Hadronic Matrix Elements in 3D

Ahmad Idilbi
Wayne State University

QCD Evolution, 2016, Nikhef

Outline

1. Review of "Recent" developments in TMD physics (unpolarized quark TMDs): Factorization theorems and the soft factor contribution

2. 3-D Hadronic matrix elements "without" factorization

3. Study case I: generalized TMDPDFs

4. Study case II: jet quenching parameter (q-hat)

5. Open questions: If we are doing the right things (gauge links and/or the soft and collinear limits of QCD)

Quark TMDPDF

$$F.T.\langle P|\bar{\psi}(x^-, x_\perp)[x^-, 0]\psi(0)|P\rangle$$

Collins and Soper, 81

- Simplest and most natural generalization of the Feynman PDF
- Ill-defined: In perturbation theory, no proper evolution and no proper matching (OPE) on the integrated counterparts
- When considering polarization effects, same story applies

Evolution

$$F.T.\langle P|\bar{\psi}(x^-, x_\perp)[x^-, 0]\psi(0)|P\rangle$$

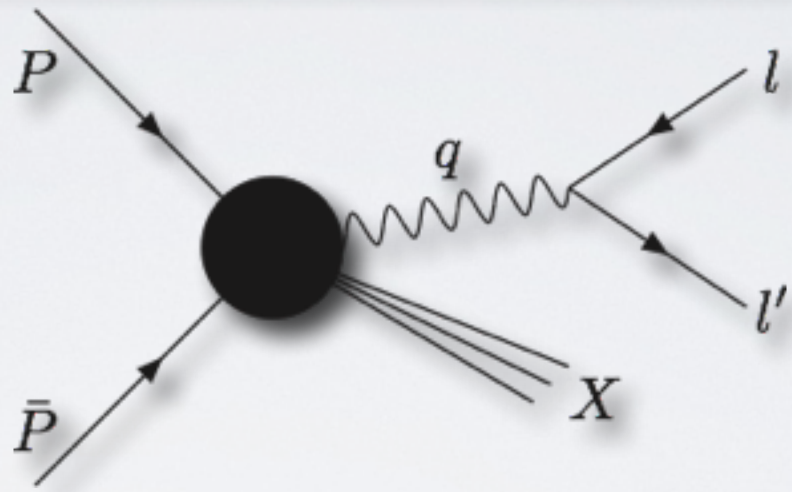
$$\frac{d\ln \tilde{J}_n^{(0)}}{d\ln \mu} = \log \Delta + ...$$

$$\frac{i(\not{p}+\not{k})}{(p+k)^2 + i\Delta^-} \longrightarrow \frac{1}{k^- + i\delta^-}, \delta^- = \frac{\Delta^-}{p^+}$$
$$\frac{i(\not{\bar{p}}-\not{k})}{(\bar{p}-k)^2 + i\Delta^+} \longrightarrow \frac{1}{-k^+ + i\delta^+}, \delta^+ = \frac{\Delta^+}{\bar{p}^-}$$

Origin: Incomplete cancelation of mixed UV-IR (Rapidity divergences) due to the 3-D Structure;

Drell-Yan q_T-spectrum

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$



$$F.T. \langle P | \bar{\psi}(x^-, x_\perp)[x^-, 0]\psi(0)|P\rangle$$

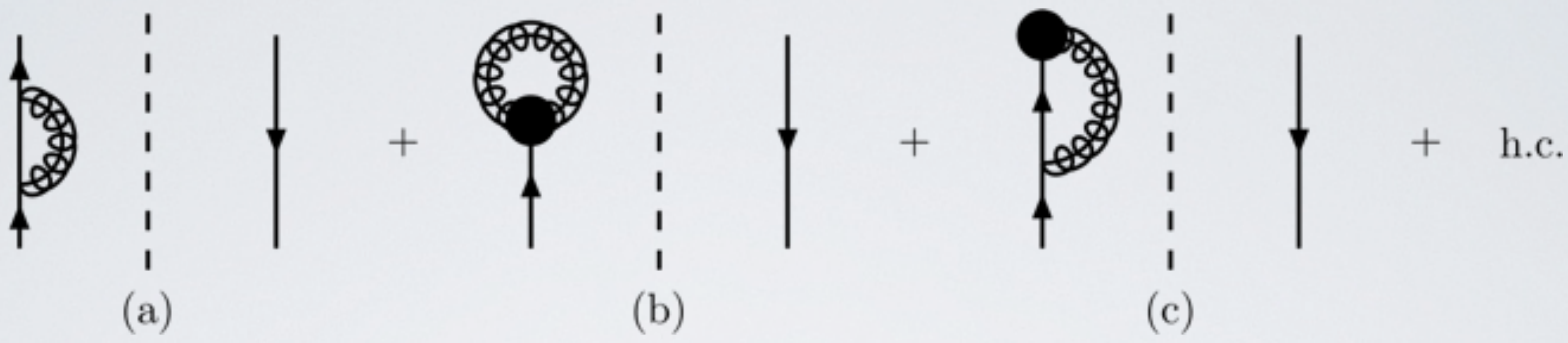
Soft Function: Low energy gluon radiation

$$k_s = Q(\lambda, \lambda, \lambda)$$

$$\lambda = q_T/Q \ll 1$$

$$\langle 0 | \mathrm{Tr}\ \bar{T}[S_n^\dagger S_{\bar{n}}^\dagger](0, 0, y_\perp)\ T[S_{\bar{n}} S_n](0)|0\rangle$$

$$F.T.\langle P|\bar{\psi}(x^-, x_\perp)[x^-, 0]\psi(0)|P\rangle$$

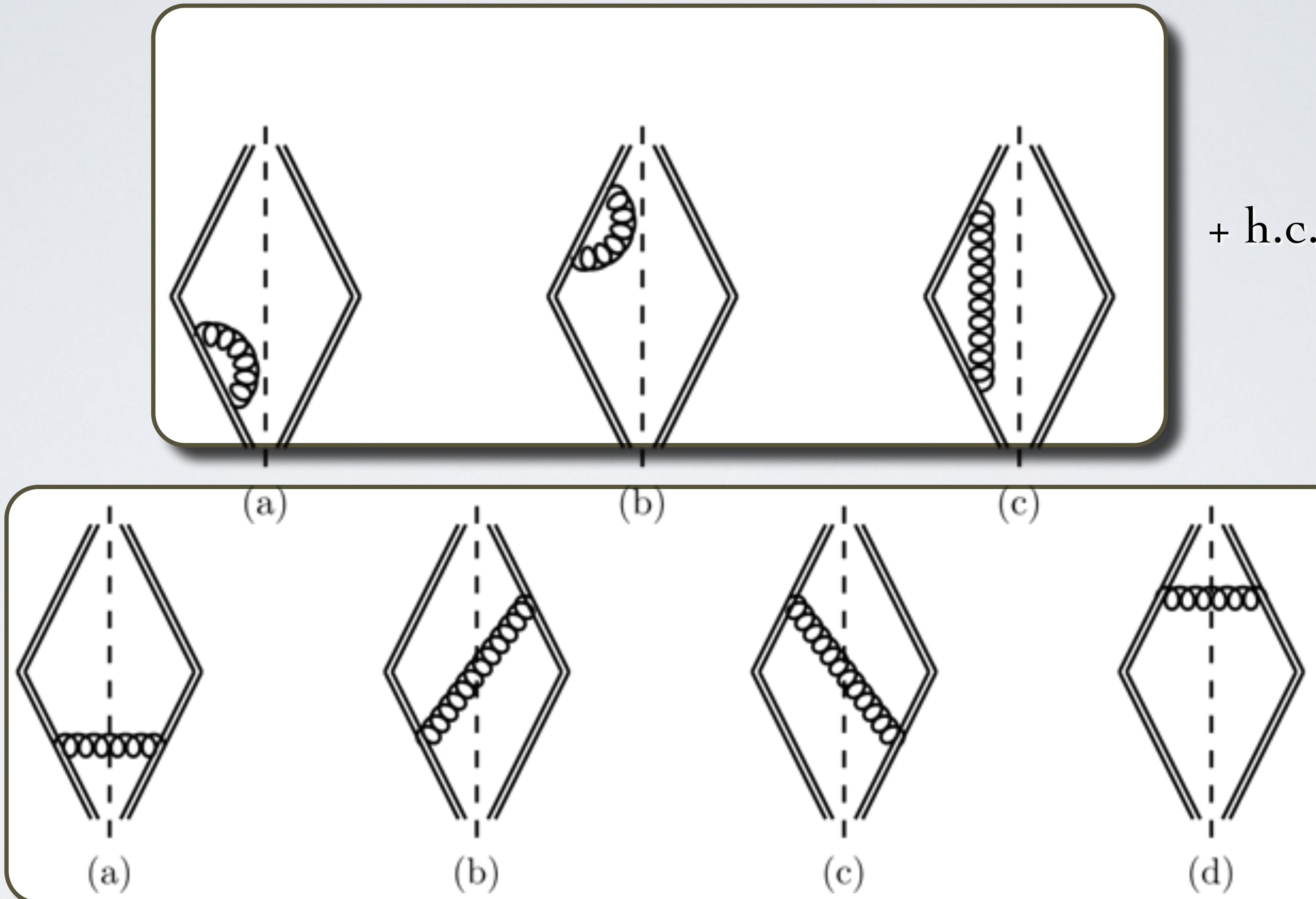


$$\tilde{J}^{(0)}_{n1}(\Delta) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon_{\mathrm{UV}}^2} - \boxed{\frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta}{\mu^2}} + \frac{3}{2\varepsilon_{\mathrm{UV}}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 + \frac{3}{2}L_T - 2L_T\ln\frac{\Delta}{\mu^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

$$\mathcal{P}_{q/q} = \left(\frac{1+x^2}{1-x}\right)_+$$

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Soft function at NLO



$$\tilde{S}_1(\Delta,\Delta) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{\mathrm{UV}}^2} + \boxed{\frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta^2}{\mu^2 Q^2}} + L_T^2 + 2L_T \ln\frac{\Delta^2}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

At $O(\alpha_s)$

$$\tilde{J}^{(0)}_{n1}(\Delta) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon^2_{\mathrm{UV}}}\boxed{-\frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta}{\mu^2}}+\frac{3}{2\varepsilon_{\mathrm{UV}}}-\frac{1}{4}-\frac{2\pi^2}{12}-L_T^2\right.\right.$$
$$\left.\left.+\frac{3}{2}L_T-2L_T\ln\frac{\Delta}{\mu^2}\right]-(1-x)\ln(1-x)-\mathcal{P}_{q/q}\ln\frac{\Delta}{\mu^2}-L_T\mathcal{P}_{q/q}\right\}$$

$$\tilde{S}_1(\Delta,\Delta) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon^2_{\mathrm{UV}}}+\boxed{\frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta^2}{\mu^2 Q^2}}+L_T^2+2L_T\ln\frac{\Delta^2}{\mu^2 Q^2}+\frac{\pi^2}{6}\right]$$

$$\tilde{F}_{n1}(x_n,b;\sqrt{\zeta_n},\mu) =$$
$$\frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x_n)\left[\frac{1}{\varepsilon^2_{\mathrm{UV}}}-\frac{1}{\varepsilon_{\mathrm{UV}}}\ln\frac{\zeta_n}{\mu^2}+\frac{3}{2\varepsilon_{\mathrm{UV}}}\right.\right.$$
$$\left.-\frac{1}{2}L_\perp^2+\frac{3}{2}L_\perp-L_\perp\ln\frac{\zeta_n}{\mu^2}-\frac{\pi^2}{12}\right]+(1-x_n)-L_\perp\mathcal{P}_{qq}$$
$$\left.-\mathcal{P}_{qq}\ln\frac{\Delta^-}{\mu^2}-\frac{1}{4}\delta(1-x_n)-(1-x_n)[1+\ln(1-x_n)]\right\}.$$

$$\boxed{\tilde{F}_n = \tilde{J}_n\sqrt{\tilde{S}}}$$

All orders in PT

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$

$$H(Q^2/\mu^2) = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{n} C_k \log^{2k}(Q^2/\mu^2) + \text{finite terms}$$

$$M = H(Q^2/\mu^2) C(x_n, x_{\bar{n}} : L_\perp, Q^2/\mu^2) \times f_n(x_n, ; \Delta^-/\mu^2) f_{\bar{n}}(x_{\bar{n}}; \Delta^+/\mu^2)$$

Pdfs

To all orders in PT:

$$\ln f_n = \mathcal{R}_1(x_n, \alpha_s) + \mathcal{R}_2(x_n, \alpha_s) \ln \frac{\Delta}{\mu^2}$$

[Raduyshkin and Korchemsky, 87]

[Manohar, 03]

Splitting to all orders in PT

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp) \ln \frac{\Delta^-}{\mu^2}$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp) \ln \frac{\Delta^+}{\mu^2}$$

$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + \mathcal{R}_{s2}(\alpha_s, L_\perp) \ln \frac{\Delta^-\Delta^+}{Q^2\mu^2}$$

$$\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)} \sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

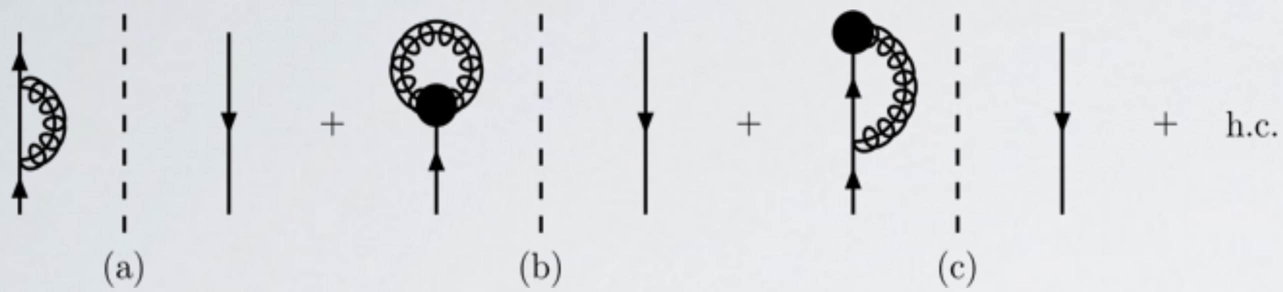
Recap

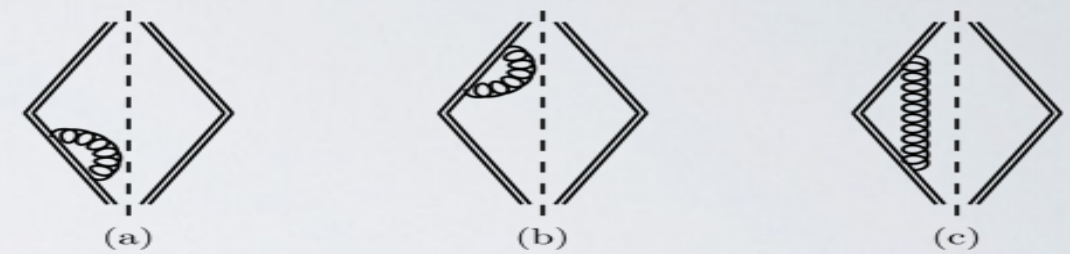
- Establishing a factorization theorem was the only tool at hand we managed to define 3-D hadronic matrix elements!

- What if we are interested (at least theoretically) in a particular 3D HME where there is no factorization at hand

  How we would be certain if it is well-defined or not
  And how to modify the definition, if any?

Revisit quark TMDPDF





$$\frac{\langle p|\bar{\psi}(x^-, x_\perp)[0, x_\perp]\psi(0)|p\rangle}{S(x_\perp)}$$

Soft (pQCD) and/or zero-bin subtraction (SCET) to avoid double counting

1. Collins (Eighties!)
2. Manohar and Stewart (06)
3. Idilbi and Mehen(08)

Not good enough

$$\frac{\langle p|\bar{\psi}(x^-, x_\perp)[0, x_\perp]\psi(0)|p\rangle}{S(x_\perp)}$$

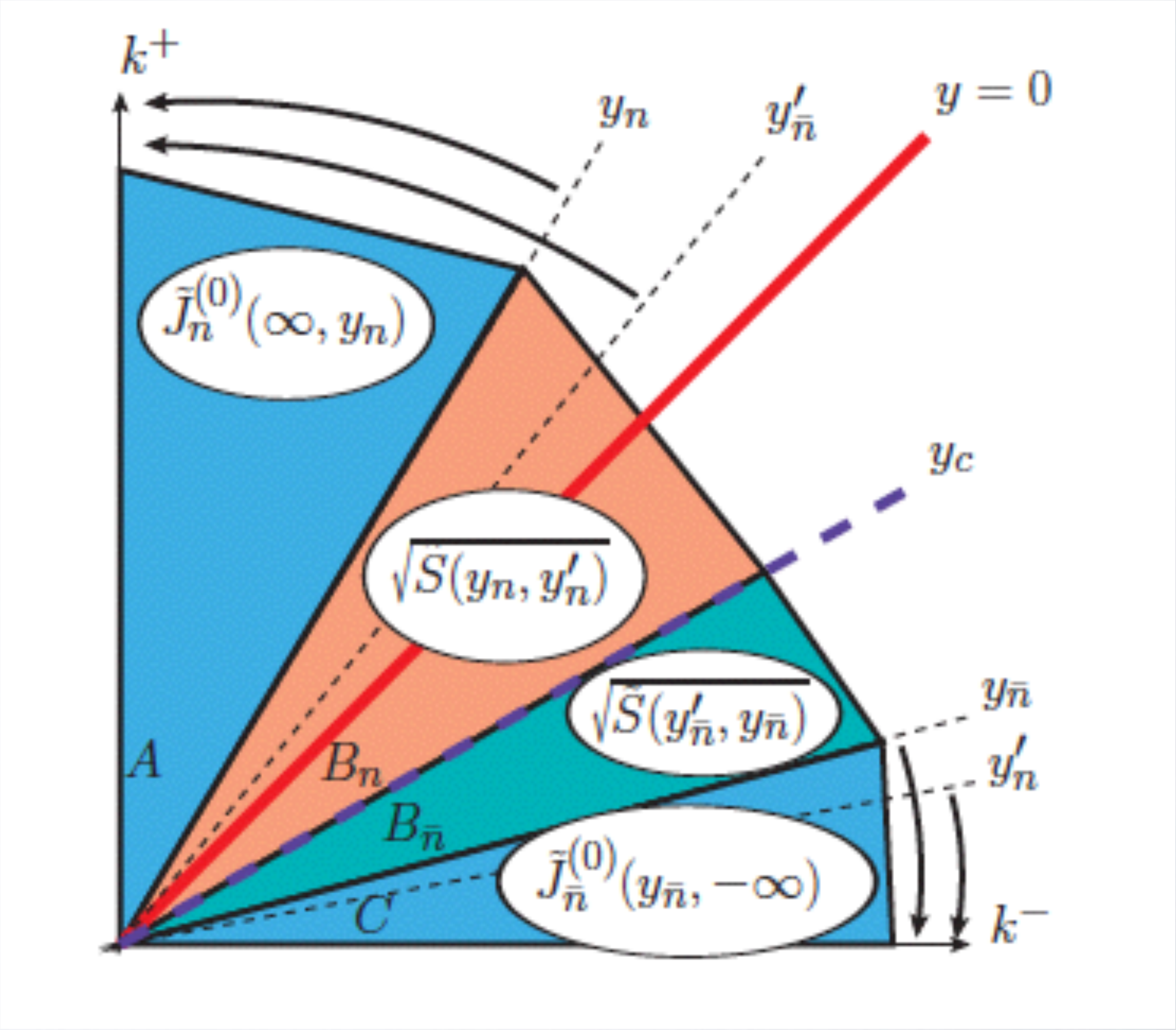
Rapidity div.

$$\begin{aligned}\tilde{J}_{n1}^{(0)} = \frac{\alpha_s C_F}{2\pi}\Big\{&\delta(1-x_n)\Big[\frac{2}{\varepsilon_{\rm UV}^2} - \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^-}{\mu^2} + \frac{3}{2\varepsilon_{\rm UV}} \\ &-\frac{1}{4} - \frac{2\pi^2}{12} - L_\perp^2 + \frac{3}{2}L_\perp - 2L_\perp\ln\frac{\Delta^-}{\mu^2}\Big] \\ &-(1-x_n)\ln(1-x_n) - \mathcal{P}_{qq}\ln\frac{\Delta^-}{\mu^2} - L_\perp\mathcal{P}_{qq}\Big\} \qquad (20)\end{aligned}$$

We are subtracting too much

$$\ln\tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + \mathcal{R}_{s2}(\alpha_s, L_\perp)\ln\frac{\Delta^-\Delta^+}{Q^2\mu^2}\,.$$

$$\tilde{F}_n = \tilde{J}_n\sqrt{\tilde{S}}$$



Generalized TMD

$$\phi = \langle p', \lambda' | \bar{\psi}(x^-, x_\perp)[0, x_\perp]\psi(0)|p, \lambda\rangle$$

- Limiting cases: Un-subtracted TMDs and GPDs

- No established factorization theorem (Not Yet)

- Is it well defined? No!

- Reason: 3-D structure

How do we fix that?

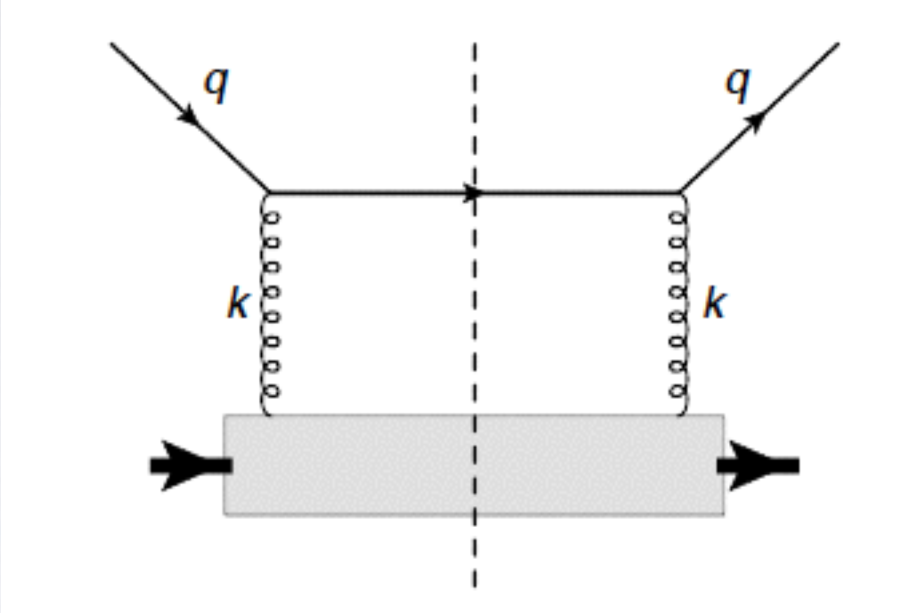
$$\phi = \langle p', \lambda' | \bar{\psi}(x^-, x_\perp)[0, x_\perp]\psi(0)|p, \lambda\rangle$$

- A square root of S is needed to eliminate all rapidity divergences. It`s the same one as for quark TMDPDF

- A new definition emerges (see Miguel`s talk)

Jet quenching parameter: q-hat as 3D object

DIS: photon probes a hot and dense medium. quark is nocked off. Final sate interactions with the medium produces transverse broadening while the ``jet" remains on-shell



[Idilbi, Majumder, 09]

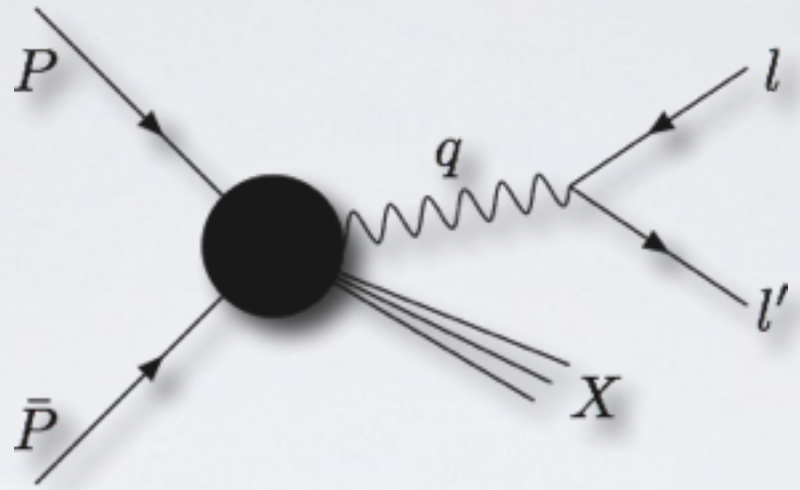
$$\hat{q} = \sum_k k_\perp^2 \frac{W(k)}{t},$$

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp d^2k_\perp}{(2\pi)^3} e^{i\frac{k_\perp^2 y^-}{2q^-} - ik_\perp \cdot y_\perp} \times \left\langle P \left| \mathrm{Tr}\left[t^a F_\perp^{a\,+\mu}(y^-, y_\perp) t^b F^{b\,+}_{\perp,\mu}\right] \right| P \right\rangle,$$

Since it is 3-D: We need soft factor!!

[Echevarria, Idilbi, Majumder: Work in progress]

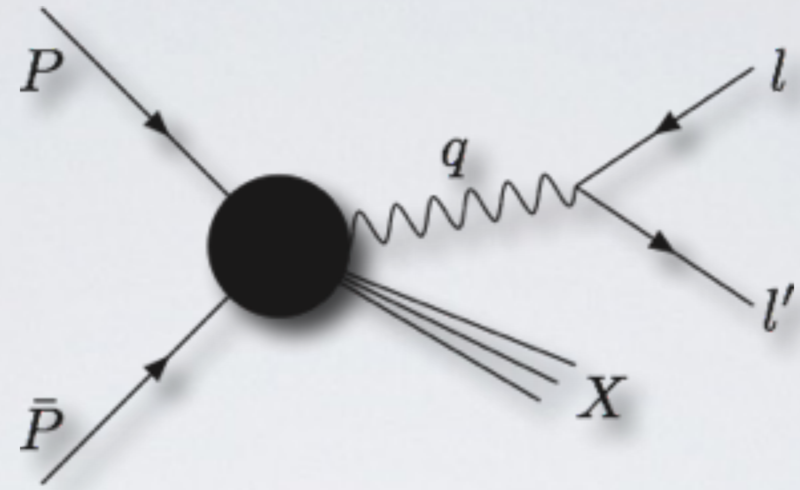
Open questions!



$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$

Esthetically: All quantities are ill-defined (in PT).
We are forced to combine two ill-defined quantities to get properly defined quantities!!

Legitimate: Can we do any better?



Is it possible to factorize q_T dependent DY without the need to combine soft and collinear?

$$H * J_n * J_{\bar{n}}$$

Each quantity is well-defined (No rapidity divergences)

Conclusions

- We know how to treat 3D hadronic matrix elements: quark or gluon TMDs, GTMDs, double TMDs, jet quenching parameter(?)
- With or without factorization theorems: prescription to define such quantities
- Open questions remain to be answered!