Double parton scattering in a multi-scale regime

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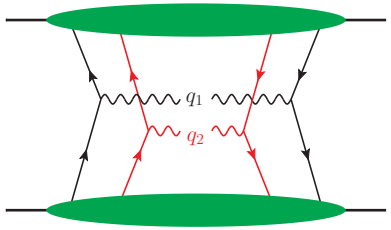
work in progress with M Buffing and T Kasemets

Double parton scattering at intermediate q_T



- production of two colour-singlet particles
  e.g. $pp \to V_1 V_2 + X$, $pp \to VH + X$, $pp \to HH + X$ $\quad V = Z, W^\pm$
  at $\Lambda \ll q_T \ll Q$ $\quad \Lambda =$ nonpert. scale, $\quad q_T \sim |\boldsymbol{q}_{1,2}|$, $\quad Q \sim Q_{1,2}$

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, dx_2\, d\bar{x}_1\, d\bar{x}_2\, d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} = \frac{1}{C} \sum_{a_1,a_2,b_1,b_2} \hat{\sigma}_{a_1 b_1}(Q_1,\mu_1)\, \hat{\sigma}_{a_2 b_2}(Q_2,\mu_2)$$
$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, d^2\boldsymbol{y}\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, W_{a_1 a_2 b_1 b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \nu)$$

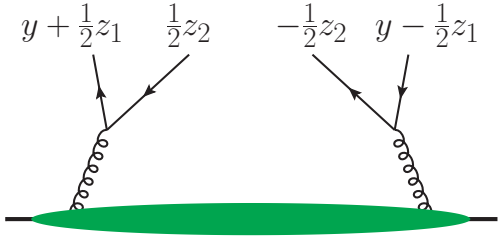
$$W_{a_1 a_2 b_1 b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \nu)$$
$$= \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \sum_R {}^R F_{b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta})\, {}^R F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

$R = 1, 8, \ldots$ colour representation

$\mu_{1,2} =$ UV renorm. scales $\qquad \zeta, \bar{\zeta} =$ rapidity regulators with $\zeta\bar{\zeta} = Q_1^2 Q_2^2$

- $|\boldsymbol{z}_{1,2}| \sim 1/q_T$ from Fourier exponents, dominant $|\boldsymbol{y}|$?

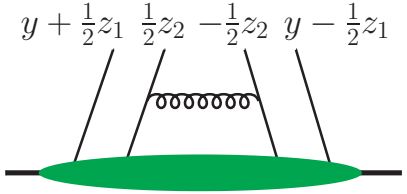
Relevant regions of $\boldsymbol{y}$



$$W_{a_1 a_2 b_1 b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \nu)$$
$$= \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \sum_R {}^R F_{b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta})\, {}^R F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

- $\boldsymbol{y}_\pm = \boldsymbol{y} \pm \frac{1}{2}(\boldsymbol{z}_1 + \boldsymbol{z}_2)$
  - $\Phi(\nu \boldsymbol{y}_\pm)$ removes UV region $|\boldsymbol{y}_\pm| \ll 1/\nu$
  - take $\nu \sim Q$
  - Φ dependence canceled by subtraction term in SPS graph

    $\rightarrow$ talk J Gaunt and slide 10 here
- region of large $|\boldsymbol{y}| \sim 1/\Lambda \gg |\boldsymbol{z}_i| \sim 1/q_T$
  - expansion $F(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) = C(\boldsymbol{z}_1 \mu_1)\, C(\boldsymbol{z}_2 \mu_2)\, F(\boldsymbol{y}) \sim \Lambda^2$
  - gives $\int d^2\boldsymbol{y}\, W(\boldsymbol{z}_i, \boldsymbol{y}) \sim \Lambda^2$



omit $x_i, \zeta, \nu, \ldots$ and labels R, a_i, b_i when not needed

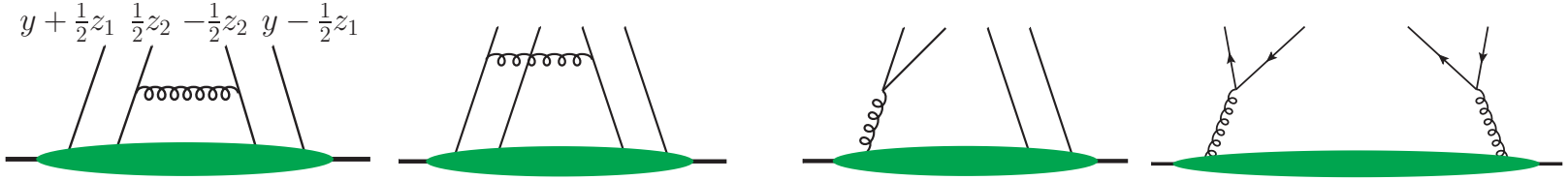
- region of small $|\boldsymbol{y}| \sim 1/q_T \sim |\boldsymbol{z}_i|$?

Relevant regions of $\boldsymbol{y}$

$$W = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \sum_R {}^R F(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta})\, {}^R F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

- region of small $|\boldsymbol{y}| \sim 1/q_T \sim |\boldsymbol{z}_i|$
  expand on collinear distributions (all fields at same trv. position)

$$F(\boldsymbol{z}_i, \boldsymbol{y}) = F_{\text{int}} + F_{\text{tw3}} + F_{\text{spl}}$$



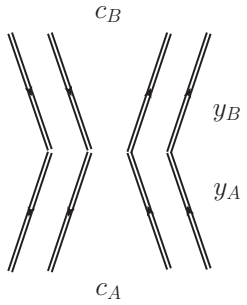
$F_{\text{int}} = G + C(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) \otimes G \sim \Lambda^2$ $\qquad$ $G =$ twist 4 dist, $C \propto \alpha_s$

F_{tw3} $\qquad$ only if chiral odd, discard

$F_{\text{spl}} \sim \dfrac{\boldsymbol{y}_+}{\boldsymbol{y}_+^2} \dfrac{\boldsymbol{y}_-}{\boldsymbol{y}_-^2}\, P \cdot f(x_1 + x_2) \sim q_T^2$ $\qquad$ $f =$ PDF, $P \propto \alpha_s$

$$\text{gives} \int d^2\boldsymbol{y}\, W(\boldsymbol{z}_i, \boldsymbol{y}) \sim \begin{cases} \alpha_s^2 q_T^2 & \text{from } F_{\text{spl}} \times F_{\text{spl}} \text{ (1vs1)} \\ \alpha_s \Lambda^2 & \text{from } F_{\text{spl}} \times F_{\text{int}} \text{ (1vs2)} \\ \Lambda^4 / q_T^2 & \text{from } F_{\text{int}} \times F_{\text{int}} \text{ (2vs2)} \end{cases}$$

Soft factor: closer look



- have $S_{c_B,c_A}(\boldsymbol{z}_i, \boldsymbol{y}; y_A - y_B)$
  $y_{A,B}$ = rapidities, $c_{A,B}$ = colour labels
- in analogy with single scattering case expect for $y_A - y_B \gg 1$

$$\frac{\partial}{\partial(y_A - y_B)} \mathbf{S}(y_A - y_B) = \mathbf{K} \cdot \mathbf{S}(y_A - y_B) = \mathbf{S}(y_A - y_B) \cdot \mathbf{K}$$

- holds if for $y_A - y_B \gg 1$

$$\mathbf{S}(y_A - y_B) = \exp\big[(y_A - y_B)\,\mathbf{K} + \mathbf{C}\big] \quad \text{with} \quad \mathbf{C} = \mathbf{0}$$

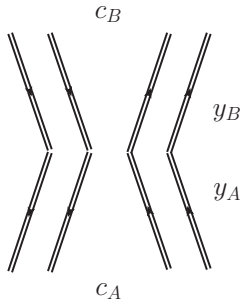
  ok perturbatively at 1 loop

$$\mathbf{S}(y_A - y_B) = \exp\big[(y_A - y_B)\tanh(y_A - y_B)\,\mathbf{K}\big] \quad \text{for all } y_A, y_B$$

  beyond: take as working hypothesis
- with square root construction for DTMDs, after projecting c_B, c_A on representations R', R:

$$\frac{\partial}{\partial \log \zeta} {}^{R}F(\boldsymbol{z}_i, \boldsymbol{y}; \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'}K(\boldsymbol{z}_i, \boldsymbol{y})\, {}^{R'}F(\boldsymbol{z}_i, \boldsymbol{y}; \zeta)$$

Evolution of DTMDs



- Collins-Soper equation:

$$\frac{\partial}{\partial \log \zeta} {}^{R}F_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \frac{1}{2} \sum_{R'} {}^{RR'}K_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i)\, {}^{R'}F_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

  a_1, a_2: quarks or gluons

- RGE for Collins-Soper kernel:

$$\frac{\partial}{\partial \log \mu_1} {}^{RR'}K_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) = \gamma_{K,a_1}(\alpha_s(\mu_1))\, \delta_{RR'}$$

  same for μ_2

- convenient to write

$${}^{RR'}K_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) = \delta_{RR'} \left[{}^{1}K_{a_1}(\boldsymbol{z}_1; \mu_1) + {}^{1}K_{a_2}(\boldsymbol{z}_2; \mu_2) \right] + {}^{RR'}M_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y})$$

  with ${}^{1}K_a$ = usual CS kernel for quarks or gluons

  - ${}^{RR'}M_{a_1 a_2}$ independent of μ_i
    computed at 1 loop for all channels ($a_1 a_2 = qq, qg, gg$)
    for qq easily diagonalised (lengthy expressions for qg)

Evolved DTMDs

- together with RGE of F get

$$
{}^{R}F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \exp\big[{}^{1}S(\boldsymbol{z}_1) + {}^{1}S(\boldsymbol{z}_2)\big]
\times \sum_{R'} {}^{RR'}\exp\left[M(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right] {}^{R'}F(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)
$$

with ${}^{1}S(\boldsymbol{z}; \mu$ and ζ scales$)$ = familiar Sudakov factor for single TMDs

$$
f(x, \boldsymbol{z}; \mu, \zeta) = \exp\big[{}^{1}S(\boldsymbol{z})\big]\, f(x, \boldsymbol{z}; \mu_0, \zeta_0)
$$

up to some rescaling $\zeta \to x_1/x_2\,\zeta$ or $x_2/x_1\,\zeta$

- ${}^{1}S$ contains all double logs
- in region $|\boldsymbol{y}| \sim |\boldsymbol{z}_i| \sim 1/q_T$
  - no suppression of colour channels $R \neq 1$ from CS evolution
  - can use short-dist. expansion of $F(\boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)$ if $\mu_{0i} \sim \sqrt{\zeta_0} \sim q_T$
  - for ${}^{R}F_{\text{spl}}$ find that $R \neq 1$ not suppressed

Put together cross section

- have in different regions

$$W_{\text{small } y} = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-)\, \exp\big[{}^1S(\boldsymbol{z}_1) + {}^1S(\boldsymbol{z}_2)\big] \sum_{RR'} {}^R F_{\text{int+spl}}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)$$
$$\times\, {}^{RR'}\exp\left[M(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{Q_1 Q_2}{\zeta_0}\right] {}^{R'} F_{\text{int+spl}}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)$$

$$W_{\text{large } y} = \Phi^2(\nu \boldsymbol{y})\, \exp\big[{}^R S(\boldsymbol{z}_1) + {}^R S(\boldsymbol{z}_2)\big] \exp\left[{}^R J(\boldsymbol{y}; \mu_{0i}) \log \frac{Q_1 Q_2}{\zeta_0}\right]$$
$$\times\, {}^R\big[C(\boldsymbol{z}_1) \otimes C(\boldsymbol{z}_2) \otimes F(\bar{x}_i, \boldsymbol{y})\big](\mu_{0i}, \zeta_0)\; {}^R\big[C(\boldsymbol{z}_1) \otimes C(\boldsymbol{z}_2) \otimes F(x_i, \boldsymbol{y})\big](\mu_{0i}, \zeta_0)$$

now S_a depends on $Q_{1,2}$ instead of ζ

- in $W = W_{\text{large } y} + W_{\text{small } y} - W_{\text{subt}}$ need double counting subtraction
  use again Collins subtraction formalism
- $W_{\text{subt}} = W_{\text{large } y}\big|_{y \ll 1/\Lambda}$ or $W_{\text{small } y}\big|_{z_i \ll y}$ gives same result
  up to possibly different scale choices in $W_{\text{small } y}$ and $W_{\text{large } y}$
  with suitable choices differences are beyond accuracy

Put together cross section

- have in different regions

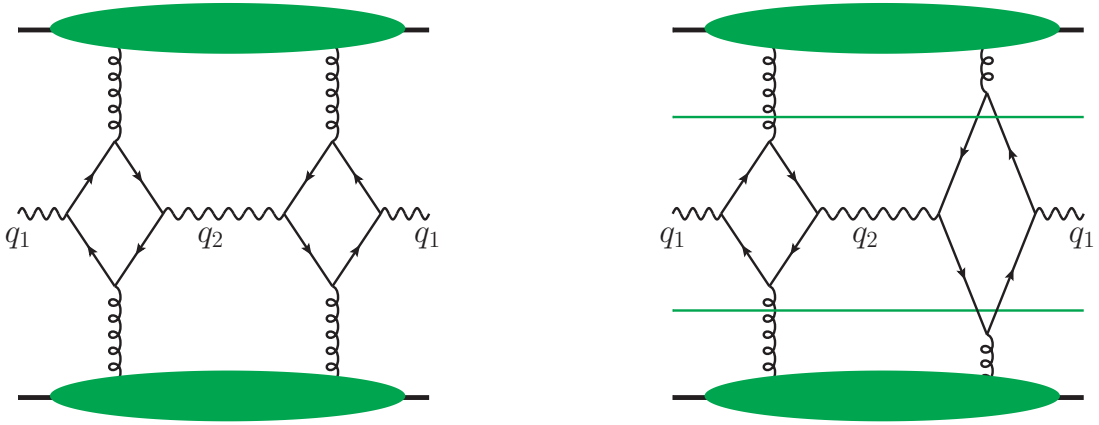
$$W_{\text{small } y} = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-)\, \exp\big[{}^1S(\boldsymbol{z}_1) + {}^1S(\boldsymbol{z}_2)\big] \sum_{RR'} {}^R F_{\text{int+spl}}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)$$
$$\times\, {}^{RR'}\exp\left[M(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{Q_1 Q_2}{\zeta_0}\right] {}^{R'} F_{\text{int+spl}}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)$$

$$W_{\text{large } y} = \Phi^2(\nu \boldsymbol{y})\, \exp\big[{}^RS(\boldsymbol{z}_1) + {}^RS(\boldsymbol{z}_2)\big] \exp\left[{}^RJ(\boldsymbol{y}; \mu_{0i}) \log \frac{Q_1 Q_2}{\zeta_0}\right]$$
$$\times\, {}^R\big[C(\boldsymbol{z}_1) \otimes C(\boldsymbol{z}_2) \otimes F(\bar{x}_i, \boldsymbol{y})\big](\mu_{0i}, \zeta_0)\; {}^R\big[C(\boldsymbol{z}_1) \otimes C(\boldsymbol{z}_2) \otimes F(x_i, \boldsymbol{y})\big](\mu_{0i}, \zeta_0)$$

- minimal non-perturbative input:
  - colour singlet DPDFs ${}^1F(x_i, \boldsymbol{y})$ with correct small-$\boldsymbol{y}$ limit
    if assume ${}^RF$ is strongly suppressed by $\exp[{}^RJ \log \ldots]$
  - twist 4 dist. ${}^RG$ in all colour channels
  - PDFs

UV region and DPS logarithms

- in cross section have $d^2\boldsymbol{y}\, d^2\boldsymbol{z}_1\, d^2\boldsymbol{z}_2\; e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}$
  $= d^2\boldsymbol{y}_+\, d^2\boldsymbol{y}_-\, d^2\boldsymbol{Z}\; e^{-i(\boldsymbol{q}_1-\boldsymbol{q}_2)(\boldsymbol{y}_+-\boldsymbol{y}_-)-i(\boldsymbol{q}_1+\boldsymbol{q}_2)\boldsymbol{Z}}$ with $\boldsymbol{Z} = (\boldsymbol{z}_1+\boldsymbol{z}_2)/2$
- $F_{\text{spl}} \times F_{\text{spl}}$ term in $W_{\text{small } y}$ builds up $\log^2(Q/q_T)$
  from $\int_{z_i}^{1/\nu} dy_+/y_+ \int_{z_i}^{1/\nu} dy_-/y_-$ with $\nu \sim Q$, $z_i \sim 1/q_T$
- ν dependence canceled by double counting subtractions in
  - SPS term (with $\nu \sim Q$ has no log)
  - and in SPS/DPS interference (with $\nu \sim Q$ has single $\log(Q/q_T)$)



- to leading $\log(Q/q_T)$ accuracy can keep only DPS term
  which resums all Sudakov logarithms

Summary

- for intermediate transverse momenta $\Lambda \ll q_T \ll Q$ in DPS:
  contributions from large $y \sim 1/\Lambda$ and small $y \sim 1/q_T$
  in addition to SPS region $y_\pm \sim 1/Q$
- different short-distance expansions for DTMDs in two regions
- Collins-Soper kernel becomes diagonal for $z_i \ll y$
  $\rightsquigarrow$ suppression of colour-nonsinglet channels (single $\log Q/q_T$)
- consistent merging of different contributions using Collins subtraction formalism
- can reduce non-perturbative functions to DPDFs
  and (for their small-y limit) twist 4 and twist 2 distributions

Sudakov exponent

- general form of Sudakov exponent S for DTMD of first parton:

$$
{}^{R}S_{a_1}(\boldsymbol{z}_1) = \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1}(\mu, \mu^2) - \gamma_{K,a_1}(\mu) \log \frac{\sqrt{x_1 \zeta / x_2}}{\mu} \right]
+ {}^{R}K_{a_1}(\boldsymbol{z}_1, \mu_{01}) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}
$$

  where

$$
\frac{\partial}{\partial \log \mu_1} \, {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \gamma_{F,a_1}(\mu_1, x_1 \zeta / x_2) \, {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)
$$

  where ζ defined w.r.t. "average" parton momentum $\sqrt{x_1 x_2}\, p$

- in cross section $\sqrt{x_1 \zeta / x_2} \to Q_1$ and $\zeta \to Q_1 Q_2$
- for second parton swap indices $1 \leftrightarrow 2$