Helicity Evolution at Small x

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arXiv:1511.06737 [hep-ph] + in preparation

Outline

- Unpolarized small-x evolution: an overview
- Helicity quark TMD at small-x

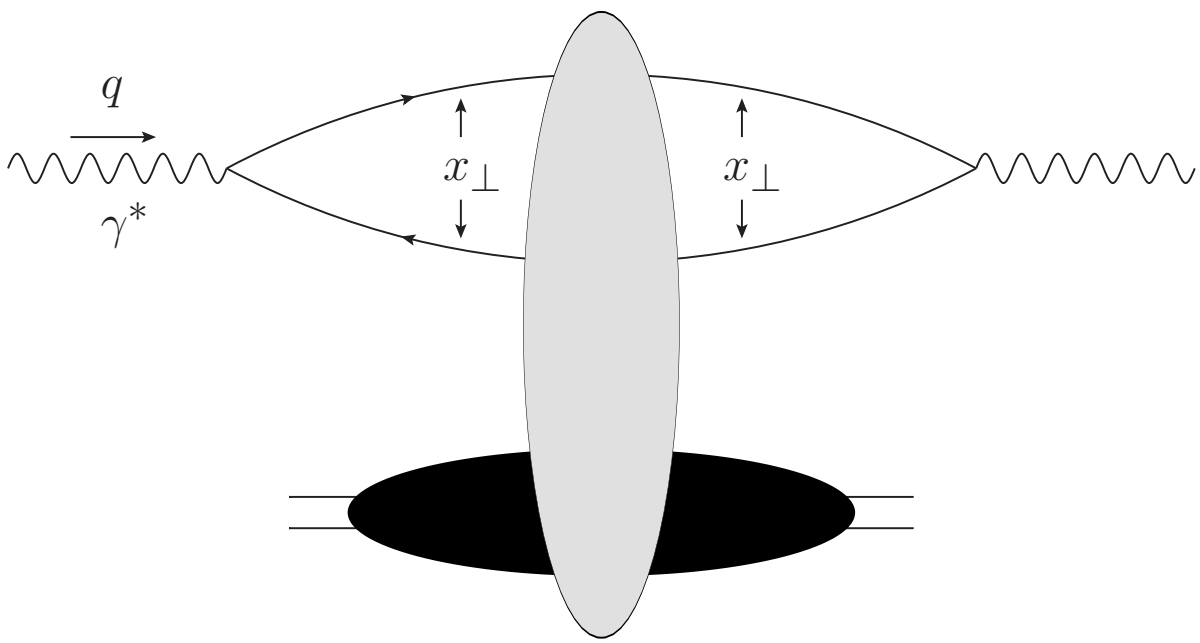
- Small-x evolution for the "polarized dipole":
  - New helicity evolution evolution equations at small x
  - Large-N_C limit
  - Large N_C & N_f limit

- Solution of the large-N_C evolution equations:
  - small-x asymptotics of the g_1 structure function, quark hPDFs and helicity TMDs.

Unpolarized DIS: Small-x Evolution

Dipole picture of DIS

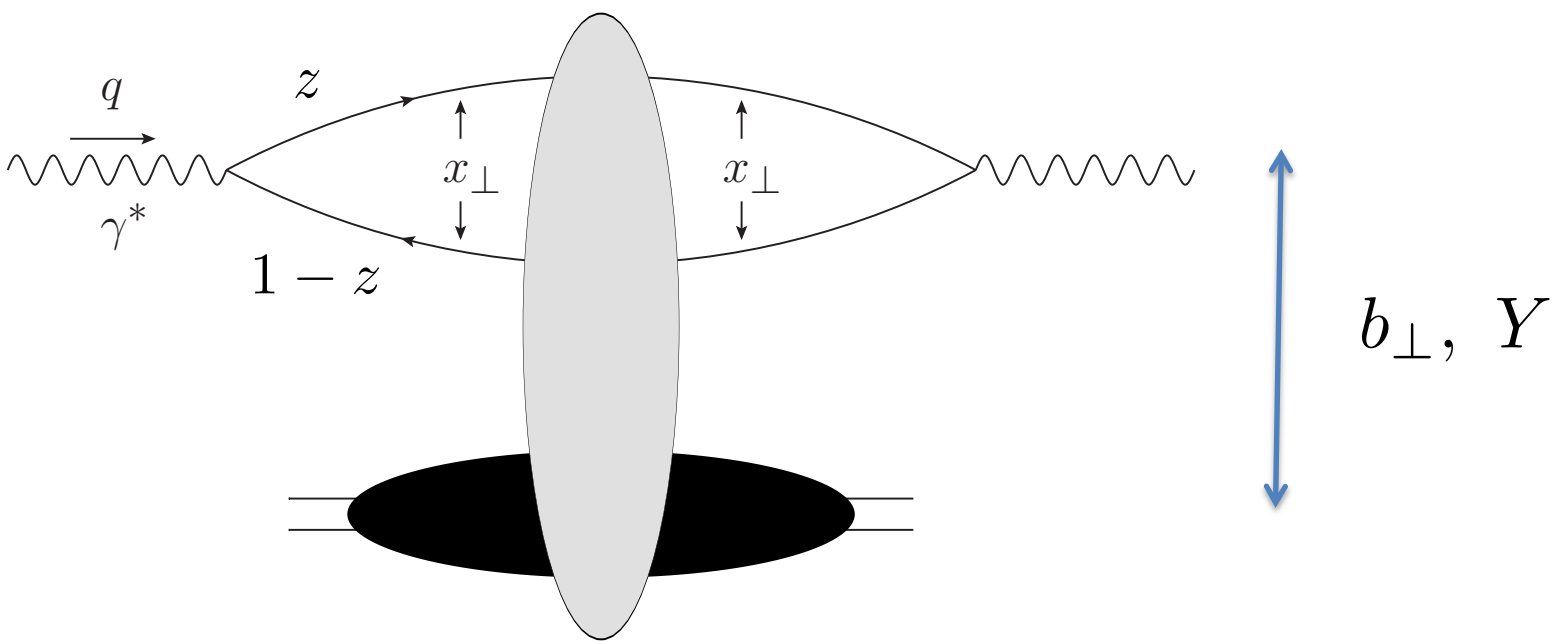
- In the dipole picture of DIS the virtual photon splits into a quark-antiquark pair, which then interacts with the target.
- The total DIS cross section and structure functions are calculated via:



Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\,\pi}\, d^2 b_\perp \int_0^1 \frac{dz}{z\,(1-z)}\, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2\; N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



Dipole Amplitude

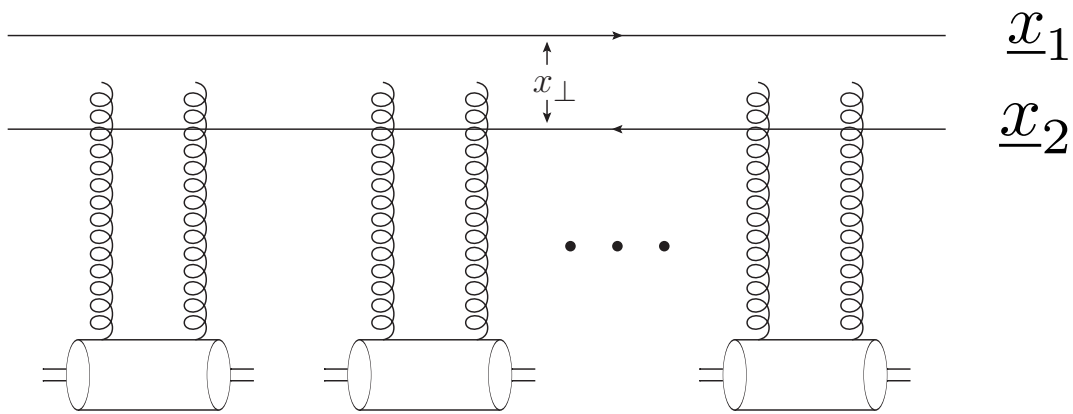
- The quark dipole amplitude is defined by

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V(\underline{x}_1)\, V^\dagger(\underline{x}_2) \right] \right\rangle$$

- Here we use the Wilson lines along the light-cone direction

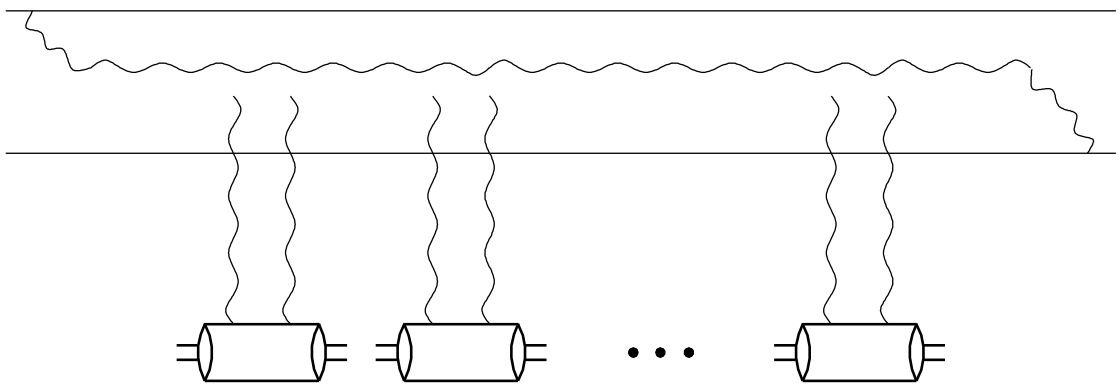
$$V(\underline{x}) = \mathrm{P} \exp \left[i\, g \int_{-\infty}^{\infty} dx^+\, A^-(x^+, x^- = 0, \underline{x}) \right]$$

- In the classical Glauber-Mueller/McLerran-Venugopalan approach the dipole amplitude resums multiple rescatterings:

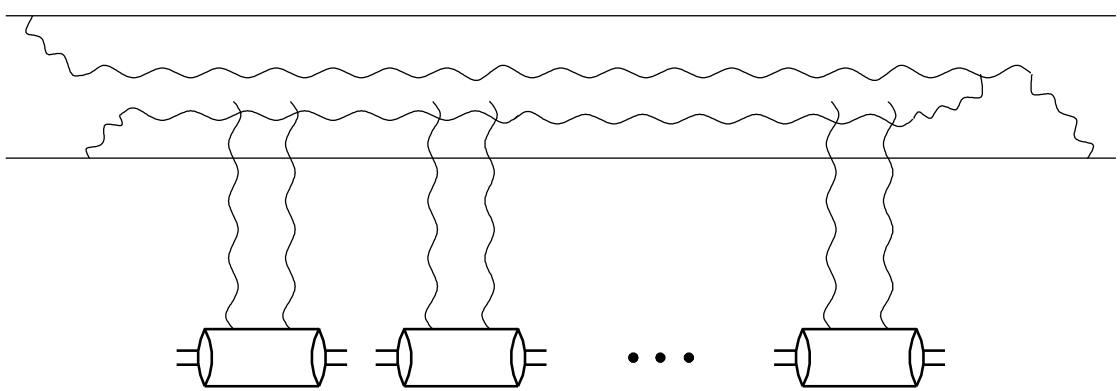


Dipole Amplitude

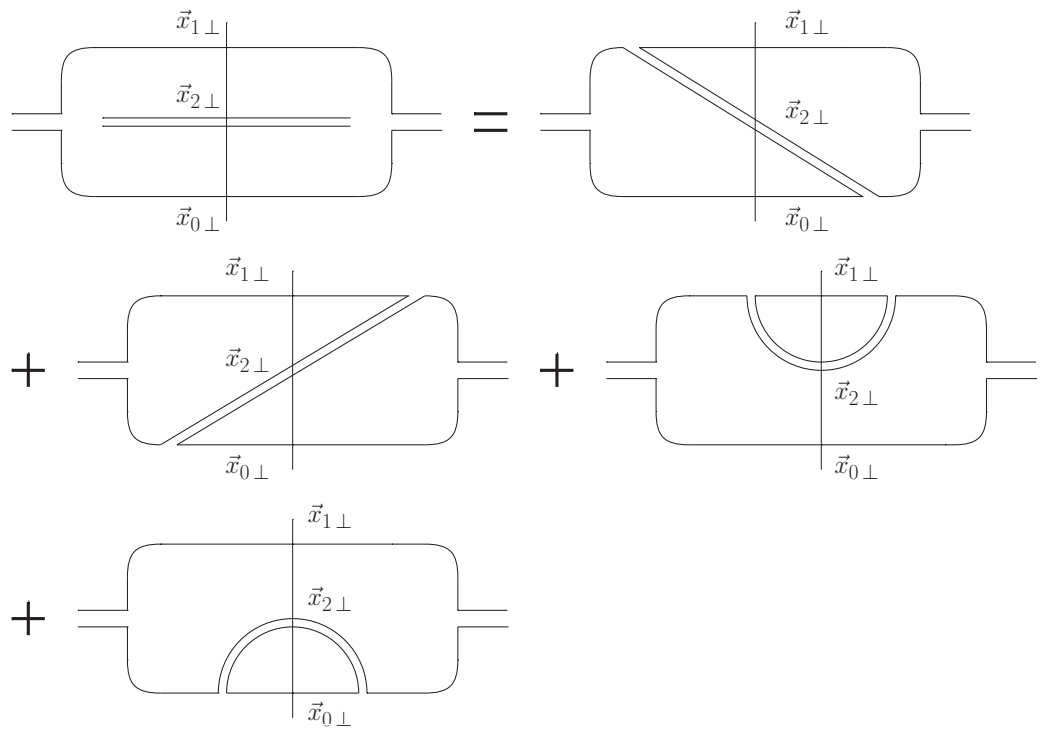
- The energy dependence comes in through nonlinear small-x BK/JIMWLK evolution, which resums the long-lived s-channel gluon corrections:



$$\alpha_s \ln \frac{1}{x} \sim \alpha_s Y \sim 1$$



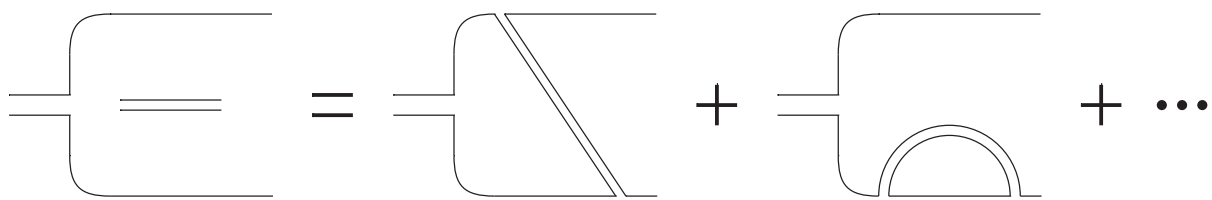
Notation (Large-N_C)



Real emissions in the amplitude squared

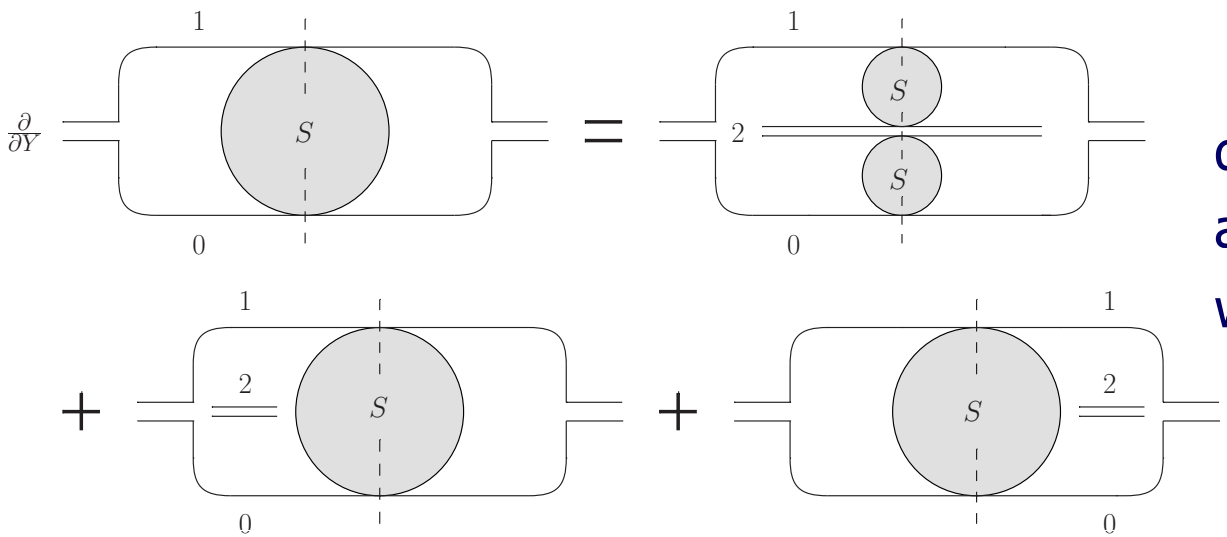
(dashed line – all Glauber-Mueller exchanges at light-cone time =0)

Virtual corrections in the amplitude (wave function)



Nonlinear Evolution

To sum up the gluon cascade at large-N_C we write the following equation for the dipole S-matrix:



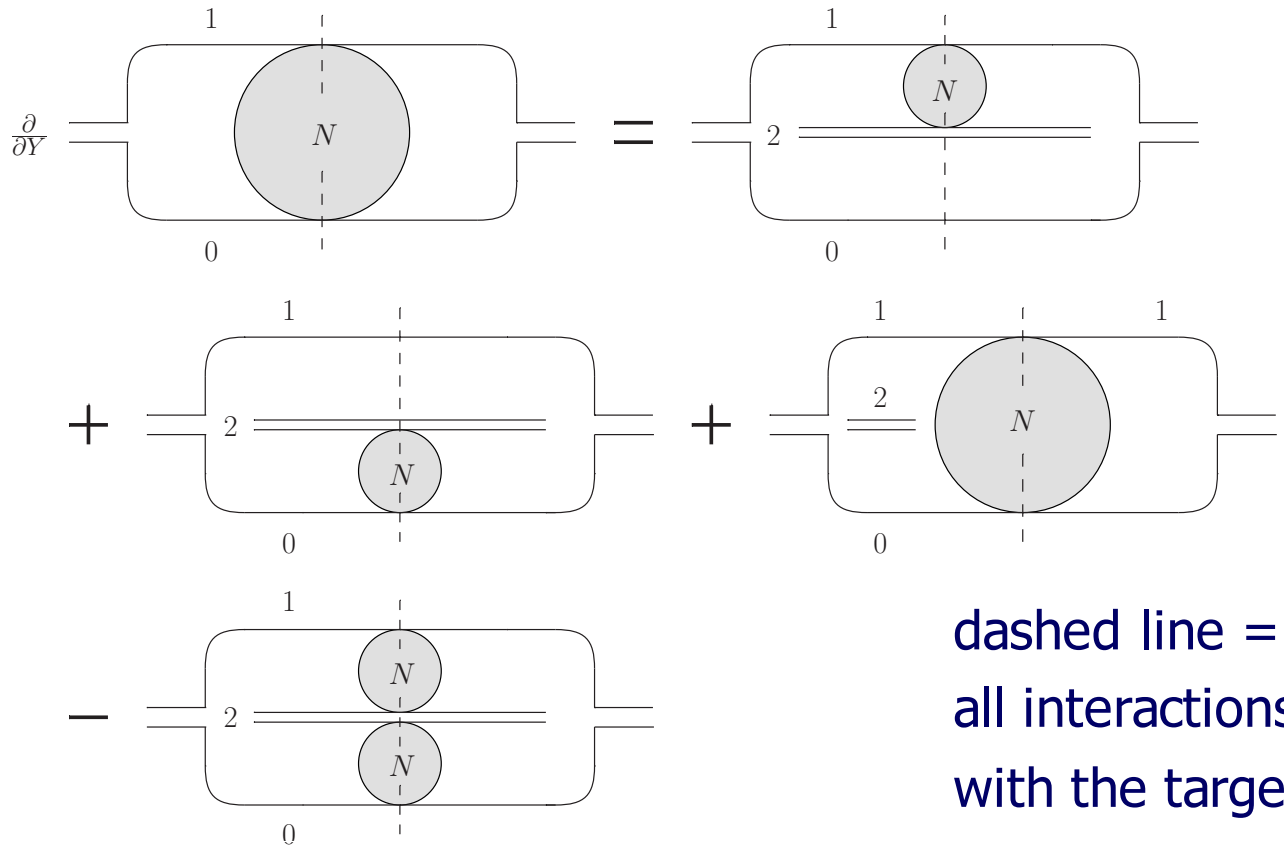
dashed line = all interactions with the target

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s\, N_c}{2\pi^2} \int d^2x_2 \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y)\right]$$

Remembering that S= 1-N we can rewrite this equation in terms of the dipole scattering amplitude N.

Nonlinear evolution at large N_c

As N=1-S we write



dashed line =
all interactions
with the target

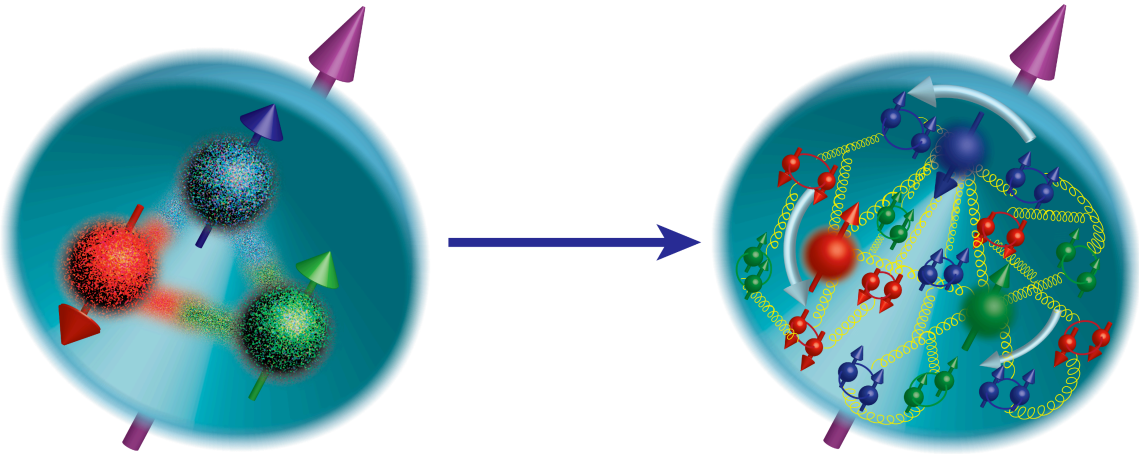
$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s\, N_c}{2\,\pi^2} \int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \left[N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y)\right]$$

Balitsky ‘96, Yu.K. ‘99

Helicity Observables

Yu.K., M. Sievert, arXiv:1505.01176 [hep-ph]
Yu.K., D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph]

Proton Spin



Our understanding of nucleon spin structure has evolved:

- In the 1980's the proton spin was thought of as a sum of constituent quark spins (left panel)

- Currently we believe that the proton spin is a sum of the spins of valence and sea quarks and of gluons, along with the orbital angular momenta of quarks and gluons (right panel)

Proton Spin Puzzle

- Helicity sum rule: $$\frac{1}{2} = S_q + L_q + S_g + L_g$$

  with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- The helicity parton distributions are (f = G, u, d, s, …)

$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

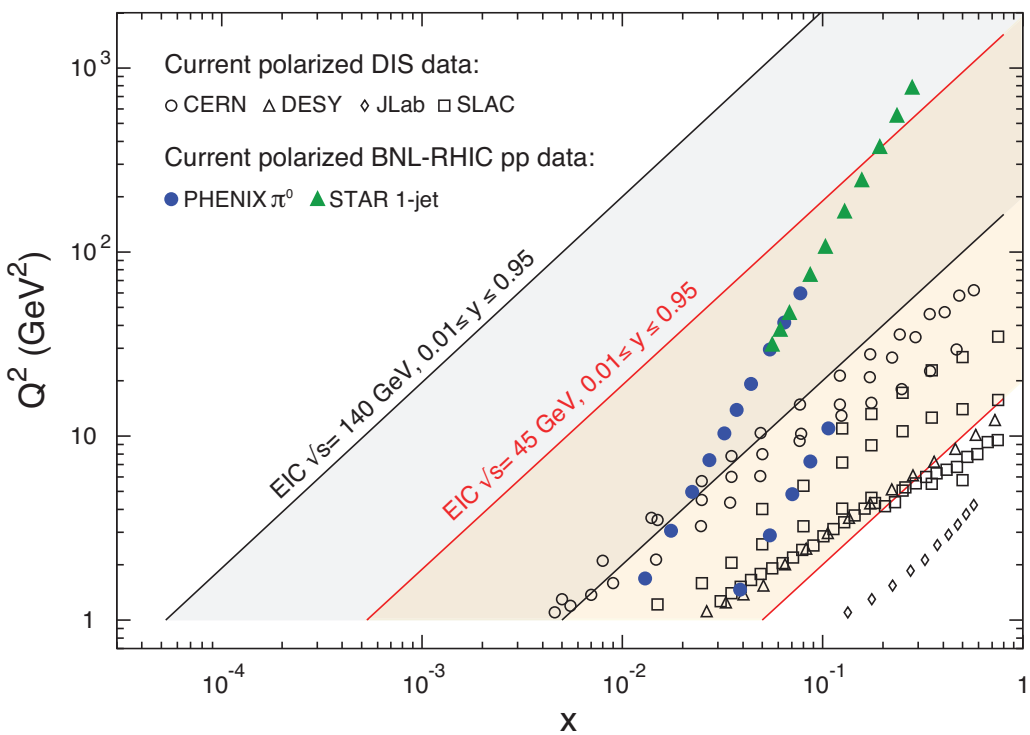
  with the net quark helicity distribution

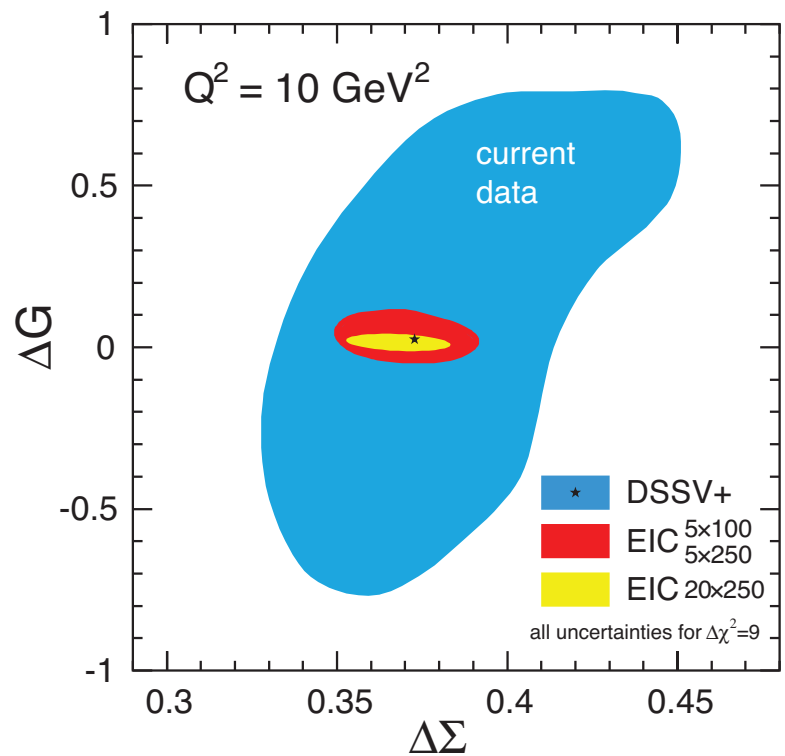
$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

- L_q and L_g are the quark and gluon orbital angular momenta

EIC & Spin Puzzle

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a spin DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton's spin:



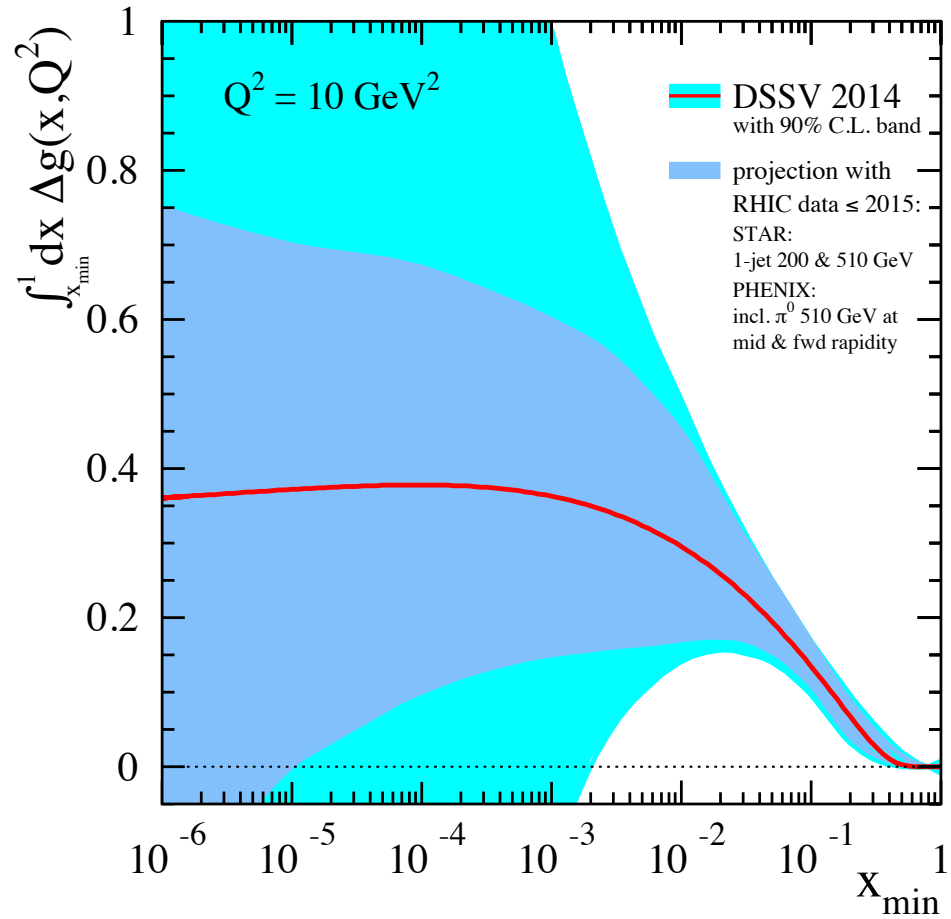


- ΔG and $\Delta\Sigma$ are integrated over x in the 0.001 < x < 1 interval.

Gluon Polarization

Recent data from RHIC appears to indicate that the net proton spin carried by the gluons S_g is non-zero, and in fact mainly comes from the small- to moderate-x region. EIC would help to measure gluon polarization at small x with unprecedented precision.

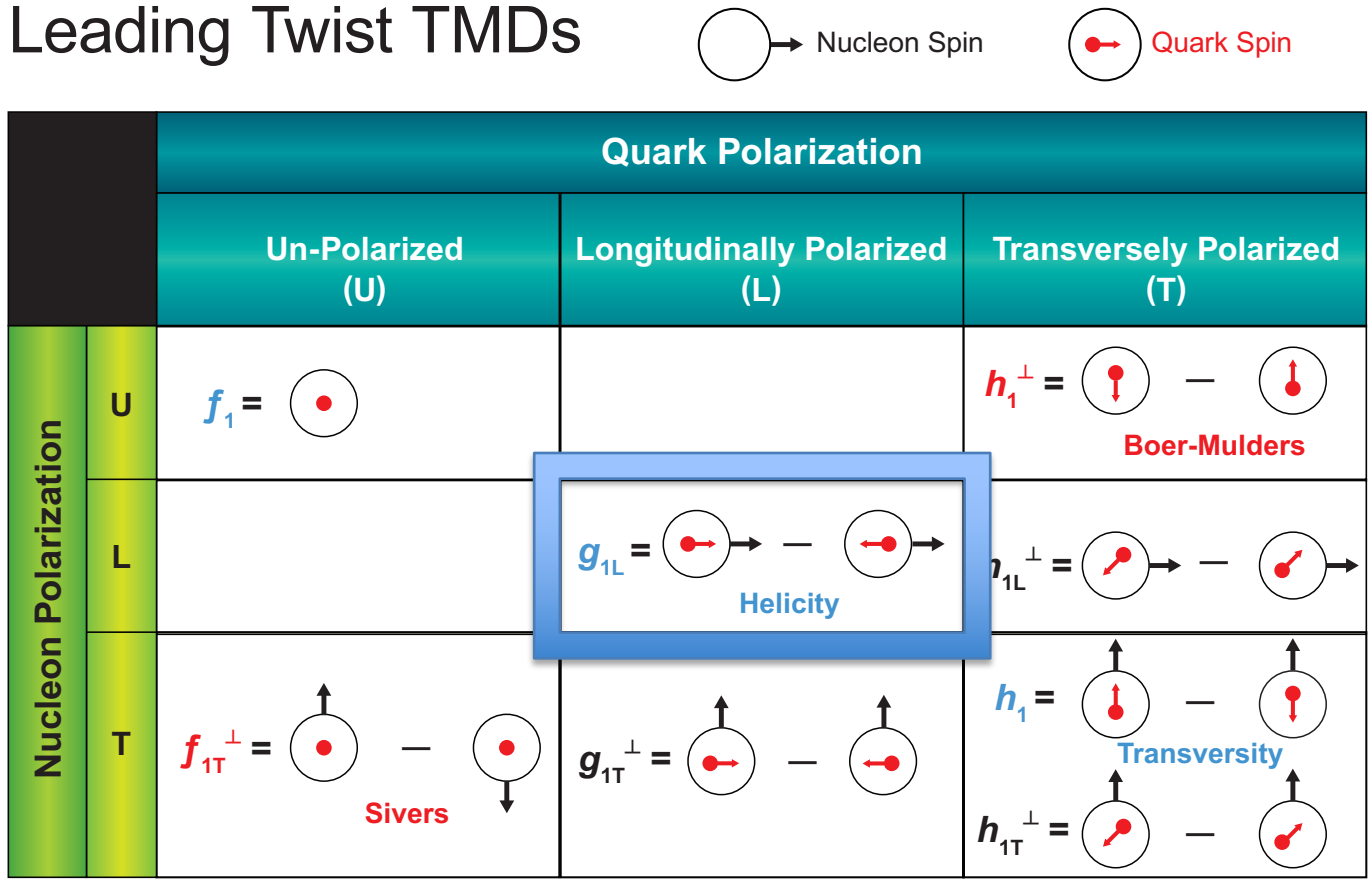
$$S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$



Transverse Momentum Distributions (TMDs)

- Here are the quark TMDs in a proton:

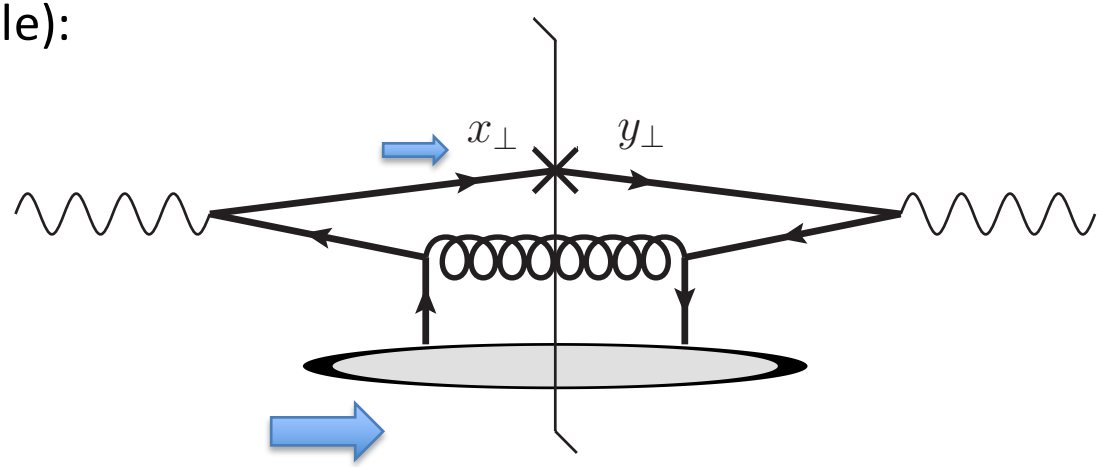
Leading Twist TMDs



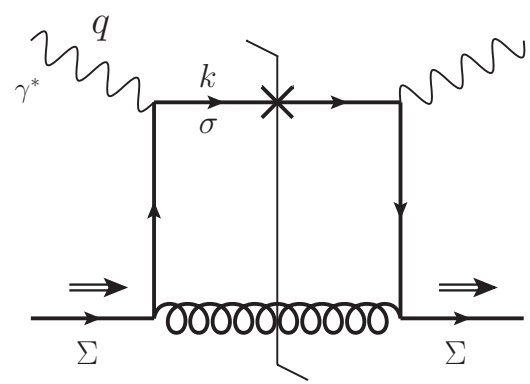
SIDIS on a Spin-Dependent Target

To transfer spin information between the polarized target and the produced quark we either need to exchange quarks in the t-channel, or non-eikonal gluons.

Here's an example of the quark exchange (we work in the $A^+=0$ light cone gauge of the projectile):

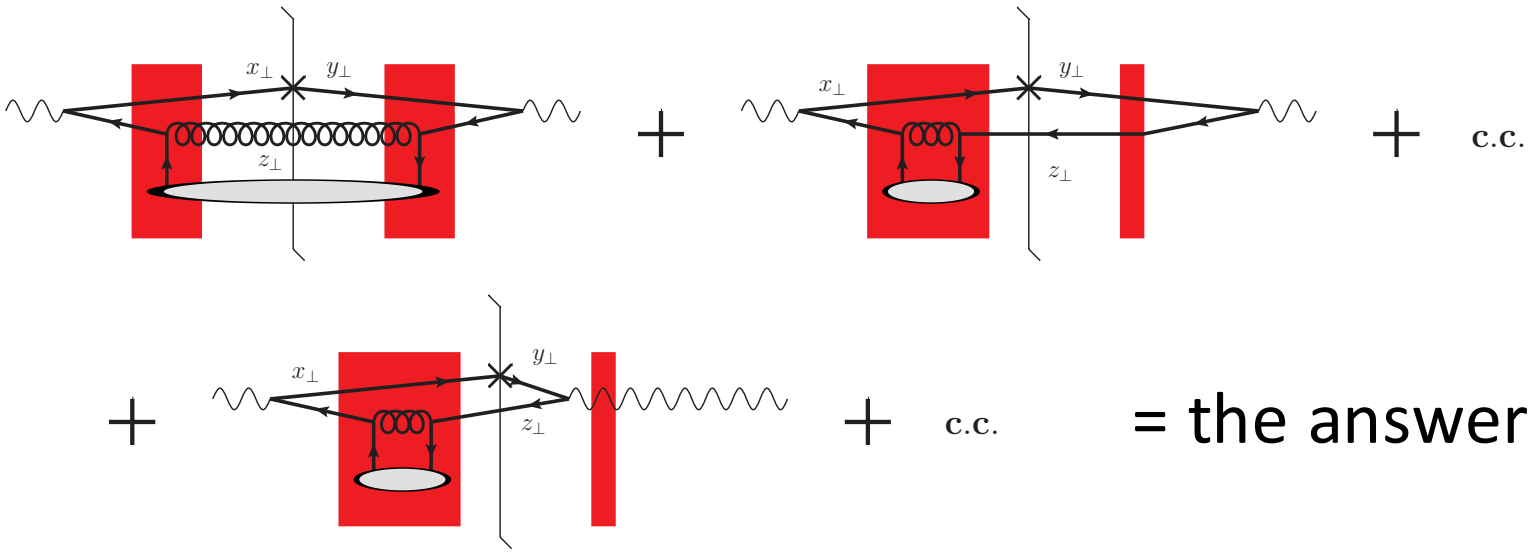


This is in addition to the standard handbag diagram which does not evolve under our small-x evolution:



Target Spin-Dependent SIDIS

It is straightforward to include multiple shock wave interactions into the polarized SIDIS cross section:

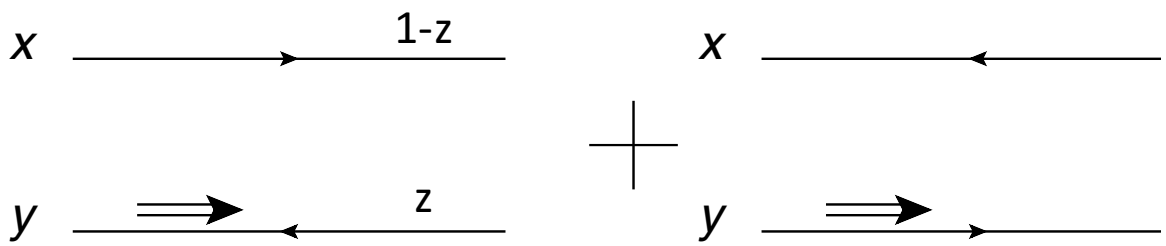


=0

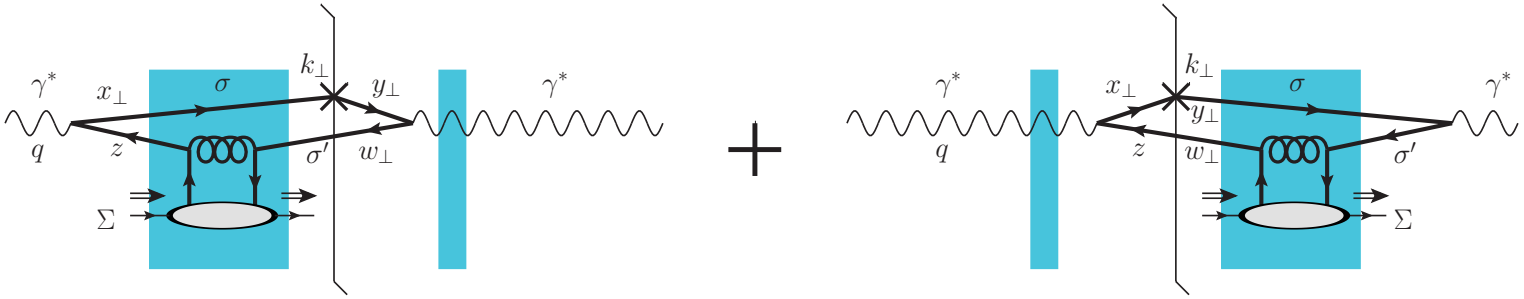
= the answer

Polarized Dipole

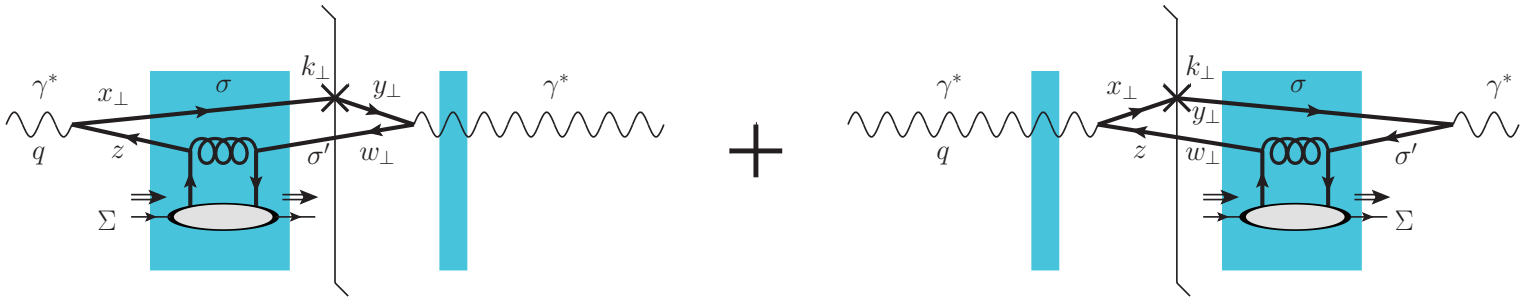
- Our goal now is to construct a small-x evolution equation for helicity.



$$\mathrm{tr}\left[V_{\underline{x}}^{unp}\, V_{\underline{y}}^{pol\,\dagger}\right] + \mathrm{tr}\left[V_{\underline{y}}^{pol}\, V_{\underline{x}}^{unp\,\dagger}\right]$$



Quark Helicity TMD at Small x



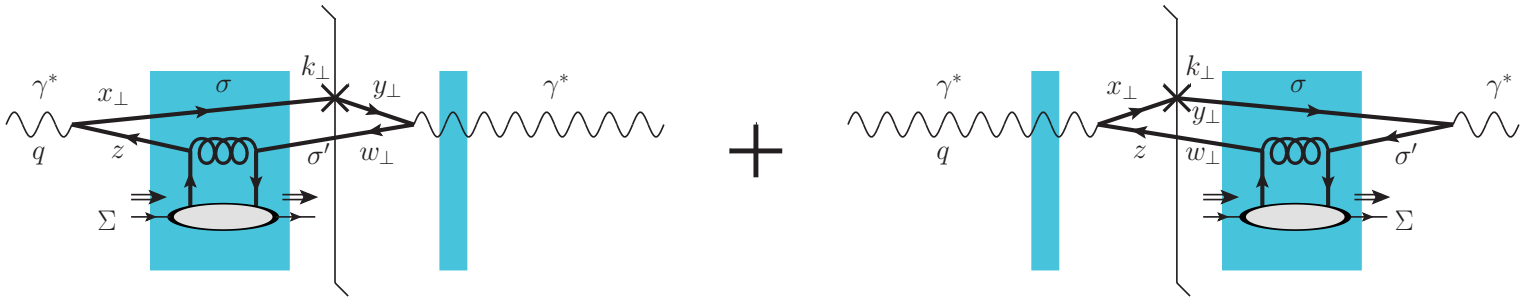
- One can show that quark helicity TMD at small-x can be expressed in terms of the polarized dipole operator:

$$g_{1L}(x,k_T) = -\frac{\Sigma}{4\pi^3}\int_{z_i}^{1}\frac{dz}{z}\int\frac{d^2x_\perp\,d^2y_\perp\,d^2w_\perp}{2(2\pi)^3}\,e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\,\frac{\underline{x}-\underline{w}}{|\underline{x}-\underline{w}|^2}\cdot\frac{\underline{y}-\underline{w}}{|\underline{y}-\underline{w}|^2}$$
$$\times\sum_{\sigma'}(-\sigma')\,\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{x}}\,V^\dagger_{\underline{w}}(\sigma')\right]+\mathrm{tr}\left[V_{\underline{w}}(\sigma')\,V^\dagger_{\underline{x}}\right]\right\rangle\!\!\right\rangle_\Sigma(z)\;\;\theta\left(\frac{1}{Q^2}-z\,|\underline{x}-\underline{w}|^2\right)\,\theta\left(\frac{1}{Q^2}-z\,|\underline{y}-\underline{w}|^2\right).$$

- Here Σ is the proton helicity, while σ' is the anti-quark helicity.
- The double brackets denote target averaging rescaled by energy:

$$\langle\ldots\rangle_\Sigma(z) = \frac{1}{z\,s}\,\langle\!\langle\ldots\rangle\!\rangle_\Sigma(z)$$

Quark Helicity PDF at Small x



- Quark hPDF can be easily constructed out of quark helicity TMD at small-x:

$$\Delta q(x,Q^2) = \int d^2k_T\, g_{1L}(x,k_T) = -\frac{\Sigma}{4\pi^3} \int\limits_{z_i}^{1} \frac{dz}{z} \int\limits_{\rho^2} \frac{d^2x_\perp\, d^2w_\perp}{4\pi} \frac{1}{|\underline{x}-\underline{w}|^2}\, \theta\left(\frac{1}{Q^2} - z\,|\underline{x}-\underline{w}|^2\right)$$

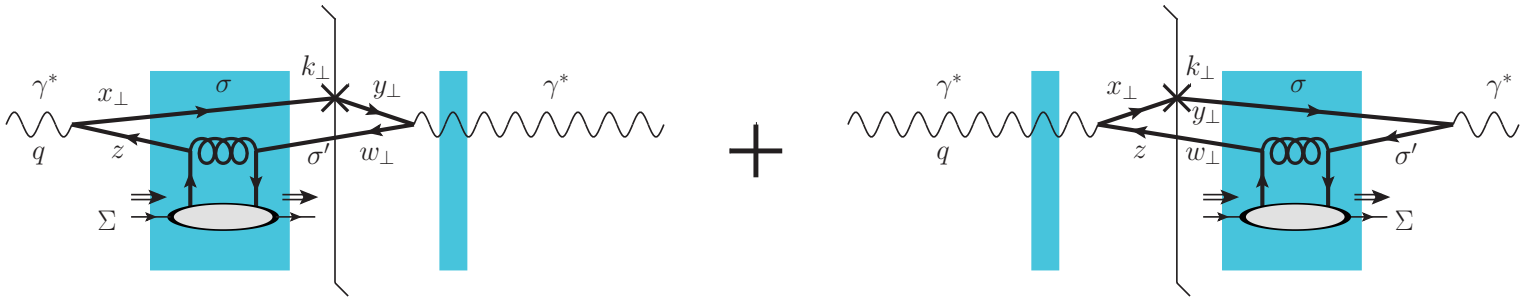
$$\times \sum_{\sigma'} (-\sigma') \left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{x}}\, V^\dagger_{\underline{w}}(\sigma')\right] + \mathrm{tr}\left[V_{\underline{w}}(\sigma')\, V^\dagger_{\underline{x}}\right] \right\rangle\!\!\right\rangle_\Sigma (z)$$

- Here and before

$$V_{\underline{w}}(\sigma') = V^{pol}_{\underline{w}}$$

(two notations for the same polarized “Wilson line”)

g_1 Structure Function at Small x



- Finally the g_1 structure function at small-x is similarly written as

$$g_1(x,Q^2) = -\frac{\Sigma}{2\pi^2\,\alpha_{EM}} \int_{z_i}^{1} \frac{dz}{z^2\,(1-z)} \int \frac{d^2x_\perp\, d^2w_\perp}{4\pi} \sum_{\sigma,\sigma',f} \sigma \left[\frac{1}{2} \sum_{\lambda} \left| \Psi_T^{\gamma^*\to q\bar{q}}(\underline{x}-\underline{w},z) \right|^2 + \left| \Psi_L^{\gamma^*\to q\bar{q}}(\underline{x}-\underline{w},z) \right|^2 \right]$$
$$\times \left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{x}}\, V_{\underline{w}}^\dagger(\sigma') \right] + \mathrm{tr}\left[V_{\underline{w}}(\sigma')\, V_{\underline{x}}^\dagger \right] \right\rangle\!\!\right\rangle_\Sigma (z)$$

- All the small-x observables (helicity TMDs and PDFs, and g_1 structure function) are expressed in terms of the same polarized dipole operator:

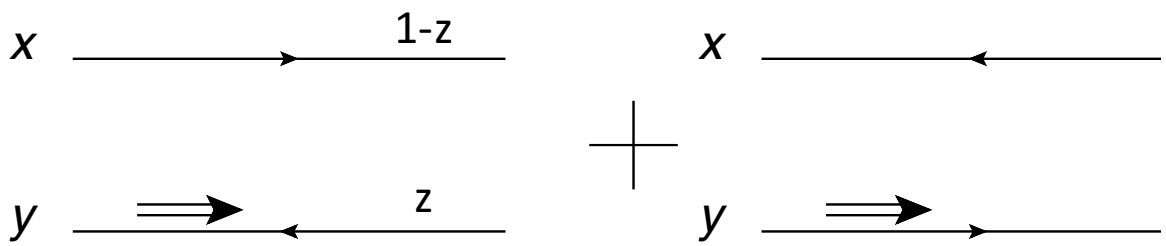
$$\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{x}}\, V_{\underline{y}}^\dagger(\sigma) \right] + \mathrm{tr}\left[V_{\underline{y}}(\sigma)\, V_{\underline{x}}^\dagger \right] \right\rangle\!\!\right\rangle_\Sigma (z)$$

Helicity Evolution at Small x

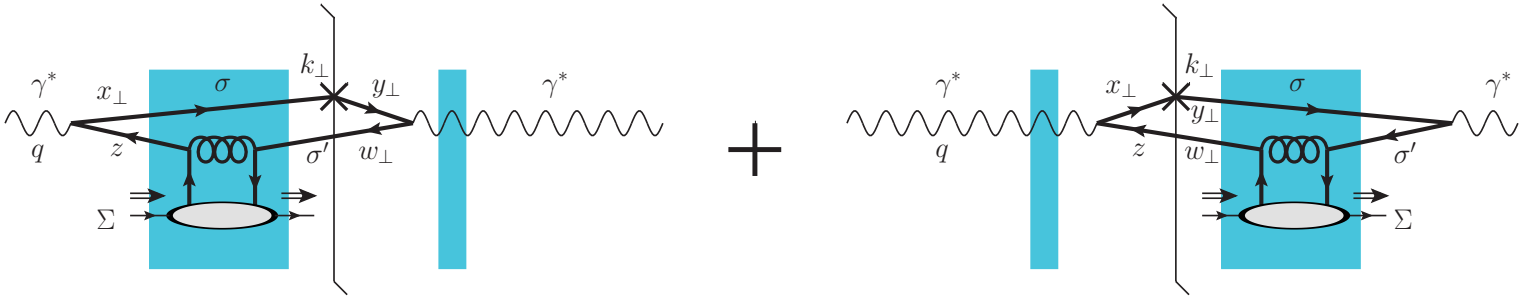
Yu.K., D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph]

Polarized Dipole

- Our goal now is to construct a small-x evolution equation for helicity.

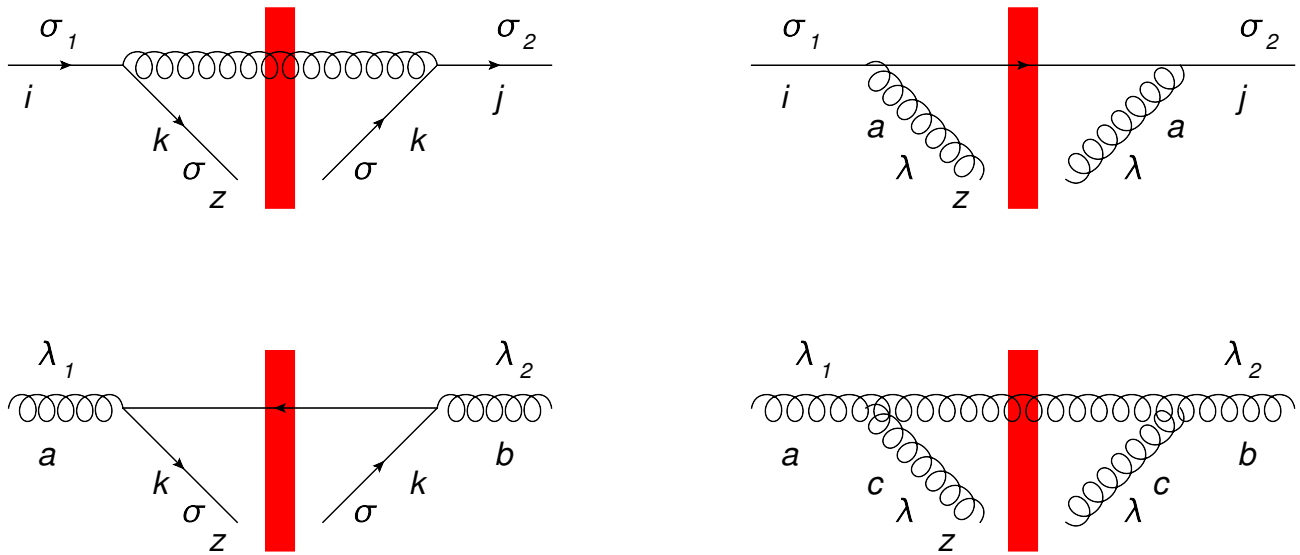


$$\mathrm{tr}\left[V_{\underline{x}}^{unp}\, V_{\underline{y}}^{pol\,\dagger}\right] + \mathrm{tr}\left[V_{\underline{y}}^{pol}\, V_{\underline{x}}^{unp\,\dagger}\right]$$



Helicity Evolution Ingredients

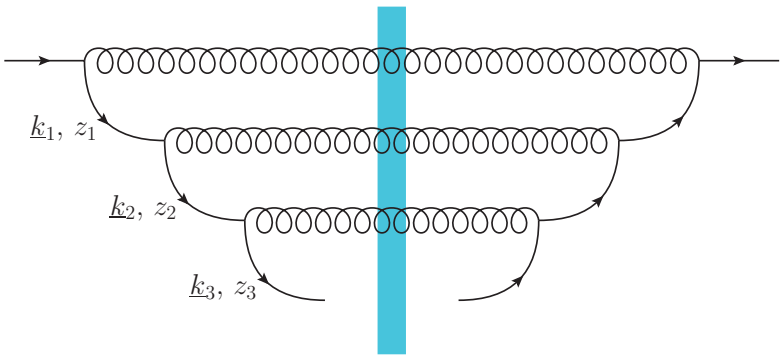
- Unlike the unpolarized evolution, in one step of helicity evolution we may emit a soft gluon or a soft quark (all in $A^+=0$ LC gauge of the projectile):

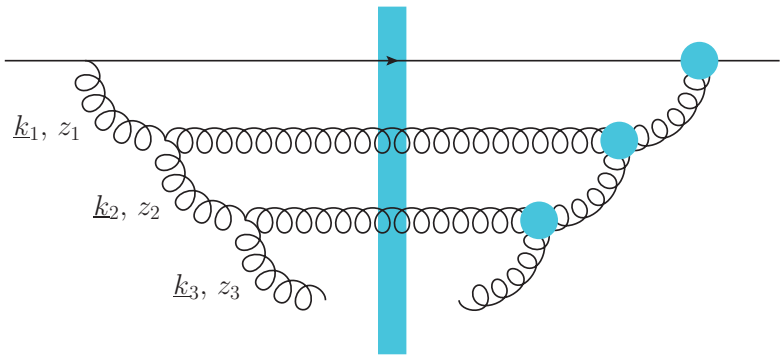


- When emitting gluons, one gluon is eikonal, while another one is soft, but non-eikonal, as is needed to transfer polarization down the cascade/ladder.

Helicity Evolution: Ladders

- To get an idea of how the helicity evolution works let us try iterating the splitting kernels by considering ladder diagrams (circles denote non-eikonal gluon vertices):



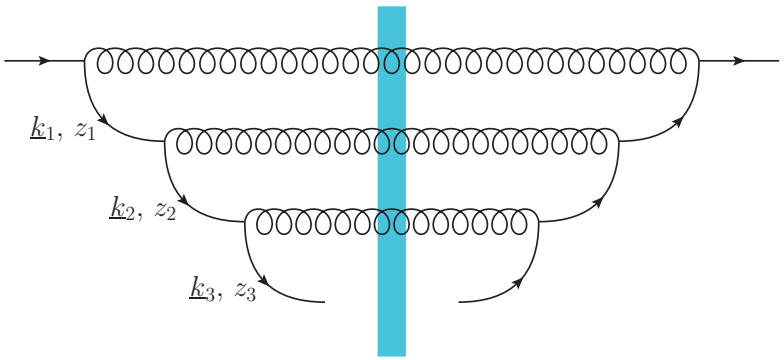


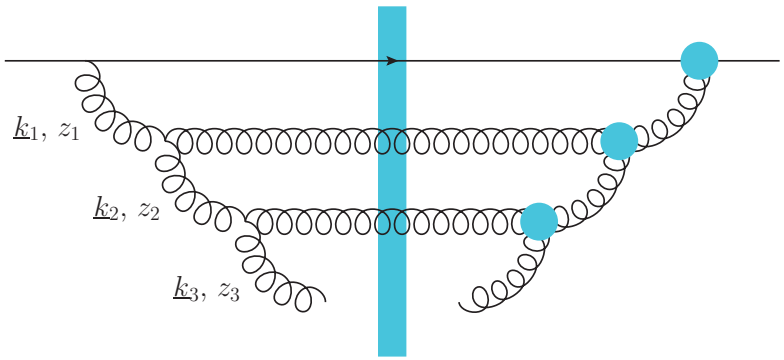
- To get the leading-energy asymptotics we need to order the longitudinal momentum fractions of the quarks and gluons (just like in the unpolarized evolution case) $1 \gg z_1 \gg z_2 \gg z_3 \gg \ldots$

obtaining a nested integral

$$\alpha_s^3 \int\limits_{z_i}^{1} \frac{dz_1}{z_1} \int\limits_{z_i}^{z_1} \frac{dz_2}{z_2} \int\limits_{z_i}^{z_2} \frac{dz_3}{z_3}\, z_3 \otimes \frac{1}{z_3\, s} \sim \frac{1}{s}\, \alpha_s^3\, \ln^3 s$$

Helicity Evolution: Ladders





- However, these are not all the logs of energy one can get here. Transverse momentum (or distance) integrals have UV and IR divergences, which lead to logs of energy as well.
- If we order transverse momenta / distances as (Sudakov-β ordering)

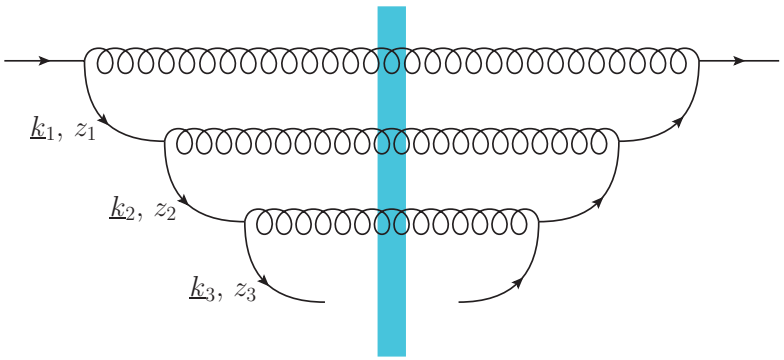
$$\frac{\underline{k}_1^2}{z_1} \ll \frac{\underline{k}_2^2}{z_2} \ll \frac{\underline{k}_3^2}{z_3} \ll \ldots \qquad z_1\, \underline{x}_1^2 \gg z_2\, \underline{x}_2^2 \gg z_3\, \underline{x}_3^2 \gg \ldots$$

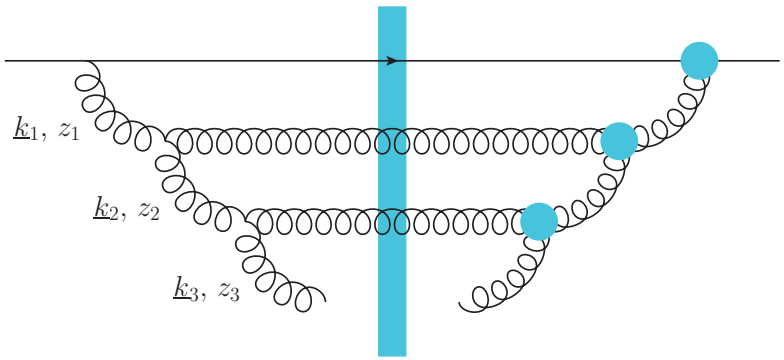
we would get integrals like

$$\int_{1/(z_n\, s)}^{x_{n-1,\perp}^2\, z_{n-1}/z_n} \frac{dx_{n,\perp}^2}{x_{n,\perp}^2}$$

also generating logs of energy.

Helicity Evolution: Ladders





- To summarize, the above ladder diagrams are parametrically of the order

$$\frac{1}{s}\,\alpha_s^3\,\ln^6 s$$

- Note two features:
  - 1/s suppression due to non-eikonal exchange
  - two logs of energy per each power of the coupling!

Resummation Parameter

- For helicity evolution the resummation parameter is different from BK or JIMWLK, which resum powers of leading logarithms (LLA)

$$\alpha_s \ \ln(1/x)$$

- Helicity evolution resummation parameter is double-logarithmic (DLA):

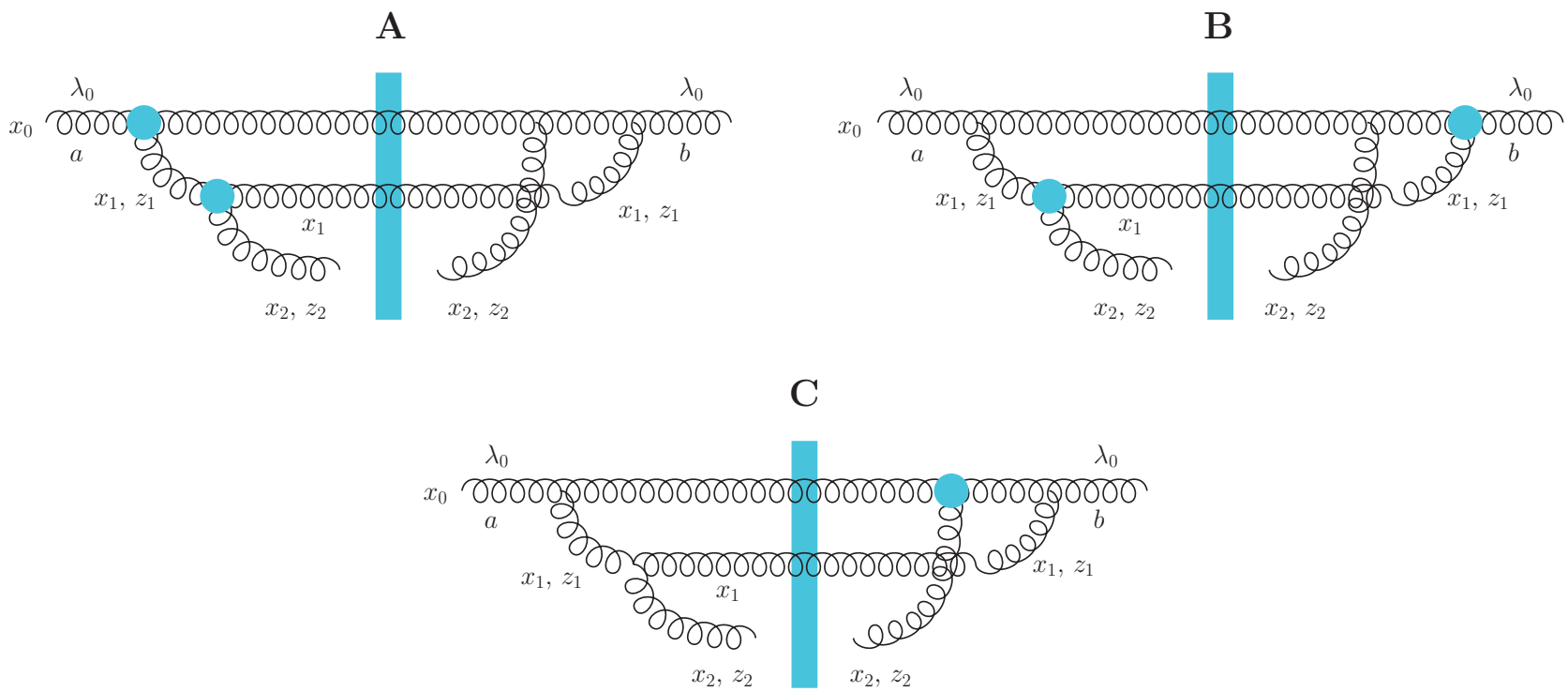
$$\alpha_s \ \ln^2 \frac{1}{x}$$

- The second logarithm of x arises due to transverse momentum integration being logarithmic both in UV and IR.

- This was known before: Kirschner and Lipatov '83; Kirschner '84; Bartels, Ermolaev, Ryskin '95, '96; Griffiths and Ross '99; Itakura et al '03; Bartels and Lublinsky '03.

Non-Ladder Diagrams

- Ladder diagrams are not the whole story. The non-ladder diagrams below are also leading-order (that is, DLA).

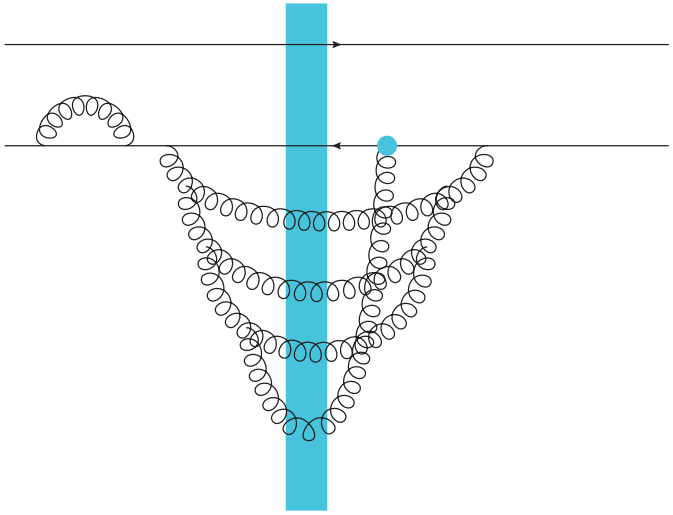


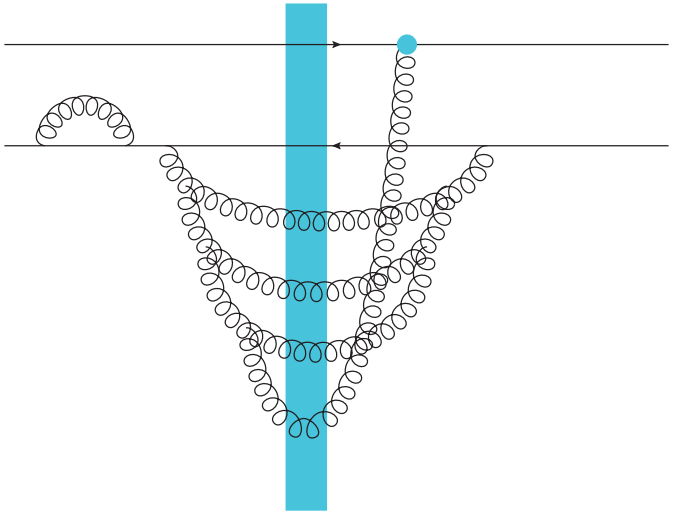
- Non-ladder soft quark emissions cancel for flavor-singlet observables we are primarily interested in. Non-ladder gluons do not cancel.

Virtual Corrections

- In addition, virtual corrections from the unpolarized LLA evolution have UV divergences, which cancel between real and virtual diagrams. Here the corrections are not cancelled, but are regulated by the cms energy.

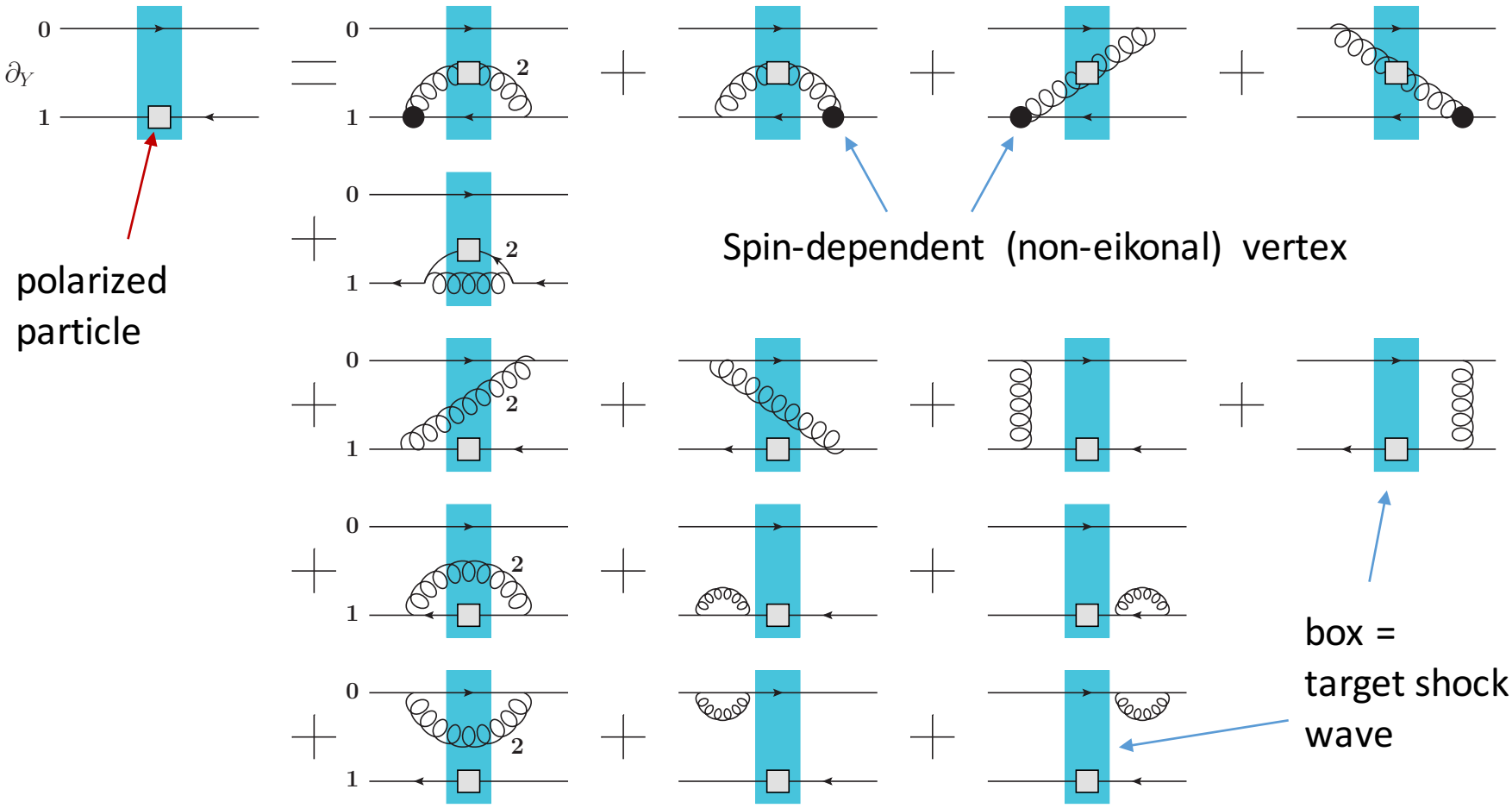
- Helicity evolution thus also contains the following types of graphs:



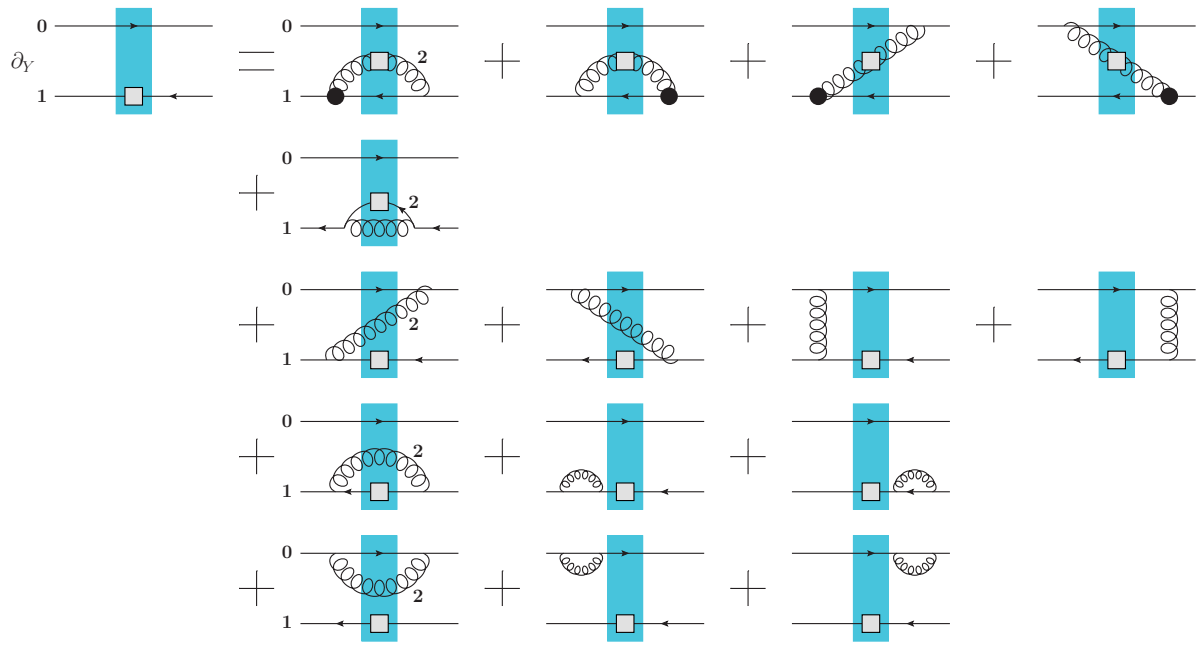


Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Evolution for Polarized Quark Dipole



$$\langle\langle \ldots \rangle\rangle = \frac{1}{z\,s}\,\langle \ldots \rangle$$

$$\rho'^{\,2} = \frac{1}{z'\,s}$$

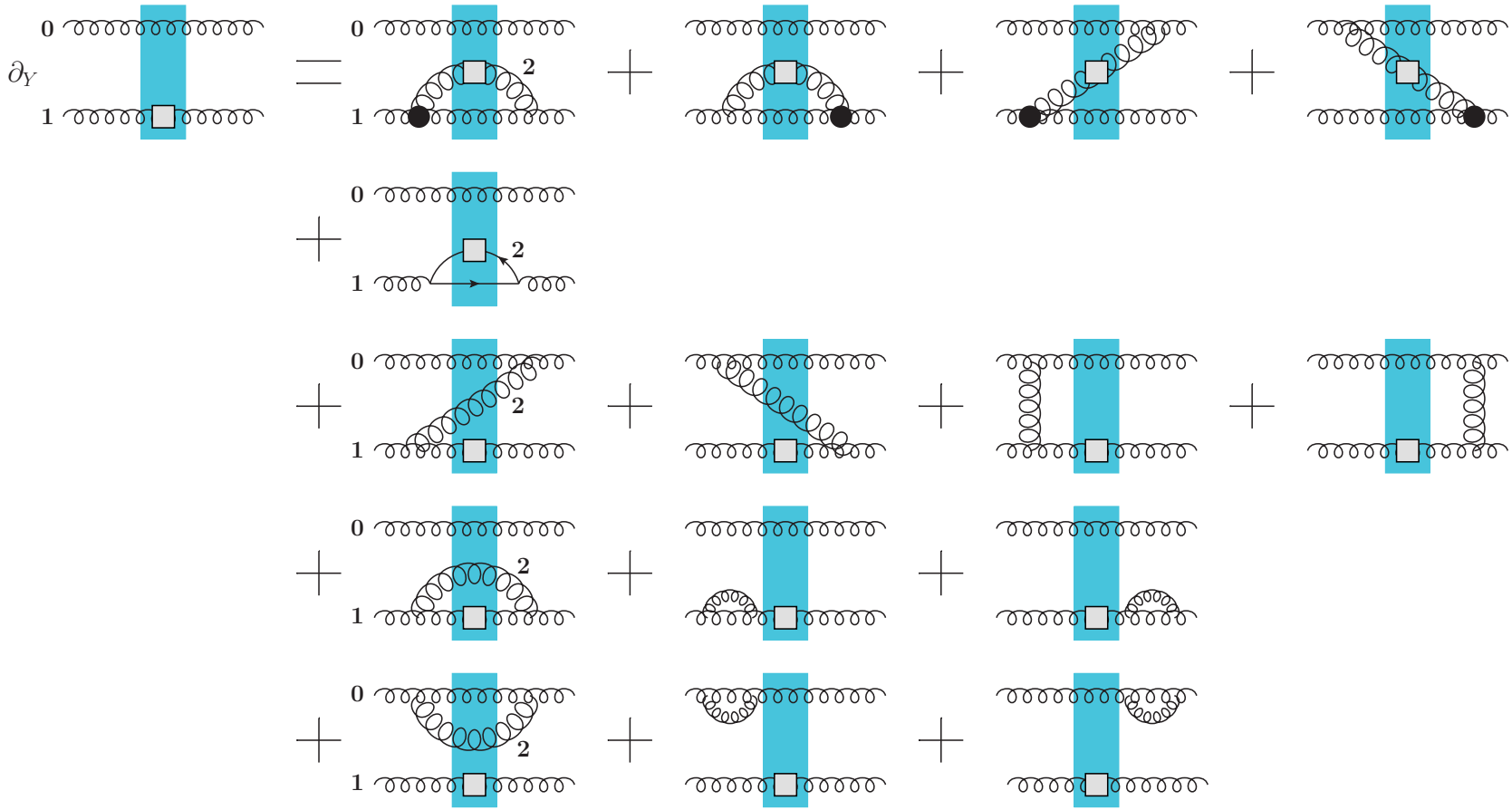
$$\frac{1}{N_c}\,\left\langle\!\!\left\langle \mathrm{tr}\left[V_0^{unp}\,V_1^{pol\,\dagger}\right]\right\rangle\!\!\right\rangle(z) = \frac{1}{N_c}\,\left\langle\!\!\left\langle \mathrm{tr}\left[V_0^{unp}\,V_1^{pol\,\dagger}\right]\right\rangle\!\!\right\rangle_0(z) + \frac{\alpha_s}{2\pi^2}\int\limits_{z_i}^{z}\frac{dz'}{z'}\int\limits_{\rho'^2}\frac{d^2x_2}{x_{21}^2}$$

$$\times\left\{\theta(x_{10}-x_{21})\,\frac{2}{N_c}\,\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\,V_0^{unp}\,t^a\,V_1^{unp\,\dagger}\right]\,U_2^{pol\,ba}\right\rangle\!\!\right\rangle(z')\right.$$

$$+\,\theta(x_{10}^2 z - x_{21}^2 z')\,\frac{1}{N_c}\,\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\,V_0^{unp}\,t^a\,V_2^{pol\,\dagger}\right]\,U_1^{unp\,ba}\right\rangle\!\!\right\rangle(z')$$

$$\left.+\,\theta(x_{10}-x_{21})\,\frac{1}{N_c}\left[\left\langle\!\!\left\langle \mathrm{tr}\left[V_0^{unp}\,V_2^{unp\,\dagger}\right]\,\mathrm{tr}\left[V_2^{unp}\,V_1^{pol\,\dagger}\right]\right\rangle\!\!\right\rangle(z') - N_c\left\langle\!\!\left\langle \mathrm{tr}\left[V_0^{unp}\,V_1^{pol\,\dagger}\right]\right\rangle\!\!\right\rangle(z')\right]\right\}$$

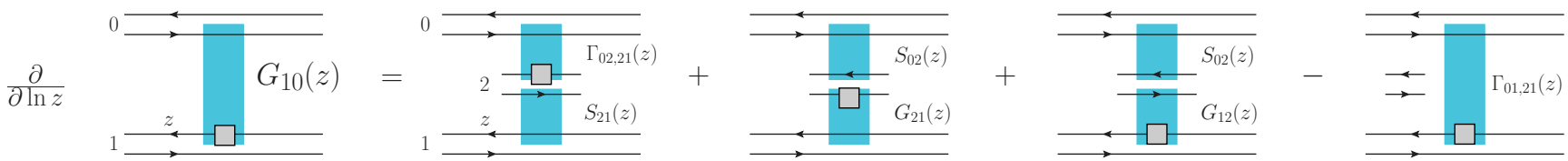
Polarized Gluon Dipole Evolution

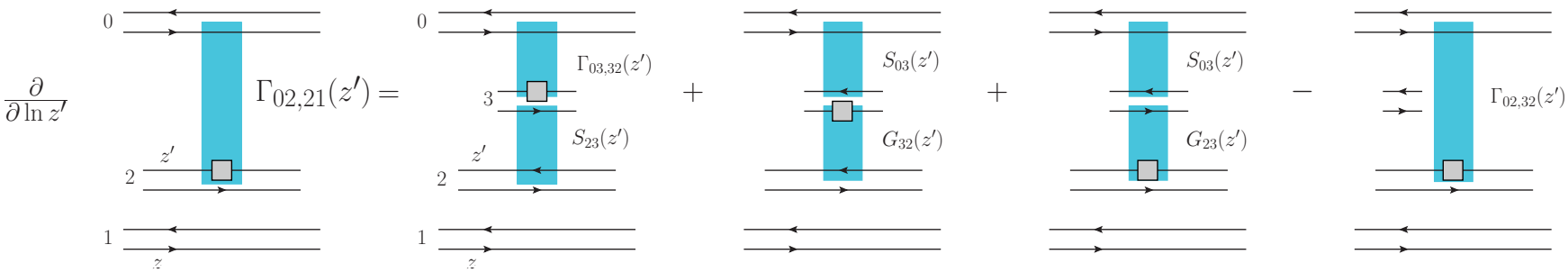


Note that at our sub-eikonal level, gluon dipole is a product of two quark dipoles color-wise, but these 'quark' dipoles evolve differently from the polarized dipole made of actual quarks.

Polarized Dipole Evolution in the Large-N_c Limit

In the large-N_c limit the equations close, leading to a closed system of 2 equations:



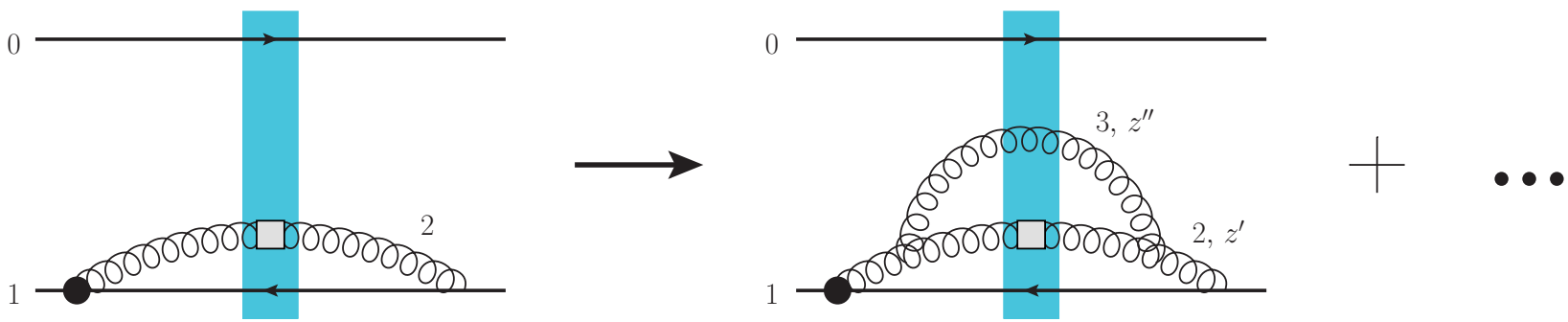


$$G_{10}(z) = G_{10}^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int_{z_i}^{z} \frac{dz'}{z'} \int_{\rho'^2}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[2\,\Gamma_{02,\,21}(z')\,S_{21}(z') + 2\,G_{21}(z')\,S_{02}(z') + G_{12}(z')\,S_{02}(z') - \Gamma_{01,\,21}(z')\right]$$

$$\Gamma_{02,\,21}(z') = \Gamma_{02,\,21}^{(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int_{z_i}^{z'} \frac{dz''}{z''} \int_{\rho''^2}^{\min\{x_{02}^2, x_{21}^2\, z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[2\,\Gamma_{03,\,32}(z'')\,S_{23}(z'') + 2\,G_{32}(z'')\,S_{03}(z'') + G_{23}(z'')\,S_{03}(z'') - \Gamma_{02,\,32}(z'')\right]$$

You friendly “neighborhood” dipole

- There is a new object in the evolution equation – **the neighbor dipole**.
- This is specific for the DLA evolution. Gluon emission may happen in one dipole, but, due to transverse distance ordering, may ’know’ about another dipole:

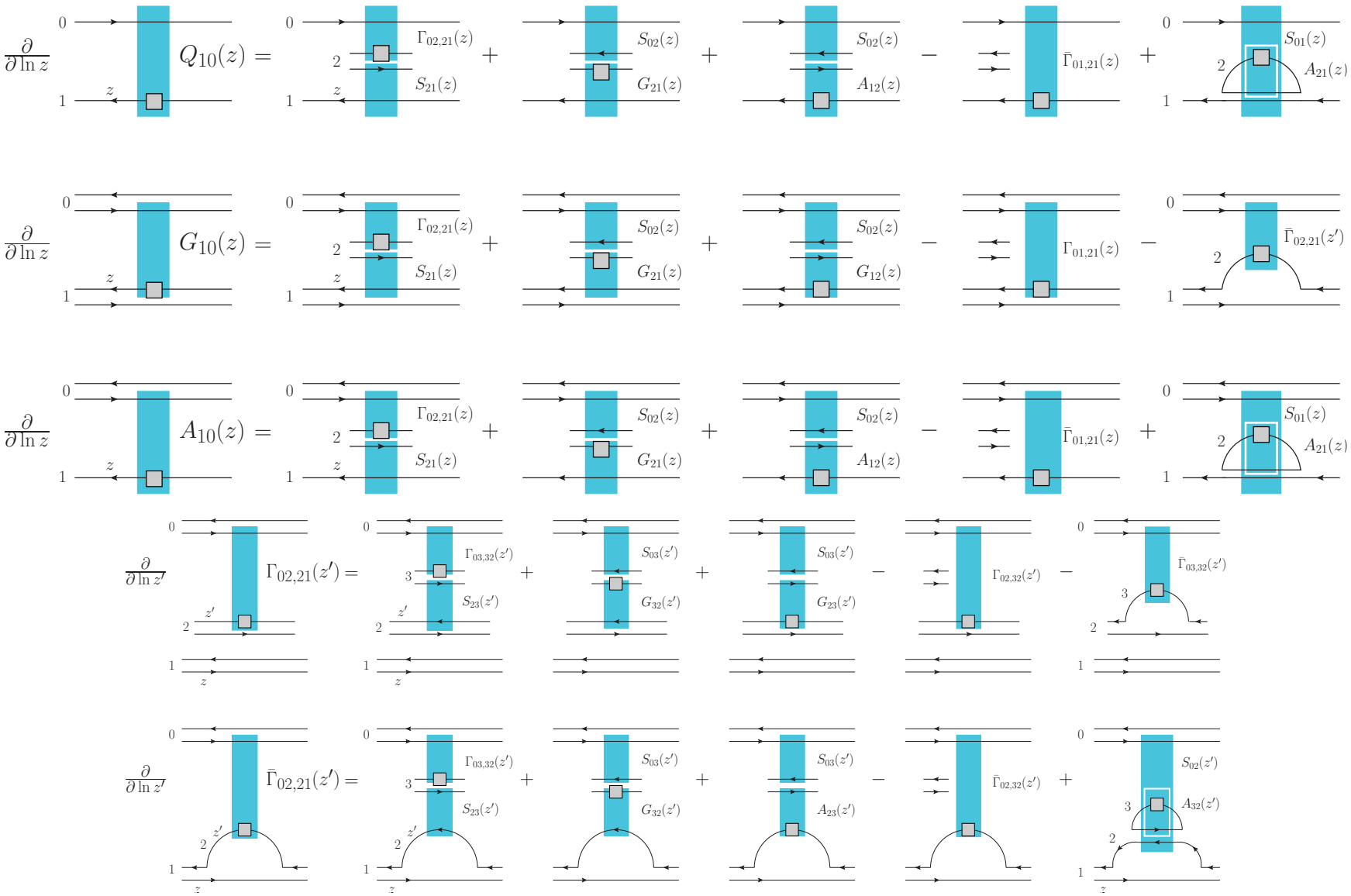


$$x_{21}^2\, z' \gg x_{32}^2\, z''$$

- We denote the evolution in the neighbor dipole 02 by $\Gamma_{02,\,21}(z')$

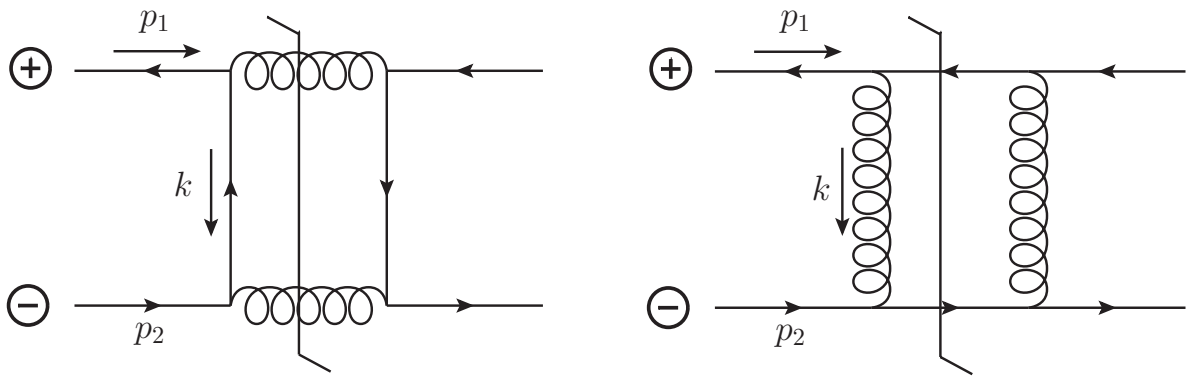
Polarized Dipole Evolution in the Large-N_c&N_f Limit

In the large-N_c&N_f limit the equations close too, leading to a closed system of 5 equations:

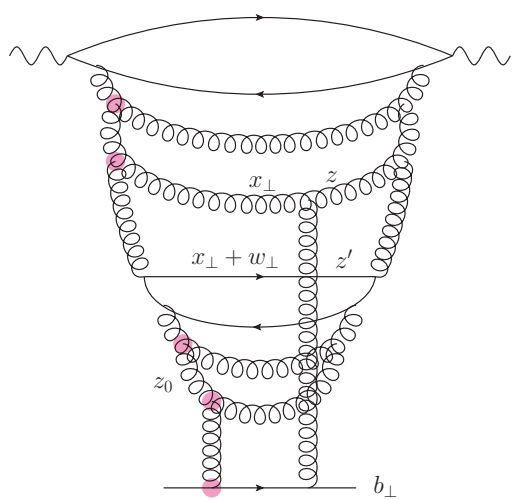


Initial Conditions

- Initial conditions for all our evolution equations should be given by Born-level interactions ("dressed" by multiple rescatterings in the saturation case):



- Note that they could be non-ladder as well:

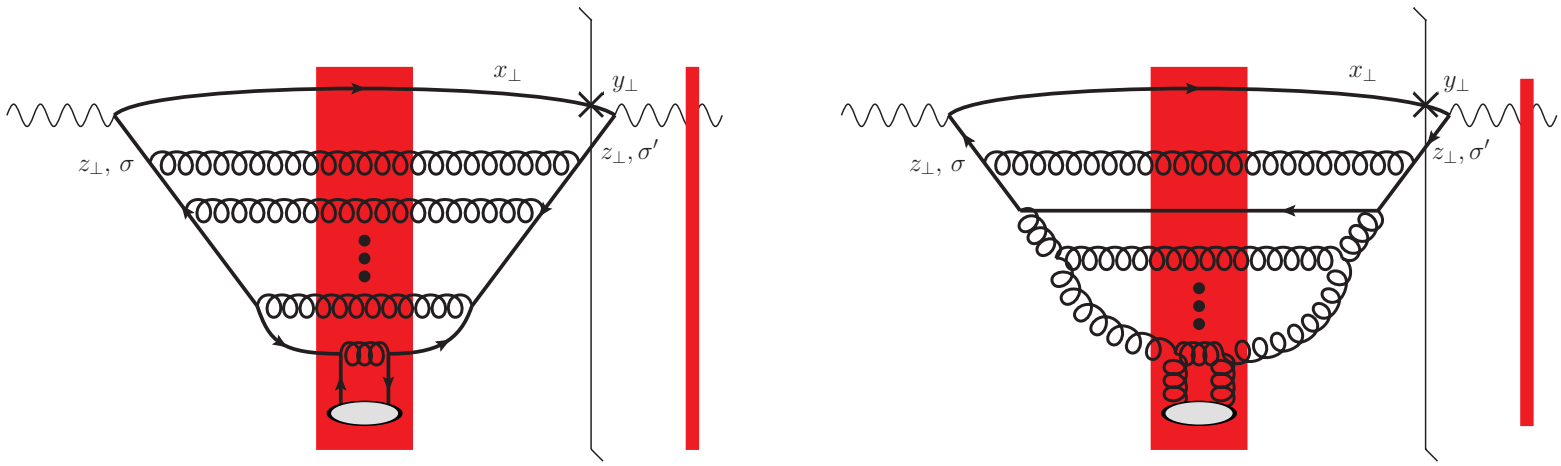


Small x Asymptotics of the Quark Helicity Distribution

Yu.K., D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph]
+ in preparation

Small-x Quark Helicity TMD Evolution: Ladders

A part of this evolution equation comes from ladder diagrams:



Interestingly the quark and non-eikonal gluon ladders mix (see the right panel), resulting in a more complicated evolution equation:

$$W^{pol}_{\underline{1}\,\underline{0}}(z) = W^{(0)\,pol}_{\underline{1}\,\underline{0}}(z) + \frac{\alpha_s}{2\,\pi} \int\limits_{z_i}^{z} \frac{dz'}{z'} \int\limits_{\rho'^2}^{x_{01}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, M\, W^{pol}_{\underline{2}\,\underline{1}}(z')$$

$$M \equiv \begin{pmatrix} C_F & 2\,C_F \\ -N_f & 4\,N_c \end{pmatrix} \qquad W^{pol}_{\underline{x}\underline{y}} = \begin{pmatrix} \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr}[V^{pol}_{\underline{x}}] + \mathrm{tr}[V^{pol\,\dagger}_{\underline{x}}] \right\rangle\!\!\right\rangle \\ \frac{1}{N_c^2-1} \left\langle\!\!\left\langle \mathrm{Tr}[U^{pol}_{\underline{x}}] + \mathrm{Tr}[U^{pol\,\dagger}_{\underline{x}}] \right\rangle\!\!\right\rangle \end{pmatrix} \begin{matrix} \text{quarks} \\ \text{gluons} \end{matrix}$$

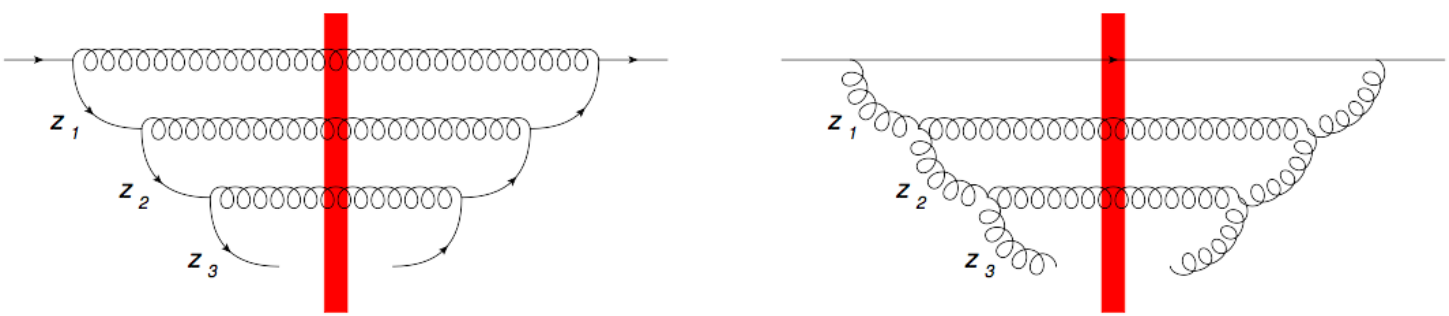
Ballpark Estimate: Ladders

- Summing up mixing quark and gluon ladders yields

$$\Delta\Sigma \sim \left(\frac{1}{x}\right)^{\omega_+} \qquad S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

with

$$\omega_+ = \sqrt{\frac{\alpha_s}{2\pi N_c}}\sqrt{9\,N_c^2 - 1 + \sqrt{(1+7\,N_c^2)^2 + 16\,N_c\,N_f\,(1-N_c^2)}}$$



- The numbers are encouraging (α_s=0.3, N_c=N_f=3): $\Delta\Sigma \sim \left(\frac{1}{x}\right)^{1.46}$

- But: need to include the non-ladder graphs.

Prior Results

- Small-x DLA evolution for the g_1 structure function was first considered by Bartels, Ermolaev and Ryskin (BER) in '96.

- Including the mixing of quark and gluon ladders, they obtained

$$\Delta\Sigma \sim g_1 \sim \left(\frac{1}{x}\right)^{z_s \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

with z_s = 3.45 for 4 quark flavors and z_S=3.66 for pure glue.

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

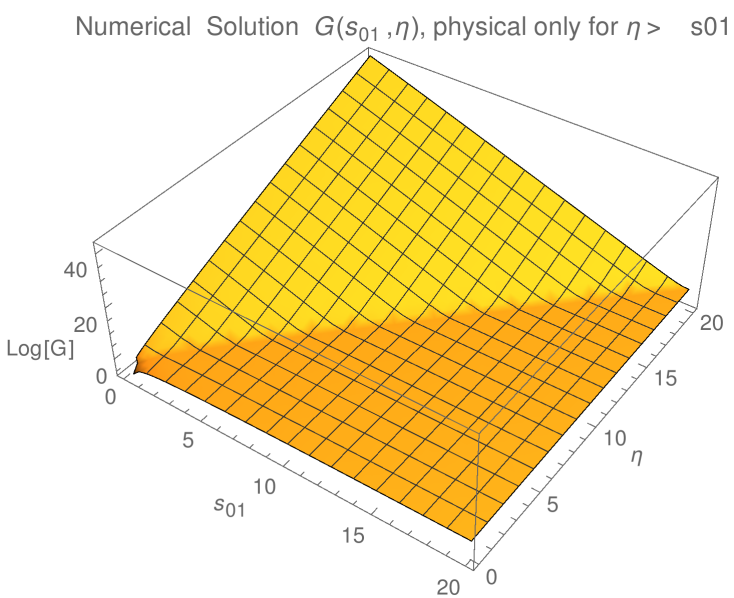
- The power is large: it becomes larger than 1 for the realistic strong coupling of the order of α_s = 0.2 – 0.3, resulting in polarized PDFs which actually grow with decreasing x fast enough for the integral of the PDFs over the low-x region to be (potentially) large.

Solution of the large-N_C Equations

- We found a numerical solution of the large-N_c evolution equations (linearized, without saturation corrections):

$$G_{01}(z) = G_{01}^{(0)}(z) + \frac{\alpha_s\, N_c}{2\pi} \int_{z_i}^{z} \frac{dz'}{z'} \int_{\rho'^2}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2}\, \left[\Gamma_{02,\,21}(z') + 3\, G_{21}(z')\right],$$

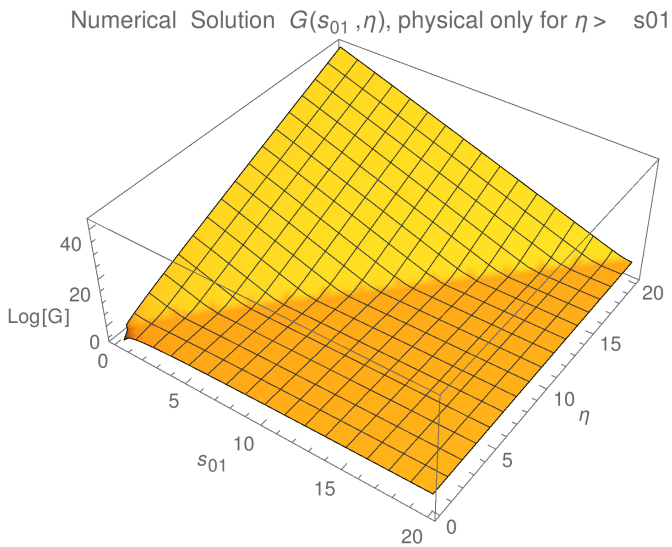
$$\Gamma_{02,\,21}(z') = \Gamma_{02,\,21}^{(0)}(z') + \frac{\alpha_s\, N_c}{2\pi} \int_{z_i}^{z'} \frac{dz''}{z''} \int_{\rho''^2}^{\min\{x_{02}^2, x_{21}^2\, z'/z''\}} \frac{dx_{32}^2}{x_{32}^2}\, \left[\Gamma_{03,\,32}(z'') + 3\, G_{23}(z'')\right]$$



$$\eta = \ln \frac{z\, s}{\Lambda^2}$$

$$s_{01} = \ln \frac{1}{x_{01}^2\, \Lambda^2}$$

Solution of the large-N_C Equations



$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

- The resulting small-x asymptotics is (about 30% smaller than BER's 3.66 all-N_C pure glue):

$$\Delta q(x, Q^2) \sim g_{1L}(x, k_T) \sim g_1^S(x, Q^2) \sim \left(\frac{1}{x}\right)^{2.34 \sqrt{\frac{\alpha_s N_C}{2\pi}}}$$

- For $\alpha_s = 0.3$ we get $x\, g_1^S \sim x^{0.11}$ --- not too much spin at small x.

- For $\alpha_s = 0.4$ we get $x\, g_1^S \sim x^{-0.02}$ --- possibly some spin at small x.

Conclusions

- We have constructed new DLA evolution equations for the polarized dipole operator, which allow us to find the small-x asymptotics of the quark helicity TMDs and PDFs and of the g_1 structure function.

- Like the B-JIMWLK hierarchy, our equations do not close in general. They close in the large-N_C and large-N_C&N_f limits.

- Solution of the evolution equations at large-N_C appears to give an intercept for helicity distributions which is about 30% smaller than the existing all-N_C one in the literature, but is still large enough to potentially generate a solid amount of spin at small-x. (D. Pitonyak, M. Sievert, YK, in preparation)

- Future work may involve including running coupling and saturation corrections + solving the large-N_C&N_f equations. All are likely to lower the intercept.