

Transverse Momentum Dependent Parton Distributions in Charmonium and Bottomonium Production

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Plan of the talk

- ◆ Gluon TMDs
- ◆ Linearly polarized gluons
- ◆ Charmonium and bottomonium production models
- ◆ Probing gluon TMDs in $pp \rightarrow J/\psi X$ and $pp \rightarrow \Upsilon X$
- ◆ Numerical Results
- ◆ Conclusion

Gluon TMDs

- ◆ Gluon TMDs play an important role in spin physics
- ◆ SIDIS data at lower energy and DY at large energy : need more data for full understanding as well as the global fit, over a wide kinematical range
- ◆ In the range $\Lambda_{QCD} \ll k_\perp \ll Q$ where Q is the large scale of the process, radiative gluon emission play an important role and they need to be resummed : evolution of TMDs
- ◆ For low k_\perp non-perturbative physics dominates

Gluon TMDs

- ◆ It is important to first fully understand the unpolarized TMDs
- ◆ Also because they sit in the denominator of the spin asymmetries
- ◆ Gluon TMDs studies in ep (eRHIC...) and pp (RHIC, AFTER, LHCb..) collisions : interesting and complimentary information : process dependence, universality..

Linearly Polarized Gluons

There are linearly polarized gluons inside an unpolarized hadron, provided they have non-zero transverse momenta.

Mulders and Rodrigues, PRD 63, 094021 (2001)

Correspond to the interference of + and – helicity states

$h_1^{\perp g}$ describes the distribution of linearly polarized gluons inside an unpolarized proton

Time reversal even, unlike the quark Boer-Mulders function;
also chiral even: allowed by the symmetries of QCD

No experimental investigation for its extraction so far

Linearly Polarized Gluons : Theoretical Proposals to Investigate

$$pp \rightarrow \gamma\gamma X$$

Qiu, Schlegel, Vogelsang, PRD (2011)

Heavy quark and dijet production
in SIDIS and hadronic collisions

Pisano, Boer, Brodsky, Buffing, Mulders,
JHEP (2013); Boer, Mulders, Pisano, PRD
(2009), Boer, Mulders, Pisano (2016)

Higgs transverse momentum
distribution

Sun, Xiao, Yuan, PRD(2011); Boer,
Dunnen, Pisano, Schlegel, Vogelsang
PRL(2013); Boer, Dunnen, Pisano,
Schlegel, PRL (2013)

C=+ heavy quarkonium
production in pp collision

Boer, Pisano, PRD (2012)

Charmonium Production Models

We propose to study the unpolarized TMDs and linearly polarized gluon TMD in J/ψ and Υ production in $p\bar{p}$ collision

Three models for charmonium production in the market. In all, cross section is factorized into a perturbative and a non-perturbative part

1. Color singlet model : Heavy quark pair produced from a color singlet state non-perturbatively; spin and color do not alter in hadronization, so the heavy quark pair must be produced in a color singlet state
2. NRQCD : Cross section is a product of short and long distance factors summed over all color, spin and angular momentum factors. Long distance factor, the transition probability from the heavy quark pair from colored state to colorless physical state is expressed in powers of the relative velocity of the heavy quark

3. Color Evaporation Model

First introduced by Halzen, PLB (1977), Halzen, Matsuda, PRD (1978); Fritsch, PLB (1978)

Heavy quark pair is first produced perturbatively with definite spin and color quantum numbers, then they hadronize by radiating soft gluons

Color of the initial heavy quark pair does not affect the color of the quarkonium produced : “color evaporation”

Cross section is a long distance factor times the perturbative cross section with the invariant mass below the threshold ; long distance factors are obtained by fitting data

Basic idea : Probability of forming a quarkonium state is independent of the color and spin quantum numbers of the heavy quark pair

Cross section in CEM

$pp \rightarrow J/\psi X$

$$\sigma = \frac{\rho}{9} \int_{2m_Q}^{2m_{Q\bar{q}}} dM \frac{d\sigma_{Q\bar{Q}}}{dM}$$

$pp \rightarrow \Upsilon X$

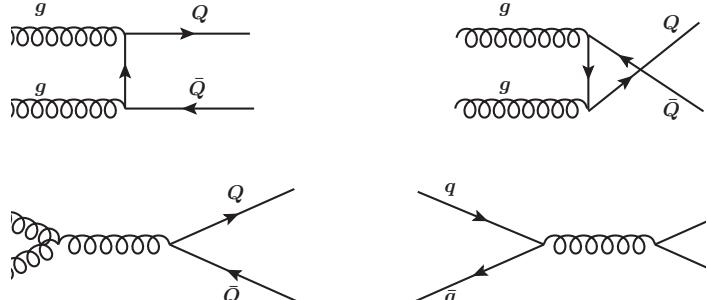
m_Q is the mass of charm/bottom quark and $m_{Q\bar{q}}$ is the mass of the lightest D or B meson , M is the invariant mass of the heavy quark pair

ρ is the long distance factor determined by fitting the cross section to experimental data and 1/9 is the probability to form a color singlet

Amundson et al PLB (1996)

ρ for J/ψ : 0.055, for Υ : 0.087

Cross section at LO $pp \rightarrow Q\bar{Q}X$



$$d\sigma = \frac{\rho}{9} \int dx_a dx_b d^2 \mathbf{k}_{\perp a} d^2 \mathbf{k}_{\perp b} \left\{ \Phi_g^{\mu\nu}(x_a, \mathbf{k}_{\perp a}) \Phi_{g\mu\nu}(x_b, \mathbf{k}_{\perp b}) d\hat{\sigma}^{gg \rightarrow Q\bar{Q}} \right. \\ + \left[\Phi^q(x_a, \mathbf{k}_{\perp a}^2) \Phi^{\bar{q}}(x_b, \mathbf{k}_{\perp b}^2) \right. \\ \left. \left. + \Phi^{\bar{q}}(x_a, \mathbf{k}_{\perp a}^2) \Phi^q(x_b, \mathbf{k}_{\perp b}^2) \right] d\hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}} \right\},$$

Gluon correlators are parametrized as, at leading twist

$$\Phi_g^{\mu\nu}(x, \mathbf{k}_{\perp}) = \frac{n_{\rho} n_{\sigma}}{(k \cdot n)^2} \int \frac{d(\lambda \cdot P) d^2 \lambda_T}{(2\pi)^3} e^{ik \cdot \lambda} \langle P | \text{Tr}[F^{\mu\rho}(0) F^{\nu\sigma}(\lambda)] | P \rangle|_{LF} \\ = -\frac{1}{2x} \left\{ g_T^{\mu\nu} f_1^g(x, \mathbf{k}_{\perp}^2) - \left(\frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{M_h^2} + g_T^{\mu\nu} \frac{\mathbf{k}_{\perp}^2}{2M_h^2} \right) h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) \right\}.$$

Note : there is also process dependent gauge link in quark and gluon correlators

Cross section calculation

$$\frac{d^4\sigma}{dydM^2d^2\mathbf{q}_T} = \frac{\rho}{18} \int dx_a dx_b d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \delta^4(p_a + p_b - q) \left\{ \frac{1}{2x_a x_b} \right. \\ \times \left[f_1^g(x_a, \mathbf{k}_{\perp a}^2) f_1^g(x_b, \mathbf{k}_{\perp b}^2) + w h_1^{\perp g}(x_a, \mathbf{k}_{\perp a}^2) h_1^{\perp g}(x_b, \mathbf{k}_{\perp b}^2) \right] \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \\ \left. + \frac{1}{4} \sum_q \left[f_1^q(x_a, \mathbf{k}_{\perp a}^2) f_1^{\bar{q}}(x_b, \mathbf{k}_{\perp b}^2) + f_1^{\bar{q}}(x_a, \mathbf{k}_{\perp a}^2) f_1^q(x_b, \mathbf{k}_{\perp b}^2) \right] \hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}}(M^2) \right\},$$

w is the weight factor

$$w = \frac{1}{2M_h^4} \left[(\mathbf{k}_{\perp a} \cdot \mathbf{k}_{\perp b})^2 - \frac{1}{2} \mathbf{k}_{\perp a}^2 \mathbf{k}_{\perp b}^2 \right].$$

* Gluon channel is dominant

Cross section (final form)

$$\hat{\sigma}^{gg \rightarrow Q\bar{Q}}, \hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}}$$



Can be calculated perturbatively

$$x_{a,b} = \frac{M}{\sqrt{s}} e^{\pm y}.$$

$$\begin{aligned} \frac{d^2\sigma^{ff}}{dy d^2\mathbf{q}_T} &= \frac{\rho}{18s} \int dM^2 \int d^2\mathbf{k}_{\perp a} \left\{ f_1^g(x_a, \mathbf{k}_{\perp a}^2) \times f_1^g(x_b, (\mathbf{q}_T - \mathbf{k}_{\perp a})^2) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \right. \\ &\quad \left. + \frac{1}{2} \sum_q \left[f_1^q(x_a, \mathbf{k}_{\perp a}^2) f_1^{\bar{q}}(x_b, (\mathbf{q}_T - \mathbf{k}_{\perp a})^2) + f_1^{\bar{q}}(x_a, \mathbf{k}_{\perp a}^2) f_1^q(x_b, (\mathbf{q}_T - \mathbf{k}_{\perp a})^2) \right] \hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}}(M^2) \right\}, \end{aligned}$$

$$\begin{aligned} \frac{d^2\sigma^{hh}}{dy d^2\mathbf{q}_T} &= \frac{\rho}{18s} \frac{1}{2M_h^4} \int dM^2 \int d^2\mathbf{k}_{\perp a} \left[(\mathbf{k}_{\perp a} \cdot (\mathbf{q}_T - \mathbf{k}_{\perp a}))^2 - \frac{1}{2} \mathbf{k}_{\perp a}^2 (\mathbf{q}_T - \mathbf{k}_{\perp a})^2 \right] \\ &\quad \times h_1^{\perp g}(x_a, \mathbf{k}_{\perp a}^2) h_1^{\perp g}(x_b, (\mathbf{q}_T - \mathbf{k}_{\perp a})^2) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2). \end{aligned}$$

Diff cross section : sum of the two

TMD models

$$f_1^{g, q}(x, \mathbf{k}_\perp^2) = f_1^{g, q}(x, Q^2) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\mathbf{k}_\perp^2 / \langle k_\perp^2 \rangle}.$$

Anselmino et al , PRD 72, 094007
(2005)

$$h_1^{\perp g}(x, \mathbf{k}_\perp^2) = \frac{M_h^2 f_1^g(x, Q^2)}{\pi \langle k_\perp^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \mathbf{k}_\perp^2 \frac{1}{r \langle k_\perp^2 \rangle}}$$

Boer and Pisano, PRD 86, 094007
(2012)

$f_1^{g, q}(x, Q^2)$ is the unpolarized pdf. We have chosen $Q^2 = M^2$

r is a parameter $0 < r < 1$

$$\frac{\mathbf{k}_\perp^2}{2M_h^2} |h_1^{\perp g}(x, \mathbf{k}_\perp^2)| \leq f_1^g(x, \mathbf{k}_\perp^2)$$

We took $r=2/3$ and $1/3$;

$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$ and 1 GeV^2

Model I: no upper limit on transverse momentum integration

Model II : $k_{\max} = \sqrt{\langle k_{\perp a}^2 \rangle}$

Anselmino et al, EPJA (2009)

TMD Evolution

Evolution is done in b space

$$\Phi(x, \mathbf{b}_\perp) = \int d^2 \mathbf{k}_\perp e^{-i \mathbf{k}_\perp \cdot \mathbf{b}_\perp} \Phi(x, \mathbf{k}_\perp)$$

$$\Phi^g(x, \mathbf{b}_\perp) = \frac{1}{2x} \left\{ g_T^{\mu\nu} f_1^g(x, \mathbf{b}_\perp^2) - \left(\frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2} - g_T^{\mu\nu} \right) h_1^{\perp g}(x, \mathbf{b}_\perp^2) \right\}$$

Cross section in b space becomes

$$\frac{d^4\sigma}{dy dM^2 d^2 \mathbf{q}_T} = \frac{\rho}{9s} \frac{1}{(2\pi)^2} \int dx_a dx_b d^2 \mathbf{b}_\perp e^{i \mathbf{q}_T \cdot \mathbf{b}_\perp} \Phi_g^{\mu\nu}(x_a, \mathbf{b}_\perp) \Phi_{g\mu\nu}(x_b, \mathbf{b}_\perp) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}$$

$$\delta \left(x_a - \frac{Me^y}{\sqrt{s}} \right) \delta \left(x_a - \frac{Me^{-y}}{\sqrt{s}} \right)$$

$$\frac{d^4\sigma}{dy dM^2 d^2 \mathbf{q}_T} = \frac{\rho}{18s} \frac{1}{2\pi} \int_0^\infty b_\perp db_\perp J_0(q_T b_\perp) \left\{ f_1^g(x_a, b_\perp^2) f_1^g(x_b, b_\perp^2) \right.$$

$$\left. + h_1^{\perp g}(x_a, b_\perp^2) h_1^{\perp g}(x_b, b_\perp^2) \right\} \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2)$$


TMD Evolution

Use the formalism of

Boer and den Dunnen, Nucl. Phys. B 886, 421 (2014)

Evolution is performed in b space. TMD pdfs also depend on ζ which is a parameter to regulate the light cone divergences. Using Collins-Soper and RG equations one can write

$$f(x, b_\perp, Q_f, \zeta) = f(x, b_\perp, Q_i, \zeta) R_{pert}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b_\perp)$$

$$Q_i = c/b_*(b_\perp), Q_f = Q \quad c = 2e^{-\gamma_\epsilon} \quad b_*(b_\perp) = \frac{b_\perp}{\sqrt{1 + \left(\frac{b_\perp}{b_{\max}}\right)^2}}$$

b^* prescription to separate the non-pert part of the evolution kernel

b^* always smaller than b_{\max}

Aybat and Rogers, PRD83, 114042 (2011)

TMD Evolution

$$R_{pert}(Q_f, Q_i, b_*) = \exp \left\{ - \int_{c/b_*}^Q \frac{d\mu}{\mu} \left(A \log \left(\frac{Q^2}{\mu^2} \right) + B \right) \right\}$$

\blacktriangleright \blacktriangleleft

$$A = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi} \right)^n A_n; B = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi} \right)^n B_n$$

Anomalous dimensions

$$R_{NP} = \exp \left\{ - \left[\frac{g_2}{2} \log \frac{Q}{2Q_0} + \frac{g_1}{2} \left(1 + 2g_3 \log \frac{10xx_0}{x_0 + x} \right) \right] b_\perp^2 \right\}$$

Aybat, Rogers, PRD (2011)

Best fit parameters

Describes SIDIS as well as DY and Z data

$g_1 = 0.201 \text{ GeV}^2, g_2 = 0.184 \text{ GeV}^2, g_3 = -0.129 \text{ GeV}^2, Q_0 = 1.6 \text{ GeV}, b_{\max} = 1.5 \text{ GeV}^{-1}, x_0 = 0.009$

$x = 0.09$

Boer, den Dunnen, Nucl. Phys. B886, 421 (2014)

Non-pert part of the evolution kernel is usually extracted by fitting data

TMD Evolution

Perturbative Sudakov factor in our case is the same for unpolarized and linearly polarized TMDPdfs

We take the same non-perturbative Sudakov factor although the Q independent part may depend on spin: difference does not affect at large Q

For small b, one can write

$$f_{g/p}(x, b_\perp, Q_i, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b_\perp, \alpha_s, \mu, \zeta) f_{i/p}(\hat{x}, c/b_*) + \mathcal{O}(b_\perp \Lambda_{QCD})$$

Coefficient function is calculated perturbatively for each pdf. One gets

$$f_1^g(x, b_\perp, Q_i, \zeta) = f_{g/p}(x, c/b_*) + \mathcal{O}(\alpha_s),$$

$$h_1^{\perp g}(x, b_\perp, Q_i, \zeta) = \frac{\alpha_s(c/b_*) C_A}{\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/p}(\hat{x}, c/b_*) + \mathcal{O}(\alpha_s^2)$$

Final Expression using TMD Evolution

$$\frac{d^2\sigma^{ff}}{dydq_T^2} = \frac{\rho}{36s} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int_0^\infty b_\perp db_\perp J_0(q_T b_\perp) f_1^g(x_a, c/b_*) f_1^g(x_b, c/b_*) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2)$$

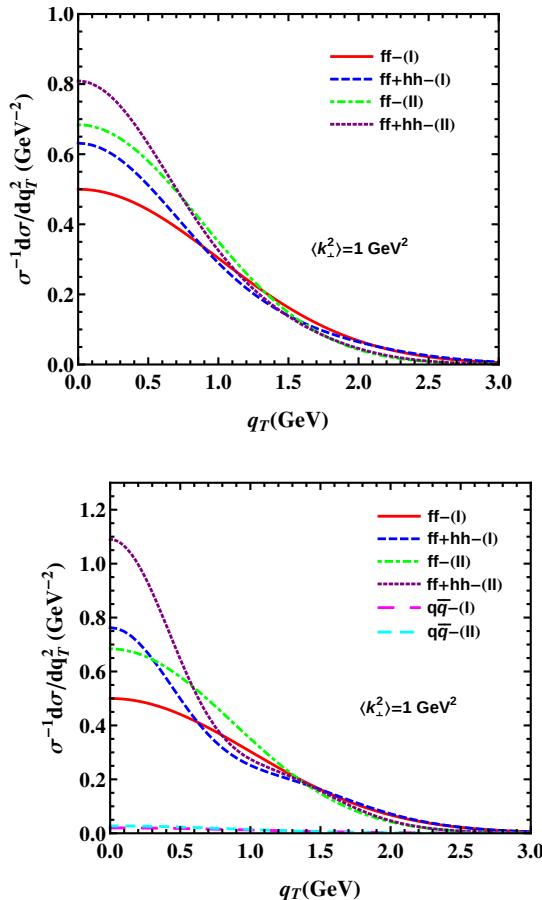
$$\exp \left\{ -2 \int_{c/b_*}^Q \frac{d\mu}{\mu} \left(\frac{\alpha_s(\mu)}{\pi} A_1 \log \left(\frac{Q^2}{\mu^2} \right) + \frac{\alpha_s(\mu)}{\pi} B_1 \right) \right\} \exp \left\{ - \left[0.184 \log \frac{Q}{2Q_0} + 0.332 \right] b_\perp^2 \right\}$$

$$\frac{d^2\sigma^{hh}}{dydq_T^2} = \frac{\rho C_A^2}{36s\pi^2} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int_0^\infty b_\perp db_\perp J_0(q_T b_\perp) \alpha_s^2(c/b_*) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2)$$

$$\int_{x_a}^1 \frac{dx_1}{x_1} \left(\frac{x_1}{x_a} - 1 \right) f_1^g(x_1, c/b_*) \int_{x_b}^1 \frac{dx_2}{x_2} \left(\frac{x_2}{x_b} - 1 \right) f_1^g(x_2, c/b_*)$$

$$\exp \left\{ -2 \int_{c/b_*}^Q \frac{d\mu}{\mu} \left(\frac{\alpha_s(\mu)}{\pi} A_1 \log \left(\frac{Q^2}{\mu^2} \right) + \frac{\alpha_s(\mu)}{\pi} B_1 \right) \right\} \exp \left\{ - \left[0.184 \log \frac{Q}{2Q_0} + 0.332 \right] b_\perp^2 \right\}$$

Numerical Results



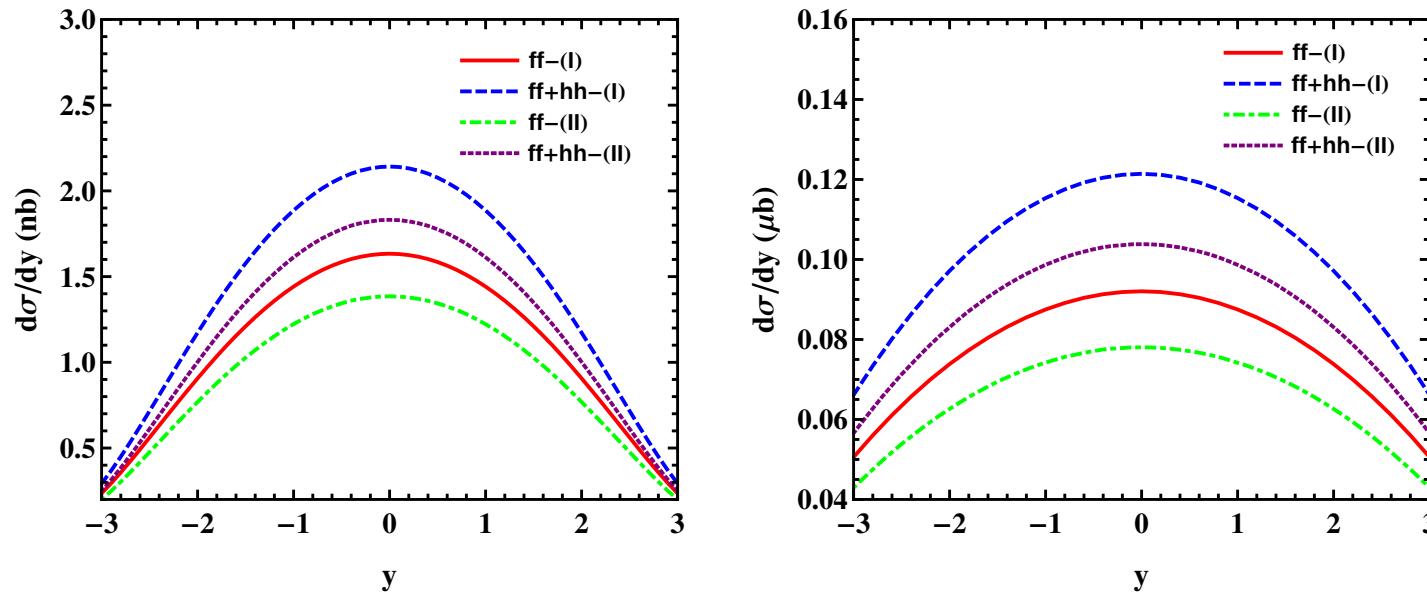
Normalized diff cross section at LHCb ($\sqrt{s} = 7$ TeV), RHIC ($\sqrt{s} = 500$ GeV), AFTER ($\sqrt{s} = 115$ GeV)

MSTW2008 pdf, without TMD evolution, used only DGLAP evolution

LHCb $2 < y < 4.5$
 RHIC $-3 < y < 3$
 AFTER $0.5 < y < 0.5$

Above $r = 2/3$, below $r = 1/3$

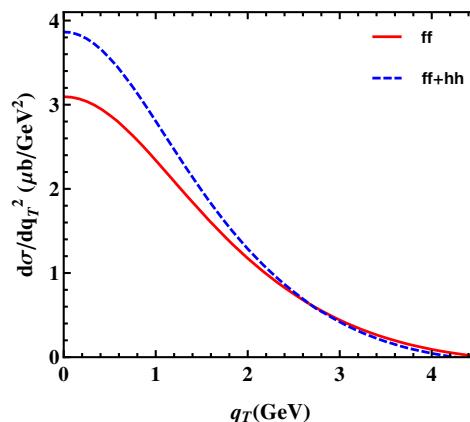
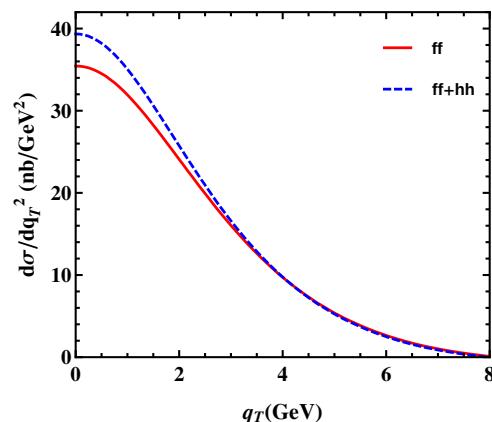
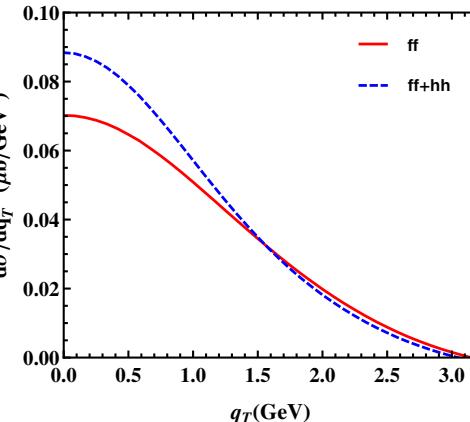
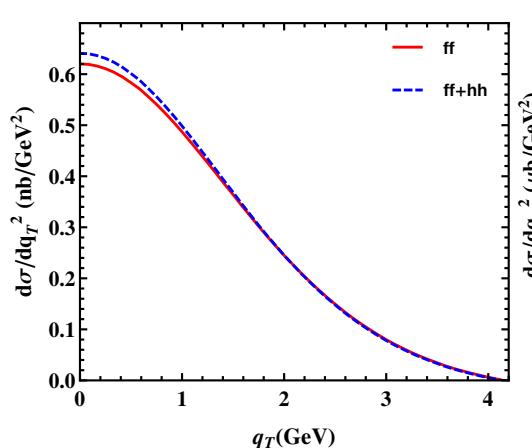
Numerical Results



Rapidity distribution at RHIC ($\sqrt{s} = 500$ GeV). LHS for J/ψ and RHS for Υ . Range of q_T integration $0 < q_T < 0.5$. DGLAP approach, $r = 1/3$, $\langle k_\perp^2 \rangle = 1$

AM and S Rajesh, PRD93, 054018 (2016)

Results with TMD Evolution



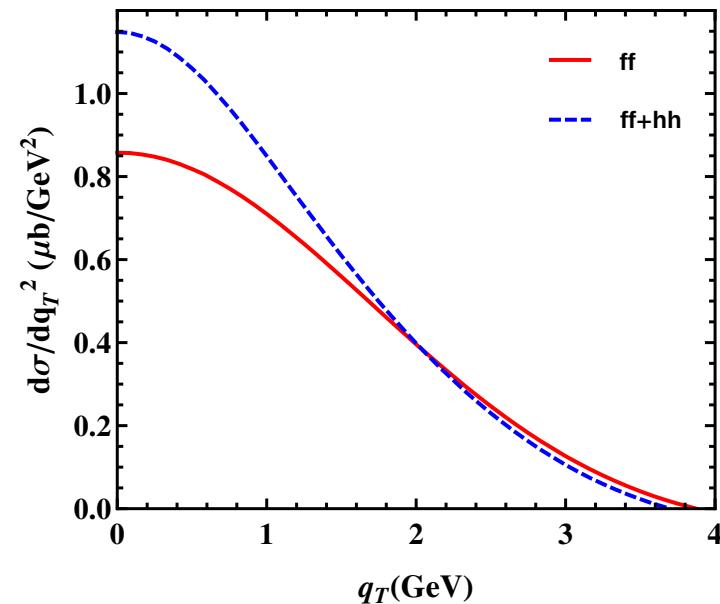
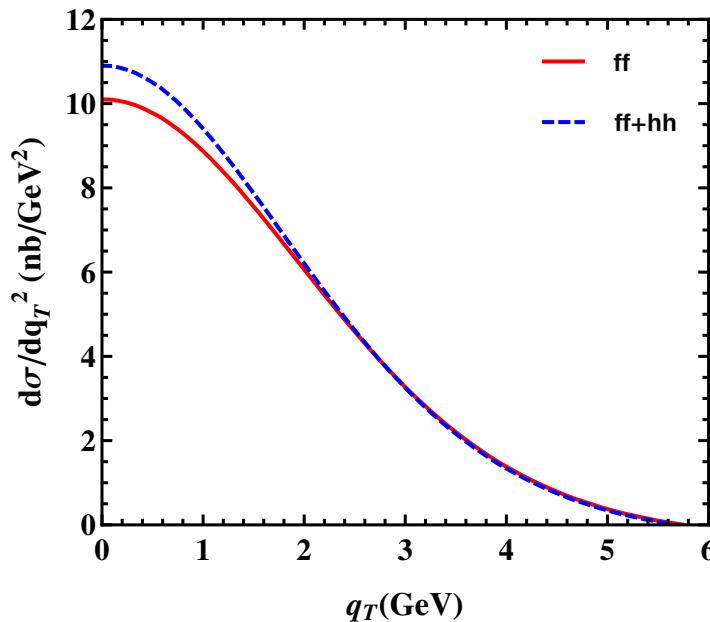
Results for AFTER (LHS Υ and RHS J/ψ)

Range of rapidity integration same as given earlier

Results for LHCb (LHS Υ and RHS J/ψ)

AM and S Rajesh,
PRD93, 054018
(2016)

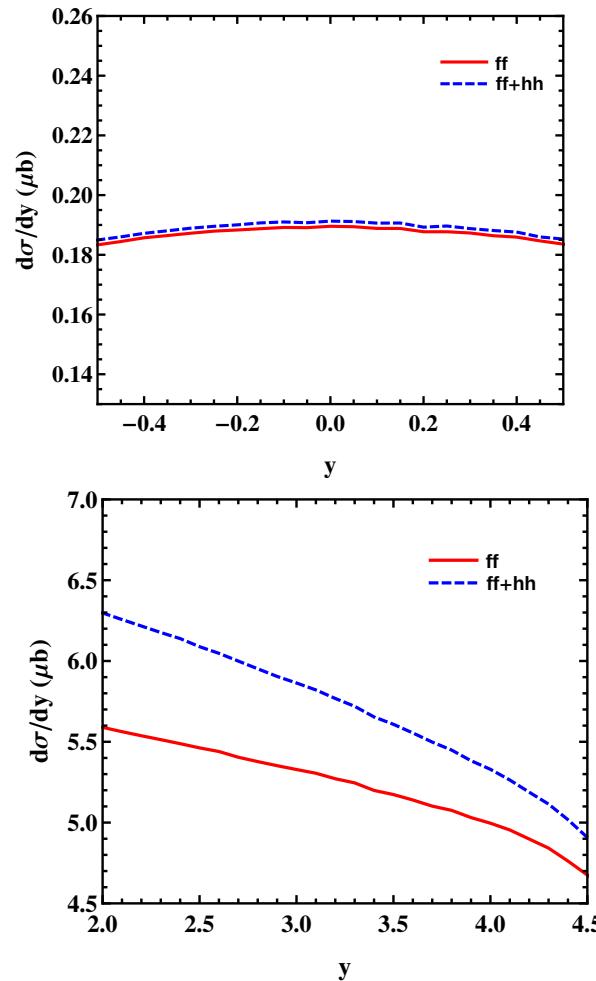
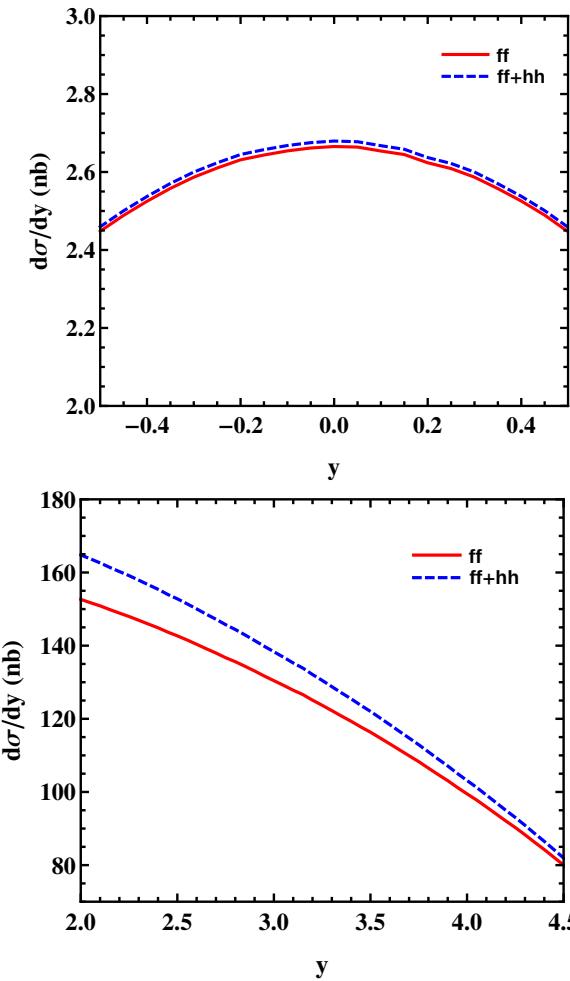
Results with TMD Evolution



Diff cross section for Υ (LHS) and J/ψ (RHS) production at RHIC

AM and S Rajesh, PRD93, 054018 (2016)

Results with TMD Evolution

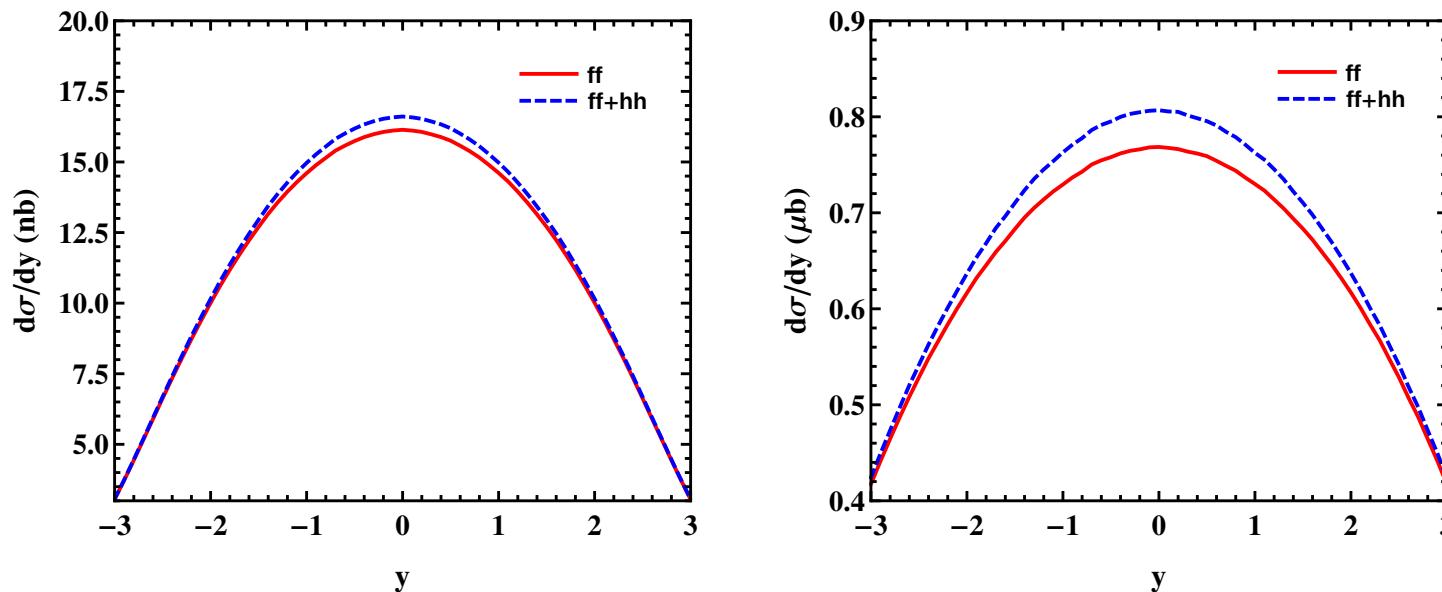


LHS Υ and RHS J/ψ at
AFTER($\sqrt{s} = 115 \text{ GeV}$)
 $0 < q_T < 4 \text{ GeV}$

LHS Υ and RHS J/ψ at
LHCb($\sqrt{s} = 7 \text{ TeV}$)
 $0 < q_T < 4 \text{ GeV}$

AM and S Rajesh,
PRD93, 054018
(2016)

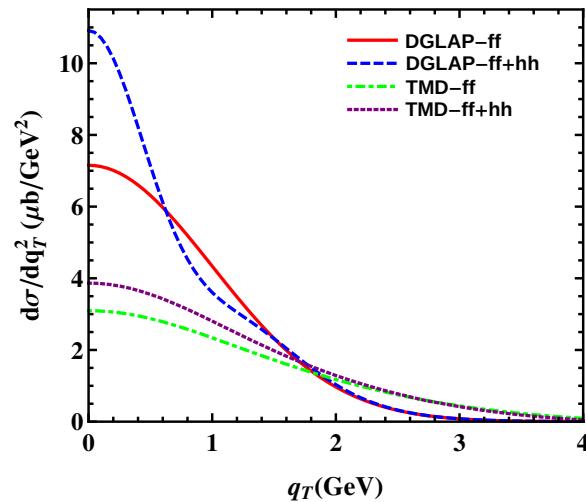
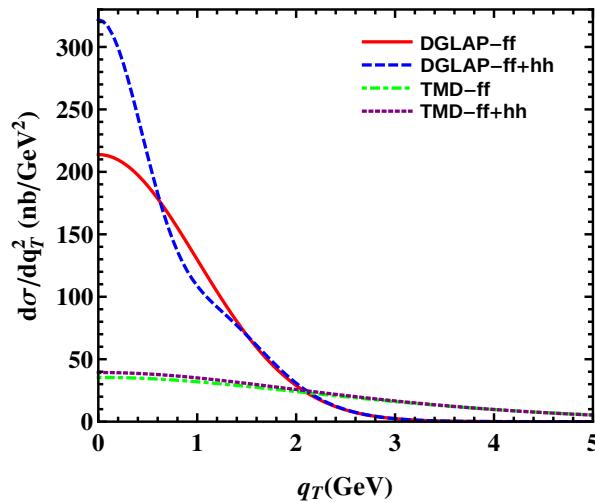
Results with TMD Evolution



Rapidity distribution at RHIC ($\sqrt{s} = 500 \text{ GeV}$). LHS γ and RHS J/ψ production. $0 < q_T < 4 \text{ GeV}$.

AM and S Rajesh, PRD93, 054018 (2016)

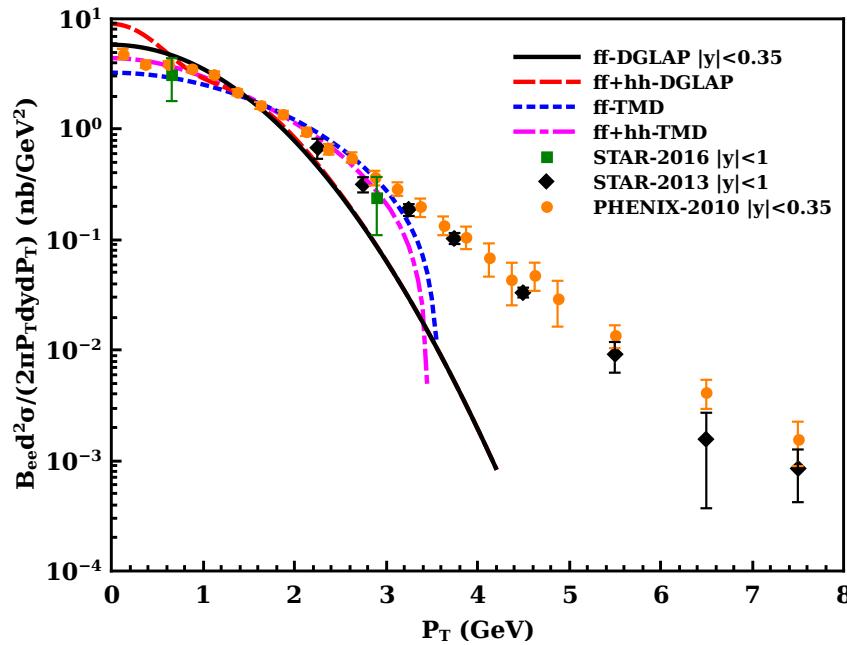
Comparison



Differential cross section for Υ (LHS) and J/ψ (RHS) production at LHCb

$r=1/3$, $\langle k_\perp^2 \rangle = 1 \text{ GeV}^2$ in Model I;
range of rapidity integration $2.0 < y < 4.5$

Comparison with Experimental data



STAR Collaboration, 1602.02212
[nucl-ex]

PHENIX Collaboration Phys. Rev.
D85, 092004 (2012)

STAR Collaboration, Phys. Lett
B722, 55 (2013)

$\sqrt{s} = 200$ GeV, B_{ee} : Branching ratio for dielectron channel decay

Agreement at higher p_T better if one uses improved version of CEM with higher order corrections

Conclusion

- ◆ Charmonium and bottomonium production in pp collision can be used as an important probe for studying the gluon TMDs; universality of T even gluon TMDs in ep and pp collisions
- ◆ Linearly polarized gluon TMDs give substantial modification at low p_T
- ◆ CEM at LO with TMD evolution, good agreement with RHIC data at low p_T , higher order corrections need to be incorporated to compare at large p_T ,
- ◆ Important to study the dependence of the result on charmonium and bottomonium production models
- ◆ Would be interesting to see how the different TMD evolution schemes affect the result