Double parton scattering in the ultraviolet:
addressing the double counting problem

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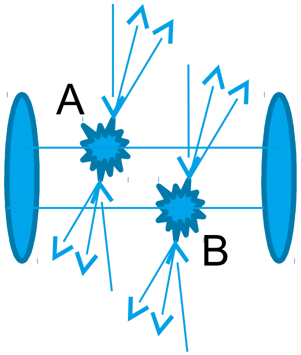
Based on work with Markus Diehl

Double Parton Scattering - Introduction



Double Parton Scattering (DPS) = when you have two separate hard interactions in a single proton-proton collision

In terms of total cross section for production of AB, DPS is power suppressed with respect to single parton scattering (SPS) mechanism: $\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \frac{\Lambda^2}{Q^2}$

Why then should we study DPS?

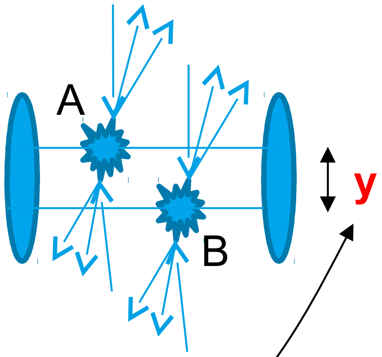
1. DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants (same sign WW, H+W production).

   JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65
   Del Fabbro, Treleani, Phys. Rev. D61 (2000) 077502
   Bandurin, Golovanov, Skachkov, JHEP 1104 (2011) 054

2. DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.
3. DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller *x* values where there is a larger density of partons.
4. DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton.

Double Parton Scattering - Introduction

Analysis of lowest order 'parton model' Feynman graphs indicates the following factorisation formula for the DPS total cross section:



Symmetry factor

$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \overbrace{F^{ik}(x_1, x_2, \mathbf{y}, Q_A, Q_B) F^{jl}(\bar{x}_1, \bar{x}_2, \mathbf{y}, Q_A, Q_B)}^{\text{Double parton distribution (DPD)}}$$

$$\times \underbrace{\hat{\sigma}_{ij}^{A}(x_1, \bar{x}_1) \hat{\sigma}_{kl}^{B}(x_2, \bar{x}_2)}_{\text{Parton level cross sections}} dx_1 d\bar{x}_1 dx_2 d\bar{x}_2 d^2\mathbf{y}$$

N. Paver, D. Treleani, Nuovo Cim. A70 (1982) 215.
M. Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

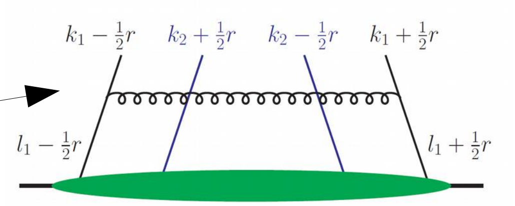
y = separation in transverse space between the two partons

This would then be added to the SPS cross section to produce AB to obtain the total cross section to produce AB...

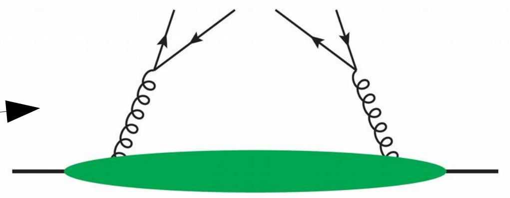
QCD evolution effects

Now we start trying to add in the effects of QCD evolution in DPS, going backwards from the hard interaction.

Some effects are similar to those encountered in SPS – i.e. (diagonal) emission from one of the parton legs. These can be treated in same way as for SPS.



However, there is a new effect possible here – when we go backwards from the hard interaction, we can discover that the two partons arose from the perturbative '1 → 2' splitting of a single parton.



This 'perturbative splitting' yields a contribution to the DPD of the following form:

Single PDF

$$F(x_1, x_2, y) \propto \alpha_s \frac{f(x_1 + x_2)}{x_1 + x_2} P\left(\frac{x_1}{x_1+x_2}\right) \frac{1}{y^2}$$

Dimensionful part

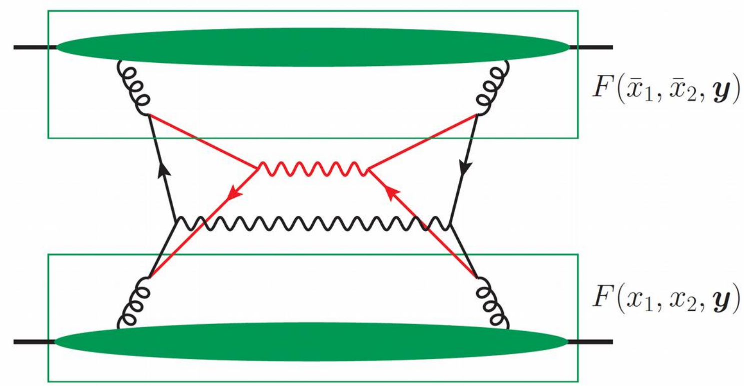
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Perturbative splitting kernel

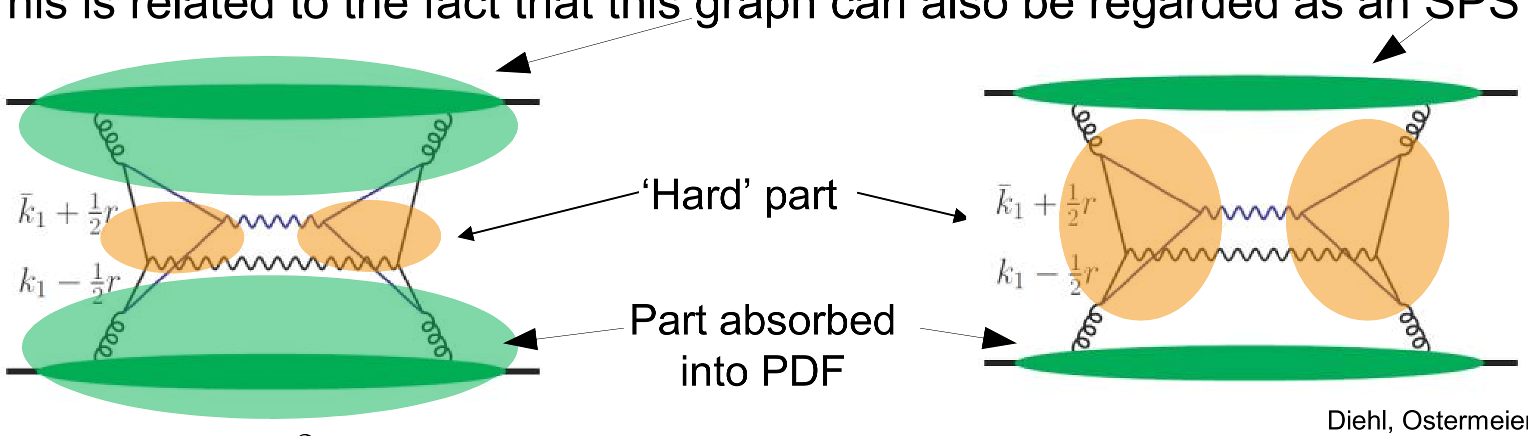
Problems...

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$



This is related to the fact that this graph can also be regarded as an SPS loop correction



$$\frac{\Lambda^2}{Q^4} \xrightarrow{\text{Power divergence!}} \frac{1}{Q^2}$$

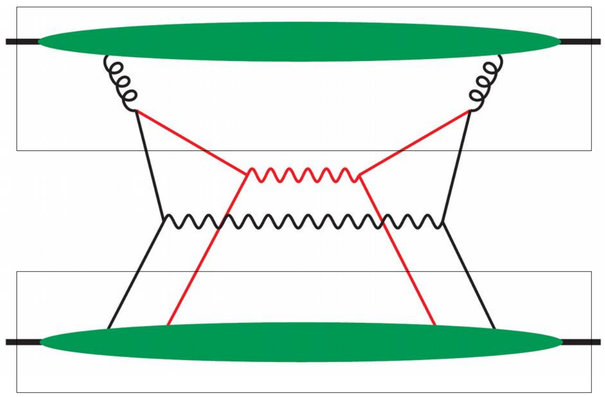
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn Phys.Lett. 713 (2012) 196
JG and Stirling, JHEP 1106 048 (2011)
Blok et al. Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

Single perturbative splitting graphs

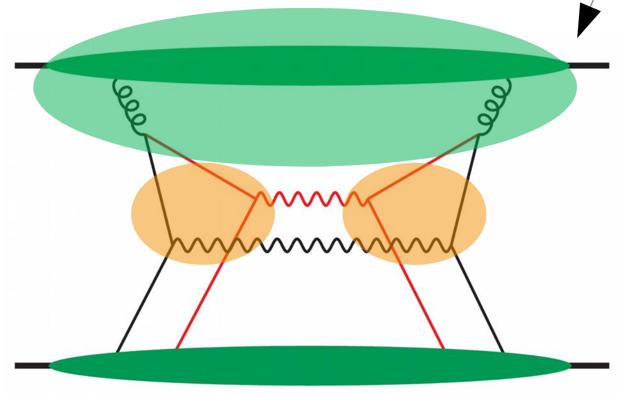
Also have graphs with perturbative $1 \to 2$ splitting in one proton only (2v1 graph).

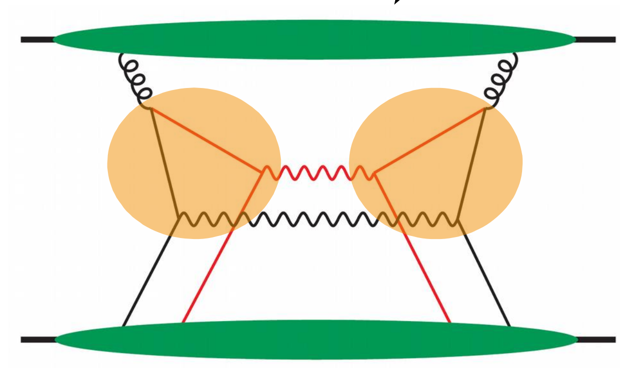
This has a log divergence:

$$\int d^2y/y^2 F_{\mathrm{intr}}(x_1, x_2; y)$$



Related to the fact that this graph can also be thought of as a twist 4 x twist 2 contribution to AB cross section





$$\frac{\Lambda^2}{Q^4} \xrightarrow{\text{Logarithmic divergence}} \frac{\Lambda^2}{Q^4}$$

Blok et al., Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
JG, JHEP 1301 (2013) 042

Previous attempts to handle these issues

Most popular suggestion previously:

Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.
JG and Stirling, JHEP 1106 048 (2011)
Blok et al. Eur.Phys.J. C72 (2012) 1963

- Completely remove 1v1 graphs from DPS cross section, and consider these as pure SPS (no natural part of these graphs to separate off as DPS).

- Put (part of) 2v1 graphs in DPS – sum logs of 1→2 splitting + DGLAP emissions in this contribution.

This scheme comes out if one chooses to regulate y integral using dim reg:

$$\int d^2y/y^4 \to \int d^{2-2\epsilon}y/y^4 = 0$$

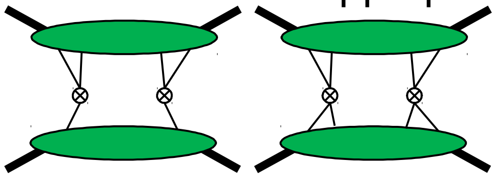
Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.

Drawback of this approach: The cross section can no longer be written as parton level cross sections convolved with overall DPD factors for each hadron.

$$\sigma^{DPS} = \cancel{\int d^2y F(y) F(y)} \to \quad \text{2v2} \quad + \quad \text{2v1} \quad + \quad \text{1v2}$$

$$(A+B)^2 \neq A^2 + AB + BA$$

No concept of the DPD for an individual hadron: appropriate hadronic operators in DPS involve both hadrons at once!



Previous attempts to handle these issues

An alternative suggestion – just add a cut-off to the y integral at y values of order 1/Q

Ryskin, Snigirev, Phys.Rev.D83:114047,2011

$$\int \frac{d^2y}{y^4} \to \int_{|y|>1/Q} \frac{d^2y}{y^4}$$

(note that technically Ryskin, Snigirev impose the cutoff in the Fourier conjugate space, but the principle is the same)

This regulates the power divergence, but:

- there is now some double counting between DPS and SPS cross sections
- in general, a sizeable contribution to the 'double perturbative splitting' part of the DPS cross section comes from y values of order 1/Q, where the DPS picture is not valid.
- strong (quadratic) dependence of result on cut-off – why take cut-off of 1/Q rather than 1/(2Q) or 2/Q?

Our solution

Diehl and JG, work in progress

[Focus for the moment only on the double perturbative splitting issue]

Insert a regulating function into DPS cross section formula:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

Requirements: $\Phi(u) \to 0$ as $u \to 0$

$\Phi(u) \to 1$ for $u \gg 1$ e.g. $\Phi(u) = \theta(u-1)$

In this way, we cut contributions with 1/y much bigger than the scale ν out of what we define to be DPS, and regulate the power divergence.

Note that the *F*s here contain both perturbative and nonperturbative splittings.

Our solution

Now we have introduced some double counting between SPS and DPS – we fix this by including a double counting subtraction:

$$\sigma_{\rm tot} = \sigma_{\rm DPS} + \sigma_{\rm SPS} - \sigma_{\rm sub}$$

The subtraction term is given by the DPS cross section with both DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

Subtraction term constructed along the lines of general subtraction formalism discussed in Collins pQCD book

Note: computation of subtraction term much easier than full SPS X sec

Straightforward extension of formalism to include twist 4 x twist 2 contribution and remove double counting with 2v1 DPS:

$$\sigma_{\rm tot} = \sigma_{\rm DPS} + \sigma_{\rm SPS} - \sigma_{\rm sub\ (1v1\ +\ 2v1)} + \sigma_{\rm tw4\times tw2}$$

Tw2 x tw 4 piece with hard part computed according to fixed order DPS expression

How the subtraction works

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$$

For small y (of order 1/Q) the dominant contribution to σ_{DPS} comes from the (fixed order) perturbative expression $\implies \sigma_{\text{DPS}} \simeq \sigma_{\text{sub}}$

& $\sigma_{\text{tot}} \simeq \sigma_{\text{SPS}}$ (as desired)

(dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub})

For large y (much larger than 1/Q) the dominant contribution to σ_{SPS} is the region of the 'double splitting' loop where DPS approximations are valid

$$\implies \sigma_{\text{SPS}} \simeq \sigma_{\text{sub}}$$

& $\sigma_{\text{tot}} \simeq \sigma_{\text{DPS}}$ (as desired)

(similar considerations hold for 2v1 part of DPS and tw4xtw2 contribution)

Summing DGLAP logarithms

DPDs are a matrix element of a product of twist 2 operators:

$$F(x_1, x_2, \boldsymbol{y}, \mu_1, \mu_2) = \langle p|\, \mathcal{O}_1(\boldsymbol{0}, \mu_1)\mathcal{O}_2(\boldsymbol{y}, \mu_2)\, |p\rangle \qquad \left[f(x, \mu) = \langle p|\, \mathcal{O}(\boldsymbol{0}, \mu)\, |p\rangle\right]$$

⇨ Separate DGLAP evolution for partons 1 and 2 $\frac{d}{d \log \mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$

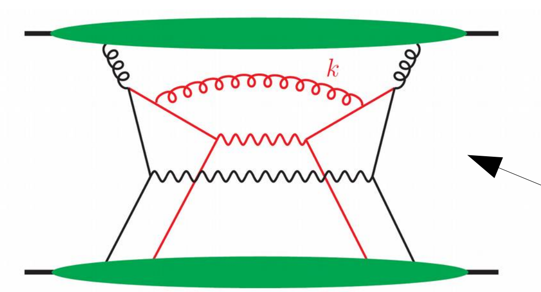
(same as for single PDF evolution)

Appropriate initial conditions for DPD are something like $F = F_{\text{split}} + F_{\text{intr}}$

$$F_{\text{split}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2\Lambda^2} \quad \text{with} \quad 1/y^{*2} = 1/y^2 + 1/y_{\text{max}}^2$$

$F_{\text{intr}}(x_1, x_1, \boldsymbol{y}, \mu_0, \mu_0)$ = NP piece, something with smooth y dependence over scales of order proton radius

(for modelling we use $f(x_1; \mu_0) f(x_2; \mu_0)\, \Lambda^2 e^{-y^2\Lambda^2}/\pi$)



Putting this information in and choosing μ_i, ν appropriately, we can sum up DGLAP logs appropriately in various scenarios

e.g. our DPS cross section contains the correct $\log^2(Q/\Lambda)$ corresponding to this 2v1 diagram if we take $\mu_1 \sim \mu_2 \sim \nu \sim Q$

Extension to measured transverse momenta

So far just discussed DPS at the total cross section level.

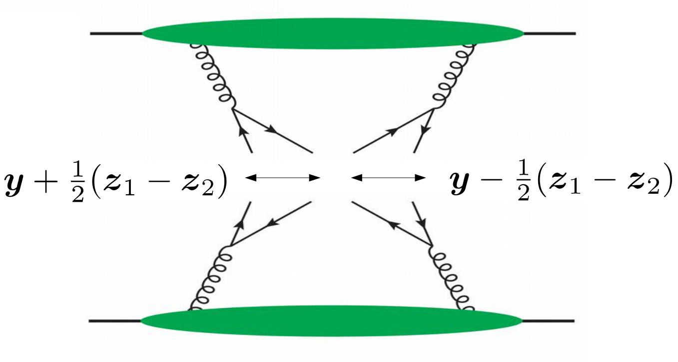
However, since DPS preferentially populates the small $\mathbf{q}_A$, $\mathbf{q}_B$ region, the transverse-momentum-differential cross section for the production of AB for small $\mathbf{q}_A$, $\mathbf{q}_B$ is also of significant interest. Need to adapt SPS TMD formalism to double scattering case.

Our scheme can be readily adapted to solve double counting issues in this case. DPS cross section involves the following regularised integral:

$$\int d^2\boldsymbol{y}\, d^2\boldsymbol{z}_1\, d^2\boldsymbol{z}_2\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, \Phi(\nu y_+)\, \Phi(\nu y_-)\, F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

Regulate (logarithmic) singularities in double perturbative splitting mechanism at the points

$$y_\pm \equiv \left|\boldsymbol{y} \pm \tfrac{1}{2}(\boldsymbol{z}_1 - \boldsymbol{z}_2)\right| = 0$$



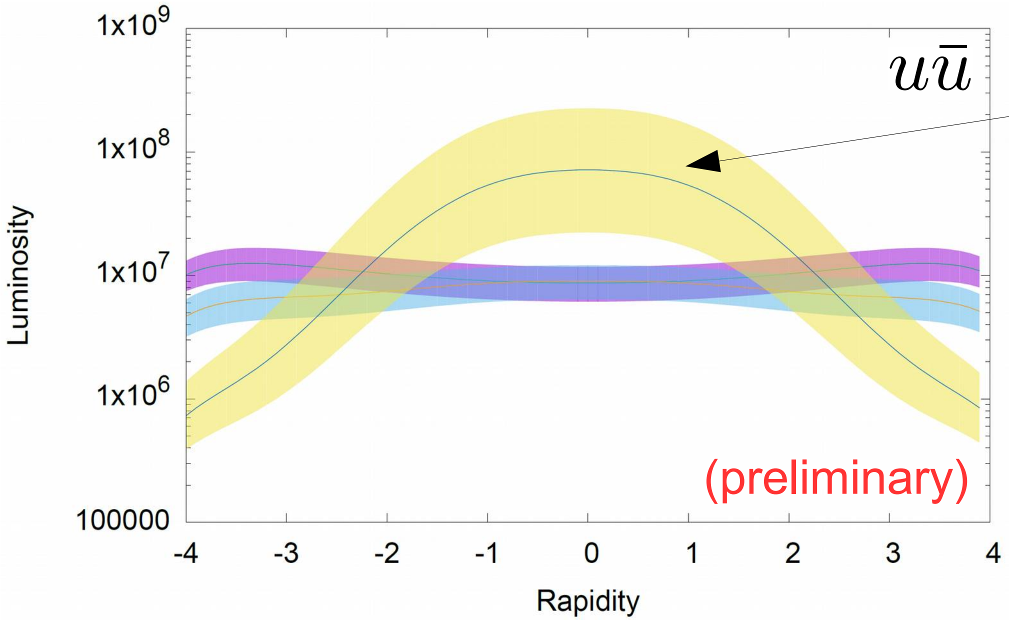
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Parton luminosities

(For illustration only, model parameters not tuned)

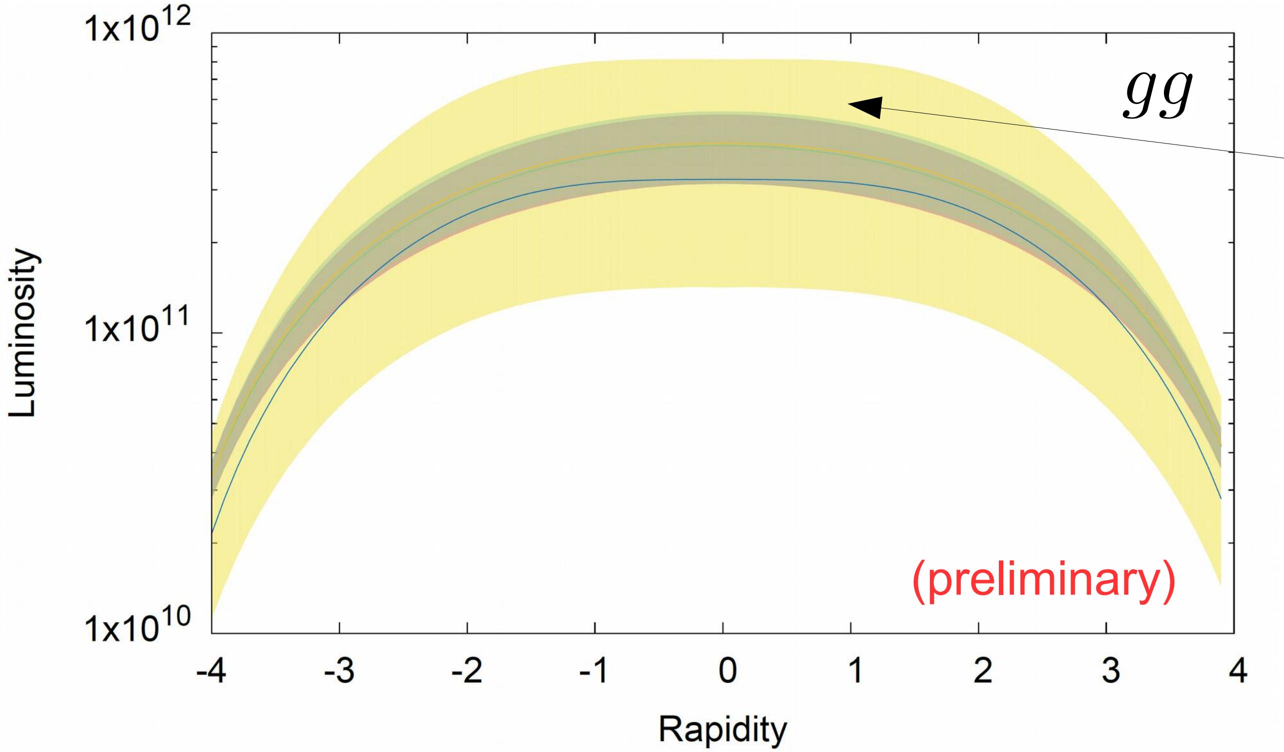
Plot $\int d^2\boldsymbol{y}\Phi^2(\boldsymbol{y})F(x_1,x_2,\boldsymbol{y})F(\bar{x}_1,\bar{x}_2,\boldsymbol{y})$ against rapidity of A with B central for $Q_A = Q_B = M_W$ and $\sqrt{s} = 14 \text{ TeV}$

Give 2v2 (purple), 2v1 (blue) and 1v1 (yellow) contributions, varying scales μ_i and ν together between $M_W/2$ and $2M_W$.



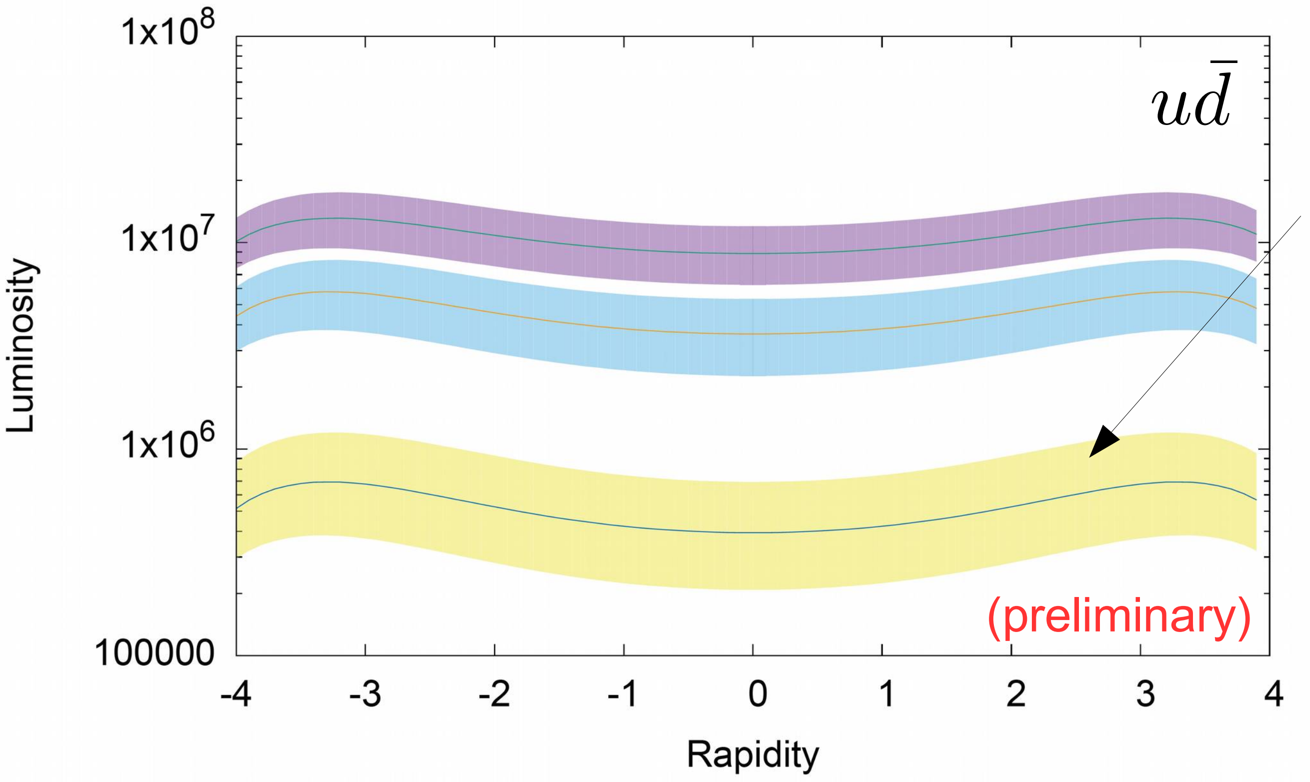
Huge 1v1 contribution, with enormous scale variation (mostly due to ν variation). Need to add SPS loop correction with subtraction to reduce ν scale dependence.

Parton luminosities

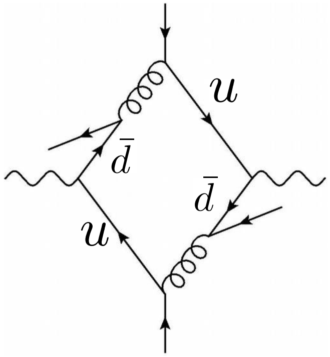


1v1 contribution still large with large uncertainty (but not quite as large as for $u\bar{u}$.)

Parton luminosities



Small 1v1 contribution, as no direct splitting yielding $u\bar{d}$

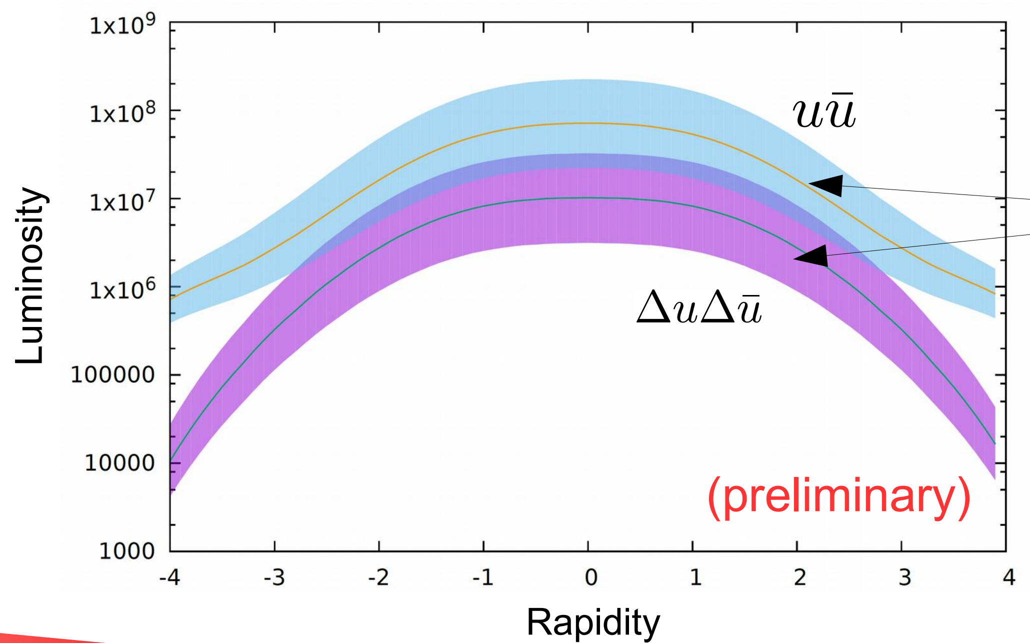


Polarised contributions

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g. $$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + \, q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - \, q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

Can use same scheme to handle SPS/DPS double counting for polarised distributions



1v1 for both $u\bar{u}$ and $\Delta u \Delta \bar{u}$ is large, with large scale dependence – should add SPS with both unpolarised and polarised subtractions.

Summary

- Power divergence in naive treatment of DPS including perturbative splittings (= 'leaking' of DPS into leading power SPS region).
- Previously-proposed solutions to handle this:
  - Consider 'double perturbative splitting' graphs as pure SPS. No concept of DPD for individual hadron, issues related to nonperturbative region.
  - Add a cut-off of order the hard scale – then considerable double counting between SPS and DPS.
- We have proposed a solution that retains the concept of a DPD for an individual hadron, and avoids double counting. Involves introduction of a regulator at the DPS cross section level, + a subtraction to remove double counting overlap between SPS and DPS.