Off-Shell Amplitudes and Four-Jet Production in kt-factorization

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QCD Evolution 2016
Amsterdam, The Netherlands
May 30 - June 3 2016

Work in collaboration with
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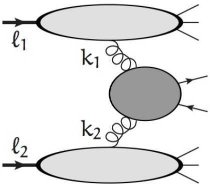
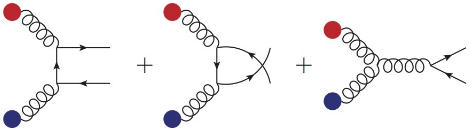
Supported by NCN grant DEC-2013/10/E/ST2/00656 of Krzysztof Kutak

1. The formal framework: off-shell amplitudes
2. BCFW recursion relations: the all-leg solution
3. 4-jet production in kt-factorization: Single and Double Parton scattering
4. Summary and perspectives

High-Energy-factorisation: original formulation

High-Energy-factorisation *(Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991)*





$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp}\, d^2k_{2\perp}\, \frac{dx_1}{x_1}\, \frac{dx_2}{x_2}\, \mathcal{F}_g(x_1,k_{1\perp})\, \mathcal{F}_g(x_2,k_{2\perp})\, \hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s}, \frac{k_{1\perp}}{m}, \frac{k_{2\perp}}{m}\right)$$

where the $\mathcal{F}_g$'s are the gluon densities, obeying BFKL, BK, CCFM evolution equations, and $\hat{\sigma}$ the **gauge invariant** parton cross section (!!!)

Non negligible transverse momentum is associated to small-x physics.

Momentum parameterisation:

$$k_1^\mu = x_1\, p_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\, p_2^\mu + k_{2\perp}^\mu \quad \text{for} \quad p_i \cdot k_i = 0 \quad k_i^2 = -k_{i\perp}^2 \quad i = 1,2$$

Gauge invariant off-shell amplitudes

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$\Rightarrow$ ordinary Feynman rules are not enough !

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on-shell amplitudes are gauge invariant, so off-shell gauge-invariant amplitudes could be got by embedding them into on-shell processes...

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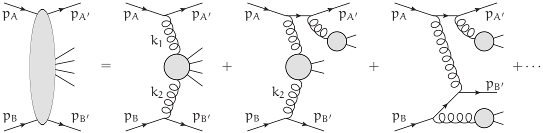
on-shell amplitudes are gauge invariant, so off-shell gauge-invariant amplitudes could be got by embedding them into on-shell processes...

...first result...: 1) For off-shell gluons: represent g^* as coming from a $\bar{q}qg$ vertex, with the quarks taken to be on-shell



- embed the scattering of the off-shell gluons in the scattering of two quark pairs carrying momenta $p_A^\mu = k_1^\mu$, $p_B^\mu = k_2^\mu$, $p_{A'}^\mu = 0$, $p_{B'}^\mu = 0$
- Assign the spinors $|p_1\rangle, |p_1]$ to the A-quark and the propagator $\frac{i\,\not{p}_1}{p_1 \cdot k}$ instead of $\frac{i\not{k}}{k^2}$ to the propagators of the A-quark carrying momentum k; same thing for the B-quark line.
- ordinary Feynman elsewhere and factor $x_1\sqrt{-k_\perp^2/2}$ to match to the collinear limit

K. Kutak, P. Kotko, A. van Hameren, JHEP 1301 (2013) 078

Prescription for off-shell gluons: derivation 1

Auxiliary vectors $p_{3,4}$ (complex in general):
$$\begin{cases} p_3^\mu = \frac{1}{2}\langle p_2|\gamma^\mu|p_1] \\ p_4^\mu = \frac{1}{2}\langle p_1|\gamma^\mu|p_2] \\ p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0 \\ p_{1,2}\cdot p_{3,4} = 0\,, \quad p_1\cdot p_2 = -p_3\cdot p_4 \end{cases}$$

Auxiliary momenta:
$$\begin{cases} p_A^\mu = (\Lambda + x_1)p_1^\mu - \frac{p_4\cdot k_{1\perp}}{p_1\cdot p_2}p_3^\mu\,, \quad p_{A'}^\mu = \Lambda p_1^\mu + \frac{p_3\cdot k_{1\perp}}{p_1\cdot p_2}p_4^\mu \\ p_B^\mu = (\Lambda + x_2)p_2^\mu - \frac{p_3\cdot k_{2\perp}}{p_1\cdot p_2}p_4^\mu\,, \quad p_{B'}^\mu = \Lambda p_2^\mu + \frac{p_4\cdot k_{2\perp}}{p_1\cdot p_2}p_3^\mu \end{cases}$$

For any Λ:
$$\begin{cases} p_A^\mu - p_{A'}^\mu = x_1\,p_1^\mu + k_{1\perp}^\mu \\ p_B^\mu - p_{B'}^\mu = x_2\,p_2^\mu + k_{2\perp}^\mu \\ p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0 \end{cases}$$

Prescription for off-shell gluons: derivation 2

Momentum flowing through a propagator of an auxiliary quark line:

$$k^\mu = (\Lambda + x_k)p_1^\mu + y_k\, p_2^\mu + k_\perp^\mu$$

Final step: remove complex components taking the $\Lambda \to \infty$ limit.

$$\frac{\not{k}}{k^2} = \frac{(\Lambda + x_k)\not{p}_1 + y_k\,\not{p}_2 + \not{k}}{2(\Lambda + x_k)y_k\,p_1 \cdot p_2 + k_\perp^2} \xrightarrow{\Lambda\to\infty} \frac{\not{p}_1}{2\,y_k\,p_1 \cdot p_2} = \frac{\not{p}_1}{2p_1 \cdot k}$$

In agreement with Lipatov's effective action

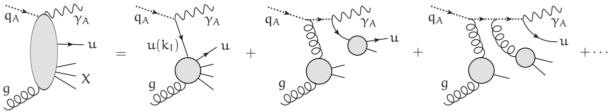
Lipatov Nucl.Phys. B452 (1995) 369-400

Antonov, Lipatov, Kuraev, Cherednikov, Nucl.Phys. B721 (2005) 111-135

Prescription for off-shell quarks

... and second result:

2) for off-shell quarks: represent q^* as coming from a $\gamma\bar{q}q$ vertex, with a 0 momentum and $\bar{q}$ on shell (and vice-versa)



- embed the scattering of the quark with whatever set of particles in the scattering of an auxiliary quark-photon pair, q_A and γ_A carrying momenta $p^\mu_{q_A} = k^\mu_1$, $p^\mu_{\gamma_A} = 0$
- Let q_A-propagators of momentum k be $\frac{i\,\not{p}_1}{p_1\cdot k}$ and assign the spinors $|p_1\rangle, |p_1]$ to the A-quark.
- Assign the polarization vectors $\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|p_1]}{\sqrt{2}\langle p_1 q\rangle}$, $\epsilon^\mu_- = \frac{\langle p_1|\gamma^\mu|q]}{\sqrt{2}[p_1 q]}$ to the auxiliary photon, with q a light-like auxiliary momentum.
- Multiply the amplitude by $x_1\sqrt{-k^2_{1\perp}/2}$ and use ordinary Feynman rules everywhere else.

K. Kutak, T. Salwa, A. van Hameren, Phys.Lett. B727 (2013) 226-233

One left issue: huge slowness for many legs

The diagrammatic approach is too slow to allow for the computation of amplitudes containing more than 4 particles in a reasonable time.

Computing scattering amplitudes in Yang-Mills theories via ordinary Feynman diagrams: soon overwhelming !

Number of Feynman diagrams at tree level on-shell:

# of gluons	4	5	6	7	8	9	10
# of diagrams	4	25	220	2485	34300	559405	10525900

And there are even more with the proposed method for amplitudes with off-shell particles due to the gauge-restoring terms.

A method to efficiently compute helicity amplitudes: **BCFW recursion relation**

Britto, Cachazo, Feng, Nucl.Phys. B715 (2005) 499-522
Britto, Cachazo, Feng, Witten, Phys.Rev.Lett. 94 (2005) 181602

BCFW recursion relation

Two very simple ideas for tree level amplitudes:

1. Cauchy's residue theorem: if the amplitude is formally treated as a function of a complex variable z and if it is rational and vanishes for $z \to \infty$, then the integral extended to an infinite contour enclosing all poles vanishes

$$\lim_{z \to \infty} \mathcal{A}(z) = 0 \Rightarrow \frac{1}{2\pi i} \oint dz \, \frac{\mathcal{A}(z)}{z} = 0$$

   implying that the value at $z = 0$ (physical amplitude) can be determined as a sum of the residues at the poles:

$$\mathcal{A}(0) = -\sum_i \frac{\lim_{z \to z_i} [(z - z_i) \, f(z)]}{z_i}$$

   where z_i is the location of the i-th pole

2. Unitarity: Poles in Yang-Mills tree level amplitudes can only be due to gluon propagators dividing the n-point amplitude into two on-shell sub-amplitudes with $k+1$ and $n-k+1$ gluons $\Rightarrow$ it is all about finding the proper way to "complexify" an amplitude.

To properly "complexify" $\mathcal{A}$: for helicities $(h_1, h_n) = (-,+)$ (no loss of generality...)

$$
\begin{array}{ccl}
|1] & \rightarrow & |\hat{1}] \equiv |1] - z\,|n] \Rightarrow p_1 \rightarrow \hat{p}_1 = |1]\langle 1| - z|1]\langle n| \\
|n\rangle & \rightarrow & |\hat{n}\rangle \equiv |n\rangle + z|1\rangle \Rightarrow p_n \rightarrow \hat{p}_n = |n]\langle n| + z|1]\langle n|
\end{array}
$$

With such a choice

- On-shellness, gauge invariance and momentum conservation preserved throughout.
- the most serious issue is the behaviour for $z \rightarrow \infty$, but either a result derived with twistor methods (*Cachazo,Svrcek and Witten JHEP 0409 (2004) 006*) or a smart choice of reference lines always allow to overcome the problem, so that $\lim_{z\rightarrow\infty} \mathcal{A}(z) = 0$ holds

BCFW applies to* color-ordered partial amplitudes*, for which the kinematics and gauge structure are factorised like

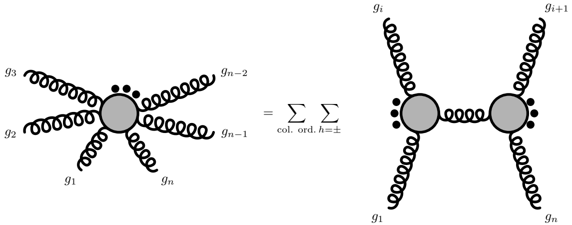
$$
\mathcal{M}_n = g^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T_{\sigma(1)} \dots T_{\sigma(n)})\, \mathcal{A}(g_{\sigma(1)}, \dots, g_{\sigma(n)})
$$

Amazingly simple recursive relation:

any tree-level color-ordered amplitude is the sum of residues of the poles it develops when it is made dependent on a complex variable as above.
Such residues are simply products of color-ordered lower-point amplitudes evaluated at the pole times an intermediate propagator.
Shifted particles are always on opposite sides of the propagator.

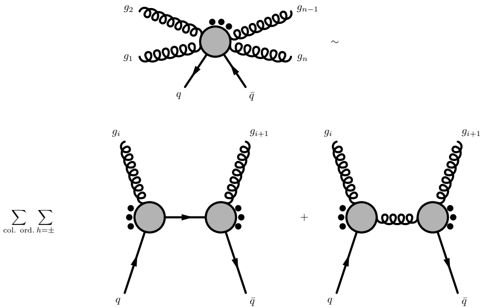
$$\mathcal{A}(g_1,\ldots,g_n)=\sum_{i=2}^{n-2}\sum_{h=+,-}\mathcal{A}(g_1,\ldots,g_i,\hat{P}^h)\,\frac{1}{(p_1+\cdots+p_i)^2}\,\mathcal{A}(-\hat{P}^{-h},g_{i+1},\ldots,g_n)$$

$$z_i=\frac{(p_1+\cdots+p_i)^2}{[1|p_1+\cdots+p_i|n\rangle}$$ location of the pole corresponding for the "i-th" partition



The inclusion of fermions and MHV amplitudes

The BCFW recursion was promptly extended to Yang-Mills theories with fermions:
M. Luo, C. Wen, JHEP 0503 (2005) 004



A couple of MHV amplitudes:

$$\mathcal{A}(g_1^+, g_2^+, \ldots, g_i^-, \ldots, g_j^-, \ldots, g_n^+) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n-1\; n\rangle\langle n1\rangle}$$

$$\mathcal{A}(q^-, g_1^-, g_2^+, \ldots, g_n^+, \bar{q}^+) = \frac{\langle q1\rangle^3\langle \bar{q}1\rangle}{\langle \bar{q}q\rangle\langle q1\rangle\langle 12\rangle \ldots \langle n\bar{q}\rangle}$$

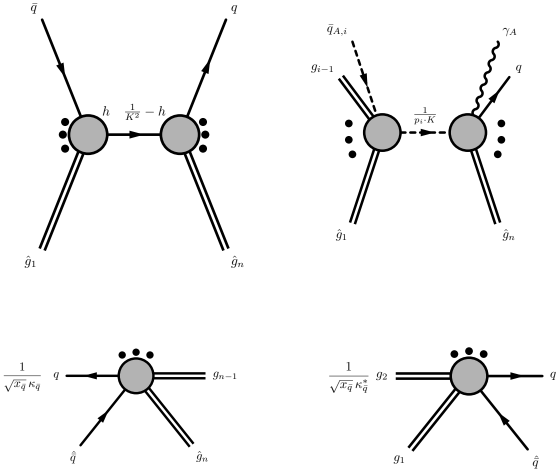
It is natural to ask whether something like a BCFW recursion relation exists with off-shell particles. For off shell, gluons, the answer was first found in *A. van Hameren, JHEP 1407 (2014) 138*

$$\mathcal{A}(0) = \sum_{s=g,f}\left(\sum_{p}\sum_{h=+,-}\mathrm{A}^{s}_{p,h} + \sum_{i}\mathrm{B}^{s}_{i} + \mathrm{C}^{s} + \mathrm{D}^{s}\right),$$

- $\mathrm{A}^{g/f}_{p,h}$ are due to the poles which appear in the original BCFW recursion for on-shell amplitudes. The pole appears because one of the intermediate virtual gluon, whose shifted momentum squared $K^2(z)$ goes on-shell.
- $\mathrm{B}^{g/f}_{i}$ are due to the poles appearing in the propagator of auxiliary eikonal quarks. This means $p_i \cdot \hat{K}(z) = 0$ for $z = -\frac{2\,p_i\cdot K}{2\,p_i\cdot e}$. $\hat{K}$ is the momentum flowing through the eikonal propagator.
- $\mathrm{C}^{g/f}$ and $\mathrm{D}^{g/f}$ show up us the first/last shifted particle is off-shell and their external propagator develops a pole.
  The external propagator for off-shell particles is necessary to ensure

$$\lim_{z\to\infty}\mathcal{A}(z) = 0$$

Classification of poles in the fermion case



General outline of the results

- It is necessary to understand which shifts are legitimate in the off-shell case, i.e. for which choices $\lim_{z\to\infty}\mathcal{A}(z)=0$. We provide a full classification of the possibilities.
- It turns out that amplitudes which are MHV in the on-shell case (2 of the partons have different helicity sign w.r.t. all the others) preserve a similar structure in the off-shell case.
- 5-point amplitudes exhibit some non-MHV structures, which have been calculated for the first time
- Numerical cross-checks are always successful. They were performed cross checked with a program implementing Berends-Giele recursion relation, *A. van Hameren, M. Bury, Comput.Phys.Commun. 196 (2015) 592-598*

Explicit results are presented and discussed thoroughly in
A. van Hameren, M.S. JHEP 1507 (2015) 010 .

The smooth results: MHV amplitudes

Transverse momentum parameterization:

$$\begin{cases} k^{\mu}_{T\,i} = -\frac{\kappa_i}{2}\,\frac{\langle p_i|\gamma^\mu|q]}{[p_i q]} - \frac{\kappa_i^*}{2}\,\frac{\langle q|\gamma^\mu|p_i]}{\langle q p_i\rangle} \\ \kappa_i \equiv \frac{\langle q|\not{k}_i|p_i]}{\langle q p_i\rangle} \qquad \kappa_i^* \equiv \frac{\langle p_i|\not{k}_i|q]}{[p_i q]} \\ q^2 = 0 \quad \text{auxiliary momentum} \end{cases}$$

Subleading contribution: it is zero in the on-shell case !

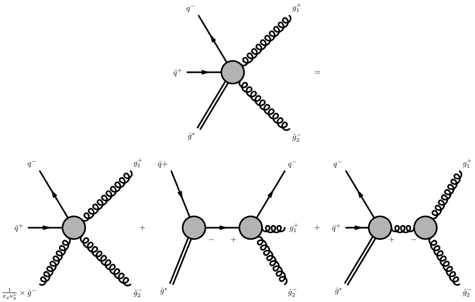
$$\mathcal{A}(g_1^+, g_2^+, \ldots, g_{n-1}^+, \bar{q}, q, g_n^+) = \frac{\langle \bar{q} q\rangle^3}{\langle 12\rangle\langle 23\rangle \ldots \langle \bar{q} q\rangle\langle q n\rangle\langle n1\rangle}$$

Structure of MHV amplitudes

$$\mathcal{A}(g_1^+, g_2^+, \ldots, g_{n-1}^+, \bar{q}^*, q^+, g_n^-) = \frac{1}{\kappa_{\bar{q}}^*}\,\frac{\langle \bar{q} n\rangle^3\langle q n\rangle}{\langle 12\rangle\langle 23\rangle \ldots \langle \bar{q} q\rangle\langle q n\rangle\langle n1\rangle}$$

$$\mathcal{A}(g^*, \bar{q}^+, q^-, g_1^+, g_2^+, \ldots, g_n^+) = \frac{1}{\kappa_g^*}\,\frac{\langle g q\rangle^3\langle g \bar{q}\rangle}{\langle g \bar{q}\rangle\langle \bar{q} q\rangle \ldots \langle n-1|n\rangle\langle n g\rangle}$$

But not everything is so smooth...

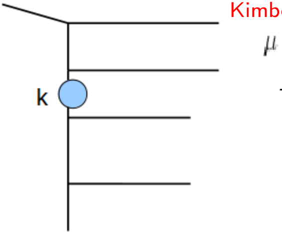


$$\mathcal{A}(g^*,\bar{q}^+,q^-,g_1^+,g_2^-) = \frac{1}{\kappa_g^*}\frac{[\bar{q}1]^3\langle 2g\rangle^4}{[\bar{q}q]\langle g|\not{p}_2+\not{k}_g|1]\langle 2|\not{k}_g\ (\not{k}_g+\not{p}_2)\,|g]\langle 2|\not{k}_g|\bar{q}]}$$

$$+\frac{1}{\kappa_g}\frac{1}{(k_g+p_{\bar{q}})^2}\frac{[g\bar{q}]^2\langle 2q\rangle^3\langle 2|\not{k}_g+\not{p}_{\bar{q}}|g]}{\langle 1q\rangle\langle 12\rangle\left\{(k_g+p_{\bar{q}})^2[\bar{q}g]\langle 2q\rangle-\langle 2|\not{k}_g+\not{p}_{\bar{q}}|g]\langle q|\not{k}_g|\bar{q}]\right\}}$$

$$+\frac{\langle gq\rangle^3[g1]^4}{\langle\bar{q}q\rangle[12][g2]\langle q|\not{p}_1+\not{p}_2|g]\langle g|\not{p}_1+\not{p}_2|g]\langle g|\not{k}_g+\not{p}_2|1]}$$

Our PDFs: the prescription



Survival probability without emissions

Kimber, Martin, Ryskin prescription, '01 :

$$T_s(\mu^2, k^2) = exp\left(- \int_{\mu^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \times \sum_{a'} \int_0^{1-\Delta} dz' P_{aa'}(z') \right)$$

$$\Delta = \frac{\mu}{\mu + k}, \quad \mu = \text{hard scale}$$

$$\mathcal{F}(x, k^2, \mu^2) \sim \partial_{\lambda^2} \left(T_s(\lambda^2, \mu^2) \, x \, g(x, \lambda^2) \right) \Big|_{\lambda^2 = k^2}$$

DLC 2016 (Double Log Coherence)

K. Kutak, R. Maciula, M.S., A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175

Conjectured formulas for 2 and 4 jets production:

$$
\begin{aligned}
\sigma_{2-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1, k_{T1}, \mu_F)\, \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\
&\quad \times \frac{1}{2\hat{s}} \prod_{l=i}^{2} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{2-jet}\, (2\pi)^4\, \delta\left(P - \sum_{l=1}^{2} k_i\right) \overline{|\mathcal{M}(i^*, j^* \to 2\,\text{part.})|^2} \\
\sigma_{4-jets} &= \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, d^2k_{T1} d^2k_{T2}\, \mathcal{F}_i(x_1, k_{T1}, \mu_F)\, \mathcal{F}_j(x_2, k_{T2}, \mu_F) \\
&\quad \times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\, (2\pi)^4\, \delta\left(P - \sum_{l=1}^{4} k_i\right) \overline{|\mathcal{M}(i^*, j^* \to 4\,\text{part.})|^2}
\end{aligned}
$$

- PDFs and matrix elements well defined.
- No factorization rigorous proof around (not even in the collinear case, actually)
- Reasonable description of data justifies this formula *a posteriori*

Our framework

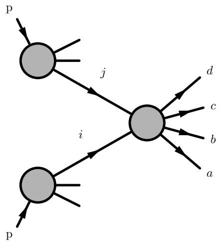
AVHLIB (A. van Hameren) : https://bitbucket.org/hameren/avhlib

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization

- **Flavour scheme**: $N_f = 5$
- **Running** α_s from the MSTW68cl PDF sets
- **Massless quarks approximation** $E_{cm} = 7/8\,TeV \Rightarrow m_{q/\bar{q}} = 0$.
- **Scale** $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2}\sum_i p_T^i$, (sum over final state particles)

We don't take into account correlations in DPS: $D(x_1, x_2, \mu) = f(x_1, \mu)\, f(x_2, \mu)$.
There are attempts to go beyond this approximation:
Golec-Biernat, Lewandowska, Snyder, M.S., Stasto, Phys.Lett. B750 (2015) 559-564
Rinaldi, Scopetta, Traini, Vento, JHEP 1412 (2014) 028

4-jet production: Single Parton Scattering (SPS)



We take into account all the (according to our conventions) 20 channels.

Here q and q' stand for different quark flavours in the initial (final) state.

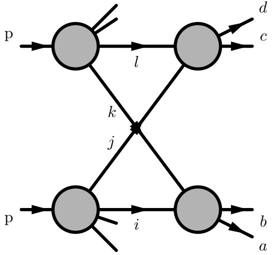
We do not introduce K factors, amplitudes@LO.

$\sim$ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,,\, gg \to q\bar{q}\,2g\,,\, qg \to q\,3g\,,\, q\bar{q} \to q\bar{q}\,2g\,,\, qq \to qq\,2g\,,\, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,,\, gg \to q\bar{q}q'\bar{q}'\,,\, qg \to qgq\bar{q}\,,\, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,,\, q\bar{q} \to q'\bar{q}'\,2g\,,\, q\bar{q} \to q\bar{q}q\bar{q}\,,\, q\bar{q} \to q\bar{q}q'\bar{q}'\,,\, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,,$$
$$q\bar{q} \to q'\bar{q}'q''\bar{q}''\,,\, qq \to qqq\bar{q}\,,\, qq \to qqq'\bar{q}'\,,\, qq' \to qq'q\bar{q}\,,$$

4-jet production: Double parton scattering (DPS)



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{\mathcal{S}}{\sigma_{eff}} \sigma(i,j \to a,b)\, \sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

$$\sigma_{eff} = 15\,mb\,, \text{(CDF, D0 and LHCb collaborations)}\,,$$

Experimental data may hint at different values of σ_{eff}; main conclusions not affected

In our conventions, 9 channels from $2 \to 2$ SPS events,

$$\begin{aligned} \#1 &= gg \to gg\,, \quad \#6 = u\bar{u} \to d\bar{d} \\ \#2 &= gg \to u\bar{u}\,, \quad \#7 = u\bar{u} \to gg \\ \#3 &= ug \to ug\,, \quad \#8 = uu \to uu \\ \#4 &= gu \to ug\,, \quad \#9 = ud \to ud \\ \#5 &= u\bar{u} \to u\bar{u} \end{aligned}$$

$\Rightarrow$ 45 channels for the DPS; only 14 contribute to $\geq 95\%$ of the cross section :

$$(1,1),(1,2),(1,3),(1,4),(1,8),(1,9),(3,3)$$
$$(3,4),(3,8),(3,9),(4,4),(4,8),(4,9),(9,9)$$

Hard jets

We reproduce all the LO results (only SPS) for $p\,p \to n\,jets\,, \quad n = 2, 3, 4$ published in
BlackHat collaboration, Phys.Rev.Lett. 109 (2012) 042001
S. Badger et al., Phys.Lett. B718 (2013) 965-978

Asymmetric cuts for hard central jets

$$p_T \geq 80\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 60\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$

PDFs set: MSTW2008LO@68cl

$$\sigma(\geq 2\,\text{jets}) = 958^{+316}_{-221} \quad \sigma(\geq 3\,\text{jets}) = 93.4^{+50.4}_{-30.3} \quad \sigma(\geq 4\,\text{jets}) = 9.98^{+7.40}_{-3.95}$$

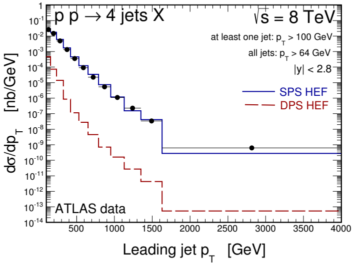
Cuts are too hard to pin down DPS and/or benefit from HEF: 4-jet case

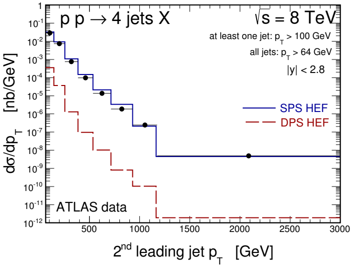
$$\text{Collinear case} \quad \begin{cases} 9.98^{+7.40}_{-3.95} & SPS \\ 0.094^{+0.06}_{-0.036} & DPS \end{cases} \qquad \text{HEF case} \quad \begin{cases} 10.0^{+6.9}_{-5.3} & SPS \\ 0.05^{+0.054}_{-0.029} & DPS \end{cases}$$

Differential cross section

Most recent ATLAS paper on 4-jet production in proton-proton collision:
ATLAS, JHEP 1512 (2015) 105

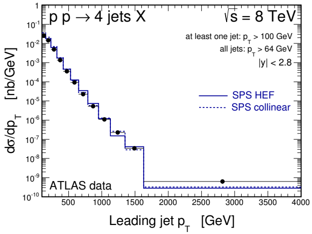
$$p_T \geq 100\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 64\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$

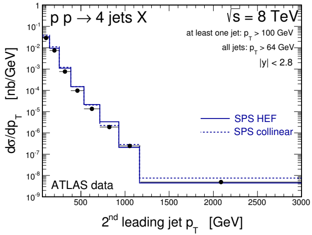


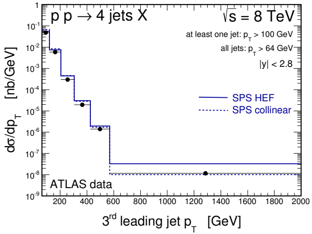


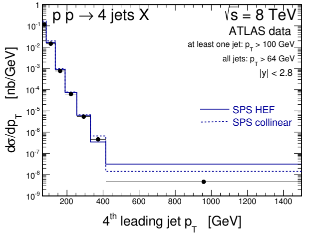
- All channels included and running α_s @ NLO
- Good agreement with data
- DPS effects are manifestly too small for such hard cuts: this could be expected.

Comparing collinear factorization and HEF









Collinear factorization performs slightly better for intermediate values and HEF does a better job for the last bins, except for the 4th jet.

DPS effects in collinear and HEF

For more formal approach to DPS $\Rightarrow$ Gaunt's, Buffing's and Diehl's talks

Inspired by Maciula, Szczurek, Phys.Lett. B749 (2015) 57-62

DPS effects are expected to become significant for lower p_T cuts, like the ones of the CMS collaboration, Phys.Rev. D89 (2014) no.9, 092010

$$p_T(1,2) \geq 50\,\text{GeV}\,, \quad p_T(3,4) \geq 20\,\text{GeV}\,, \quad |\eta| \leq 4.7\,, \quad R = 0.5$$

CMS collaboration :	$\sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\; nb$
LO collinear factorization :	$\sigma_{SPS} = 697\; nb\,, \quad \sigma_{DPS} = \mathbf{125\,nb}\,, \quad \sigma_{tot} = 822\; nb$
LO HEF k_T-factorization :	$\sigma_{SPS} = 548\; nb\,, \quad \sigma_{DPS} = \mathbf{33\,nb}\,, \quad \sigma_{tot} = 581\; nb$

In HE factorization DPS gets suppressed and does not dominate at low p_T

Counterintuitive result from well-tested perturbative framework
$\Rightarrow$ phase space effect ? ($\Rightarrow$ see Colferai's talk)

Higher order corrections to 2-jet production

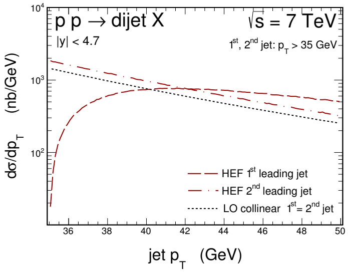


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: Frixione, Ridolfi, Nucl.Phys. B507 (1997) 315-333

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in Eur.Phys.J. C71 (2011) 1763; theoretical predictions from Phys.Rev.Lett. 109 (2012) 042001

#jets	ATLAS	LO	NLO
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

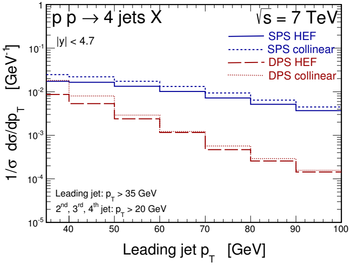
Reconciling HE and collinear factorisation: asymmetric p_T cuts

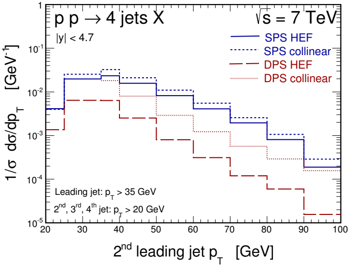
In order to open up wider region of soft final states and thereof expected that the DPS contribution increases

$$p_T(1) \geq 35\text{GeV}\,, \quad p_T(2,3,4) \geq 20\text{ GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$$

$$\text{LO collinear factorization} : \sigma_{SPS} = 1969\,nb\,, \quad \sigma_{DPS} = 514\,nb\,, \quad \sigma_{tot} = 2309\,nb$$

$$\text{LO HEF } k_T\text{-factorization} : \sigma_{SPS} = 1506\,nb\,, \quad \sigma_{DPS} = 297\,nb\,, \quad \sigma_{tot} = 1803\,nb$$



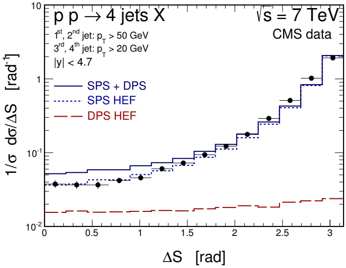


DPS dominance pushed to even lower p_T but restored in HE factorization as well

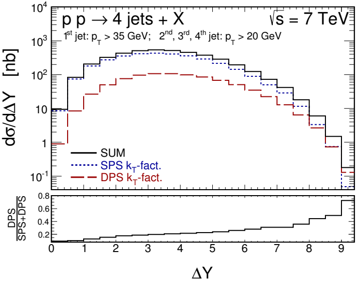
An interesting variable: do we see DPS ?

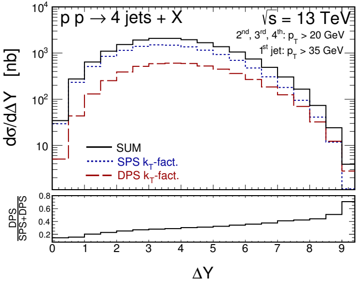
$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|}\right), \quad \vec{p}_T(j_i, j_k) = p_{T,i} + p_{T,j}$$

We roughly describe the data via pQCD effects within our HEF approach which are (equally partially) described by parton-showers and soft MPIs by CMS.
CMS collaboration Phys.Rev. D89 (2014) no.9, 092010



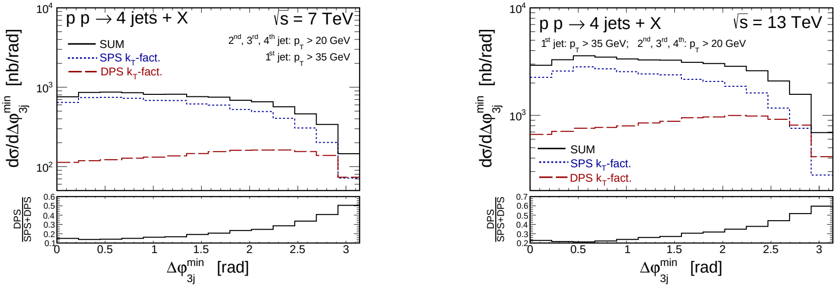
Pinning down double parton scattering: large rapidity separation





- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for $\Delta Y > 6$ the total cross section is dominated by DPS.

Pinning down double parton scattering: $\Delta\phi_3^{min}$ - azimuthal separation



- Definition: $\Delta\phi_3^{min} = min_{i,j,k[1,4]} \left(\left|\phi_i - \phi_j\right| + \left|\phi_j - \phi_k\right| \right)$, $\quad i \neq j \neq k$
- Proposed by ATLAS in *JHEP 12 105 (2015)* for high p_T analysis
- High values favour configurations closer to back-to-back, i.e. DPS
- For $\Delta\phi_3^{min} \geq \pi/2$ the total cross section is dominated by DPS

Summary and conclusions

- The problem of the recursive computation of tree-level amplitudes in kt-factorization was completely solved for any number of legs in massless QCD
- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMDs via KMR procedure obtained from NLO collinear PDFs
- HE factorisation reproduces well ATLAS data @ 7 and 8 TeV for hard central inclusive 4-jet production. Essential agreement with collinear predictions.
- HE factorisation smears out the DPS contribution to the cross section for less central jet, pushing the DPS-dominance region to lower p_T, but asymmetric cuts are in order: initial state transverse momentum generates asymmetries in the p_T of final state jet pairs.
- It would be interesting to have an experimental analysis with cuts which are *asymmetric and soft*.
- We have proposed a set of observables which would help pinning down DPS more effectively
- Further insight into HE factorisation prediction will come with progress in NLO results and with the addition of final state paton showers. **Work in progress...**

Just for the formalism: Weyl spinors

High energy limit $\Rightarrow$ massless particles $\Rightarrow$ Weyl basis for spinors.
If $p^2 = 0$, it can be cast in the Pauli matrices language,

$$p \cong p^\mu \, \sigma_\mu = \begin{pmatrix} p^0 - p^3 & -p^1 + i\,p^2 \\ -p^1 - i\,p^2 & p^0 + p^3 \end{pmatrix} = |p]\langle p|$$

$$|p] \quad = \quad \begin{pmatrix} L(p) \\ \mathbf{0} \end{pmatrix} \qquad L(p) = \frac{1}{\sqrt{|p^0 + p^3|}} \begin{pmatrix} -p^1 + i\,p^2 \\ p^0 + p^3 \end{pmatrix}$$

$$|p\rangle \quad = \quad \begin{pmatrix} \mathbf{0} \\ R(p) \end{pmatrix} \qquad R(p) = \frac{\sqrt{|p^0 + p^3|}}{p^0 + p^3} \begin{pmatrix} p^0 + p^3 \\ p^1 + i\,p^2 \end{pmatrix}$$

and the charge-conjugated spinors

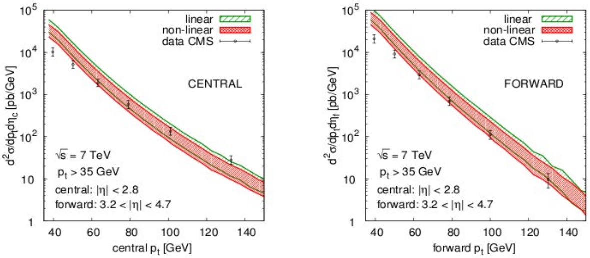
$$[p| = \left((\mathcal{E} L(p))^T \, , \, \mathbf{0} \right) \qquad \langle p| = \left(\mathbf{0} \; (\mathcal{E}^T R(p))^T \right) \qquad \text{where} \quad \mathcal{E} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Example: central-forward dijets production

Hybrid factorization, (Deak, Hautmann, Jung, Kutak, '09):

$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp}\,dx_1\,dx_2\,\mathcal{F}(x_1,k_{1\perp},\mu)\,f(x_2,\mu)\,\hat{\sigma}\;(x_1,x_2,k_{1\perp},\mu)$$

Kutak, Sapeta, '12:



- Reasonable agreement with data
- No traditional parton showers: the Unintegrated PDF as a parton shower.
- Hybrid factorization formula for dijet production (fully differential) can be derived from Color-Glass-Condensate P. Kotko, K. Kutak, C. Marquet, E. Petreska, A. van Hameren, JHEP 1509 (2015) 106