Do fragmentation functions in factorization theorems correctly treat non-perturbative effects?

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- Formal proofs of factorization are by analysis of leading power behavior in every order of perturbation theory.
- How well do we know factorization from full QCD, beyond PT?
- I'll show a mechanism in hadronization that is not covered by factorization proofs.
- What are the implications?

Factorization and its uses

Basis:

- Factorization at high Q:

$$\sigma = \text{hard sc.} \otimes \text{pdfs and/or ffs} \; + \; \text{power-suppressed}$$

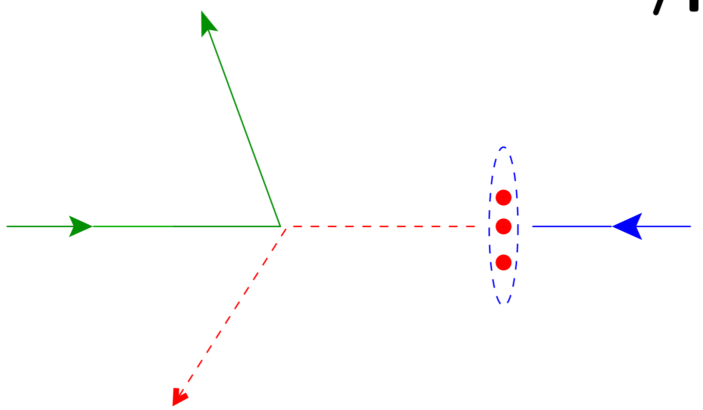
  and generalizations.

- Evolution equations for pdfs, ffs and α_s

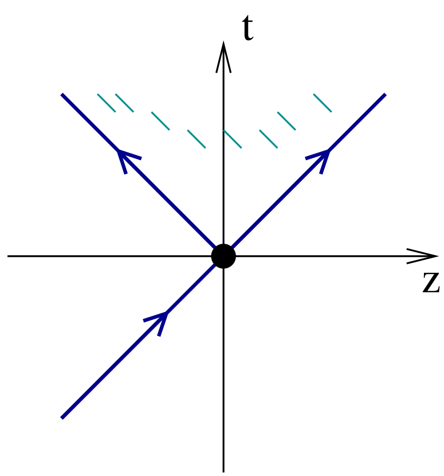
Predictive power:

- pQCD calculation of hard scattering, DGLAP kernels, etc
- Measurement of pdfs, ffs, $\Lambda_{\rm QCD}$ (etc) from a limited set of data.
- Universality of pdfs, ffs, etc gives predictions for many other processes at all (high enough) Q.

Non-perturbative reasoning in coordinate-space motivates factorization/partons in DIS at high Q



Short distance collision of electron and constituent of fast moving hadron

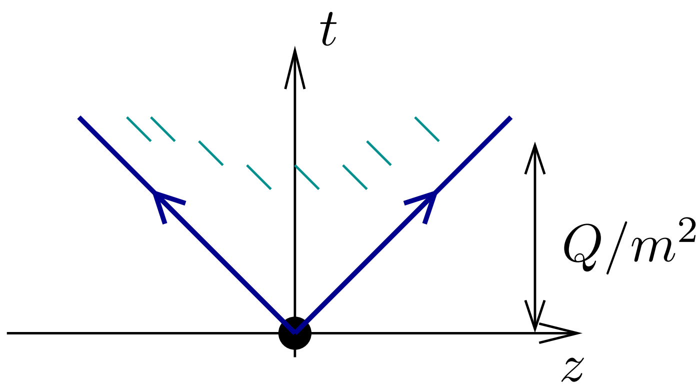


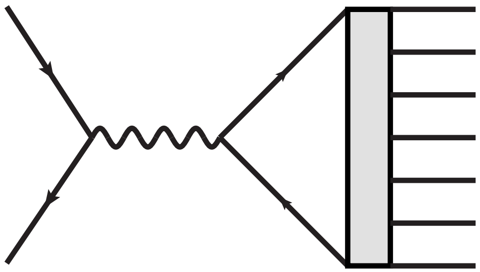
Use of

- Relativity
- Unitarity on final-state interactions

- $\mathrm{d}\sigma =$ Hard sc. $\otimes$ pdf
- (Extend to SIDIS with fragmentation)
- Coordinate space reasoning critical here

Fragmentation $e^+ + e^- \to h_1 + h_2 + X$ at high Q





Space-like separation of “valence hadronization” *and* string-like fragmentation implies independent fragmentation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_1\,\mathrm{d}z_2} = \text{Hard sc.} \otimes \mathsf{ff}_1(z_1) \otimes \mathsf{ff}_2(z_2) \ + \ \text{power suppressed}$$

But rapidity gap filled in, so Feynman graphical structures couple the jets.

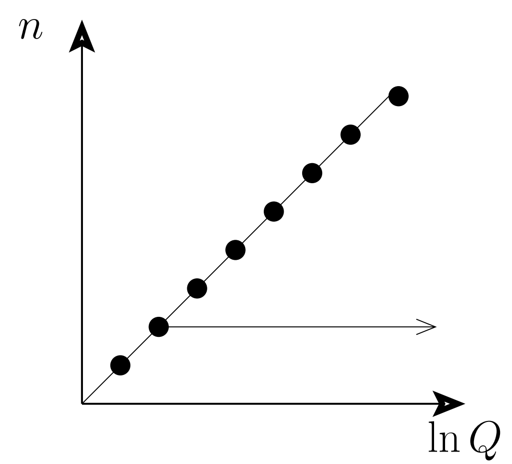
Standard factorization proofs

Given some process with a large scale Q:

- Extract leading (usually) power large Q asymptote of each individual graph for process.
- Organized in factorized form after sum over graphs

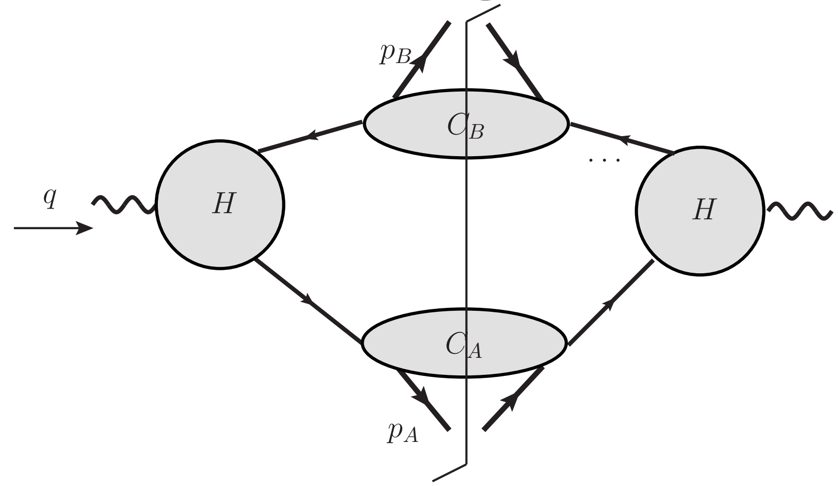
Issues in going beyond order-by-order perturbation theory:

- Perturbation theory is not literally convergent.
- Non-perturbative effects exist in QCD.
- Asymptote and infinite sum might not commute.

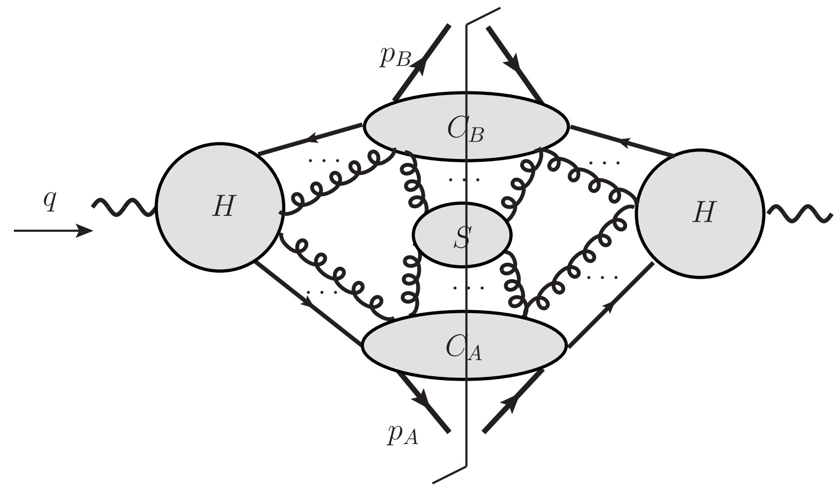


Feynman graphs, momentum regions

Basic “parton model”



But also have leading graphs



Canonical momentum regions in $(+,-,T)$ coordinates are, e.g.,

$$\text{Coll. A}: \left(Q, m^2/Q, m\right), \qquad \text{S}: (m,m,m) \text{ or } \left(m^2/Q, m^2/Q, m^2/Q\right)$$

All intermediate regions also contribute!

Steps in standard factorization proofs

1. Extract leading (usually) power large Q asymptote of each individual graph for process. (Libby-Sterman analysis)

2. Apply approximations. Aims:
   - Suitable for separation into factors
   - See item 4

3. Subtractions

4. *Ward identities. (Critical to disentangle gluons connecting subgraphs for different regions.)*

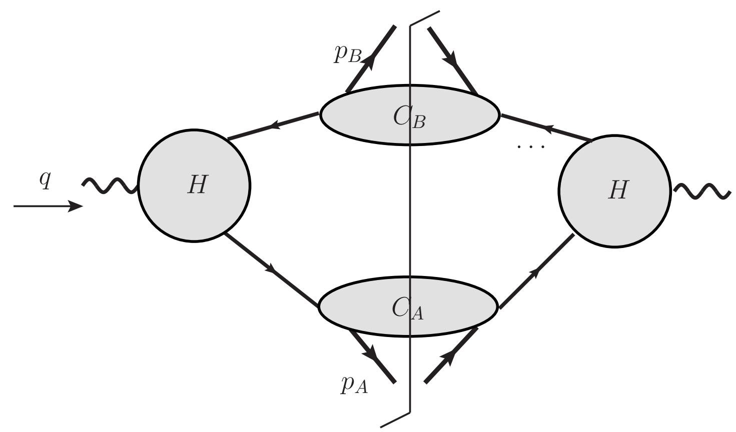
5. Final-state unitarity cancellations, etc.

6. Deduce factorized form after sum over graphs.

7. Similarly derive evolution equations (RG, DGLAP, CSS, etc).

Non-perturbative-compatible structures in factorization proof

- Overall leading-power analysis à la Libby-Sterman gives result that matches (extended) parton-model view, in coordinate space, etc.
- Approximations can be applied block-by-block rather than just individual-subgraph-by-individual-subgraph.

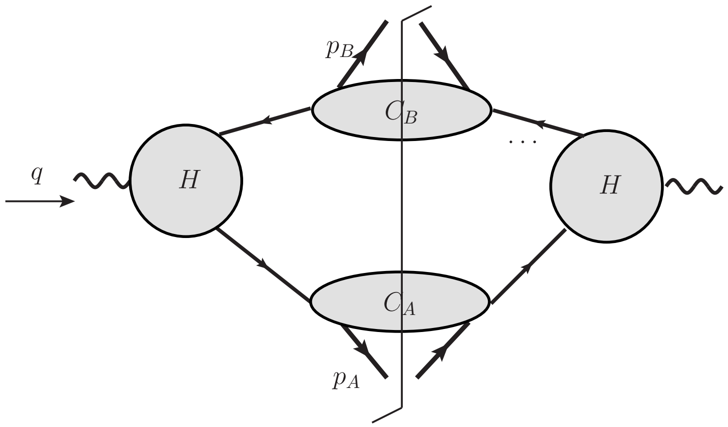


- Cancellation of spectator-spectator interactions in Drell-Yan was originally fully non-perturbative (De Tar, Ellis, Landshoff), with parton-model assumptions pre-QCD

  CSS updated it and included proper QCD interface. (. . .)

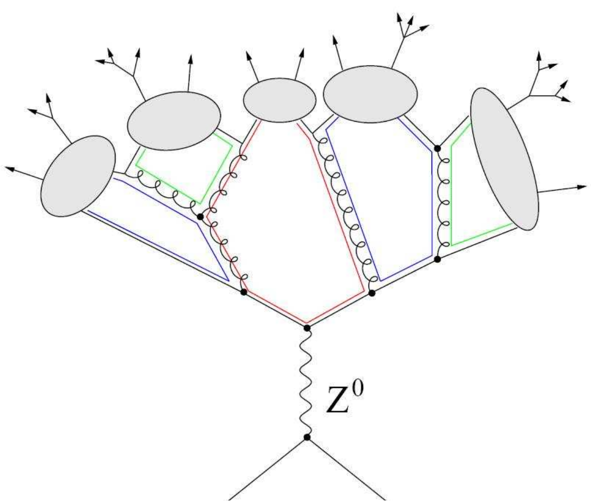
Filled-in rapidity gap

Parton-model-like graphs have large rapidity gap and fractionally charged particles:



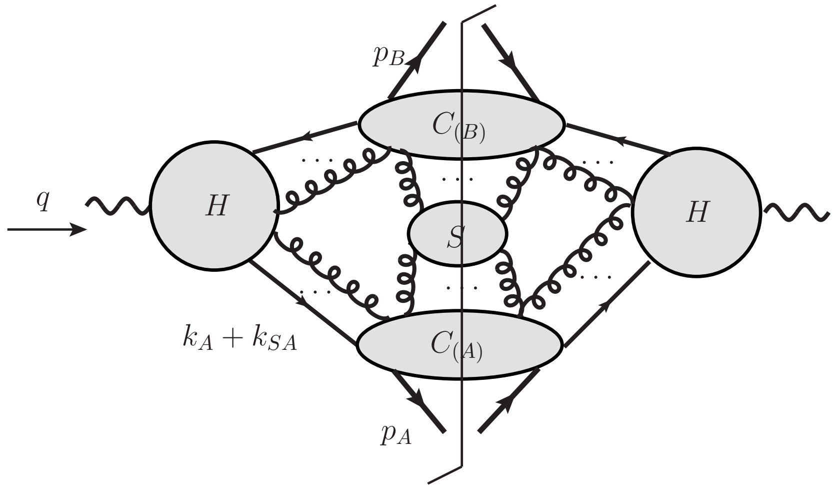
How does string-like (including cluster hadronization) match perturbative analysis?

Minimal Feynman graph model for string-like hadronization is cluster hadronization:

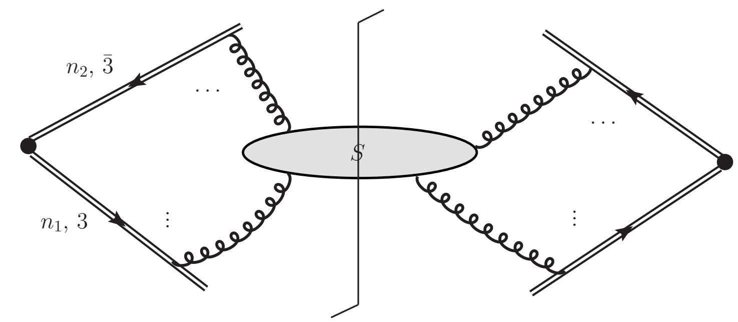


with order of graph $\propto \ln(Q^2/m^2)$

But usual approximations need large rapidity differences



$\Longrightarrow$



E.g., for canonical soft gluon k attaching to collinear-A subgraph

$$k = (m, m, m) \text{ or } \left(m^2/Q, m^2/Q, m^2/Q\right) \qquad \text{Coll. A : } \left(Q, m^2/Q, m\right),$$

we use

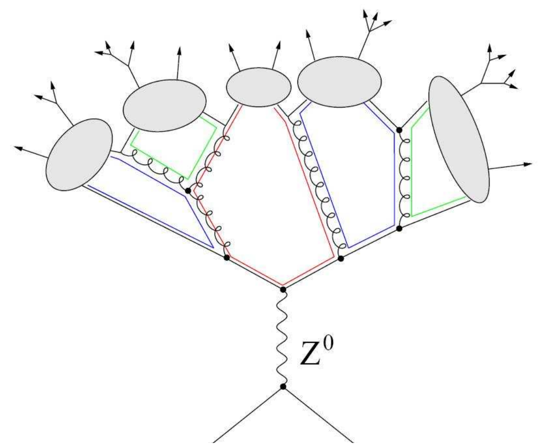
$$A(k, \dots)^\mu S(k, \dots)_\mu \simeq A(\hat{k}, \dots)^+ S(k, \dots)^- = A(\hat{k}, \dots) \cdot \hat{k} \frac{1}{k^-} S(k, \dots)^-$$

where $\hat{k} = (0, k^-, \mathbf{0}_\mathsf{T})$

Ward identities on $A(\hat{k}, \dots) \cdot \hat{k}$ etc convert soft factor to Wilson line matrix element.

Etc (soft-to-B, collinear-to-H)

Soft-to-collinear approximation fails in string-compatible graphs



Define

- Δy to be total available rapidity range ($\simeq \ln(Q^2/m^2)$);
- δy to be typical cluster separation, i.e., $\Delta y/\#$clusters.
- Experimentally δy is typically small.

Then

- Errors are now a power of $e^{-\delta y}$, not m/Q
- Order of relevant graphs increases with Q.

Results

Overall issues:

- How well do we know that factorization (and generalizations) is valid?
- What new phenomena could gaps in derivations uncover?

Here we have:

- String-type hadronization doesn't match perturbative derivation, even in perturbative model.
- Problem applies everywhere with final-state detection.

But factorization is not (yet) falsified

- The opposite ends of a Lund string are at space-like separation, so can still expect independent hadronization (except for QM entanglement).
- That already leads to some kind of fragmentation function, but without an explicit formal definition.
- There may be modified physics relative to order-by-order perturbatively derived factorization.

Future

- Sort out whether these issues actually do impact factorization formulae with fragmentation functions. (Collinear and TMD)
- If they do, find an improved formulation, perhaps involving a proper systematic interface between standard pQCD constructs and string-like constructs.
- Find a better way of analyzing graphs and amplitudes in coordinate space.
- What other implications are there?
- What other problems?