Probing Gluon TMDs at a future EIC

Cristian Pisano

With D. Boer, P.J. Mulders, J. Zhou

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3DSPIN MAPPING THE PROTON IN 3D

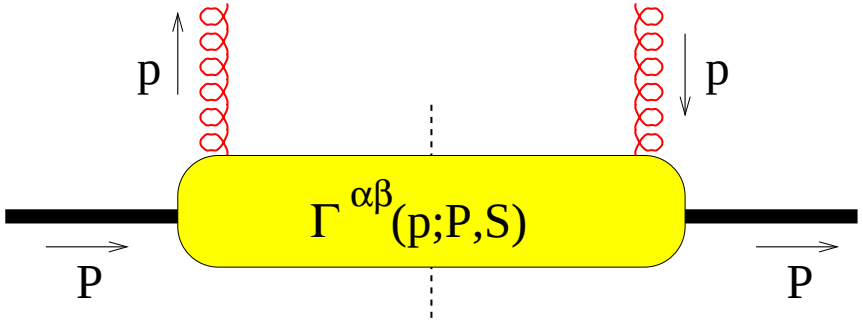
UNIVERSITÀ DI PAVIA

Outline

- Definition of gluon TMDs inside the proton
- Azimuthal asymmetries in heavy quark pair and dijet production in DIS
  - Close analogy with study of quark TMDs in SIDIS
  - Maximal allowed asymmetries
  - Small-x behavior
- Related observables at LHC/RHIC: study of process dependence of TMDs
- Sign change test of gluon Sivers function and other T-odd TMDs

Boer, Mulders, CP, Zhou, ArXiv:1605.07934

The gluon correlator describes the hadron → gluon transition



Gluon momentum $p^\alpha = x\,P^\alpha + p_T^\alpha + p^- n^\alpha$, with $n^2{=}0$ and $n{\cdot}P \neq 0$

Definition of $\Gamma^{\alpha\beta}$ for a spin-1/2 hadron

$$\Gamma^{\alpha\beta} = \frac{n_\rho\, n_\sigma}{(P{\cdot}n)^2} \int \frac{\mathrm{d}(\xi{\cdot}P)\,\mathrm{d}^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\, \langle P,S|\,\mathrm{Tr}\big[\,F^{\alpha\rho}(0)\,U_{[0,\xi]}\,F^{\beta\sigma}(\xi)\,U'_{[\xi,0]}\,\big]\,|P,S\rangle\,\big|_{\xi\cdot n=0}$$

Mulders, Rodrigues, PRD 63 (2001) 094021

U, U': process dependent gauge links

transverse projectors: $g_T^{\alpha\beta} \equiv g^{\alpha\beta} - P^\alpha n^\beta - n^\alpha P^\beta$, $\epsilon_T^{\alpha\beta} \equiv \epsilon^{\alpha\beta\gamma\delta} P_\gamma n_\delta$

Spin vector: $S^\alpha = S_L\left(P^\alpha - M_h^2\, n^\alpha\right) + S_T$, with $S_L^2 + \boldsymbol{S}_T^2 = 1$

Parametrization of $\Gamma^{\alpha\beta}$ (at "Leading Twist" and omitting gauge links)

$$\Gamma_U^{\alpha\beta}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\alpha\beta} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\alpha p_T^\beta}{M_h^2} + g_T^{\alpha\beta}\frac{\boldsymbol{p}_T^2}{2M_h^2}\right) h_1^{\perp g}(x,\boldsymbol{p}_T^2)\right\} \qquad \text{[unp. hadron]}$$

$$\Gamma_T^{\alpha\beta}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{g_T^{\alpha\beta}\frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_h} f_{1T}^{\perp g}(x,\boldsymbol{p}_T^2) + i\epsilon_T^{\alpha\beta}\frac{p_T\cdot S_T}{M_h} g_{1T}^g(x,\boldsymbol{p}_T^2)\right.$$
$$\left. - \frac{p_{T\rho}\epsilon_T^{\rho\{\alpha} S_T^{\beta\}} + S_{T\rho}\epsilon_T^{\rho\{\alpha} p_T^{\beta\}}}{4M_h} h_{1T}^g(x,\boldsymbol{p}_T^2) + \frac{p_{T\rho}\,\epsilon_T^{\rho\{\alpha} p_T^{\beta\}}}{2M_h^2}\frac{p_T\cdot S_T}{M_h} h_{1T}^{\perp g}(x,\boldsymbol{p}_T^2)\right\}$$

[transv. pol. hadron]

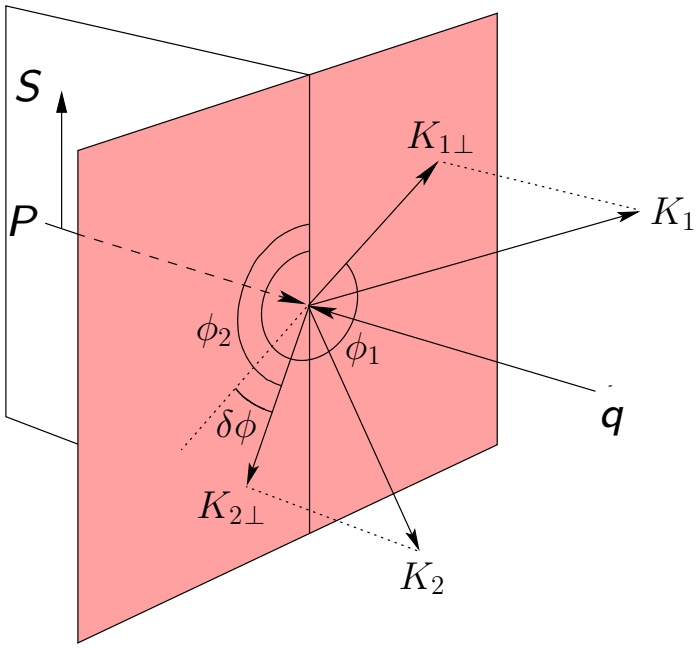
Mulders, Rodrigues, PRD 63 (2001) 094021
Meissner, Metz, Goeke, PRD 76 (2007) 034002

- f_1^g : unpolarized TMD gluon distribution
- $h_1^{\perp g}$: T-even distribution of linearly polarized gluons inside an unp. hadron
- $f_{1T}^{\perp g}$: T-odd distributions of unp. gluons inside a transversely pol. hadron
- h_{1T}^g, $h_{1T}^{\perp g}$: helicity flip distributions like h_{1T}^q, $h_{1T}^{\perp q}$, but T-odd, chiral even!

Transversity $h_1^q \equiv h_{1T}^q + \frac{\boldsymbol{p}_T^2}{2M_p^2} h_{1T}^{\perp q}$ survives under p_T integration, unlike h_1^g

Gluon TMDs probed directly in $e(\ell) + p(P,S) \to e(\ell') + Q(K_1) + \overline{Q}(K_2) + X$

- the $Q\overline{Q}$ pair is almost back to back in the plane $\perp$ to q and P
- $q \equiv \ell - \ell'$: four-momentum of the exchanged virtual photon γ^*

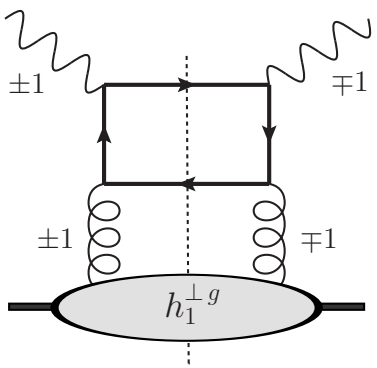


$$\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp}$$

$$\boldsymbol{K}_\perp \equiv (\boldsymbol{K}_{1\perp} - \boldsymbol{K}_{2\perp})/2$$

$\Longrightarrow$ Correlation limit: $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$, $\quad |\boldsymbol{K}_\perp| \approx |\boldsymbol{K}_{1\perp}| \approx |\boldsymbol{K}_{2\perp}|$

Heavy quark pair production in DIS
Angular structure of the cross section



y_1 (y_2) rapidities of Q ($\bar{Q}$) in the $\gamma^* p$ cms; x_B, y: DIS variables

$\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp} = |\boldsymbol{q}_T|(\cos\phi_T, \sin\phi_T)$

$\boldsymbol{K}_\perp \equiv (\boldsymbol{K}_{1\perp} - \boldsymbol{K}_{2\perp})/2 = |\boldsymbol{K}_\perp|(\cos\phi_\perp, \sin\phi_\perp)$

$\boldsymbol{S}_T = S_T(\cos\phi_S, \sin\phi_S)$ in a frame where $\phi_\ell = \phi_{\ell'} = 0$

At LO in pQCD: only $\gamma^* g \to Q\overline{Q}$ contributes

$$\mathrm{d}\sigma(\phi_S, \phi_T, \phi_\perp) = \mathrm{d}\sigma^U(\phi_T, \phi_\perp) + \mathrm{d}\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

Angular structure of the unpolarized cross section for $ep \to e' Q\overline{Q}X$, $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$\frac{\mathrm{d}\sigma^U}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}y\,\mathrm{d}x_B\,\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}^2\boldsymbol{K}_\perp} \propto \left\{A_0^U + A_1^U\cos\phi_\perp + A_2^U\cos 2\phi_\perp\right\} f_1^g(x, \boldsymbol{q}_T^2) + \frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp g}(x, \boldsymbol{q}_T^2)$$

$$\times \left\{B_0^U\cos 2\phi_T + B_1^U\cos(2\phi_T - \phi_\perp) + B_2^U\cos 2(\phi_T - \phi_\perp) + B_3^U\cos(2\phi_T - 3\phi_\perp) + B_4^U\cos 2(\phi_T - 2\phi_\perp)\right\}$$

The different contributions can be isolated by defining

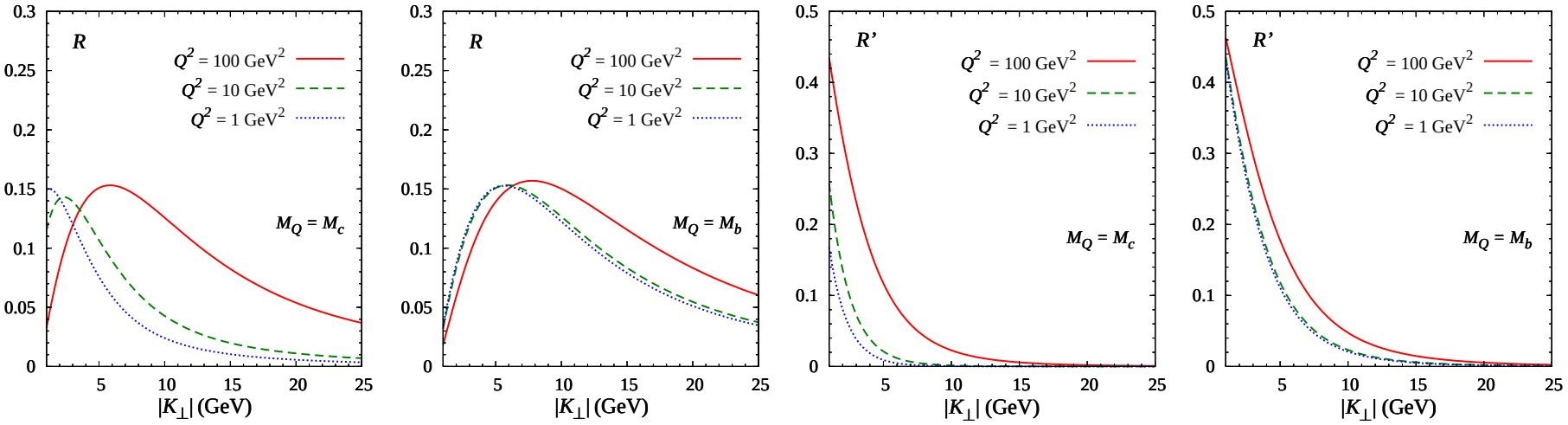
$$\langle W(\phi_\perp, \phi_T)\rangle = \frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, W(\phi_\perp, \phi_T)\,\mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\,\mathrm{d}\sigma}, \quad W = \cos 2\phi_T,\ \cos 2(\phi_\perp - \phi_T), \ldots$$

Positivity bound for $h_1^{\perp g}$: $\quad |h_1^{\perp g}(x, \boldsymbol{p}_T^2)| \leq \frac{2M_p^2}{\boldsymbol{p}_T^2} \, f_1^g(x, \boldsymbol{p}_T^2)$

It can be used to estimate maximal values of the asymmetries

Asymmetries usually larger when Q and $\overline{Q}$ have same rapidities

Upper bounds on $R \equiv |\langle \cos 2(\phi_T - \phi_\perp) \rangle|$ and $R' \equiv |\langle \cos 2\phi_T \rangle|$ at $y = 0.01$



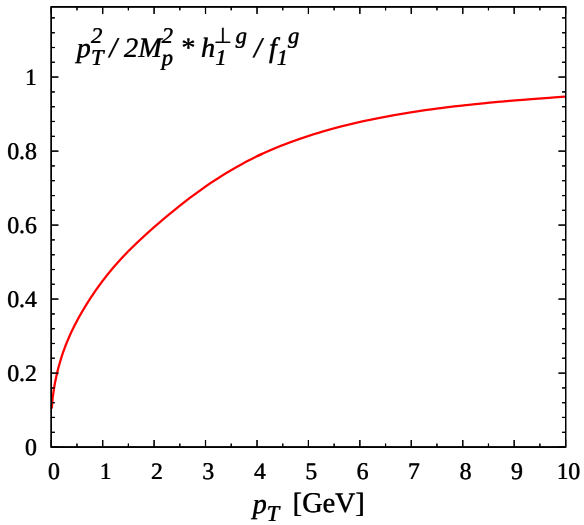
CP, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013) 024
Boer, Brodsky, Mulders, CP, PRL 106 (2011) 132001

TMDs in this process are WW-type at small-x (future pointing [++] gauge links)

WW f_1^g and $h_1^{\perp g} \sim \ln 1/x$ as $x \to 0$: computable in a saturation (MV) models

Dominguez *et al*, PRD **85** (2012) 045003
Metz, Zhou, PRD **84** (2011) 051503
McLerran, Venugopalan, PRD **49** (1994) 2233

Positivity bound of $h_1^{\perp g}$ saturated only at large p_T, $\neq$ dipole case ([+−])



$Q_{s0}^2 = C_A/C_F \times 0.35$ GeV^2 at $x = 0.01$

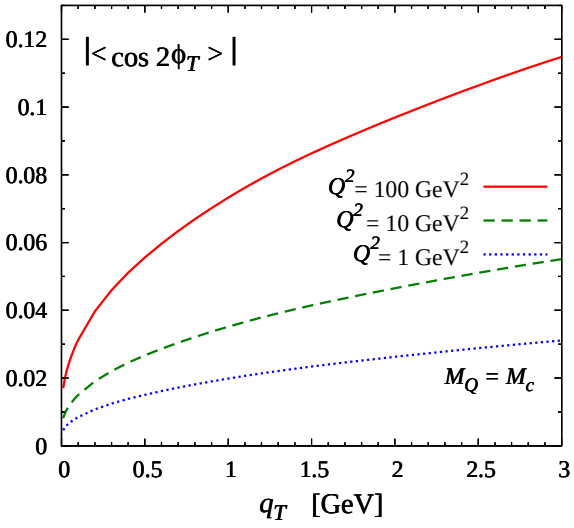
Based on a GBW model fit of HERA data
Golec-Biernat, Wüsthoff, PRD **59** (1998) 014017

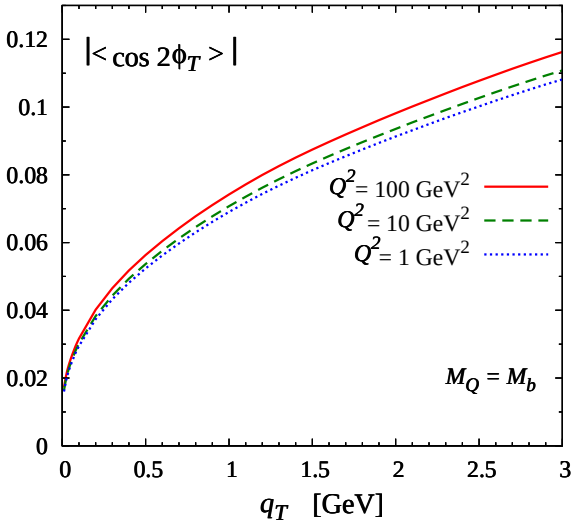
$\Lambda_{QCD} = 0.2$ GeV

Numerically it has to be $Q_{s0}^2 \geq 1.1 \times C_A \Lambda_{QCD}^2$ to satisfy the bound

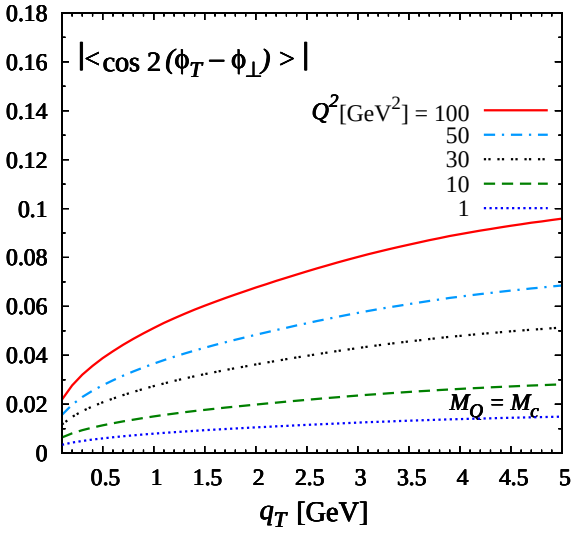
$h_1^{\perp g}$ in $ep \to e' Q\overline{Q}X$
MV model

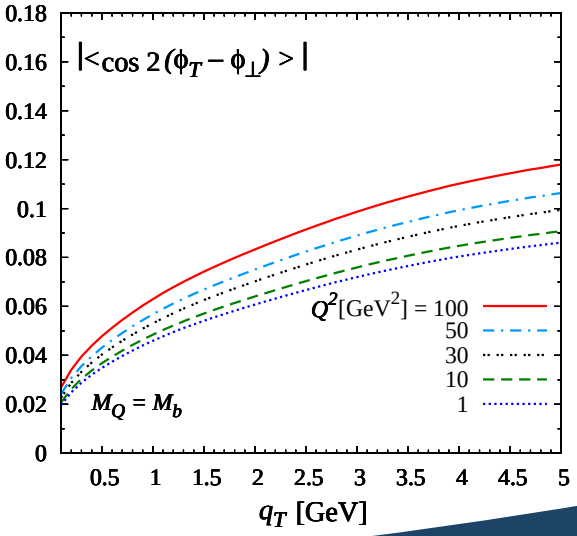
- $|\langle \cos 2\phi_T \rangle|$ *vs* $|\boldsymbol{q}_T|$ at $|\boldsymbol{K}_\perp| = 6$ GeV and $y = 0.1$





- $|\langle \cos 2(\phi_T - \phi_\perp) \rangle|$ *vs* $|\boldsymbol{q}_T|$ at $|\boldsymbol{K}_\perp| = 10$ GeV and $y = 0.3$





SSAs in $ep^\uparrow \to e' Q\overline{Q} X$

Angular structure of the single polarized cross section for $ep^\uparrow \to e' Q\overline{Q} X$, $\quad |\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$
\begin{aligned}
\mathrm{d}\sigma^T \propto \; & \sin(\phi_S - \phi_T)\Big[A_0^T + A_1^T \cos\phi_\perp + A_2^T \cos 2\phi_\perp\Big] f_{1T}^{\perp\, g} + \cos(\phi_S - \phi_T)\Big[B_0^T \sin 2\phi_T \\
& + B_1^T \sin(2\phi_T - \phi_\perp) + B_2^T \sin 2(\phi_T - \phi_\perp) + B_3^T \sin(2\phi_T - 3\phi_\perp) + B_4^T \sin(2\phi_T - 4\phi_\perp)\Big] h_{1T}^{\perp\, g} \\
& + \Big[B_0'^{\,T} \sin(\phi_S + \phi_T) + B_1'^{\,T} \sin(\phi_S + \phi_T - \phi_\perp) + B_2'^{\,T} \sin(\phi_S + \phi_T - 2\phi_\perp) \\
& \quad + B_3'^{\,T} \sin(\phi_S + \phi_T - 3\phi_\perp) + B_4'^{\,T} \sin(\phi_S + \phi_T - 4\phi_\perp)\Big] h_{1T}^{g}
\end{aligned}
$$

The ϕ_S dependent terms can be singled out by means of azimuthal moments A_N^W

$$
A_N^{W(\phi_S,\phi_T)} \equiv 2\, \frac{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, W(\phi_S,\phi_T)\, \mathrm{d}\sigma_T(\phi_S,\, \phi_T,\, \phi_\perp)}{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, \mathrm{d}\sigma_U(\phi_T,\phi_\perp)}
$$

$$
A_N^{\sin(\phi_S-\phi_T)} \propto \frac{f_{1T}^{\perp\, g}}{f_1^g} \qquad A_N^{\sin(\phi_S+\phi_T)} \propto \frac{h_1^g}{f_1^g} \qquad A_N^{\sin(\phi_S-3\phi_T)} \propto \frac{h_{1T}^{\perp\, g}}{f_1^g}
$$

Same modulations as in SIDIS for quark TMDs ($\phi_T \to \phi_h$)

Boer, Mulders, PRD D57 (1998) 5780

Omitted factors in $A_N^{\sin(\phi_S-3\phi_T)}$, $A_N^{\sin(\phi_S+\phi_T)}$ $\to 0$ if $y \to 1$ ($x \to 0$ if $s/Q^2 \to \infty$)

SSAs in $ep^\uparrow \to e' Q\overline{Q}X$

$$\frac{A_N^{\sin(\phi_S - 3\phi_T)}}{A_N^{\sin(\phi_S + \phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\, g}}{h_1^g}$$ direct probe of the relative magnitude of the two TMDs

T-odd WW TMDs vanish at small-x (no MV), suppressed comp. to dipole ones

Boer, Echevarria, Mulders, Zhou, PRL **116** (2016) 122001

Subleading terms important at moderate x, computed perturbatively in the collinear twist-three approach. Different scenarios for ratio of TMDs

Boer, Mulders, CP, Zhou, ArXiv:1605.07934

Alternatively, azimuthal angles defined w.r.t. $\phi_\perp$ instead of ϕ_ℓ (integrated over) $A_N^{W_\perp}$ with $W_\perp = \sin(\phi_S^\perp - \phi_T^\perp)$, $\sin(\phi_S^\perp + \phi_T^\perp)$, $\sin(\phi_S^\perp - 3\phi_T^\perp)$

Positivity bounds of T-odd gluon TMDs used to estimate maximal SSAs

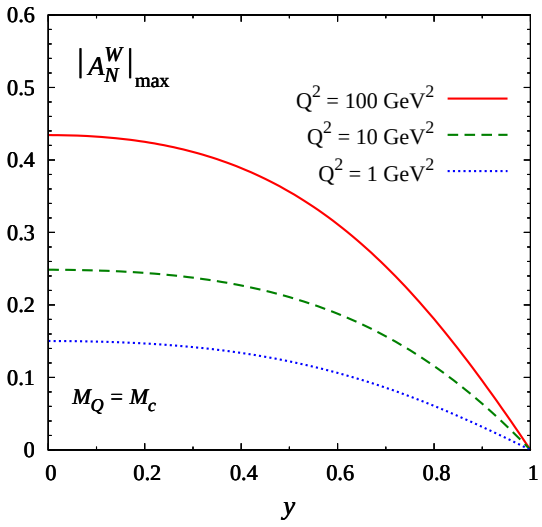
$$\frac{|\boldsymbol{p}_T|}{M_p}\,|f_{1T}^{\perp\, g}| \le f_1^g \qquad \frac{|\boldsymbol{p}_T|}{M_p}\,|h_1^g| \le f_1^g \qquad \frac{|\boldsymbol{p}_T|^3}{2M_p^3}\,|h_{1T}^{\perp\, g}| \le f_1^g$$

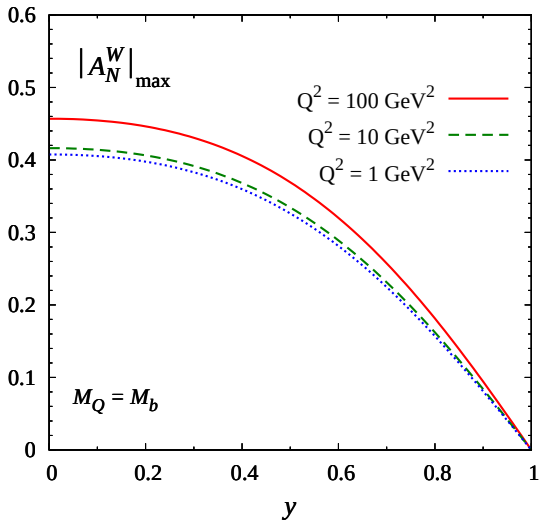
Upper bound of the Sivers asymmetries is 1

SSAs A_N^W and $A_N^{W_\perp}$ in $ep^\uparrow \to e' Q\overline{Q}X$

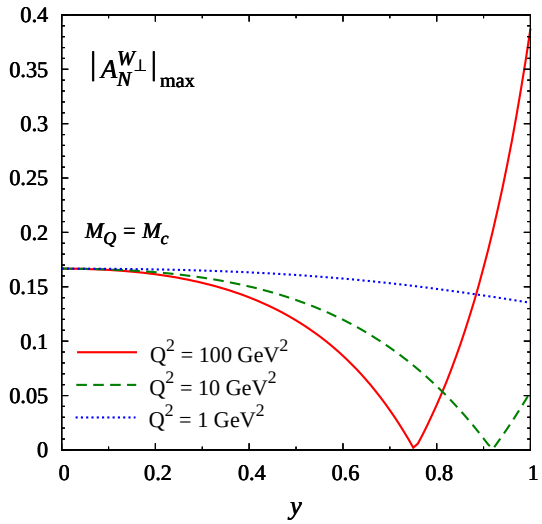
Upper bounds

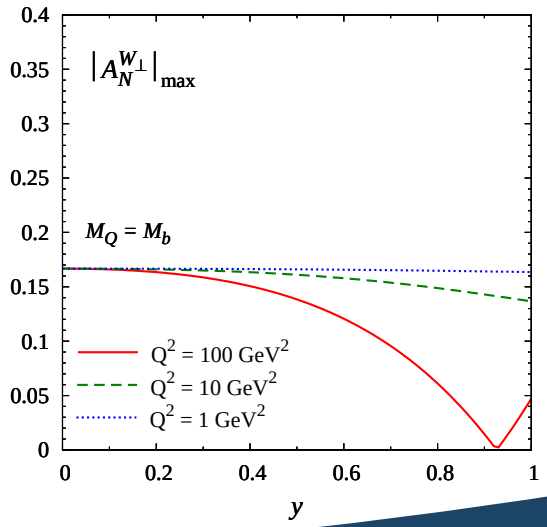
- Maximal values for $|A_N^W|$, $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$ $\quad (|\boldsymbol{K}_\perp| = 1$ GeV$)$





- $|A_N^{W_\perp}|$ do not vanish when $y \to 0$, zero crossing

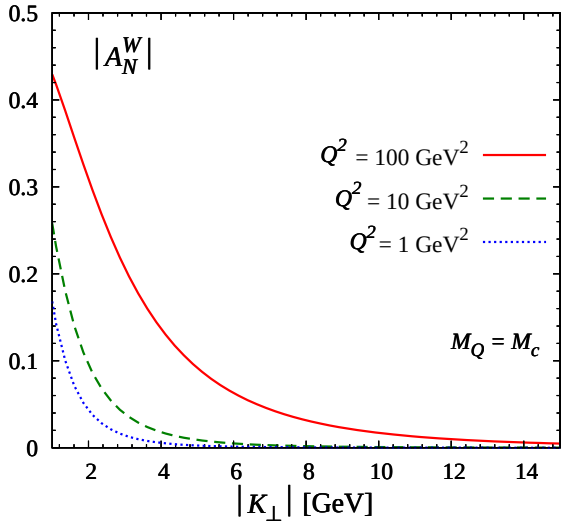


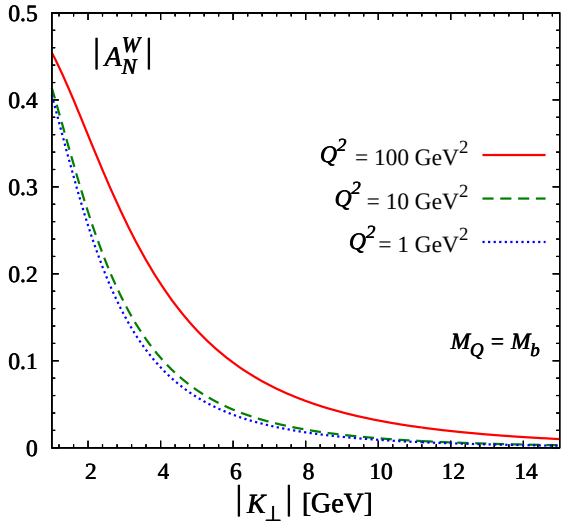


SSAs A_N^W and $A_N^{W_\perp}$ in $ep^\uparrow \to e' Q\overline{Q}X$

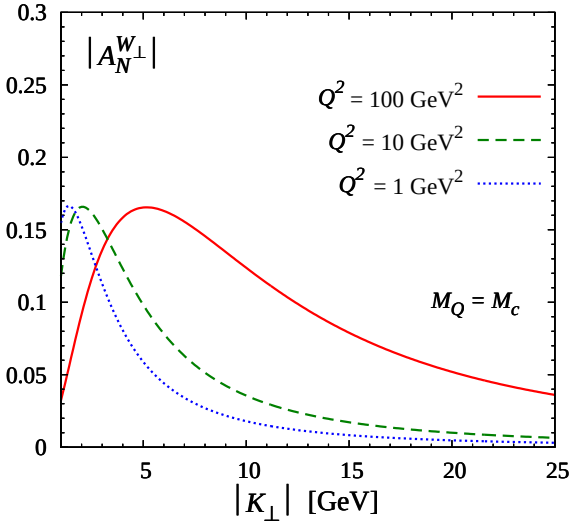
Upper bounds

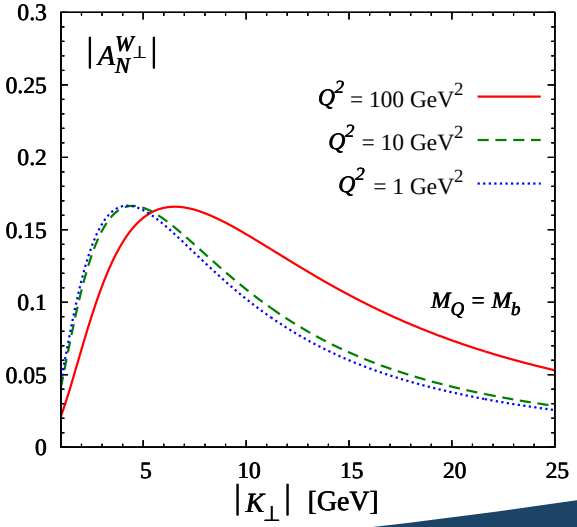
- $|A_N^W|$ *vs* $|\boldsymbol{K}_\perp|$ at $y = 0.1$; same upper bounds as $|\langle \cos 2\phi_T \rangle|$





- $|A_N^{W_\perp}|$ *vs* $|\boldsymbol{K}_\perp|$ at $y = 0.1$; same upper bounds as $|\langle \cos 2(\phi_T - \phi_\perp) \rangle|$



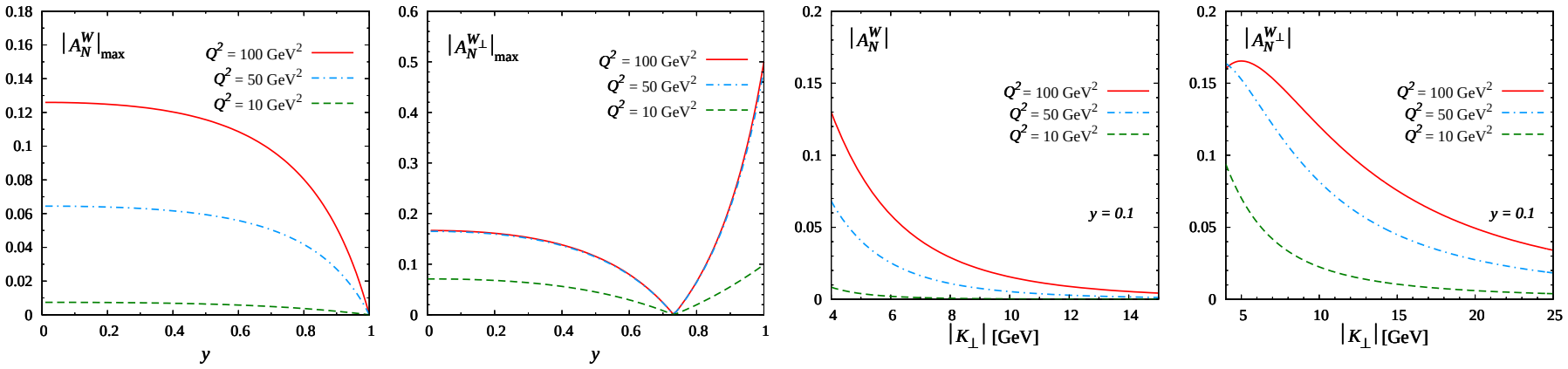


Results can be obtained by taking $M_Q = 0$ in the expressions for $ep \to e' Q \bar{Q} X$

Contribution to the denominator also from $\gamma^* q \to gq$, negligible at small-x

$\langle \cos 2\phi_T \rangle$ and $\langle \cos 2(\phi_T - \phi_\perp) \rangle$ in the MV model similar to $c\bar{c}$ for $Q^2 \geq 10$ GeV^2

Upper bounds for A_N^W for $K_\perp \geq 4$ GeV



Asymmetries much smaller than in $c\bar{c}$ case for $Q^2 \leq 10$ GeV^2

Complementary Processes

$ep \to e' Q\overline{Q} X$, $ep \to e'\,\mathrm{jet}\,\mathrm{jet}\,X$ probe gluon TMDs with $[++]$ gauge links (WW)

$pp \to \gamma\,\mathrm{jet}\,X$ probes an entirely independent gluon TMD: $[+-]$ links (dipole)

Related Processes

In $pp \to \gamma\gamma X$ and/or other CS final state: gluon TMDs have $[--]$ gauge links

Qiu, Schlegel, Vogelsang, PRL **107** (2011) 062001

Analogue of the sign change of $f_{1T}^{\perp\, q}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp\, g}$)

$$f_{1T}^{\perp\, g\,[e\, p^\uparrow \to e'\, Q\overline{Q}\, X]} = -f_{1T}^{\perp\, g\,[p^\uparrow\, p \to \gamma\, \gamma\, X]}$$

Motivation to study the gluon Sivers effect at RHIC and AFTER@LHC

Talk by J.P. Lansberg

T-even gluon TMDs probed in DIS are the same as in $pp \to H/\eta_{c,b}/...X$

$$h_1^{\perp\, g\,[e\, p \to e'\, Q\overline{Q}\, X]} = h_1^{\perp\, g\,[p\, p \to H\, X]}$$

TMD observables at EIC and LHC can be either related or complementary

Conclusions

- Azimuthal asymmetries in heavy quark pair and dijet production in DIS could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)
- Asymmetries maximally allowed by positivity bounds of gluon TMDs can be sizeable in specific kinematic region
- Study of TMDs in the small-x region: effects of linearly polarized gluons still sizeable, ratio of T-odd TMDs can test our model expectations
- Different behaviour of WW and dipole gluon TMDs accessible at RHIC could be tested experimentally
- Such observables could be part of both the *spin* and the *small-x* program at a future EIC