Factorization and Resummation for Massive Quark Effects in Exclusive Drell-Yan

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(University of Vienna)

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universität wien

Doktoratskolleg Particles and Interactions

Outline

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Motivation

- m_b-effects frequently relevant at the LHC (e.g. Higgs production)
- systematic treatment for many inclusive processes available
- often missing for exclusive processes
- here: quark mass effects in p_T-spectrum for Drell-Yan + 0 jets (i.e. small p_T)

Drell-Yan at small p_T

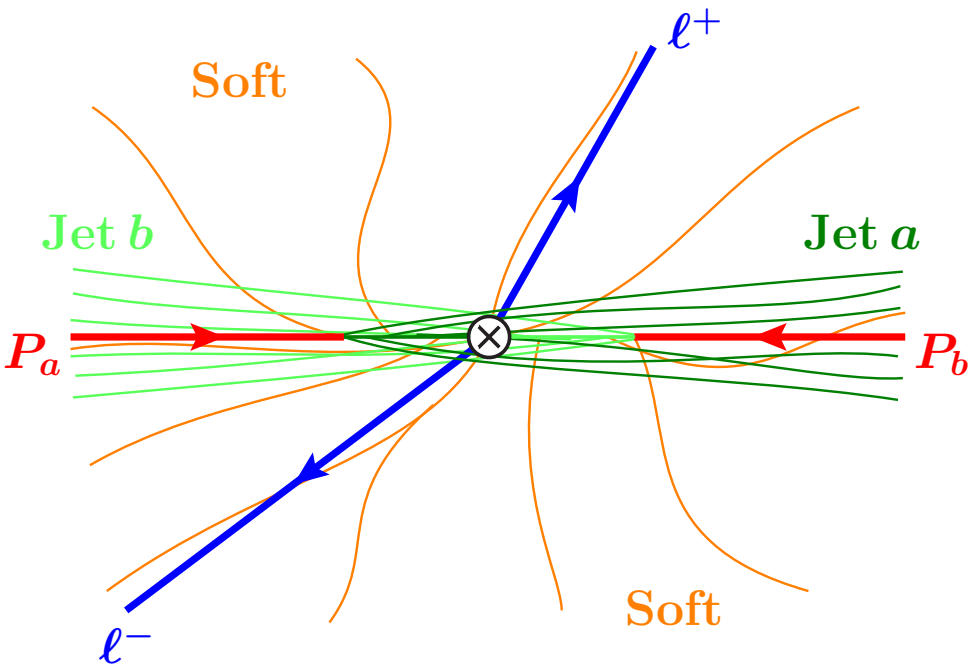
- Drell-Yan + 0 Jets
  different jet vetoes, here: $p_T \equiv |\vec{p}_T^{\,\ell\bar{\ell}}| \ll Q$

- p_T spectrum of Z-boson measured
  with high precision

- NNLL′ analyses available,
  ingredients for N^{3}LL known

- no systematic theoretical description of
  b-quark mass effects yet

- discrepancies between MC and
  experiment in low p_T region



from: [Stewart, Tackmann, Waalewijn (2010)]

our project: systematic treatment of quark mass effects at NNLL′ accuracy for transverse momentum spectrum using EFTs (also for beam thrust)

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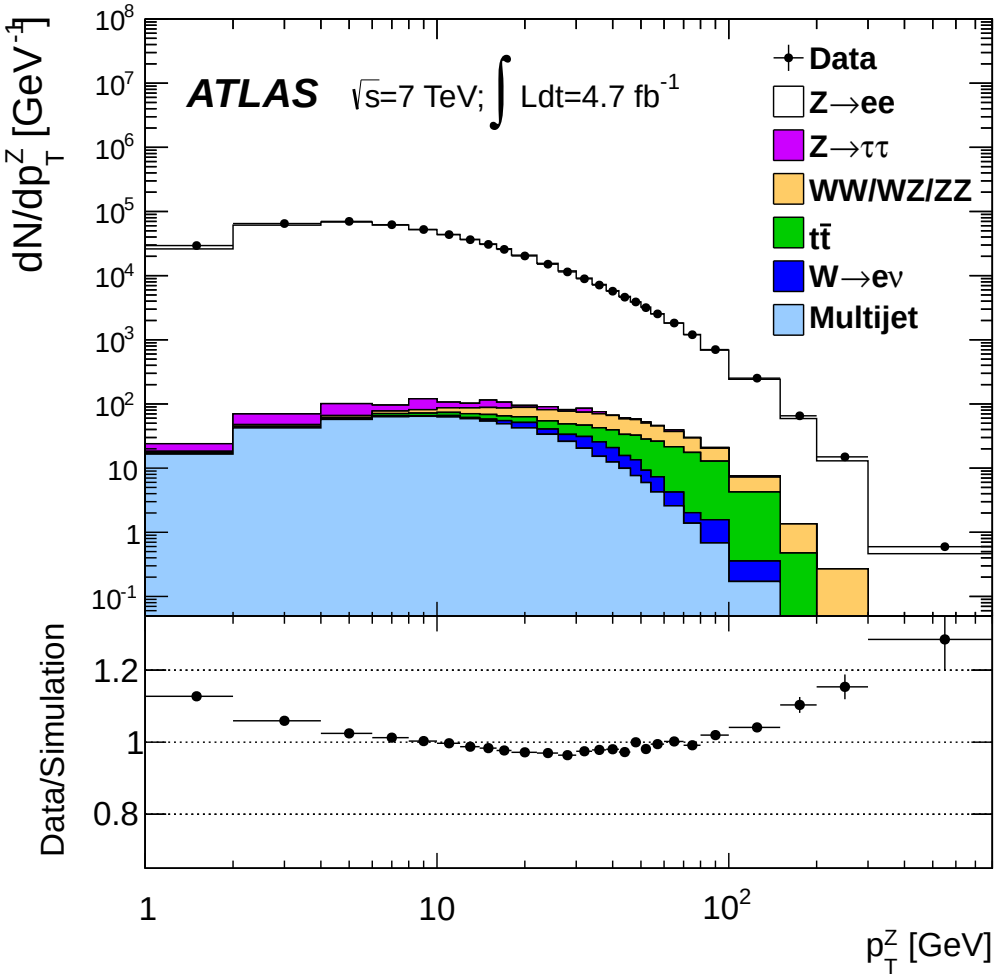

[ATLAS Collaboration (2014)]

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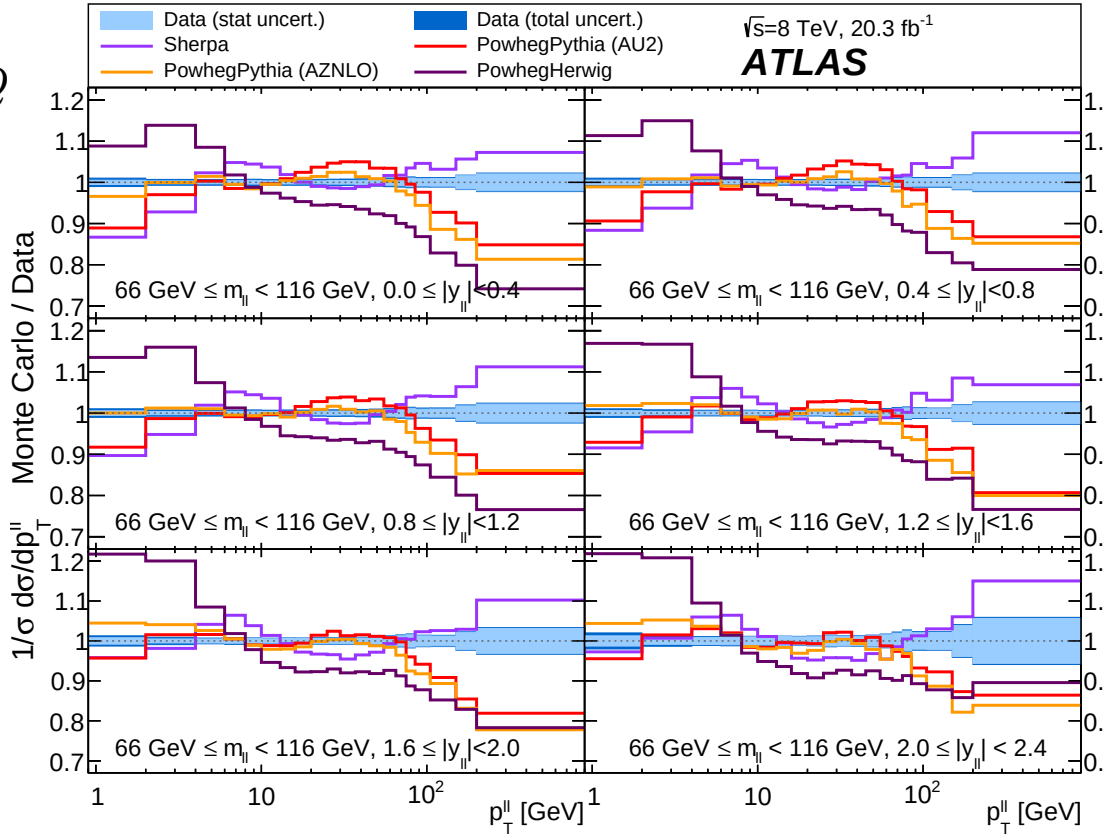

[ATLAS Collaboration (2015)]

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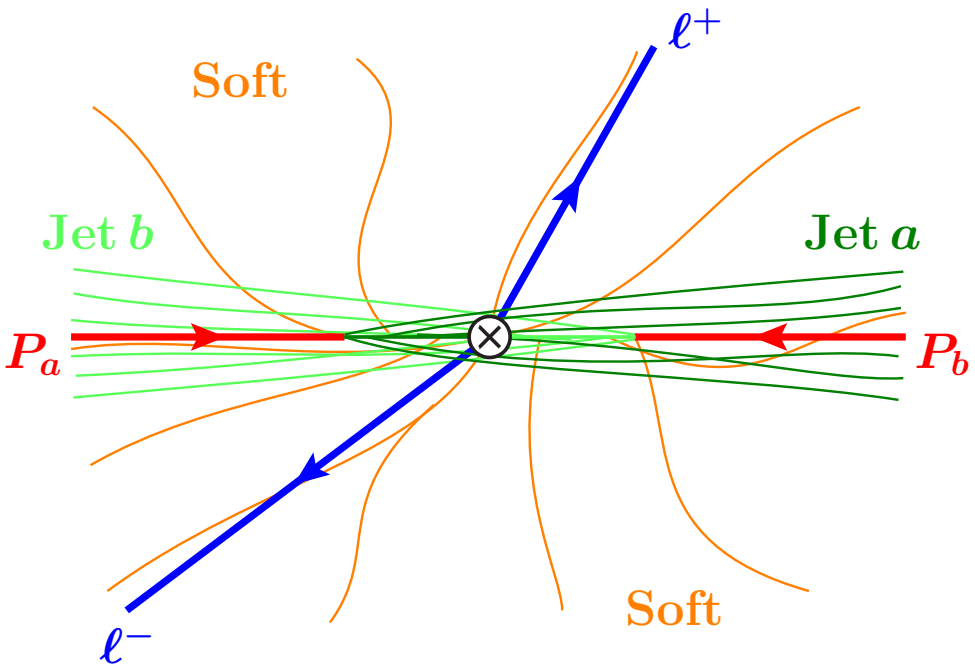
Outline

Massless Factorization

[Collins, Soper, Sterman (1985); Bozzi, Catani, de Florian, Grazzini (2001), Becher, Neubert, Wilhelm (2011);

Echevarria, Idilbi, Scimemi (2011); Chiu, Jain, Neill, Rothstein (2012); . . .]

- involved scales
  - hard process $\mu \sim Q$
  - collinear ISR $\mu \sim p_T$
  - soft ISR $\mu \sim p_T$
  - non pert. collinear proton $\mu \sim \Lambda_{\rm QCD}$

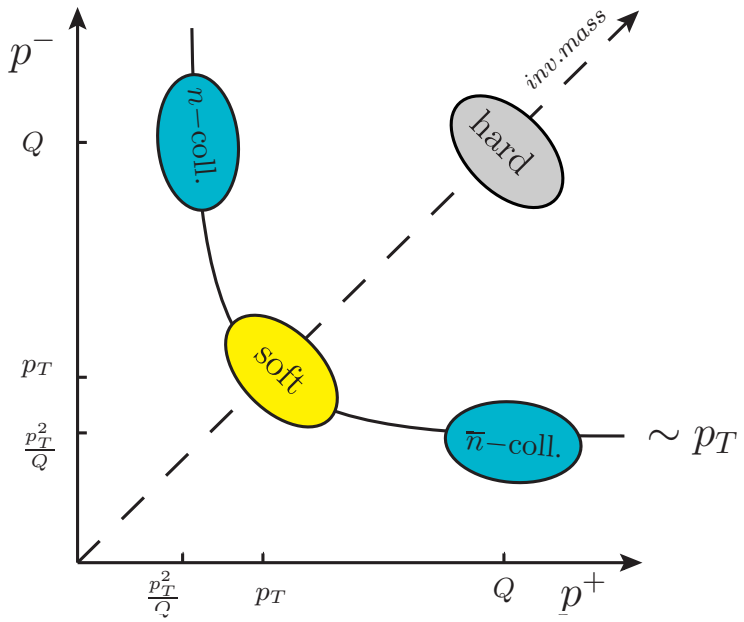


from: [Stewart, Tackmann, Waalewijn (2010)]

- large hierarchies: $Q \gg p_T \gg \Lambda_{\rm QCD}$
  large logarithms: $\log(Q^2/p_T^2),\ \log(p_T^2/\Lambda_{\rm QCD}^2)$

- we use methods of SCET to derive Factorization theorem

Massless Factorization



(non-pert. modes (PDF) not shown)

hard $\mu \sim Q$:

hard function $H^{(n_l)}(Q)$

$n-$coll. $\mu \sim p_T$:

TMD/beam function

$$B_i^{(n_l)}(\vec{p}_T, x) = \sum_{k\in\{q,g\}} \mathcal{I}_{ik}^{(n_l)}(\vec{p}_T, x) \otimes f_k^{(n_l)}(x)$$

soft $\mu \sim p_T$:

soft function $S^{(n_l)}(\vec{p}_T)$

rapidity divergences cancel between soft and beam functions/TMDs.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T}{Q}\right)$$

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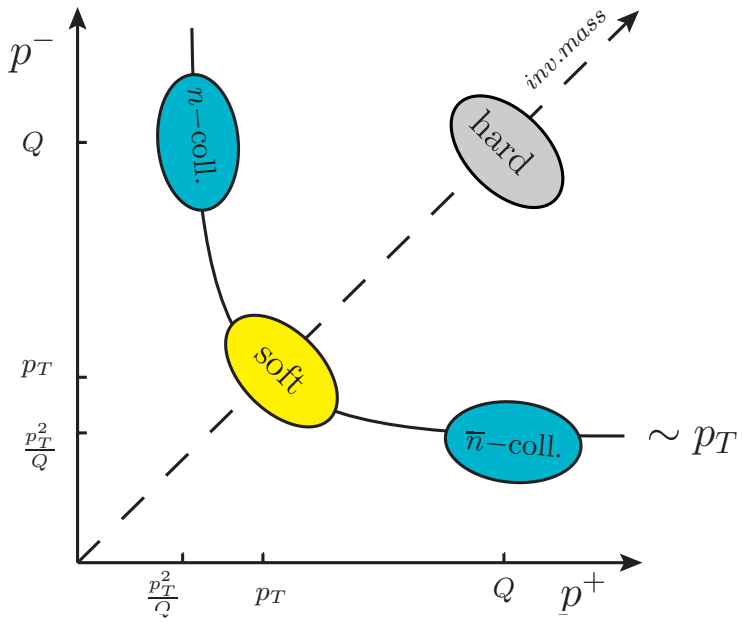

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hard $\mu \sim Q$:

hard function $H^{(n_l)}(Q)$

matching between QCD and SCET current

$$\left(J^\mu_{\rm QCD}\right)^{(n_l)} = C(Q) \times \left(J^\mu_{\rm SCET}\right)^{(n_l)}$$

$$H(Q) = |C(Q)|^2$$

$$J^\mu_{\rm QCD} = \overline{\psi}\,\Gamma^\mu\,\psi$$

$$J^\mu_{\rm SCET} = \overline{\chi}_{\bar{n}}\,\Gamma^\mu\,\chi_n$$

rapidity divergences cancel between soft and beam functions/TMDs.

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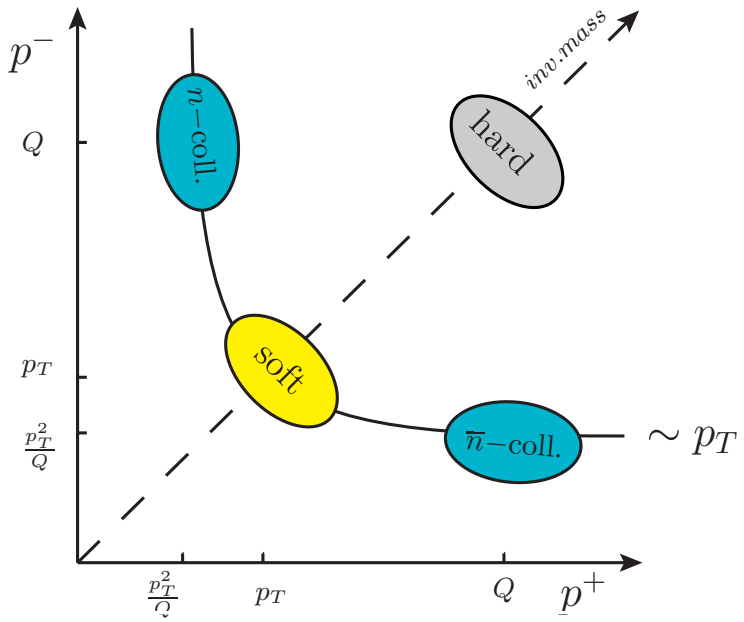

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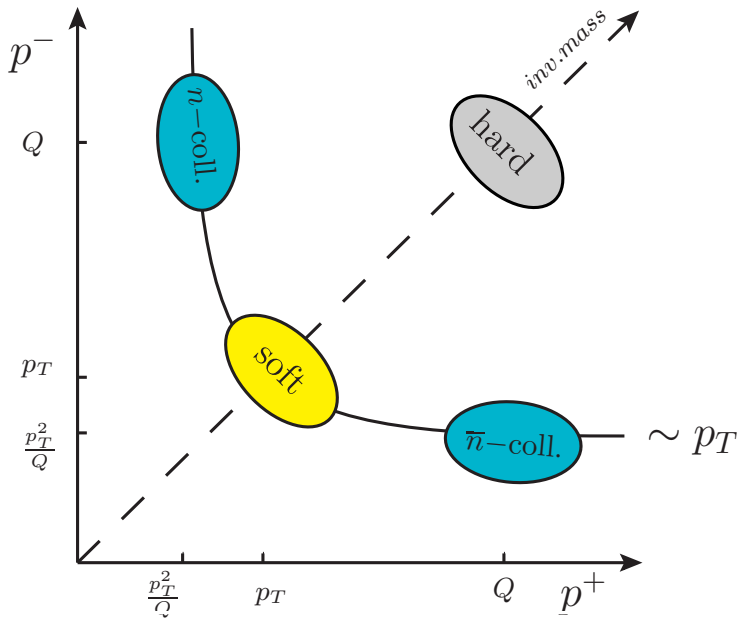

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$$B_q(\vec{p}_T, x) = \theta(x)\langle P|\overline{\chi}_n(0)\frac{\not{\bar{n}}}{2}\left[\delta(p^- x - \mathcal{P}^-)\delta^{(2)}(\vec{p}_T - \vec{\mathcal{P}}_T)\chi_n(0)\right]|P\rangle$$

$$f_q(x) = \theta(x)\langle P|\overline{\chi}_n(0)\frac{\not{\bar{n}}}{2}\left[\delta(p^- x - \mathcal{P}^-)\chi_n(0)\right]|P\rangle$$

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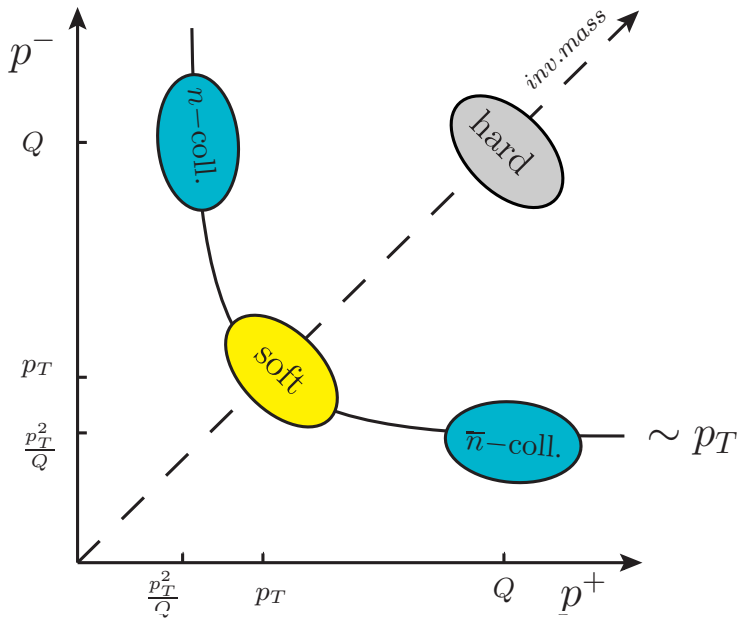

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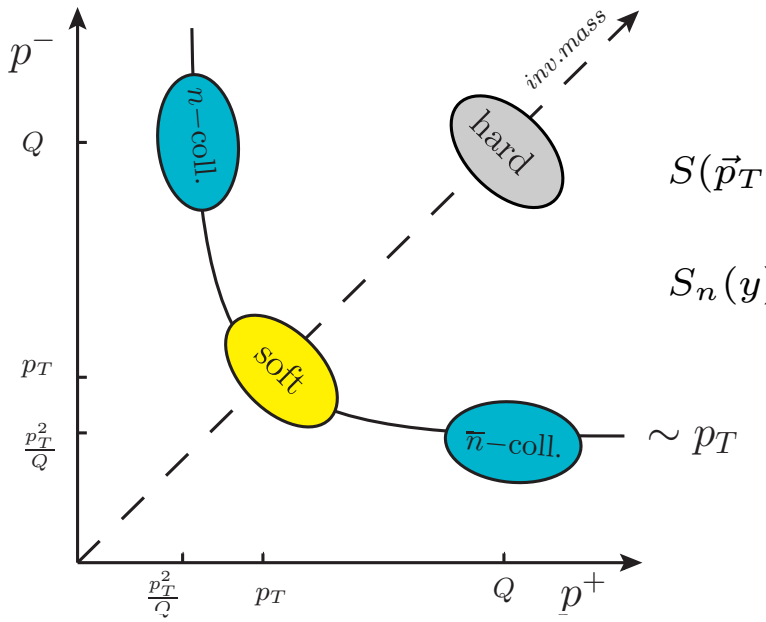

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$$S(\vec{p}_T) = \frac{1}{N_C}\mathrm{tr}\langle 0|\overline{\mathrm{T}}\left[S_n^\dagger(0)S_{\bar{n}}(0)\right]\delta^{(2)}(\vec{p}_T - \vec{\mathcal{P}}_T)\mathrm{T}\left[S_{\bar{n}}^\dagger(0)S_n(0)\right]|0\rangle$$

$$S_n(y) = \mathrm{Pexp}\left[ig_s\int_{-\infty}^{y}\mathrm{d}s\, n\cdot A_s(sn)\right]$$

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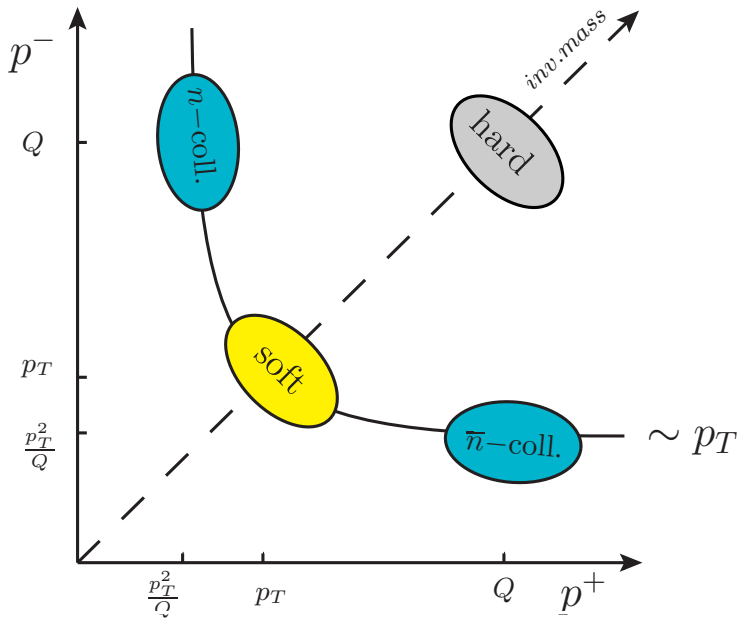

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Outline

Massive Quarks

- massive quarks introduce a new scale into the calculation that leads to new logarithms: $\log\left(\mu_m^2/\mu_i^2\right)$
- to resum those logs we set up a VFNS

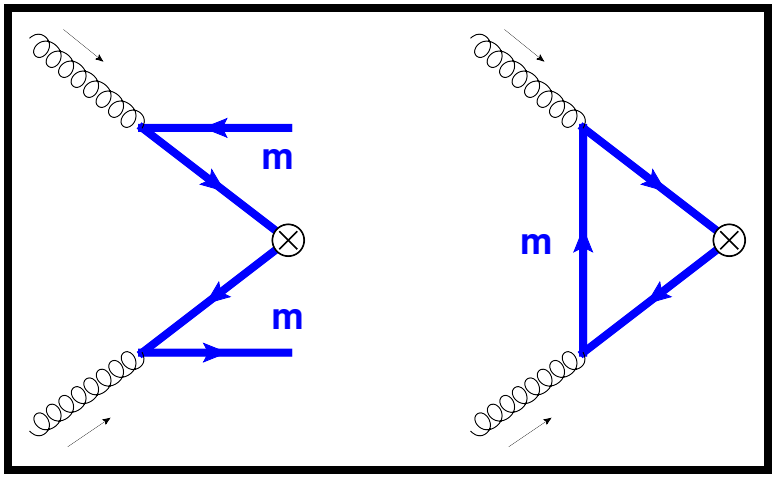
  [Aivazis, Collins, Olness, Tung (1994)]
- massive particles contribute at scales above the mass, are integrated out for scales below the mass
- VFNS for all components: H, B, S

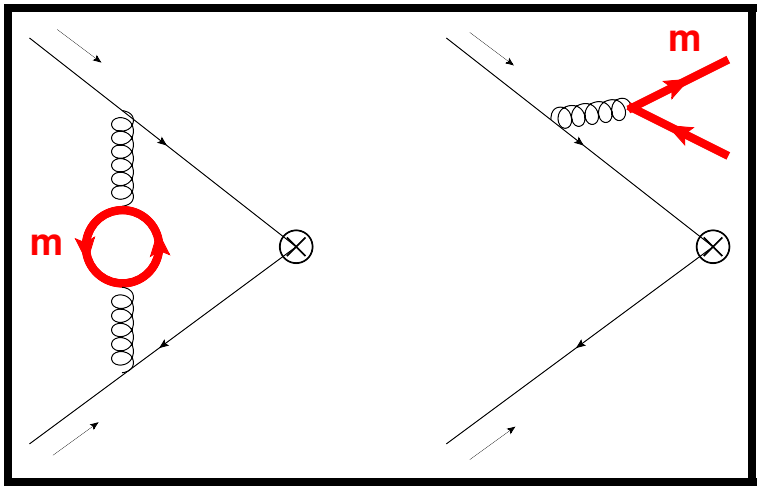
  $e^+e^- \to 2$ jets: [Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz (2014)]

  DIS at large x: [Hoang, Pietrulewicz, D.S. (2016)]
- mass dependent matching factors calculated perturbatively

Massive Quarks

primary and secondary massive quarks.

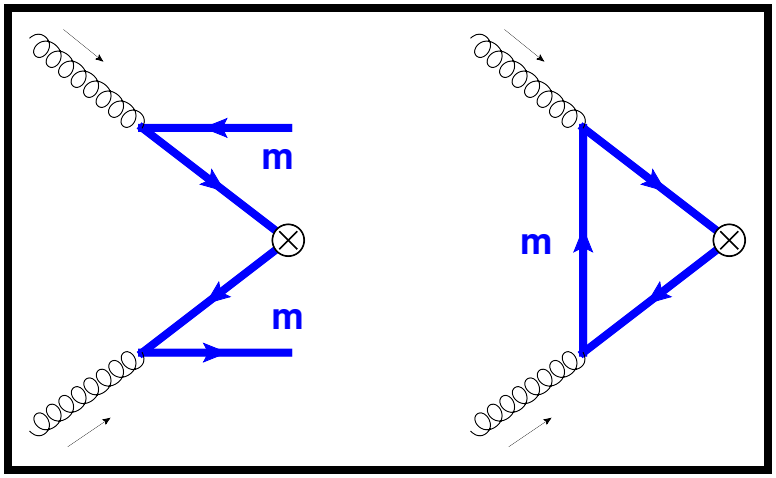


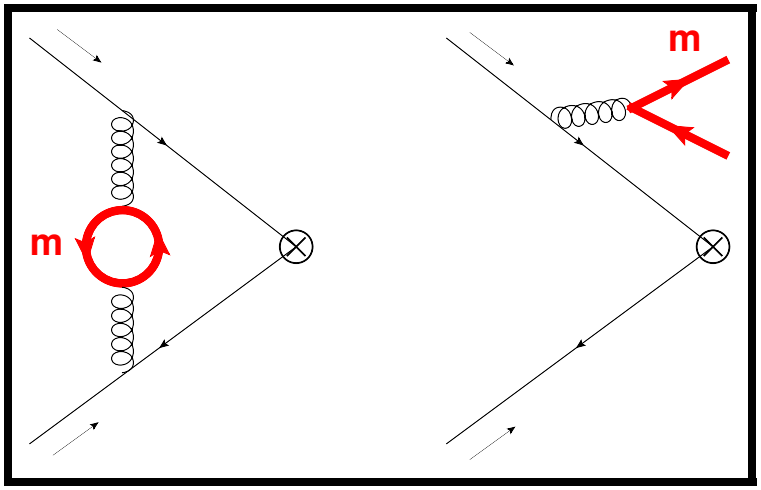


- start at $\mathcal{O}\left(\alpha_s^2\right)$, relevant for NNLL′ resummation
- rapidity logarithms due to (secondary) massive quarks
- secondary massive quarks can contribute to all components: H, B_q, S, change structure of rapidity divergences
- heavy flavor TMDs/PDFs for primary massive quarks for $m \lesssim p_T$: $B_Q\left(f_Q\right)$

Massive Quarks

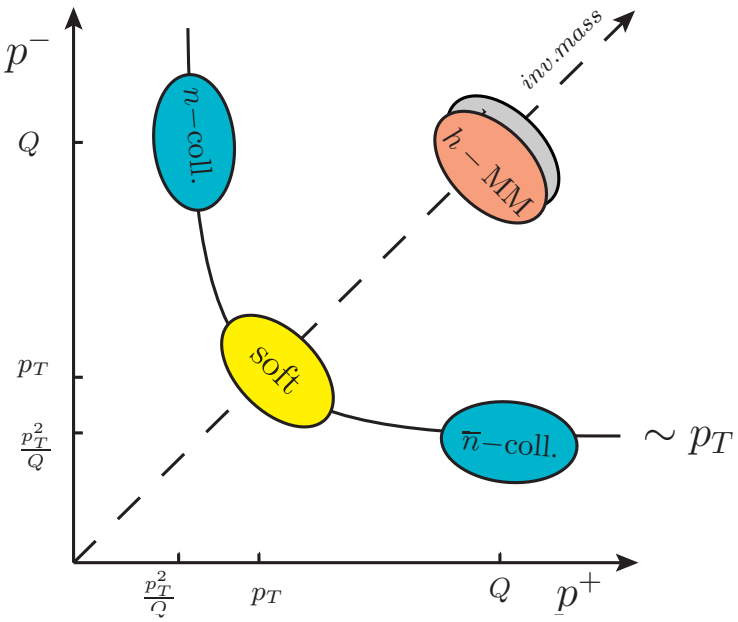
primary and secondary massive quarks.





- introduce new mass-modes: fluctuations around the the mass shell
- integrate out mass-modes at their natural scale $\mu \sim m$
  $\Rightarrow$ additional mass dependent structures in the factorization theorem
- different hierarchies between the mass and the other scales possible
- first assume large hierarchies to derive factorization theorem
- include power corrections between the different theories if necessary

$m \sim Q$



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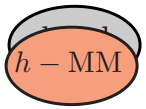


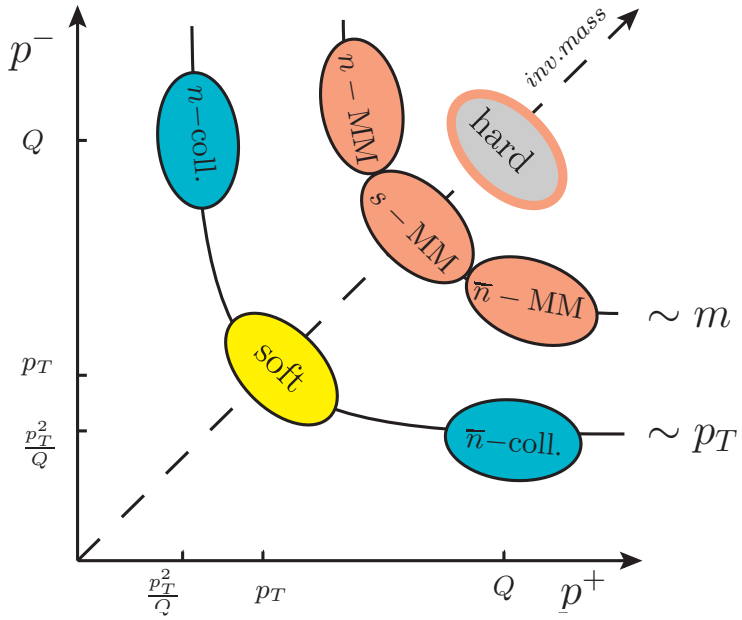
$\mu \sim Q \sim m$:

$$\left(J^{\mu}_{\mathrm{QCD}}\right)^{(n_l+1)} = C(Q,m) \times \left(J^{\mu}_{\mathrm{SCET}}\right)^{(n_l)}$$

hard function with contributions from primary and secondary massive quarks

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}(m) \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

$Q \gg m \gg p_T$



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$\mu \sim Q$:

$$\left(J^\mu_{\rm QCD}\right)^{(n_l+1)} = C(Q) \times \left(J^\mu_{\rm SCET}\right)^{(n_l+1)}$$

hard function with $(n_l + 1)$ massless flavors

$\mu \sim m$:

$$\left(J^\mu_{\rm SCET}\right)^{(n_l+1)} = C_n(m) \times C_{\bar{n}}(m) \times C_s(m) \times \left(J^\mu_{\rm SCET}\right)^{(n_l)}$$

$$H_i(m) = |C_i(m)|^2$$

[Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz (2014)]

[Hoang, Pietrulewicz, D.S. (2016)]

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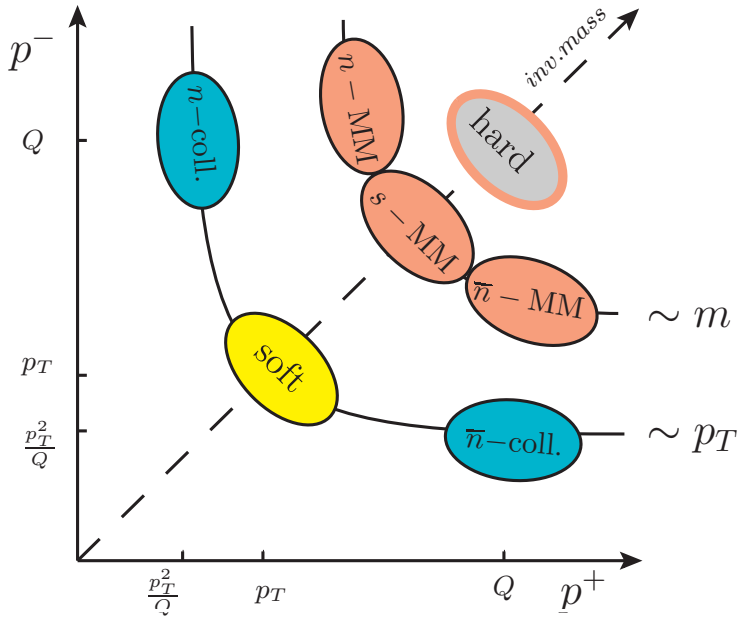

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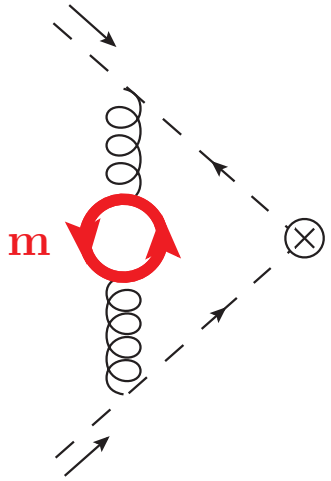

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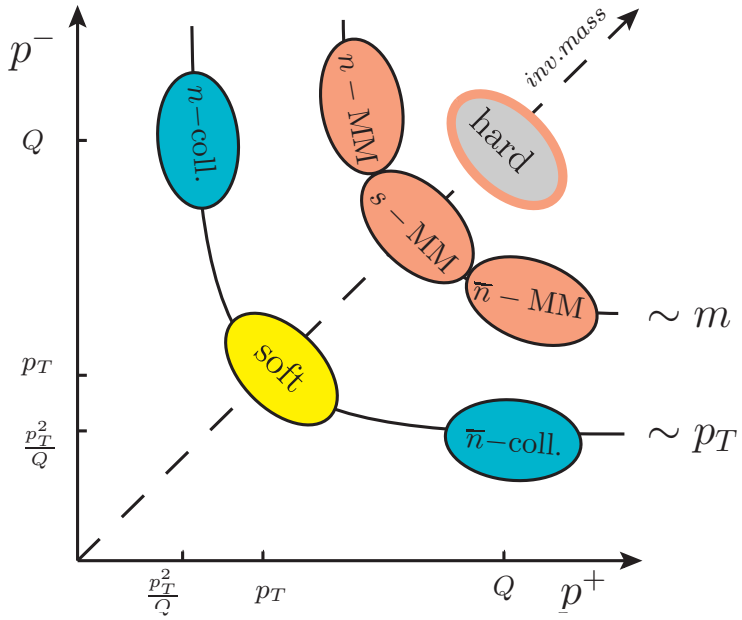

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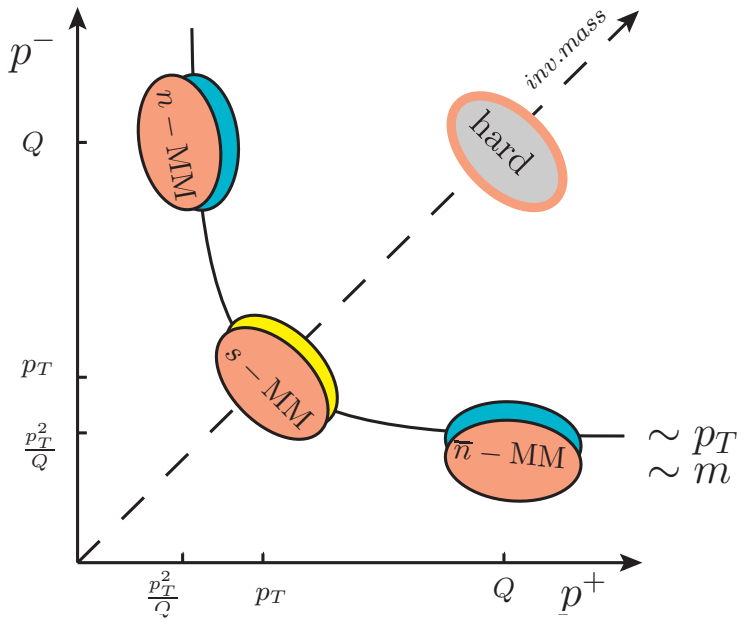

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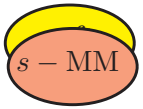
$i \in \{q, Q\}$

one-loop primary massive: $\mathcal{I}_{Qg}(\vec{p}_T, x, m)$ new

two-loop secondary massive: $\mathcal{I}_{qq}(\vec{p}_T, x, m)$ new



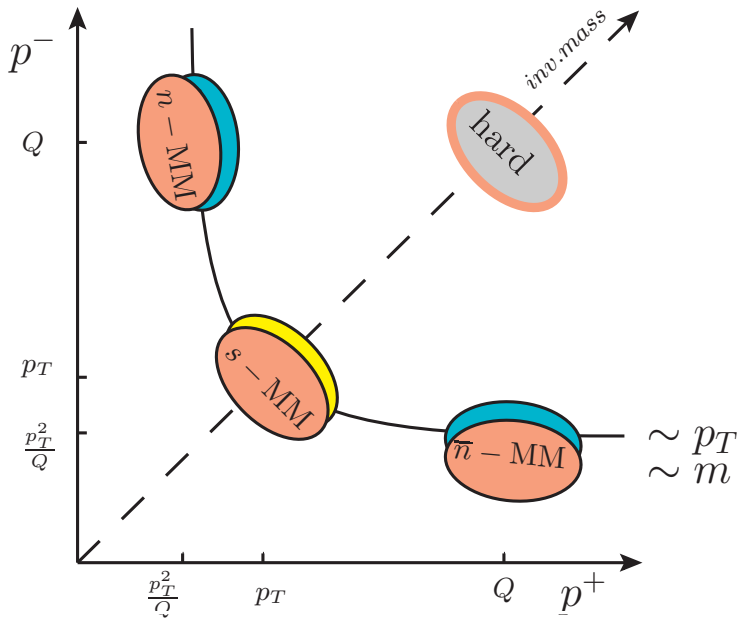
$\mu \sim p_T \sim m$:

$$S^{(n_l+1)}(\vec{p}_T, m)$$

two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$m \sim p_T$



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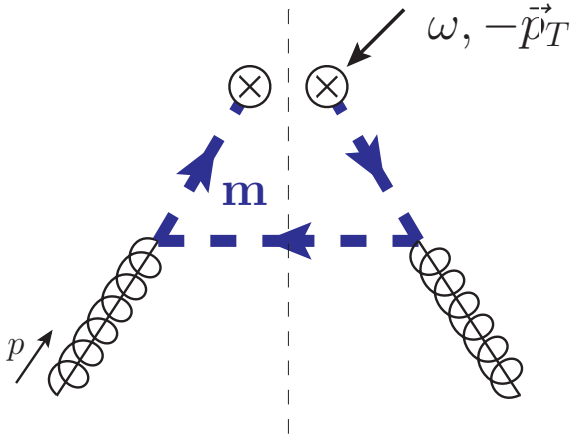
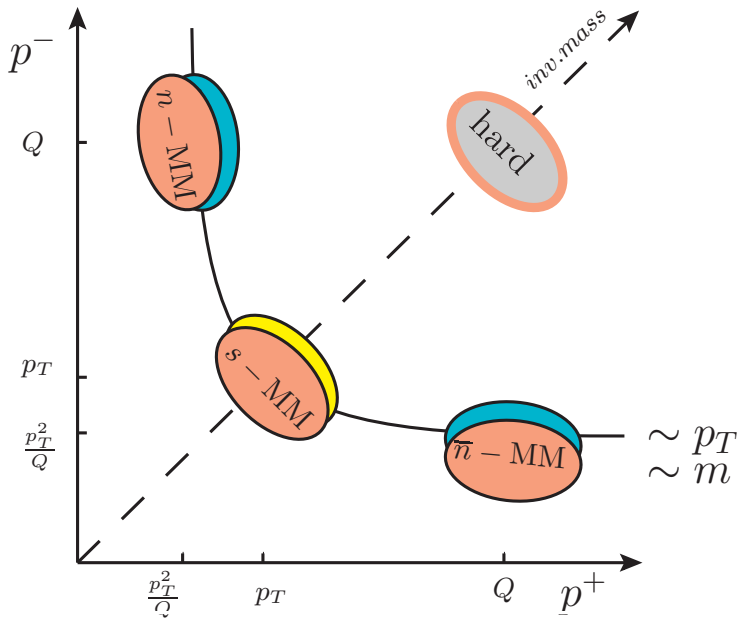


$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$m \sim p_T$



(non-pert. modes (PDF) not shown)

$\mu \sim p_T \sim m$:

$$B_i^{(n_l+1)}(\vec{p}_T, x, m) = \sum_{k\in\{q,g\}} \mathcal{I}_{ik}(\vec{p}_T, x, m) \otimes f_k^{(n_l)}(x)$$

$i \in \{q, Q\}$

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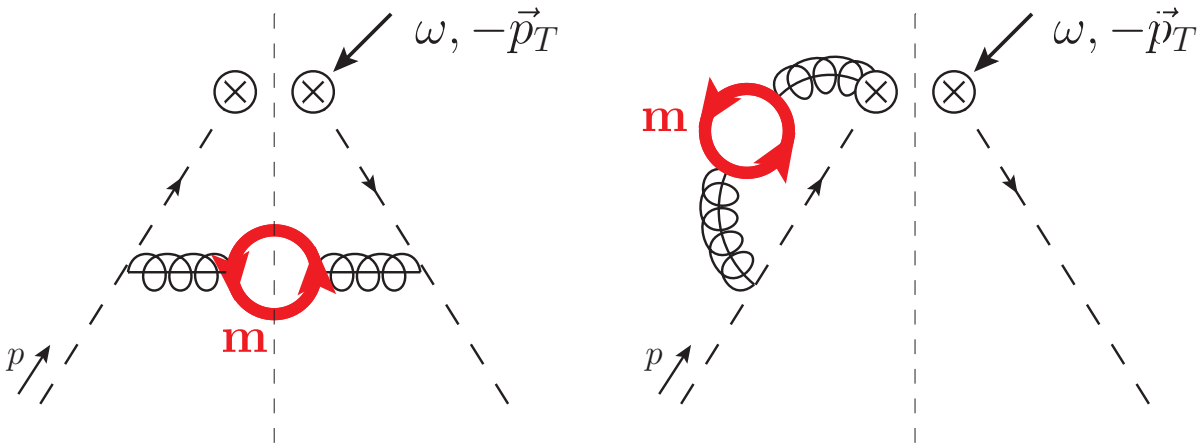

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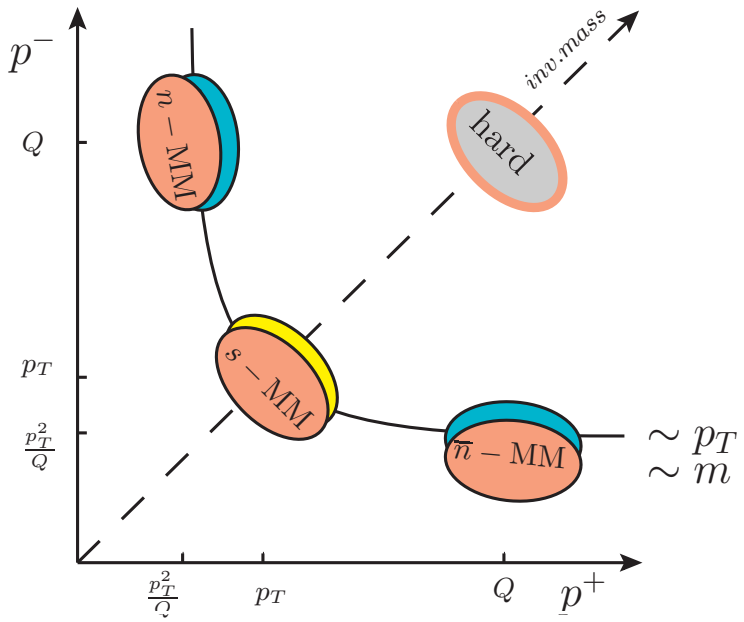

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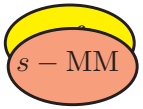


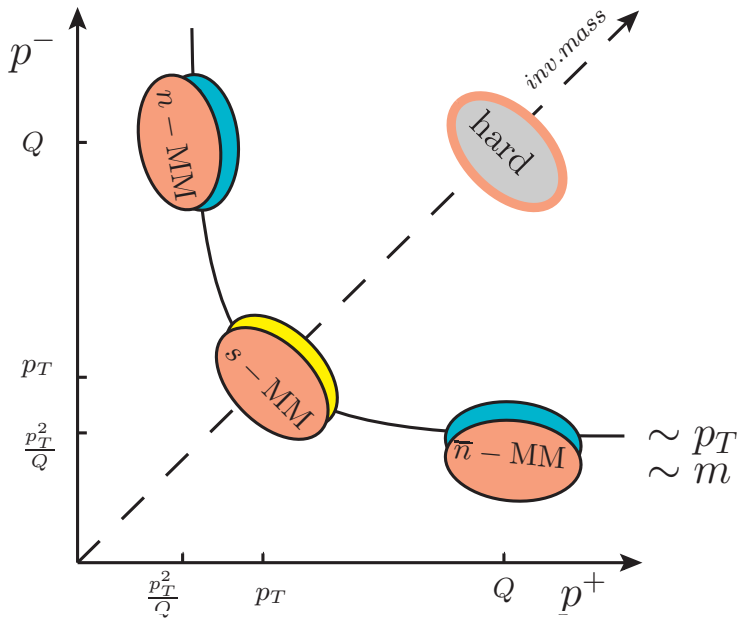
$\mu \sim p_T \sim m$:

$$S^{(n_l+1)}(\vec{p}_T, m)$$

two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$m \sim p_T$



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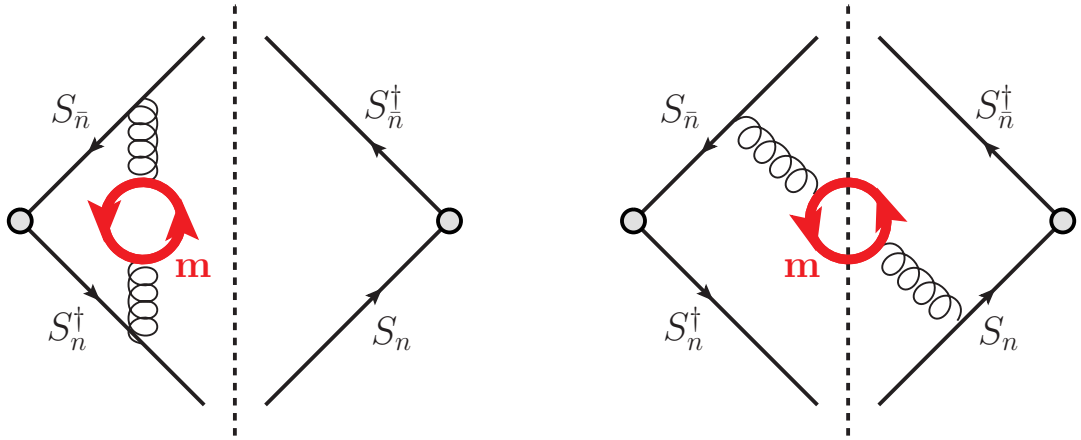

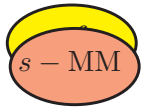

$\mu \sim p_T \sim m$:

$S^{(n_l+1)}(\vec{p}_T, m)$

two-loop secondary massive

new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

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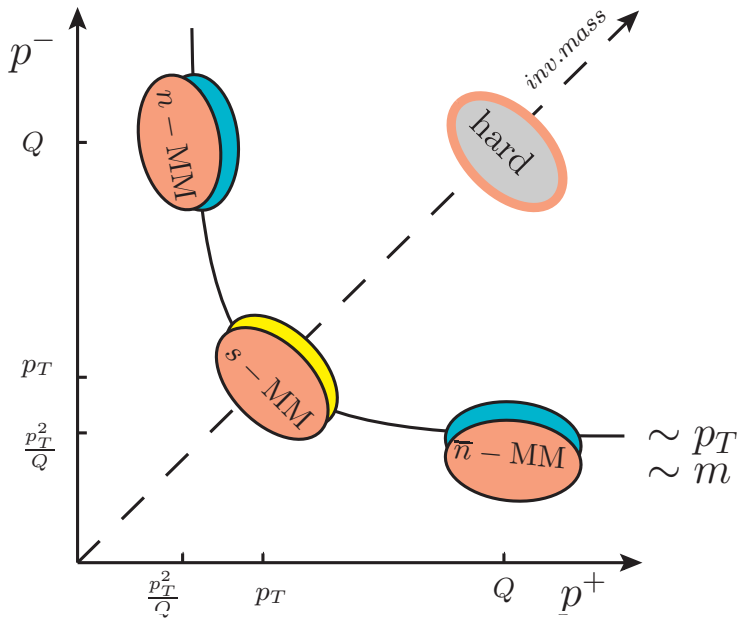

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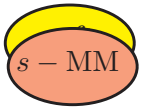

$\mu \sim p_T \sim m$:

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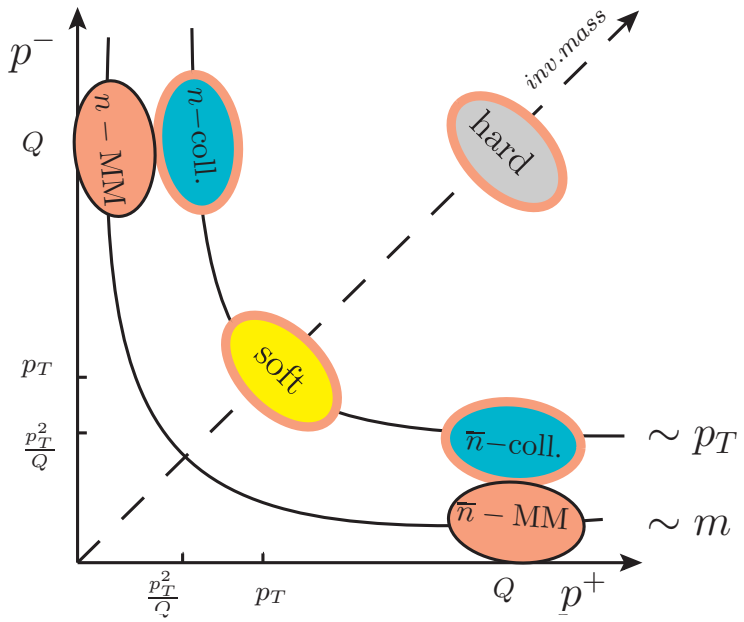
two-loop secondary massive new

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

$m \ll p_T$



(non-pert. modes (PDF) not shown)

$n-$coll. $\mu \sim p_T$:

$$B_i^{(n_l+1)}(\vec{p}_T, x) = \sum_{k\in\{q,Q,g\}} \mathcal{I}_{ik}^{(n_l+1)}(\vec{p}_T, x) \otimes f_k^{(n_l+1)}(x)$$

beam function with (n_l+1) massless flavors

soft $\mu \sim p_T$:

$$S^{(n_l+1)}(\vec{p}_T)$$

soft function with (n_l+1) massless flavors

$n-$MM $\mu \sim m$:

$$f_i^{(n_l+1)}(x, m) = \sum_{k\in\{q,g\}} \mathcal{M}_{ik}(x, m) \otimes f_k^{(n_l)}(x)$$

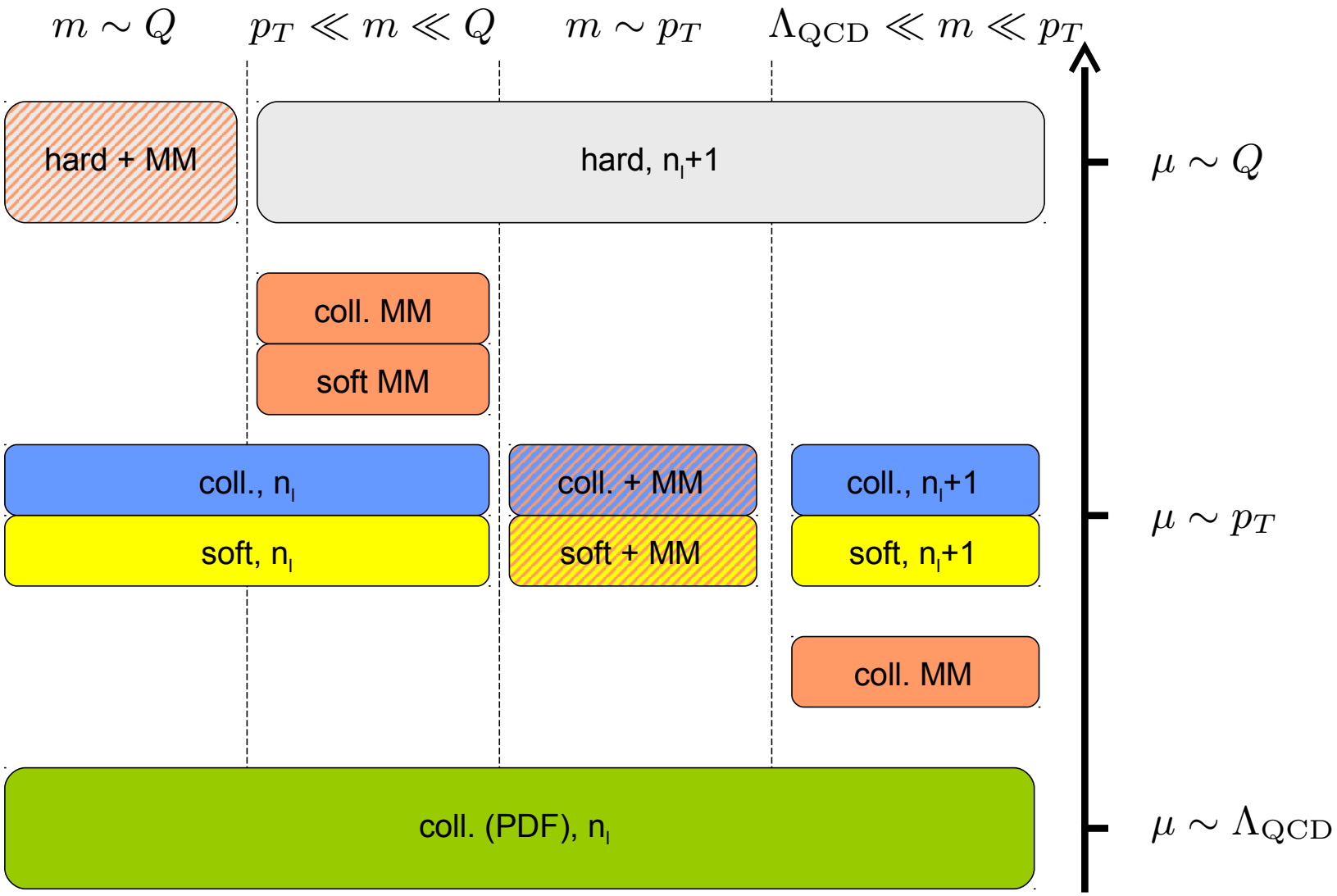
$i \in \{q, Q\}$

one-loop primary massive: $\mathcal{M}_{Qg}(x, m)$
two-loop secondary massive: $\mathcal{M}_{qq}(x, m)$

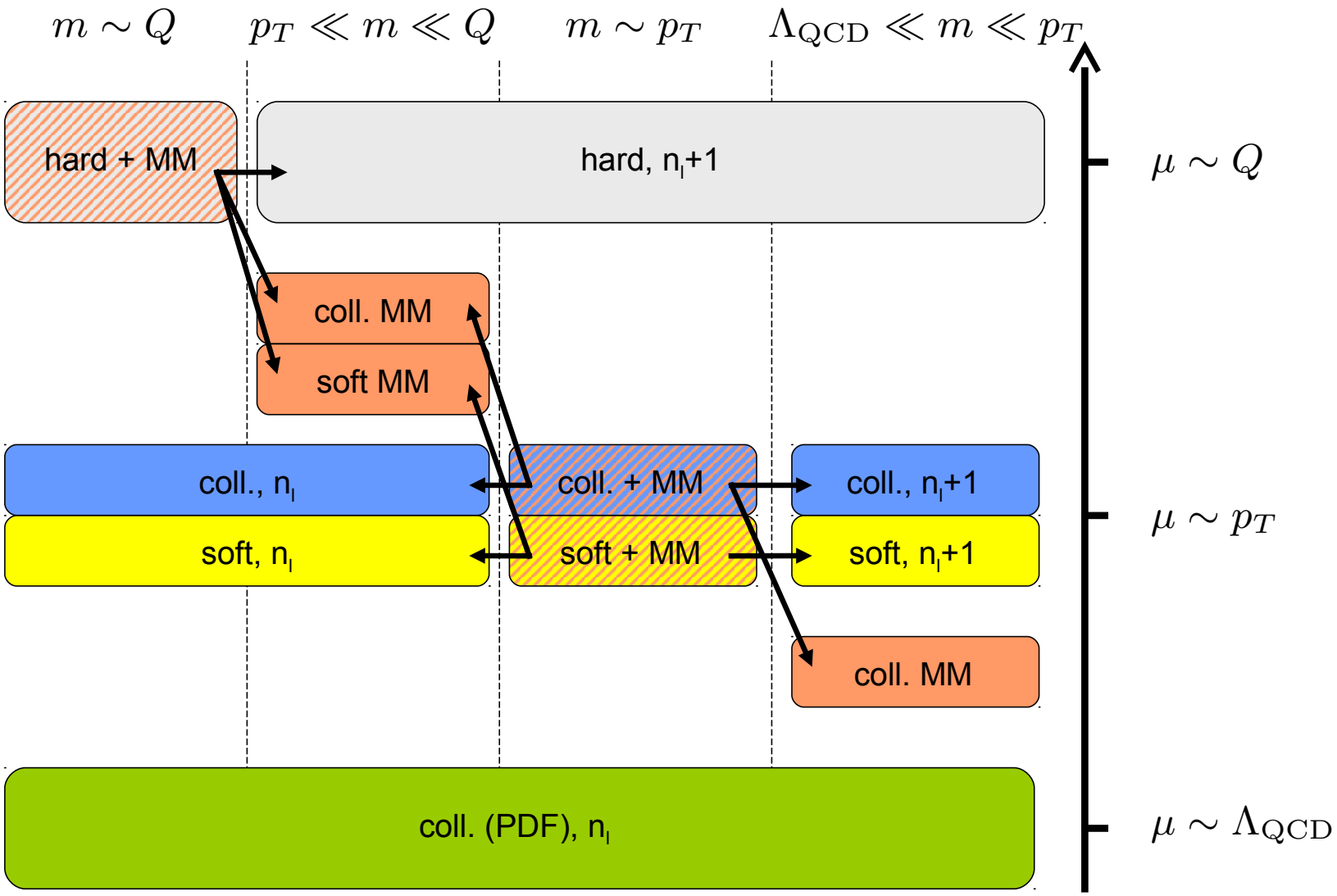
[Buza, Matiounine, Smith, van Neerven (1998)]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{p_T^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

Summary of all Modes



Summary of all Modes



Relations between Hierarchies

components for the different hierarchies are related.

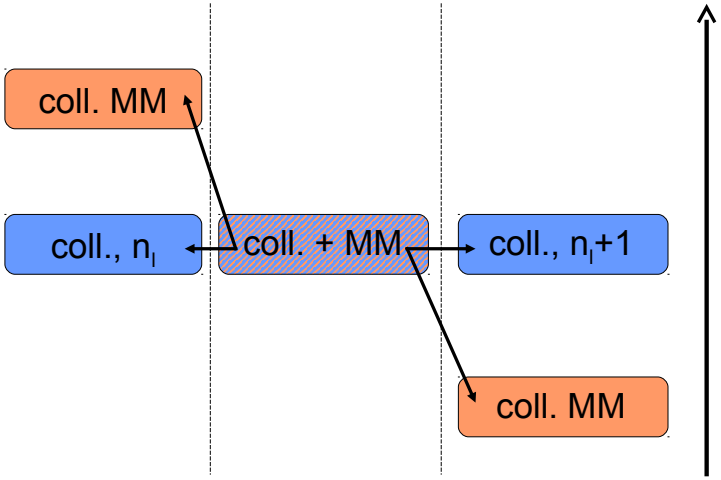
e.g. beam function matching coefficients:

$$\mathcal{I}_{ik}(m) = \mathcal{I}_{ik}^{(n_l)} \times H_n(m) \times \left[1 + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)\right]$$

$$\mathcal{I}_{ik}(m) = \sum_{j\in\{q,Q,g\}} \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \times \left[1 + \mathcal{O}\left(\frac{m^2}{p_T^2}\right)\right]$$

can be used to systematically include all power corrections between the different theories.

$\Rightarrow$ GM-VFNS



GM-VFNS for $m \lesssim p_T$:

Relations between ingredients:

$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \left[1 + \mathcal{O}\Big(\frac{m^2}{p_T^2}\Big)\right], \quad S^{(n_l+1)}(m) = S^{(n_l+1)}\left[1 + \mathcal{O}\Big(\frac{m^2}{p_T^2}\Big)\right]$$

checked explicitly at $\mathcal{O}(\alpha_s^2)$ with known massless results

[Gehrmann, Luebbert, Yang (2012); Luebbert, Oredsson, Stahlhofen (2016)]

define:

$$\mathcal{I}_{ik}^{(n_l+1)}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) + \Big(\mathcal{I}_{ik}(m) - \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m)\Big|_{\mathrm{FO}}\Big)$$

Factorization theorem for $m \lesssim p_T$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l+1)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l+1)}(m)$$

$\Rightarrow$ resums $\ln(m^2/p_T^2)$ & includes all power corrections of $\mathcal{O}(m^2/p_T^2)$

Outline

Resummation of Logs from Massive Quarks

- logs of the form $\ln \frac{\mu_m}{\mu_i}$ with $i = H, B, S$ are resummed in the evolution of the matching factors

- e.g. hard function for $\mu_H \gg \mu_m \gg \mu_B, \quad \mu < \mu_m$:

$$U_H^{n_l+1}(\mu_H, \mu) \times \underbrace{U_{H_n}(\mu_m, \mu) \times U_{H_{\bar{n}}}(\mu_m, \mu) \times U_{H_S}(\mu_m, \mu)}_{U_H^{n_l}(\mu_m,\mu) \times \left(U_H^{n_l+1}(\mu_m,\mu)\right)^{-1}}$$

$$= U_H^{n_l+1}(\mu_H, \mu_m) \times U_H^{n_l}(\mu_m, \mu)$$

additional flavor in the running above μ_m resums $\ln \frac{\mu_m}{\mu_H}$.

- secondary massive quarks introduce new rapidity logarithms

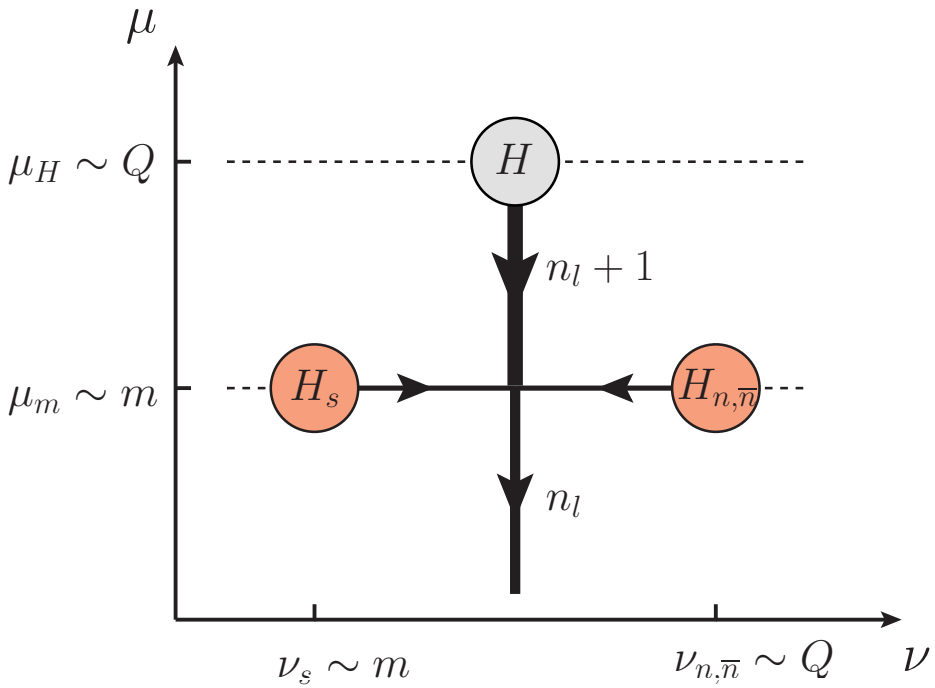
- rapidity logarithms resummed via rapidity RGE

  [Chiu, Jain, Neill, Rothstein (2012)]

- e.g. hard matching functions H_i, $i = n, \bar{n}, s$:

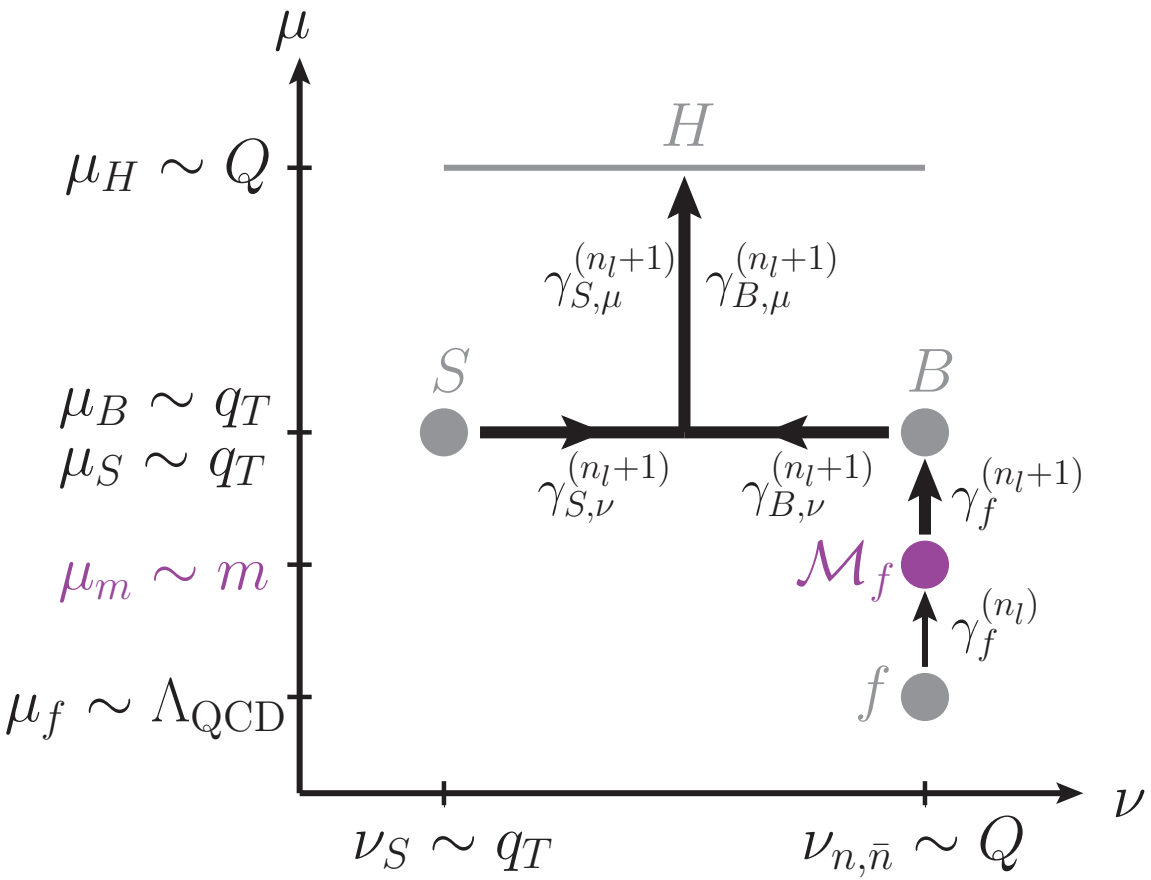
$$H_i(m, \nu) = V_{H_i}(\nu, \nu_i) \times H_i(m, \nu_i)$$

  [Hoang, Pathak, Pietrulewicz, Stewart (2015)]



Resummation for $m \lesssim p_T$

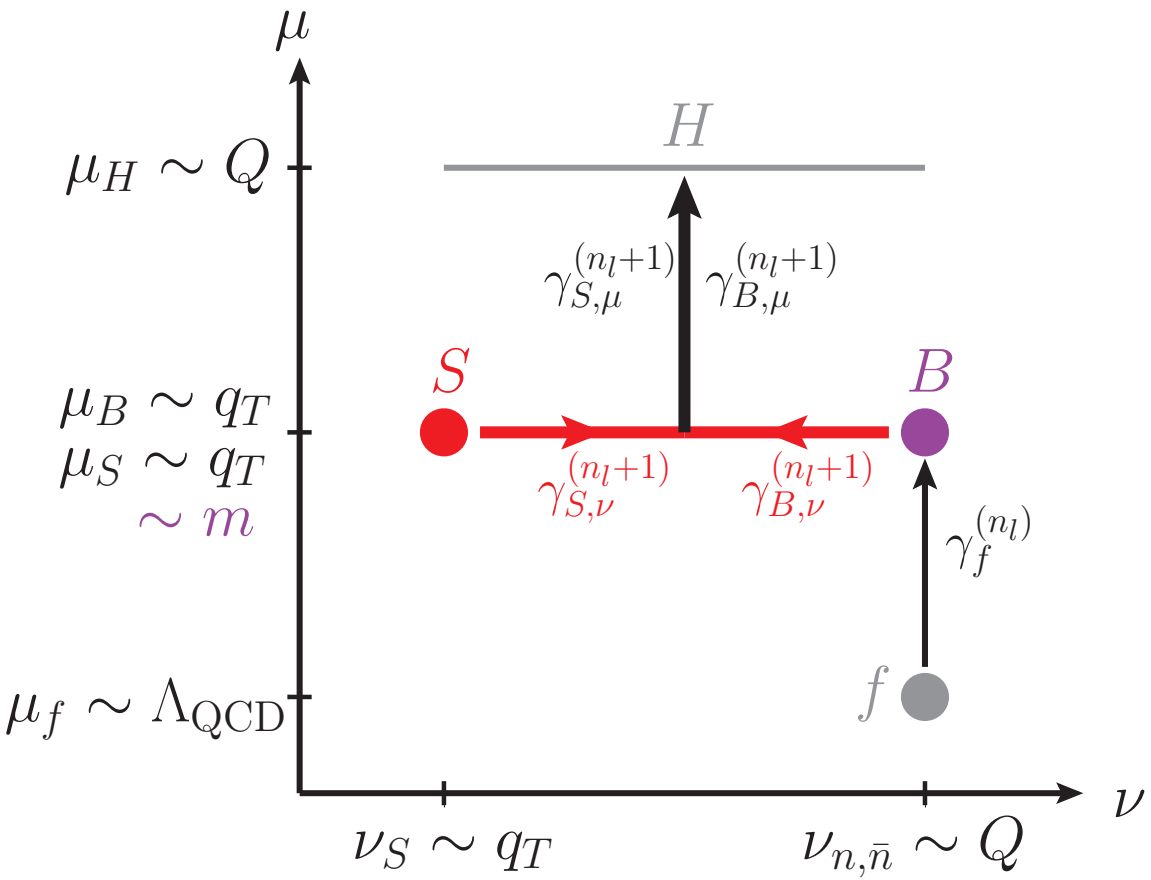
$$\Lambda_{\mathrm{QCD}} \ll m \ll p_T \ll Q:$$



- μ-evolution with $n_l = 4$ quark flavors below the mass scale
- μ-evolution with $n_l + 1 = 5$ quark flavors above the mass scale

Resummation for $m \lesssim p_T$

$$\Lambda_{\rm QCD} \ll m \sim p_T \ll Q:$$



- μ-evolution with $n_l = 4$ quark flavors below the mass scale
- μ-evolution with $n_l + 1 = 5$ quark flavors above the mass scale
- ν-evolution affected by quark mass for $p_T \sim m$ (due to secondary effects)

Resummation of Rapidity Logarithms

- solution of rapidity RGE straight forward for local matching functions.
- large logarithms of μ in rapidity anomalous dimension γ_ν for TMD and soft function
- can be resummed in impact parameter space ($\vec{p_T} \leftrightarrow \vec{b}$)

$$\tilde{\gamma}_\nu(b, m, \mu) = \int_{\ln \mu_0}^{\ln \mu} \mathrm{d} \ln \mu' \, \frac{\mathrm{d}\tilde{\gamma}_\mu(b, \mu', \nu)}{\mathrm{d} \ln \nu} \; + \; \tilde{\gamma}_\nu^{\mathrm{FO}}(b, m, \mu_0)$$

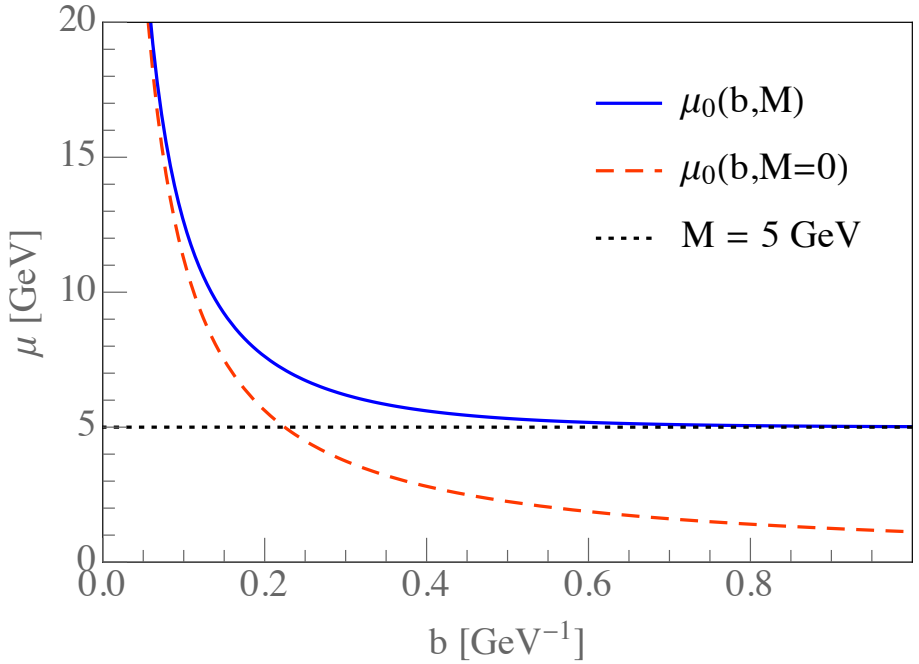
- choose μ_0 such that logs in $\tilde{\gamma}_\nu^{\mathrm{FO}}$ are minimized.

Resummation of γ_ν

one-loop soft function with massive gauge boson:

massless: $\tilde{\gamma}_\nu^{\mathrm{FO}}(b,\mu_0) = -\frac{\alpha_s(\mu_0)C_F}{2\pi^3}\ln\left(\frac{b^2\mu_0^2 \mathrm{e}^{2\gamma_E}}{4}\right)$ $\Rightarrow \mu_0 \sim \frac{2\mathrm{e}^{-\gamma_E}}{b}$

massive: $\tilde{\gamma}_\nu^{\mathrm{FO}}(b,M,\mu_0) = \frac{\alpha_s(\mu_0)C_F}{2\pi^3}\left(\ln\frac{M^2}{\mu_0^2} + 2K_0(b\,M)\right)$ $\Rightarrow \mu_0 \sim M\mathrm{e}^{K_0(b\,M)}$



$$\mu_0(M, b\to 0) \to \frac{2\mathrm{e}^{-\gamma_E}}{b}$$
$$\mu_0(M, b\to \infty) \to M$$

mass introduces IR cutoff

no Landau pole for $b \to \infty$

similar behavior for effects of secondary massive quarks.

correct scheme choice for α_s required in the two limits $b \to 0, b \to \infty$.

Outline

Outlook and Conclusions

Conclusions:

- included massive quarks into the factorization theorem for small p_T region of the spectrum for Drell-Yan + 0 jets
- resummation of all mass related logarithms at NNLL′ accuracy (one and two loop matrix elements with massive quarks)
- same also for beam thrust

Outlook:

- phenomenological analysis of m_b effects in p_T-spectrum
- application to other processes, e.g. $b\bar{b}H$ production

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Thank you for your attention!

Rapidity Anomalous Dimension

soft function rapidity anomalous dimensions at 1-loop:

massless:

$$\gamma_\nu(\vec{p}_T,\mu) = \frac{\alpha_s(\mu)C_F}{4\pi} \times 16\,\mathcal{L}_0(\vec{p}_T,\mu) \qquad \mathcal{L}_0(\vec{p}_T,\mu) = \frac{1}{2\pi\mu^2}\left[\frac{\mu^2}{p_T^2}\right]_+$$

massive gluon:

$$\gamma_\nu(\vec{p}_T,M,\mu) = \frac{\alpha_s(\mu)C_F}{4\pi} \times 8\left[\delta^{(2)}(\vec{p}_T)\ln\frac{M^2}{\mu^2} + \frac{1}{\pi\left(p_T^2+M^2\right)}\right]$$