse moment" κ_T^q

$$\int \tilde{E}_T(x,\xi,t)\,dx = 0$$

No direct interpretation of E_T .

6 helicity amps for π^0
after Compton Form Factors



Selecting transversity

$$f_{10}^{++} \propto \Delta\left(2\widetilde{\mathcal{H}}_T + (1+\xi)\mathcal{E}_T - (1+\xi)\widetilde{\mathcal{E}}_T\right)$$

$$f_{10}^{+-} \propto \mathcal{H}_T + \frac{t_0 - t}{4M^2}\widetilde{\mathcal{H}}_T - \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T$$

$$f_{10}^{-+} \propto \Delta^2\,\widetilde{\mathcal{H}}_T$$

$$f_{10}^{--} \propto \Delta\left(2\widetilde{\mathcal{H}}_T + (1-\xi)\mathcal{E}_T + (1-\xi)\widetilde{\mathcal{E}}_T\right),$$

$$2\tilde{H}_T + E_T \equiv \bar{E}_T$$

Compare also f_{long}^{odd} & with chiral even f_{long}^{even}

$$f_{00}^{+-} = g_\pi^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\frac{\sqrt{t_0 - t}}{2M}$$

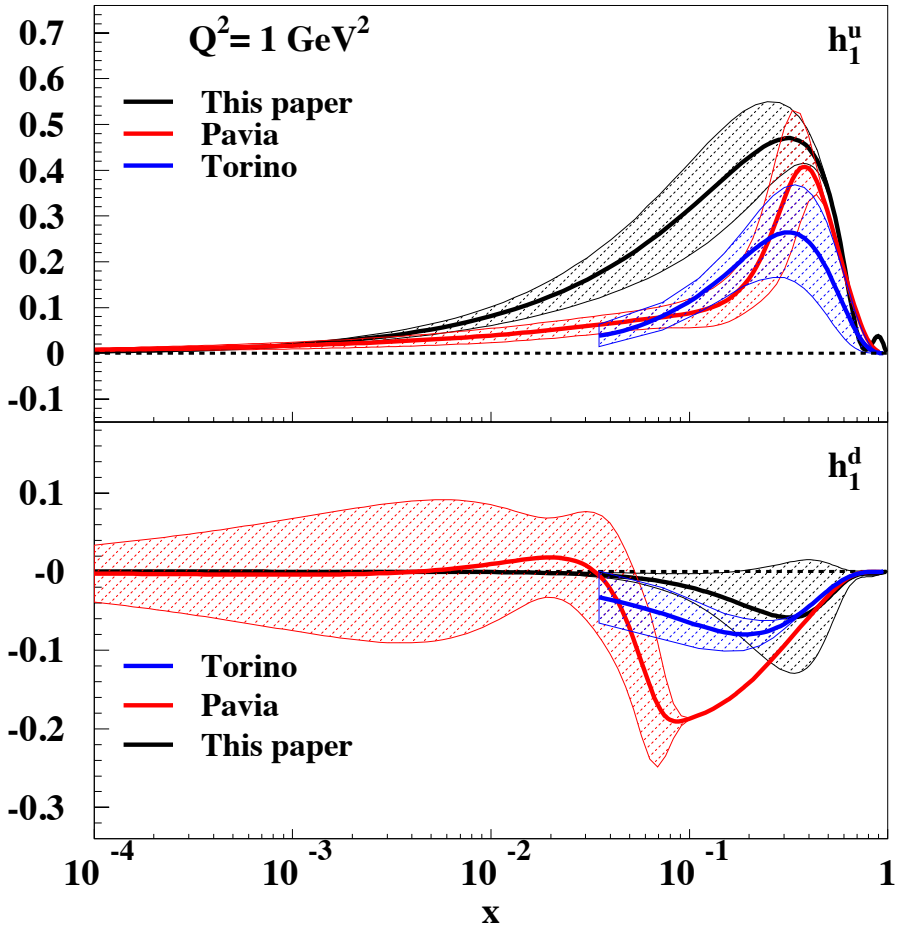
$$f_{00}^{++} = -g_\pi^{A,odd}(Q)\left(\frac{\sqrt{t_0 - t}}{2M}\right)^2\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right].$$

$$f_{00}^{+-,\,even} = \frac{\zeta}{\sqrt{1-\zeta}}\frac{1}{1-\zeta/2}\frac{\sqrt{t_o - t}}{2M}\widetilde{\mathcal{E}},$$

$$f_{00}^{++,\,even} = \frac{\sqrt{1-\zeta}}{1-\zeta/2}\widetilde{\mathcal{H}} + \frac{-\zeta^2/4}{(1-\zeta/2)\sqrt{1-\zeta}}\widetilde{\mathcal{E}},$$

Chiral odd GPDs →

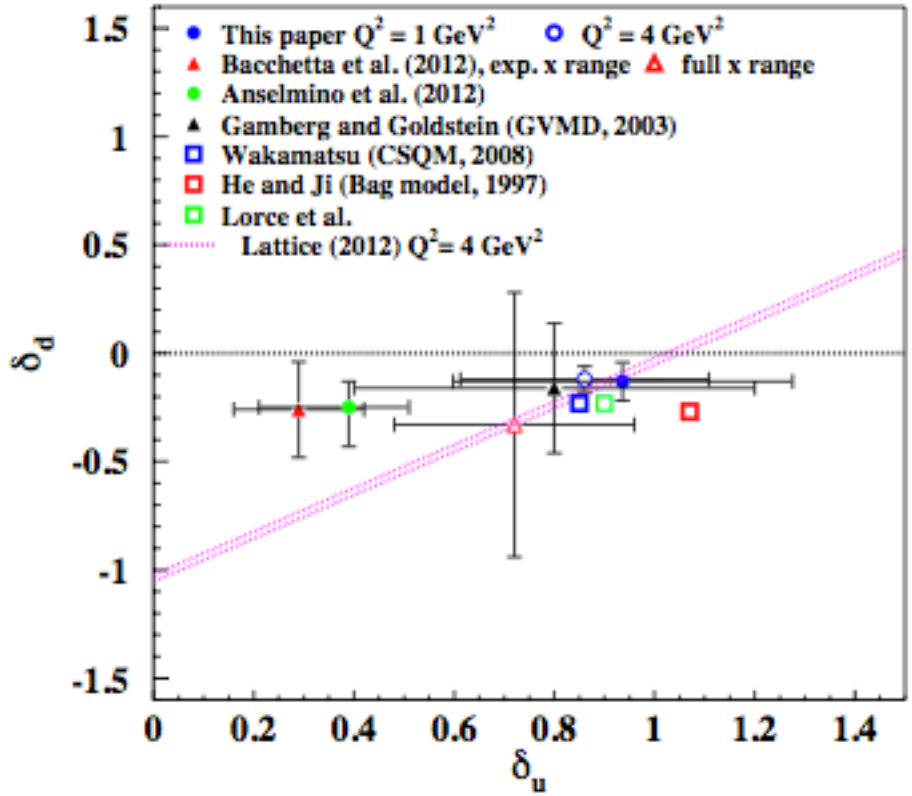
Transversity → pdf's: $h_1^q(x,Q^2)$



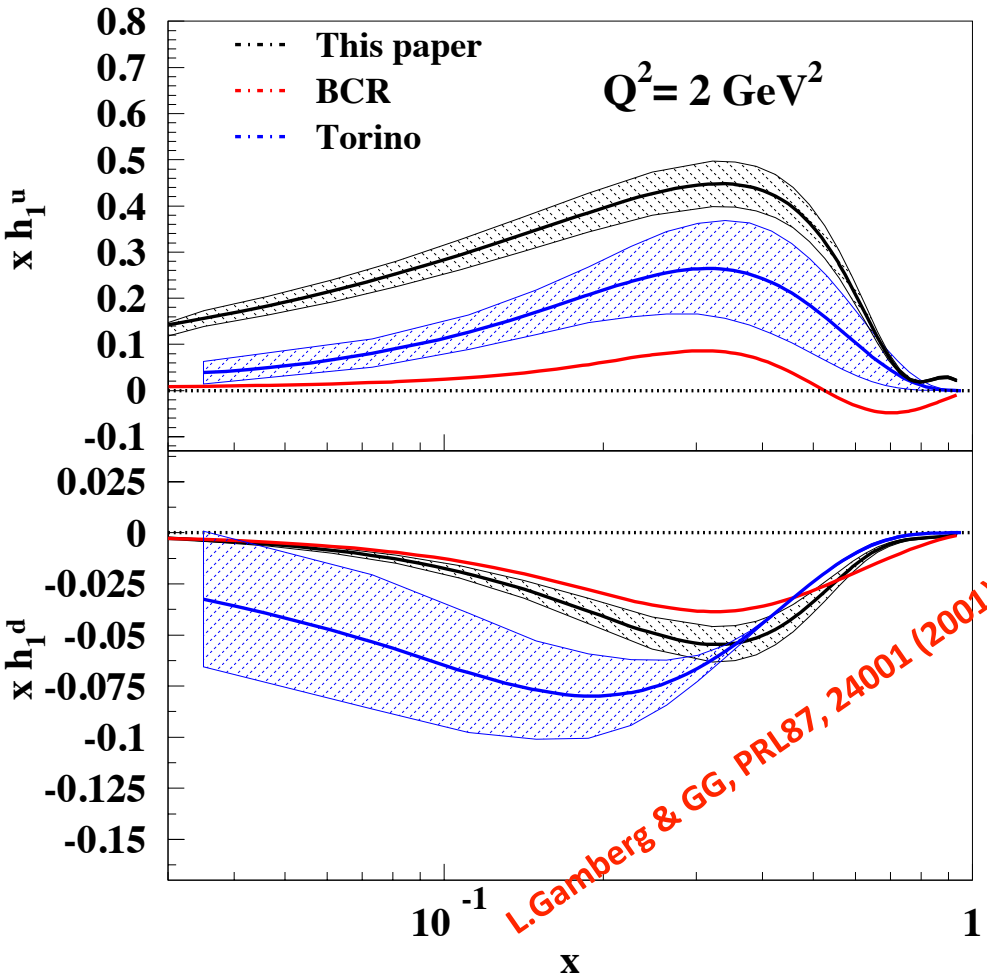
GG, Gonzalez, Liuti,
arXiv:1311.0483 [hep-ph]
1401.0438 PRD91, 114013 (2015)

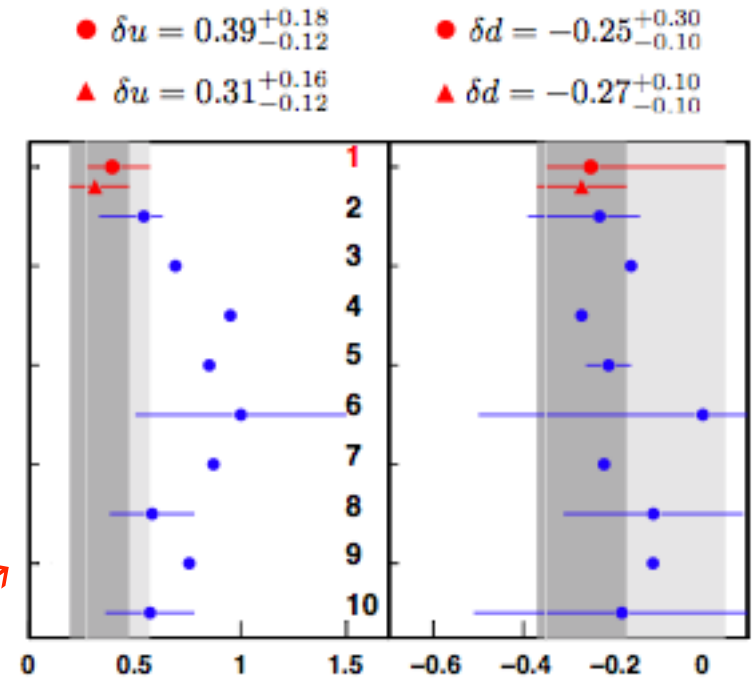
Extraction of tensor charge–

GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438



Extraction of tensor charge-- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438



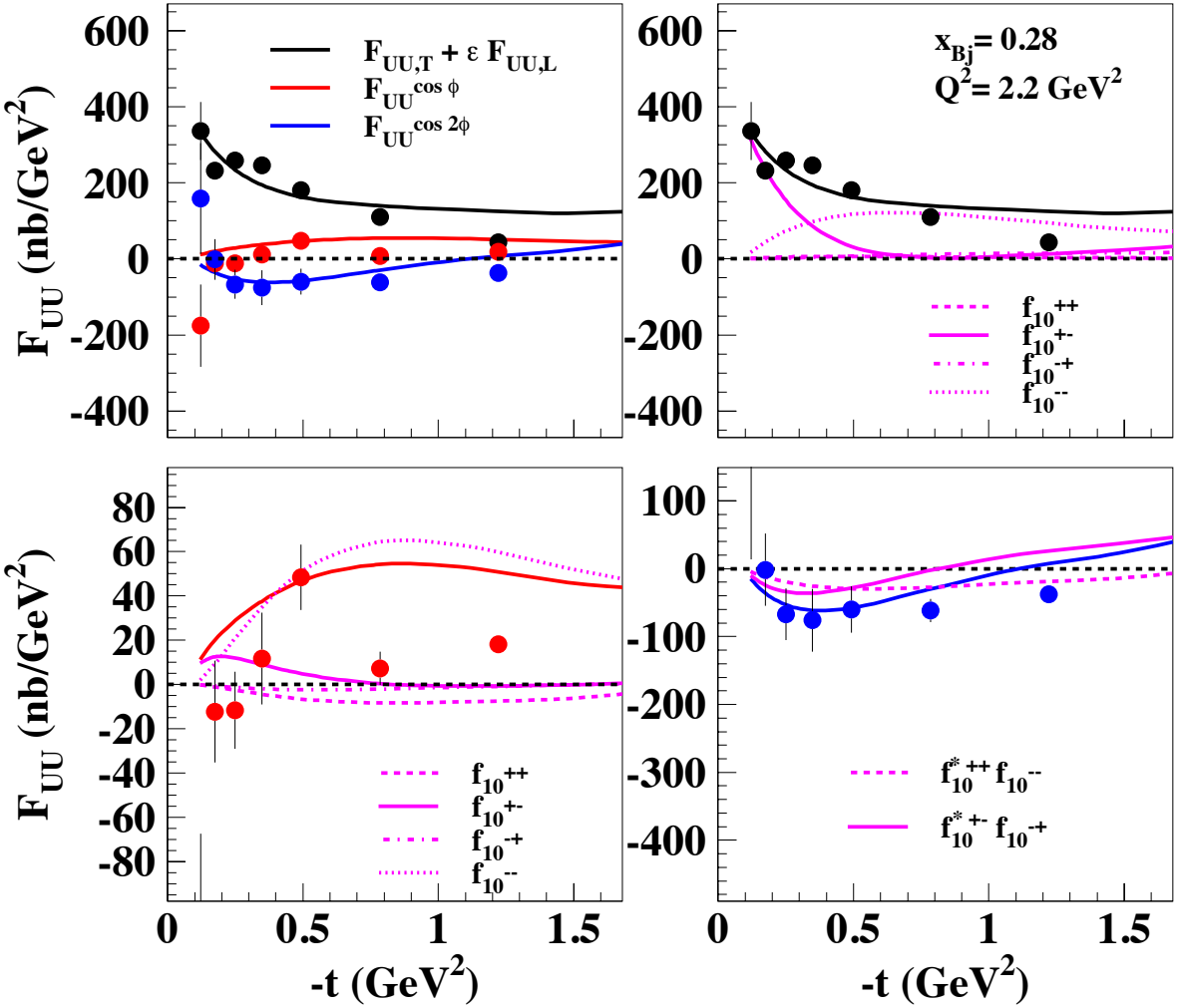


Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

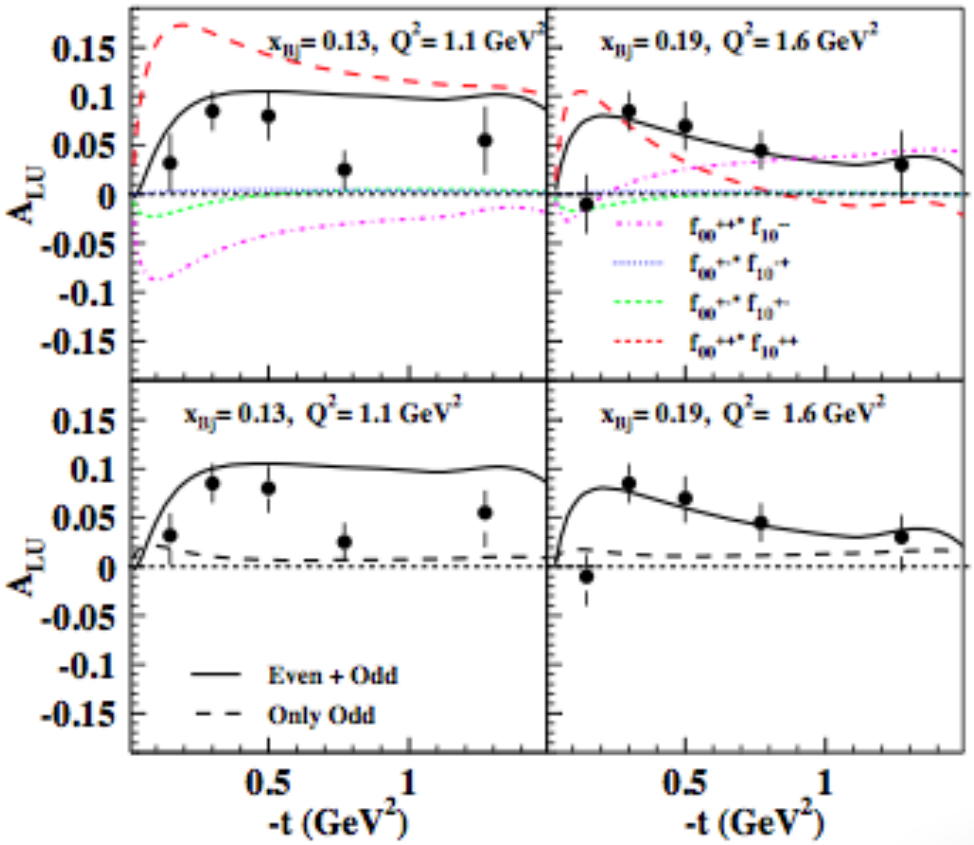
From our Reggeized form
$\delta u \approx 1.2$ $\delta d \approx -0.08$
Closer to QCD sum rule values

How well do the parameters fixed with DVCS data reproduce π^0 electroproduction data?

Hall B data, Bedlinskii, et al. PRL 109, 112001 (2012)

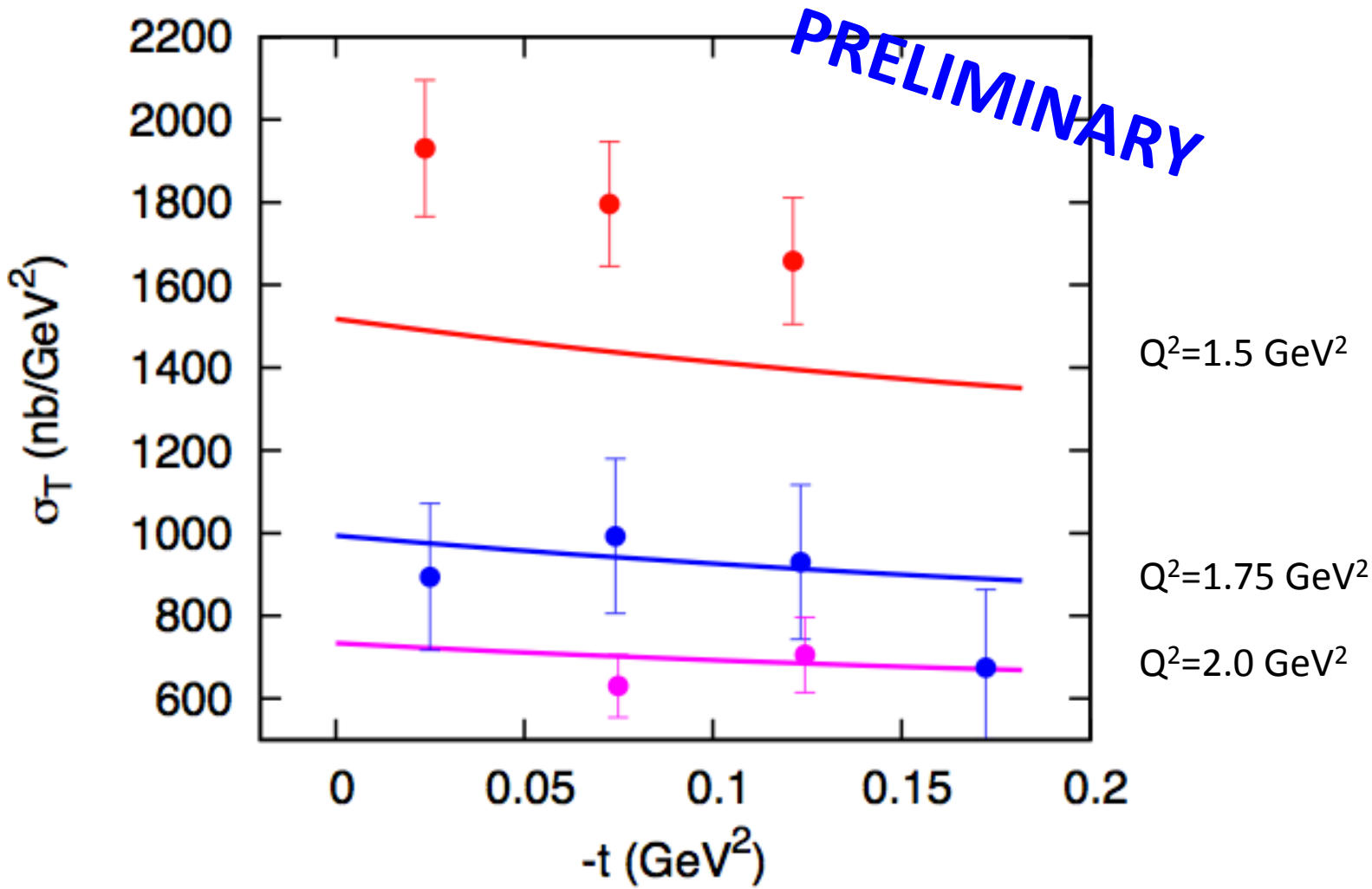


Beam spin asymmetry
shows importance of *H* chiral even (CLAS data –DeMasi, et al.)

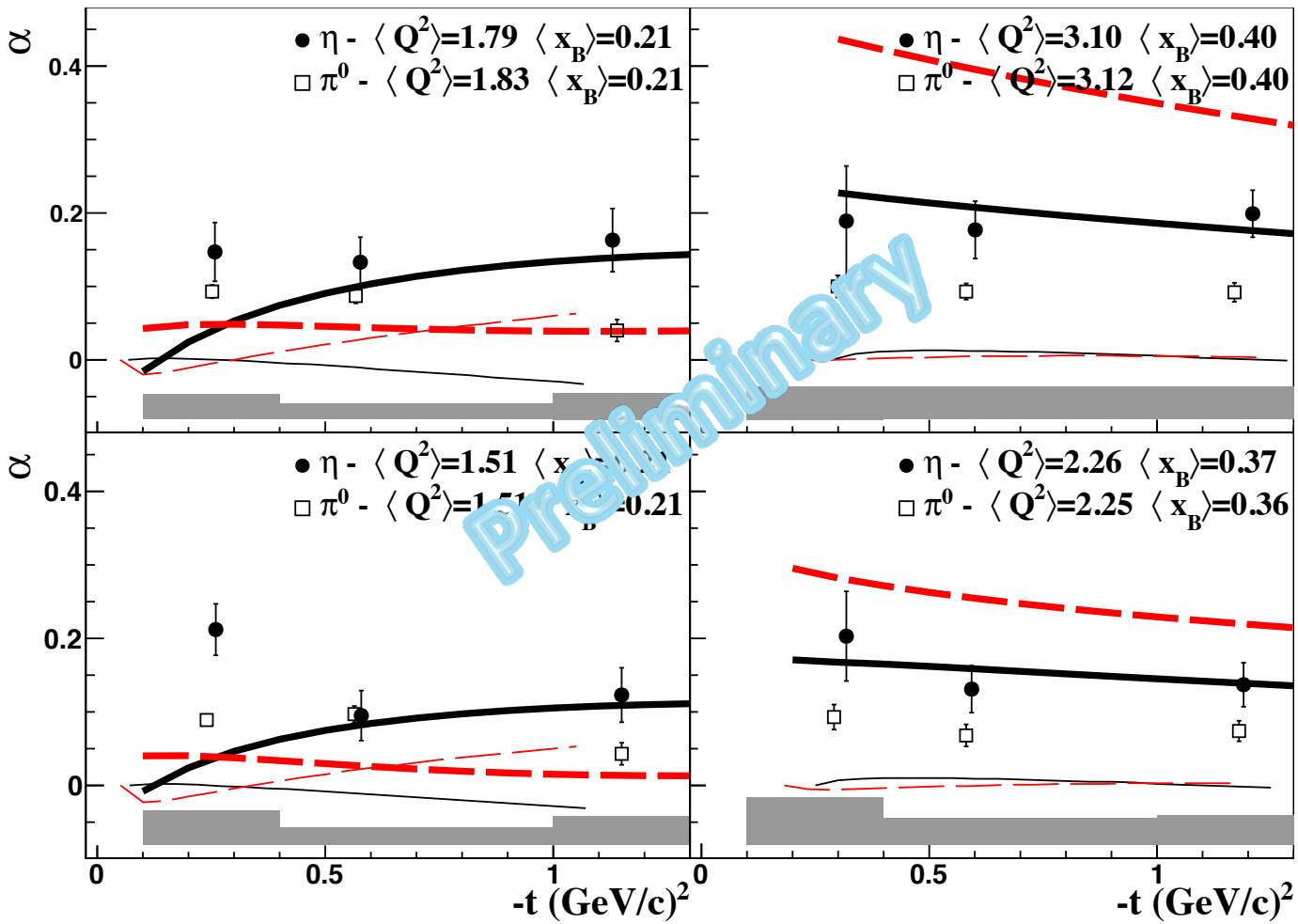


Hall A data x_B=0.36

courtesy F. Sabatie & M. Defurne



Ratio of unpolarized η / π^0 for flavor decomposition (CLAS data in progress, Andrey Kim, Harut Avakian)



Summary

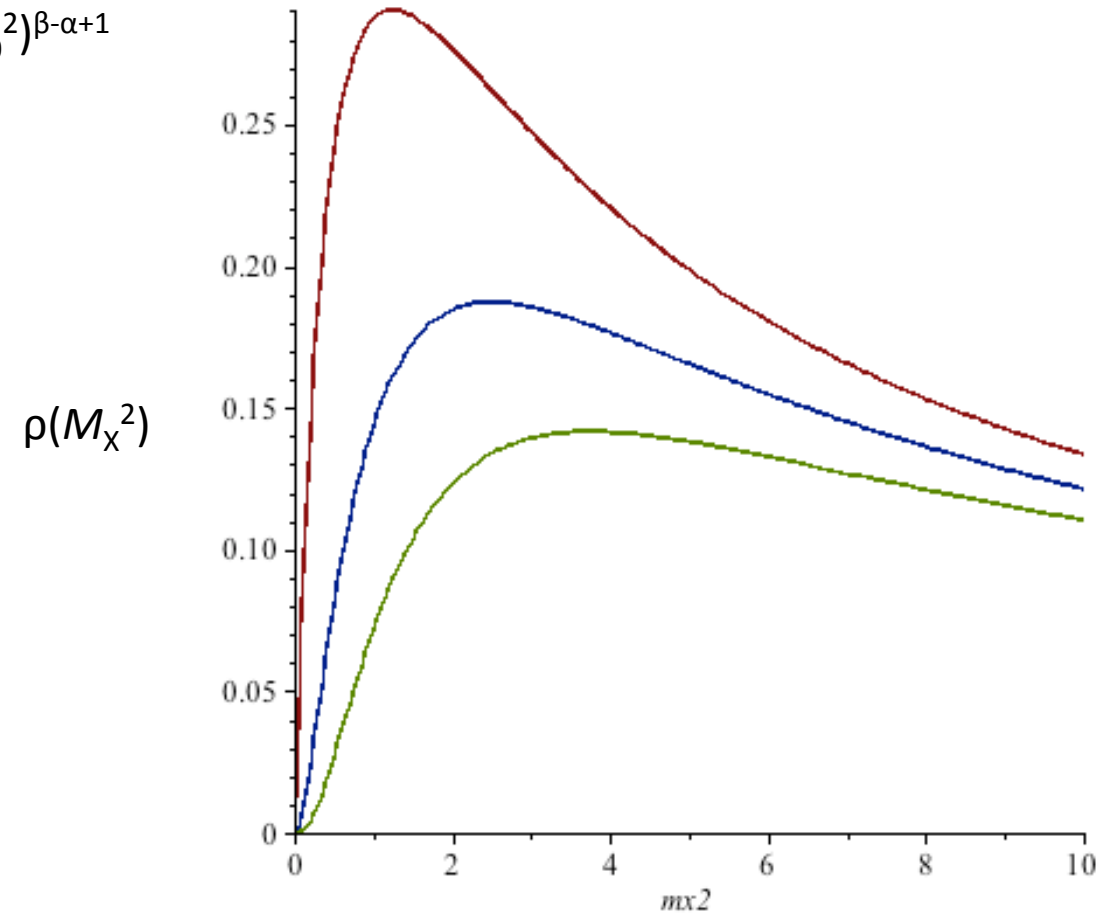
- Flexible parameterization for chiral even valence quarks from form factors, pdfs & DVCS R✖Dq
- Extended R✖Dq to chiral odd sector
- DVMP – π^0 many dσ 's & Asymmetries measure *Transversity*
- **Compared to new Hall A data – showed agreement within error bands.**
- **New Extension to gluons & the sea**
- **More phenomenology**

Backup Slides

Spectral distribution
of form $\rho(M_X^2,k^2) \approx \rho(M_X^2)\ \beta(k^2)$

$\rho(M_X^2) = (M_X^2/M_0^2)^{\beta}/(1+M_X^2/M_0^2)^{\beta-\alpha+1}$

$\beta(k^2)$ chosen to give large
k_T^2 falloff behavior



E_u & E_d , etc.
Gonzalez Hernandez, GG, Liuti
& GG, Liuti - QCD-Evol'np.Rel'n

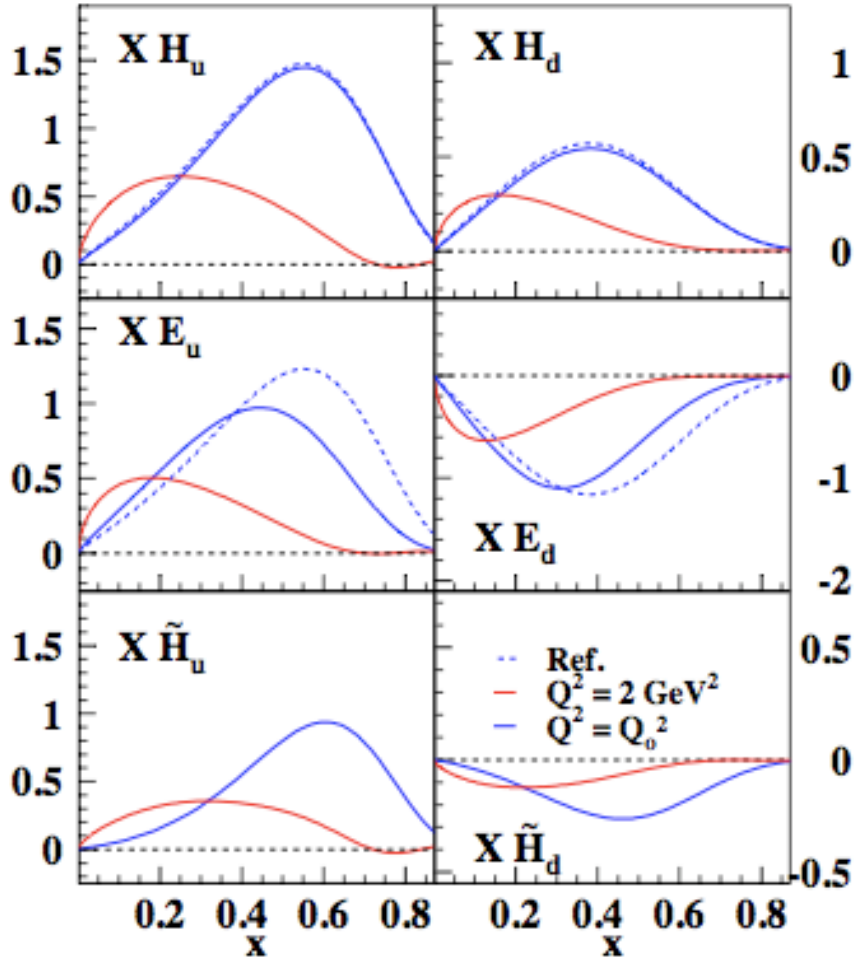


FIG. 6: (color online) GPDs $F_q(X,0,0) \equiv \{H_q, E_q, \widetilde{H}_q\}$, for $q = u$ (left) and $q = d$ (right), evaluated at the initial scale, $Q_o^2 = 0.0936$ GeV^2, and at $Q^2 = 2$ GeV^2, respectively. The dashed lines were calculated using the model in Refs. [24], [25] at the initial scale.

Compton Form Factors
→ Real & Imaginary Parts

$$\mathcal{H}_q(\zeta,t,Q^2) = \int_{-1+\zeta}^{+1} dX H_q(X,\zeta,t,Q^2) \times \left(\frac{1}{X-\zeta+i\epsilon} + \frac{1}{X-i\epsilon}\right)$$

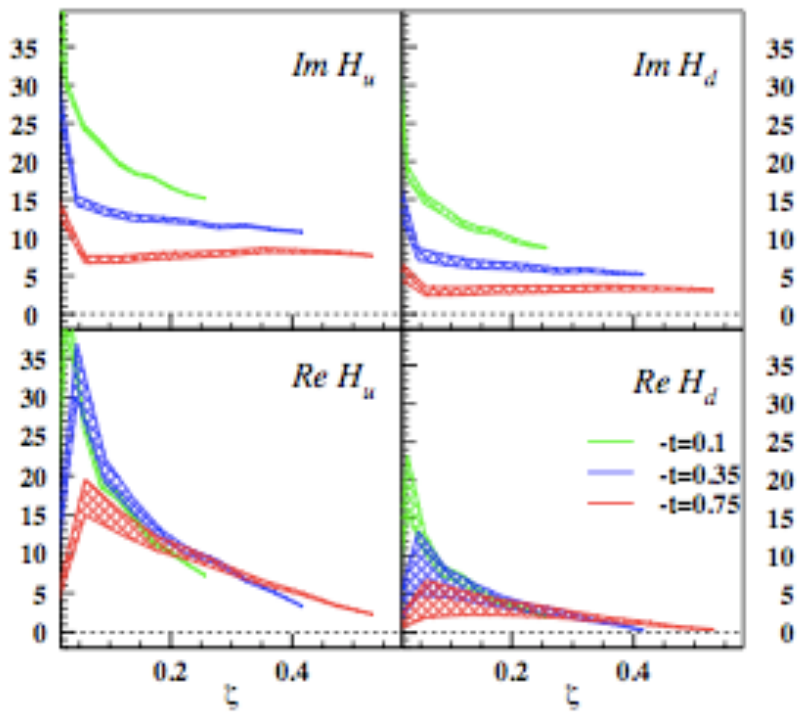
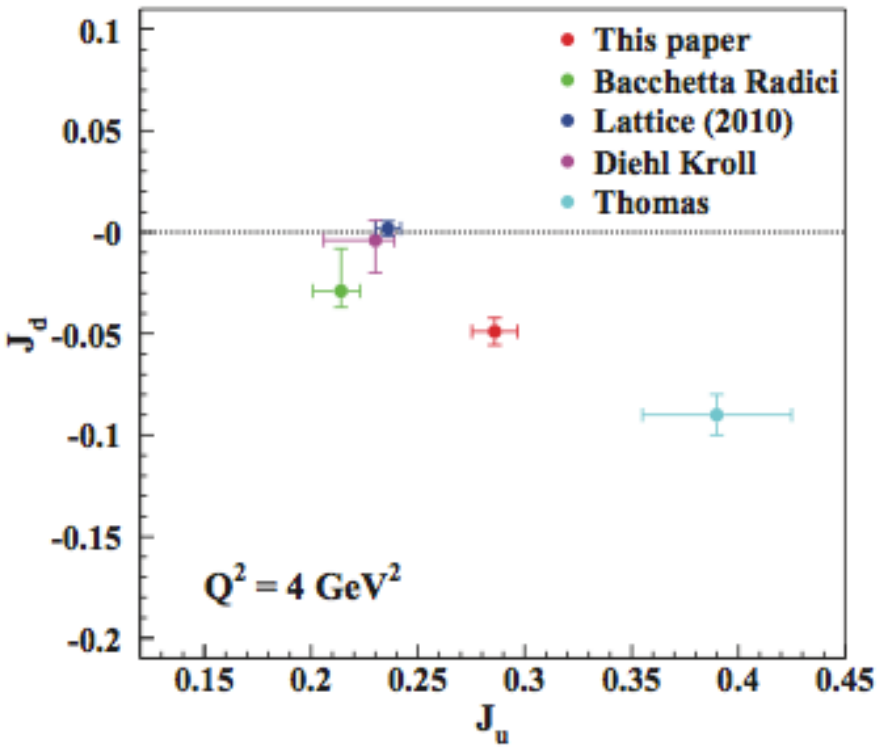


FIG. 9: (color online) Real and imaginary parts of the CFFs, $\mathcal{H}_i(\zeta,t)$, entering Eqs.(85). The CFFs are plotted vs. $x_{Bj} \equiv \zeta$, for different values of t, at $Q^2 = 2$ GeV^2. They are shown with the theoretical uncertainty from the parameters in TableI. Similar results are obtained for E and $\tilde{H}$.

Valence quark angular momenta – from "flexible" chiral even model applied to EM form factors, pdf's & some cross section & asymmetry data

Gonzalez Hernandez, Liuti, GRG, Kathuria

PHYSICAL REVIEW C **88**, 065206 (2013)



Improved precision based on EM Form Factor measurements
G. D. Cates, et al,, Phys. Rev. Lett. **106**, 252003 (2011).

Observables expressed in bilinears of helicity amps – 6 amps for π^0

Compton Form Factors

Couplings $g_\pi^{V\ \&/\text{or}\ A}(Q^2)$

f_1

$$f_{10}^{++} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1+\xi)\left(\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1}{2-\zeta}E_T + \frac{1}{2-\zeta}\widetilde{\mathcal{E}}_T\right],$$

f_2

$$f_{10}^{+-} = \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

$$= \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta}\mathcal{E}_T + \frac{\zeta/2}{1-\zeta}\widetilde{E}_T\right]$$

f_3

$$f_{10}^{-+} = -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

$$= -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\,\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

f_4

$$f_{10}^{--} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1-\xi)\left(\mathcal{E}_T - \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta}\mathcal{E}_T + \frac{1-\zeta}{2-\zeta}\widetilde{\mathcal{E}}_T\right]$$

Also Chiral Even CFFs

f_5

$$f_{00}^{+-} = g_\pi^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

f_6

$$f_{00}^{++} = -g_\pi^{A,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

Asymmetries & helicity amps

structure functions for the unpolarized beam and single transversely polarized target,

$$F_{UT,T}^{\sin(\phi-\phi_S)} = \Im m\, F_{11}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{10}^{-\Lambda'} = \Im m \left[f_{10}^{++*} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--} \right]$$

$$F_{UT,L}^{\sin(\phi-\phi_S)} = \Im m\, F_{00}^{+-} = \Im m \sum_{\Lambda'} f_{00}^{+\Lambda'*} f_{00}^{-\Lambda'} = \Im m \left[f_{00}^{++*} f_{00}^{-+} + f_{00}^{+-*} f_{00}^{--} \right]$$

$$F_{UT}^{\sin(\phi+\phi_S)} = \Im m\, F_{1-1}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{-10}^{-\Lambda'} = \Im m \left[-f_{10}^{++*} f_{10}^{+-} + f_{10}^{+-*} f_{10}^{++} \right]$$

$$F_{UT}^{\sin(3\phi+\phi_S)} = \Im m\, F_{1-1}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda'*} f_{-10}^{+\Lambda'} = \Im m \left[f_{10}^{-+*} f_{10}^{--} - f_{10}^{--*} f_{10}^{-+} \right]$$

$$F_{UT}^{\sin\phi_S} = \Im m\, F_{10}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{00}^{-\Lambda'} = \Im m \left[f_{10}^{++*} f_{00}^{-+} + f_{10}^{+-*} f_{00}^{--} \right]$$

$$F_{UT}^{\sin(2\phi-\phi_S)} = \Im m\, F_{10}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda'*} f_{00}^{+\Lambda'} = \Im m \left[f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-} \right],$$

and three for the longitudinally polarized lepton and transversely polarized target,

$$F_{LT}^{\cos(\phi-\phi_S)} = \Re e\, F_{11}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{10}^{-\Lambda'} = \Re e \left[f_{10}^{++*} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--} \right]$$

$$F_{LT}^{\cos\phi_S} = \Re e\, F_{10}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{00}^{-\Lambda'} = \Re e \left[f_{10}^{++*} f_{00}^{+-} + f_{10}^{+-*} f_{00}^{--} \right]$$

$$F_{LT}^{\cos(2\phi-\phi_S)} = \Re e\, F_{10}^{-+} = \Re e \sum_{\Lambda'} f_{10}^{-\Lambda'*} f_{00}^{+\Lambda'} = \Re e \left[f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-} \right].$$