

# Multi-differential resummation with SCET+

---

Wouter Waalewijn



UNIVERSITY OF AMSTERDAM



QCD evolution 2016

A nighttime photograph of the Amsterdam skyline, showing the iconic Dutch buildings with their characteristic gabled roofs reflected in the water of the canals. The buildings are illuminated from within, creating a warm glow against the dark sky.

# Motivation

---

- Resummation has mostly been restricted to single variables
- Classic example of joint resummation: threshold and transverse momentum [Li; Laenen, Sterman, Vogelsang; ...]
- In this talk, joint resummation of:

- beam thrust

$$\mathcal{T}_0 = \sum_i p_{iT} e^{-|y_i|} = \sum_i \min\{p_i^+, p_i^-\}$$

- transverse momentum

$$\vec{p}_T = \sum_i \vec{p}_{iT}$$

- threshold

$$1 - z = 1 - \frac{Q^2}{\hat{s}}$$

Drell-Yan invariant mass  
partonic c.o.m. energy

- Achieved by adding modes to SCET = SCET+
- Consider  $pp \rightarrow \text{color-singlet} + X$  for definiteness

# Outline

---

1. Resummation in SCET
2.  $p_T$  and beam thrust resummation
3.  $p_T$  and threshold resummation
4. Conclusions

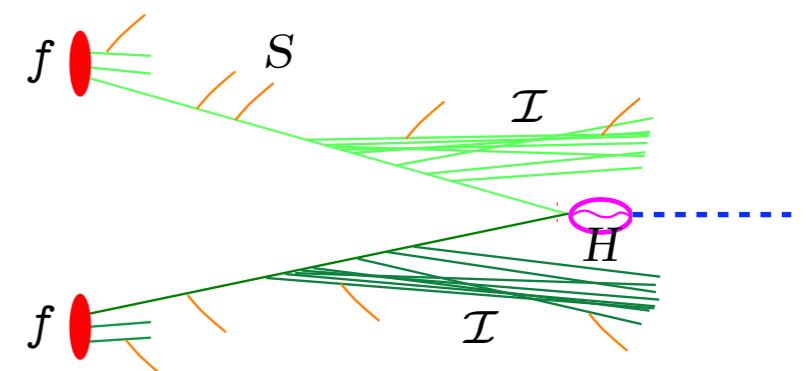
# 1. Resummation in SCET

# Factorization in SCET

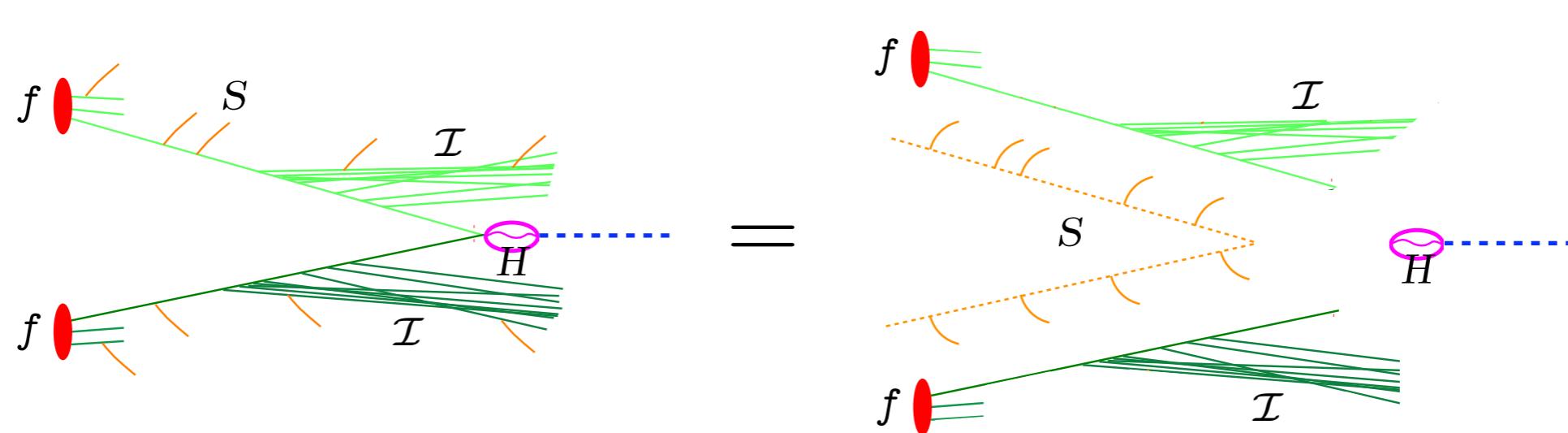
- Effective theory of QCD for **collinear** and **soft** radiation

[Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart, ....]

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{coll}} + \mathcal{L}_{\text{coll}} + \mathcal{L}_{\text{soft}} + \sum_i C_i O_i$$



- Hard virtual corrections are **integrated out**
- Factorize collinear and soft modes in Lagrangian



$$\mathcal{T}_0 = \mathcal{T}_0^{\text{coll}} + \mathcal{T}_0^{\text{coll}} + \mathcal{T}_0^{\text{soft}}$$

# Beam thrust factorization

---

$$\frac{d\sigma}{d\mathcal{T}_0} = H(Q, \mu) \int d\mathcal{T}_0^{\text{coll}} B_a(Q\mathcal{T}_0^{\text{coll}}, x_a, \mu) \int d\mathcal{T}_0^{\text{coll}} B_b(Q\mathcal{T}_0^{\text{coll}}, x_b, \mu)$$
$$\times \int d\mathcal{T}_0^{\text{soft}} S(\mathcal{T}_0^{\text{soft}}, \mu) \delta(\mathcal{T}_0 - \mathcal{T}_0^{\text{coll}} - \mathcal{T}_0^{\text{coll}} - \mathcal{T}_0^{\text{soft}})$$

- Hard function contains hard virtual corrections at the scale  $Q$
- Beam functions describe the contribution from collinear radiation at scale  $\sqrt{Q\mathcal{T}_0}$  and parton distribution functions

$$B_i(t = Q\mathcal{T}_0^{\text{coll}}, x, \mu) = \sum_{i'} \int \frac{dx'}{x'} \mathcal{I}_{ii'} \left( t, \frac{x}{x'}, \mu \right) f_{i'}(x', \mu)$$

[Stewart, Tackmann, WW]

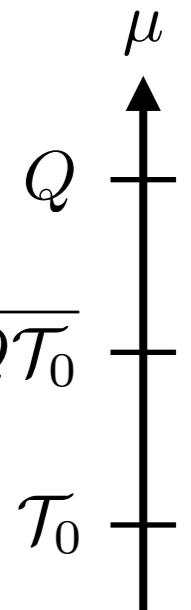
- Soft function captures the soft radiation effects at scale  $\mathcal{T}_0$

# Resummation in SCET

---

- Achieve resummation by evaluating each ingredient at its natural scale and RG evolving to a common scale

$$\mu \frac{d}{d\mu} H(Q, \mu) = \gamma_H(Q, \mu) H(Q, \mu)$$
$$\mu \frac{d}{d\mu} B_i(t, x, \mu) = \int dt' \gamma_B^i(t - t', \mu) B_i(t', x, \mu)$$
$$\dots$$

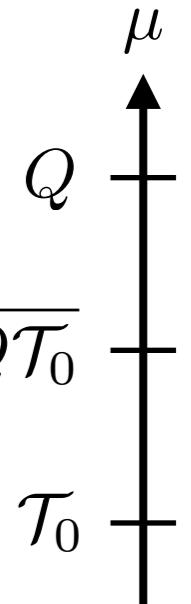


# Resummation in SCET

- Achieve resummation by evaluating each ingredient at its natural scale and RG evolving to a common scale

$$\mu \frac{d}{d\mu} H(Q, \mu) = \gamma_H(Q, \mu) H(Q, \mu)$$
$$\mu \frac{d}{d\mu} B_i(t, x, \mu) = \int dt' \gamma_B^i(t - t', \mu) B_i(t', x, \mu)$$

...



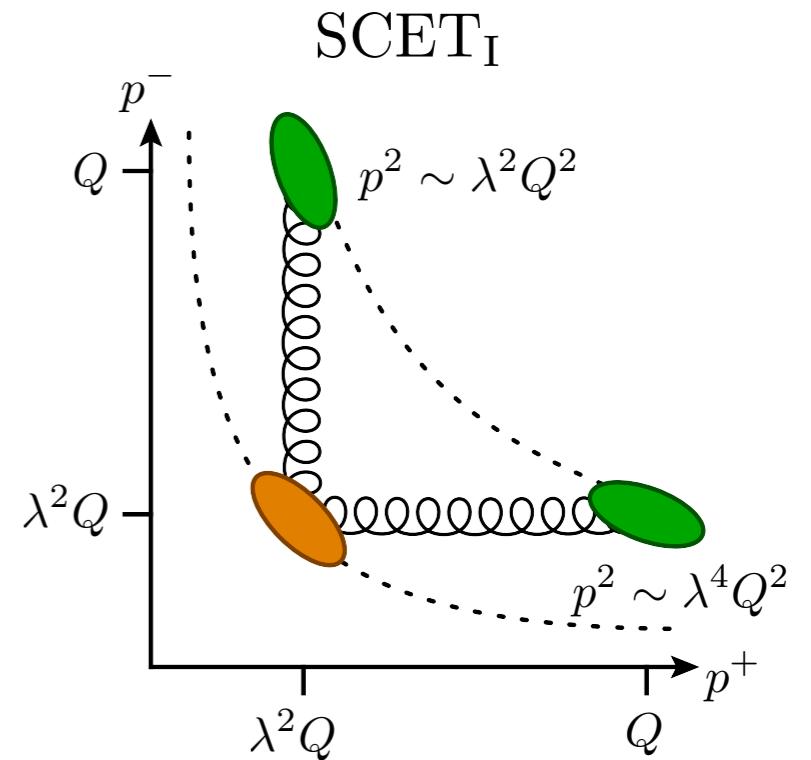
- Features of SCET:
  - Manifest gauge invariance
  - Matrix element definitions of ingredients. E.g.

$$S(\mathcal{T}_0^{\text{soft}}, \mu) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T}[Y_n^\dagger Y_{\bar{n}}] \delta(\mathcal{T}_0^{\text{soft}} - \hat{\mathcal{T}}_0) T[Y_{\bar{n}}^\dagger Y_n] | 0 \rangle$$

Wilson lines  
measurement

# Modes and power counting

|                      | SCET <sub>I</sub>                    |
|----------------------|--------------------------------------|
| $n$ -collinear       | $Q(\lambda^2, 1, \lambda)$           |
| $\bar{n}$ -collinear | $Q(1, \lambda^2, \lambda)$           |
| soft                 | $Q(\lambda^2, \lambda^2, \lambda^2)$ |

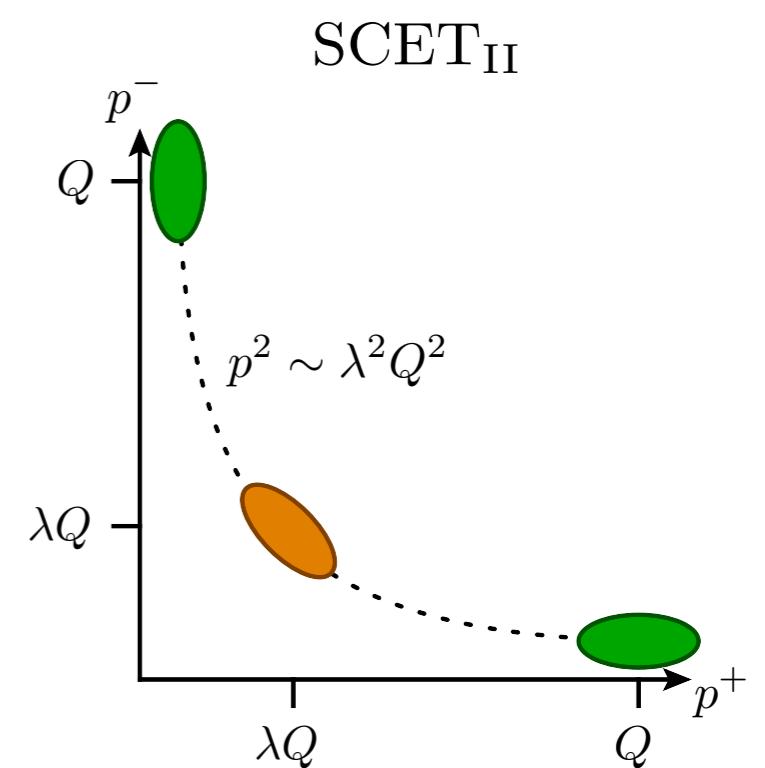
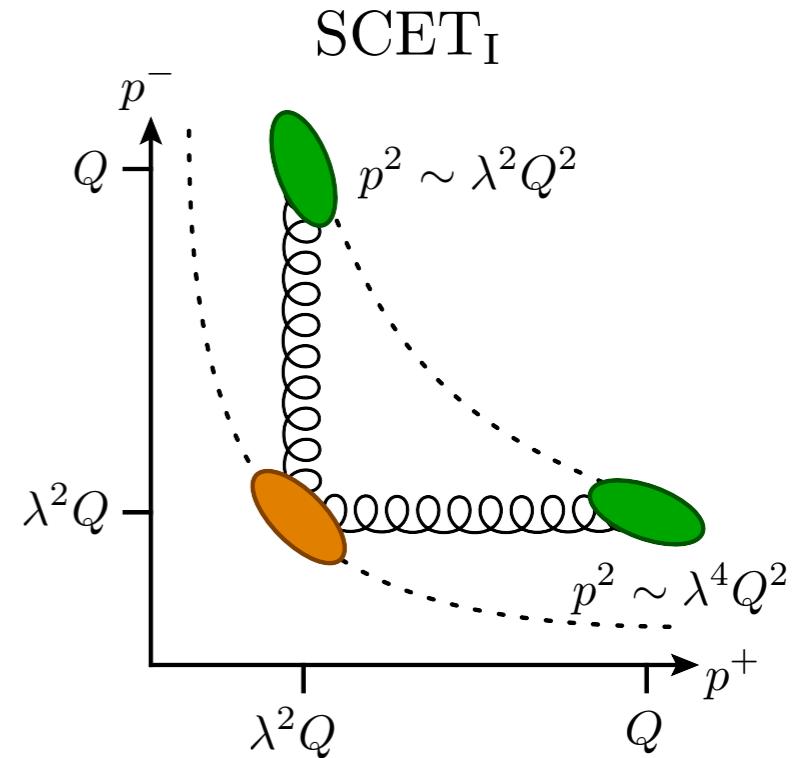


- Light cone coordinates  
 $p^\mu = (p^+, p^-, p_\perp^\mu) = (n \cdot p, \bar{n} \cdot p, p_\perp^\mu)$
- Measurement determines modes:
  - Beam thrust is described by SCET<sub>I</sub> with  $\lambda^2 = \mathcal{T}_0/Q$

# Modes and power counting

|                      | SCET <sub>I</sub>                    | SCET <sub>II</sub>             |
|----------------------|--------------------------------------|--------------------------------|
| $n$ -collinear       | $Q(\lambda^2, 1, \lambda)$           | $Q(\lambda^2, 1, \lambda)$     |
| $\bar{n}$ -collinear | $Q(1, \lambda^2, \lambda)$           | $Q(1, \lambda^2, \lambda)$     |
| soft                 | $Q(\lambda^2, \lambda^2, \lambda^2)$ | $Q(\lambda, \lambda, \lambda)$ |

- Light cone coordinates  
 $p^\mu = (p^+, p^-, p_\perp^\mu) = (n \cdot p, \bar{n} \cdot p, p_\perp^\mu)$
- Measurement determines modes:
  - Beam thrust is described by SCET<sub>I</sub> with  $\lambda^2 = \mathcal{T}_0/Q$
  - $p_T$  is a SCET<sub>II</sub> observables,  $\lambda^2 = p_T^2/Q^2$



# Transverse momentum resummation

---

- SCET<sub>II</sub> involves rapidity divergences

$$S^{(1)}(p_T) \propto \alpha_s \frac{\mu^{2\epsilon}}{p_T^{1+2\epsilon}} \int dy$$

# Transverse momentum resummation

---

- SCET<sub>II</sub> involves rapidity divergences

$$S^{(1)}(p_T) \propto \alpha_s \frac{\mu^{2\epsilon} \nu^\eta}{p_T^{1+2\epsilon+\eta}} \int dy |2 \sinh y|^{-\eta}$$

[Chiu, Jain, Neill, Rothstein]

- Many other choices for rapidity regulator

[Collins; Chiu, Fuhrer, Hoang, Kelley, Manohar; Becher, Bell; ...]

# Transverse momentum resummation

- SCET<sub>II</sub> involves **rapidity** divergences

$$S^{(1)}(p_T) \propto \alpha_s \frac{\mu^{2\epsilon} \nu^\eta}{p_T^{1+2\epsilon+\eta}} \int dy |2 \sinh y|^{-\eta}$$

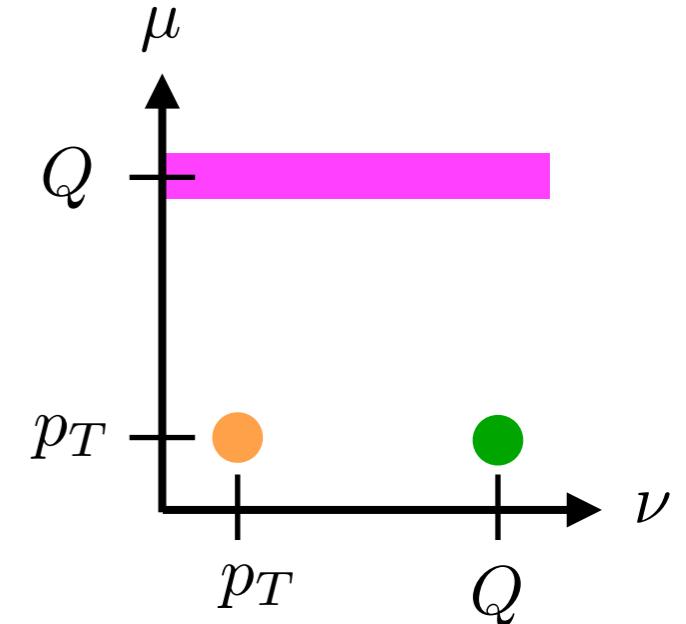
[Chiu, Jain, Neill, Rothstein]

- Many other choices for rapidity regulator  
[Collins; Chiu, Fuhrer, Hoang, Kelley, Manohar; Becher, Bell; ...]

- Rapidity resummation encoded in rapidity RG evolution

$$\begin{aligned} \frac{d\sigma}{d\vec{p}_T} = & H(Q, \mu) \int d\vec{p}_T^{\text{coll}} B_a(\vec{p}_T^{\text{coll}}, x_a, \mu, \nu) \int d\vec{p}_T^{\text{coll}} B_b(\vec{p}_T^{\text{coll}}, x_b, \mu, \nu) \\ & \times \int d\vec{p}_T^{\text{soft}} S(\vec{p}_T^{\text{soft}}, \mu, \nu) \delta(\vec{p}_T - \vec{p}_T^{\text{coll}} - \vec{p}_T^{\text{coll}} - \vec{p}_T^{\text{soft}}) \end{aligned}$$

- E.g.  $\nu \frac{d}{d\nu} B_i(\vec{p}_T, x, \mu, \nu) = \int d\vec{p}_T' \gamma_B^\nu(\vec{p}_T - \vec{p}_T', \mu) B_i(\vec{p}_T', x, \mu, \nu)$



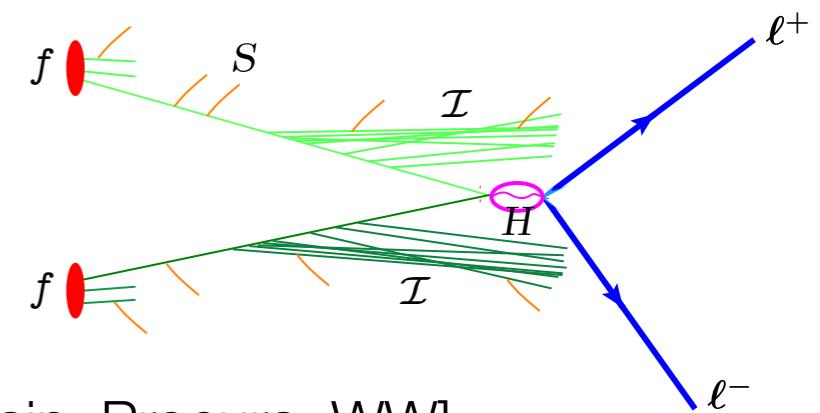
## 2. $p_T$ and beam thrust resummation

Based on arXiv:1410.6483 (Procura, WW, Zeune)

# Beam thrust and transverse momentum resummation

- Make  $\mathcal{T}_0$  or  $p_T$  factorization formulas more differential

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(Q\mathcal{T}_0, \vec{p}_T) \otimes B(Q\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad [\text{Jain, Procura, WW}]$$



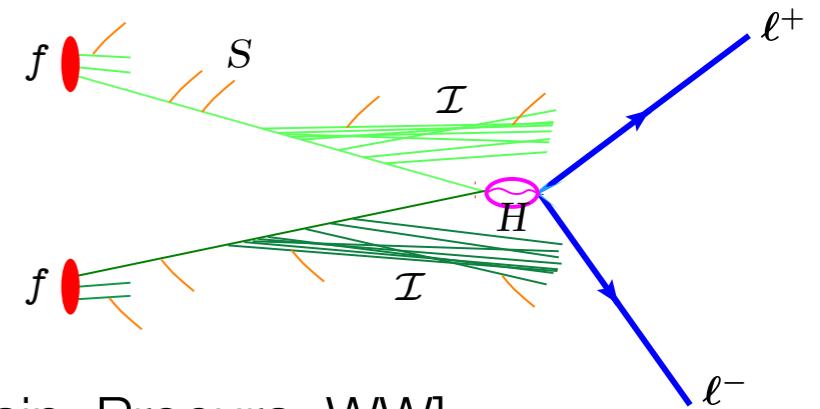
- Structure dictated by power counting

$$\text{SCET}_I : \vec{p}_T = \vec{p}_T^{\text{coll}} + \vec{p}_T^{\text{coll}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad Q\mathcal{T}_0 \sim \vec{p}_T^2$$

# Beam thrust and transverse momentum resummation

- Make  $\mathcal{T}_0$  or  $p_T$  factorization formulas more differential

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(Q\mathcal{T}_0, \vec{p}_T) \otimes B(Q\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad [\text{Jain, Procura, WW}]$$



$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S(\mathcal{T}_0, \vec{p}_T) \quad [\text{Larkoski, Moult, Neill}]$$

- Structure dictated by power counting

$$\text{SCET}_{\text{I}} : \vec{p}_T = \vec{p}_T^{\text{coll}} + \vec{p}_T^{\text{coll}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad Q\mathcal{T}_0 \sim \vec{p}_T^2$$

$$\text{SCET}_{\text{II}} : \mathcal{T}_0 = \mathcal{T}_0^{\text{soft}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad \mathcal{T}_0^2 \sim \vec{p}_T^2$$

# 0-jettiness and transverse momentum resummation

---

- Make  $\mathcal{T}_0$  or  $p_T$  factorization formulas more differential

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(Q\mathcal{T}_0, \vec{p}_T) \otimes B(Q\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad [\text{Jain, Procura, WW}]$$

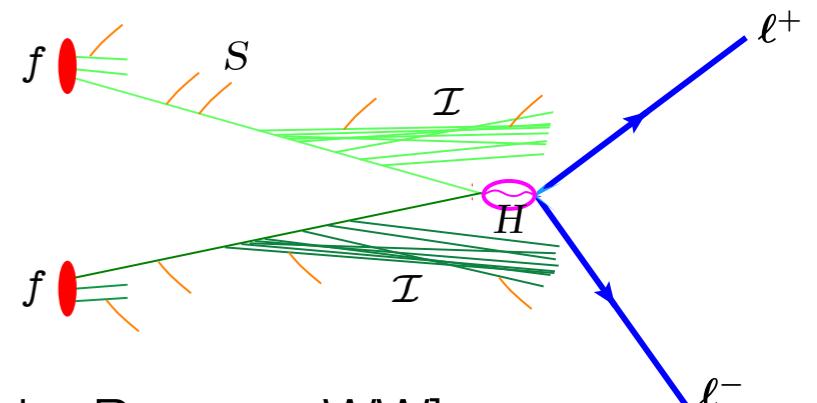
$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S(\mathcal{T}_0) \quad \text{????}$$

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q)B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S(\mathcal{T}_0, \vec{p}_T) \quad [\text{Larkoski, Moult, Neill}]$$

- Structure dictated by power counting

$$\text{SCET}_{\text{I}} : \vec{p}_T = \vec{p}_T^{\text{coll}} + \vec{p}_T^{\text{coll}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad Q\mathcal{T}_0 \sim \vec{p}_T^2$$

$$\text{SCET}_{\text{II}} : \mathcal{T}_0 = \mathcal{T}_0^{\text{soft}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad \mathcal{T}_0^2 \sim \vec{p}_T^2$$



# Beam thrust and transverse momentum resummation

---

- Make  $\mathcal{T}_0$  or  $\vec{p}_T$  factorization formulas more differential

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(Q\mathcal{T}_0, \vec{p}_T) \otimes B(Q\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad [\text{Jain, Procura, WW}]$$

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad [\text{Procura, WW, Zeune}]$$

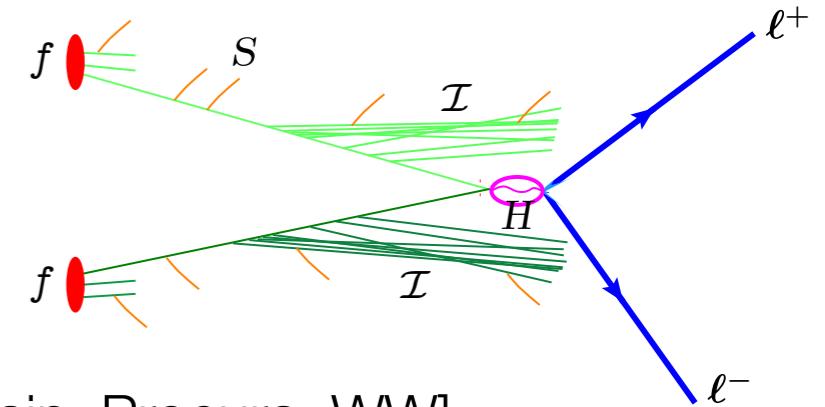
$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S(\mathcal{T}_0, \vec{p}_T) \quad [\text{Larkoski, Moult, Neill}]$$

- Structure dictated by power counting

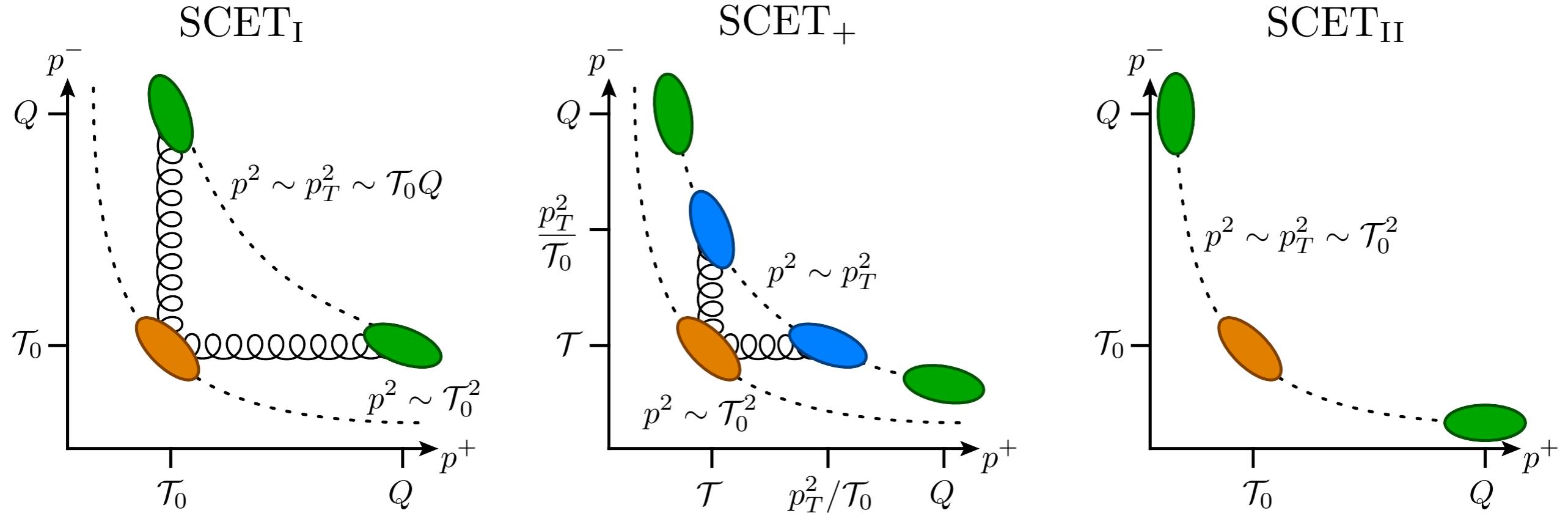
$$\text{SCET}_{\text{I}} : \vec{p}_T = \vec{p}_T^{\text{coll}} + \vec{p}_T^{\text{coll}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad Q\mathcal{T}_0 \sim \vec{p}_T^2$$

$$\text{SCET}_{\text{II}} : \mathcal{T}_0 = \mathcal{T}_0^{\text{soft}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad \mathcal{T}_0^2 \sim \vec{p}_T^2$$

- Intermediate regime requires additional collinear-soft functions

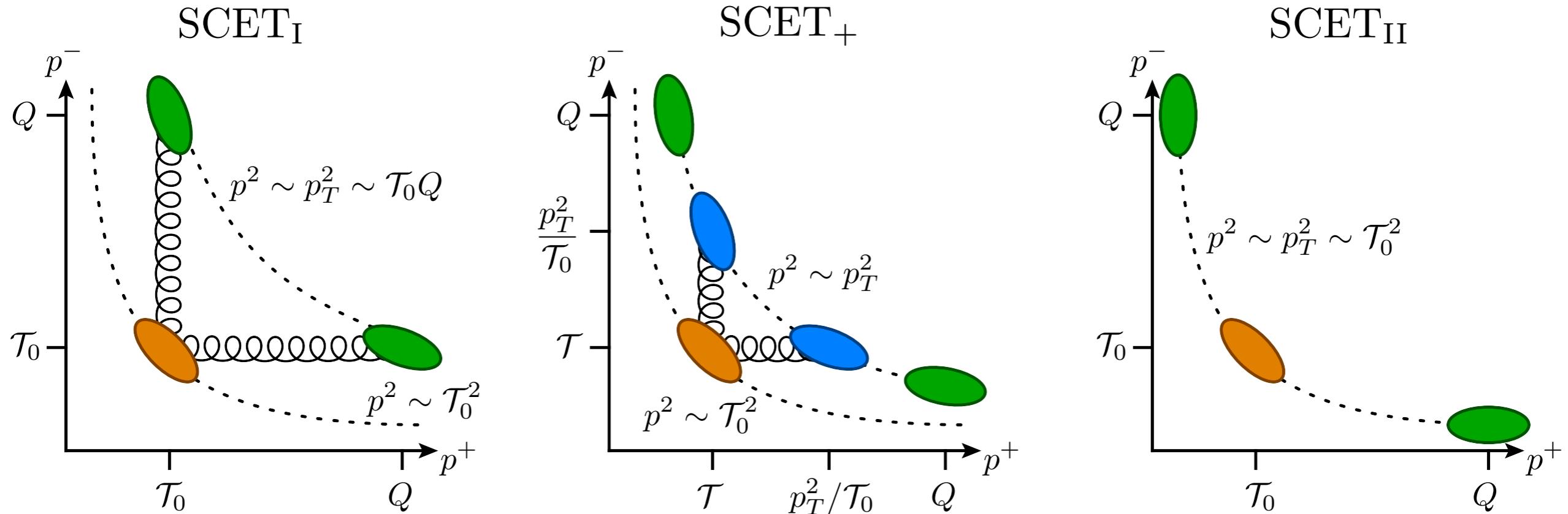


# Modes in SCET+



- Collinear-soft mode uniquely fixed by  $p_T, \mathcal{T}_0$  measurement

# Modes in SCET<sub>+</sub>



- Collinear-soft mode uniquely fixed by  $p_T, \mathcal{T}_0$  measurement
- $n$ -collinear-soft modes behaves for factorization like a
  - soft mode with respect to the  $n$ -collinear mode
  - collinear mode with respect to the rest



[Bauer, Tackmann, Walsh, Zuberi]

# Ingredients of factorization formulas

---

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(Q\mathcal{T}_0, \vec{p}_T) \otimes B(Q\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \quad Q\mathcal{T}_0 \sim \vec{p}_T^2$$

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0)$$

$$\frac{d\sigma}{d\vec{p}_T d\mathcal{T}_0} = H(Q) B(\vec{p}_T) \otimes B(\vec{p}_T) \otimes S(\mathcal{T}_0, \vec{p}_T) \quad \mathcal{T}_0^2 \sim \vec{p}_T^2$$

- $B(\vec{p}_T, x, \mu, \nu)$  transverse momentum beam function
- $S(\mathcal{T}_0, \mu)$  beam thrust soft function
- $B(Q\mathcal{T}_0, \vec{p}_T, x, \mu)$  double differential beam function
- $S(\mathcal{T}_0, \vec{p}_T, \mu, \nu)$  double differential soft function
- $S_+(\mathcal{T}_0, \vec{p}_T, \mu, \nu)$  collinear-soft function

Note:  $p_T$  beam function without  $p_T$  soft function

# Collinear-soft function

---

- Matrix-element of eikonal collinear-soft Wilson lines

$$S_+(\mathcal{T}_0^{\text{csoft}}, \vec{p}_T^{\text{csoft}}) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_{\bar{n}}] \delta(\mathcal{T}_0^{\text{soft}} - \hat{p}^+) \delta(\vec{p}_T^{\text{csoft}} - \hat{\vec{p}}_T) T [V_n^\dagger X_n] | 0 \rangle$$

- Differs from soft function  $S(\mathcal{T}_0^{\text{soft}}, \vec{p}_T^{\text{soft}})$  because the collinear-soft radiation goes into one hemisphere,  $\hat{\mathcal{T}}_0 \rightarrow \hat{p}^+$

# Collinear-soft function

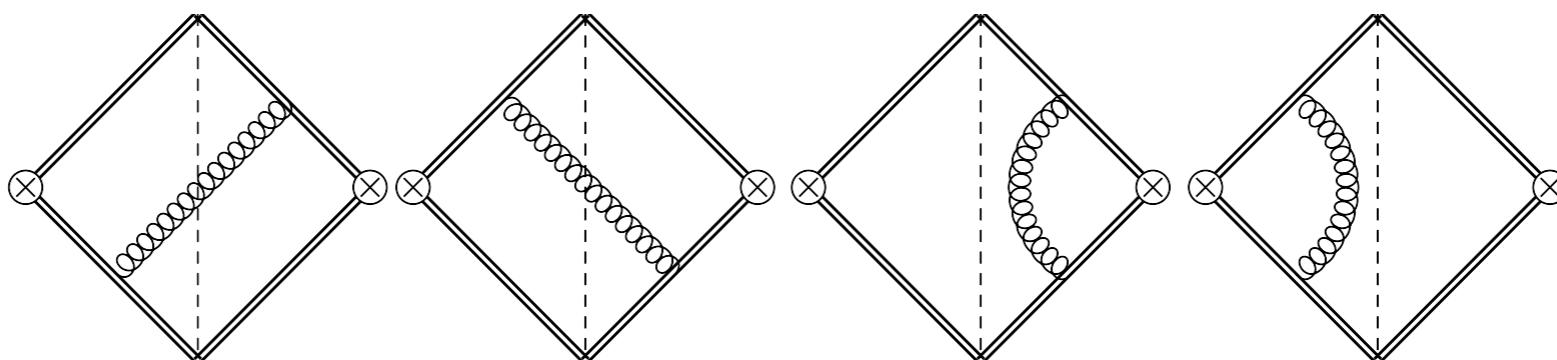
---

- Matrix-element of eikonal collinear-soft Wilson lines

$$S_+(\mathcal{T}_0^{\text{csoft}}, \vec{p}_T^{\text{csoft}}) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_{\bar{n}}] \delta(\mathcal{T}_0^{\text{soft}} - \hat{p}^+) \delta(\vec{p}_T^{\text{csoft}} - \hat{\vec{p}}_T) T [V_n^\dagger X_n] | 0 \rangle$$

- Differs from soft function  $S(\mathcal{T}_0^{\text{soft}}, \vec{p}_T^{\text{soft}})$  because the collinear-soft radiation goes into one hemisphere,  $\hat{\mathcal{T}}_0 \rightarrow \hat{p}^+$
- At one loop

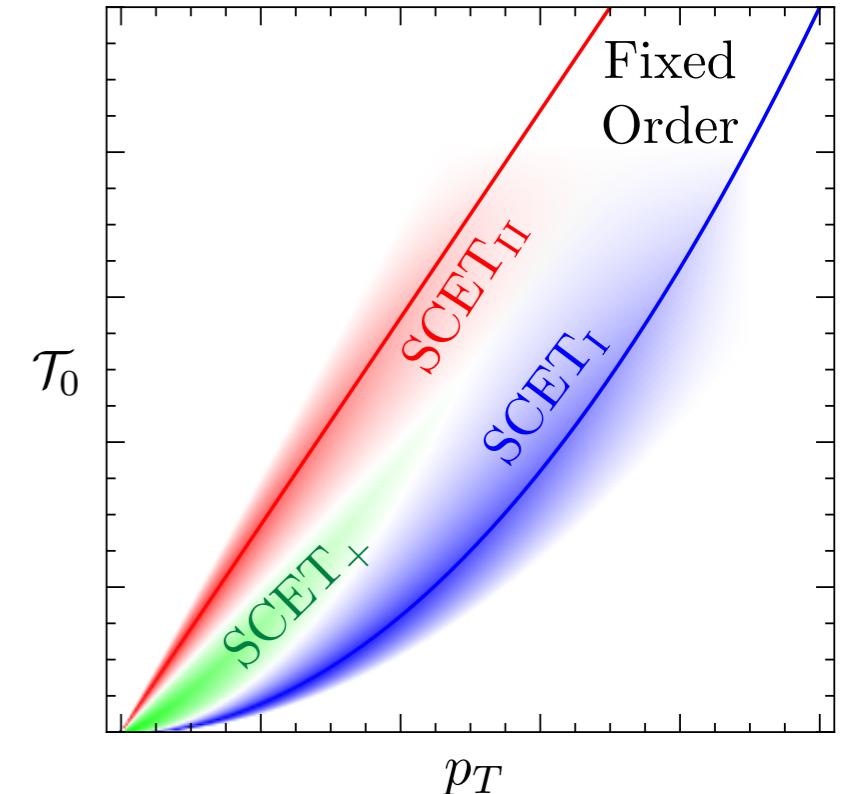
$$\begin{aligned} S_+^{(1)}(k^+, \vec{k}_T, \mu, \nu) = & \frac{\alpha_s C_F}{\pi^2} \left\{ \frac{1}{\mu^3} \frac{1}{(k^+/\mu)_+} \frac{1}{(k_T^2/\mu^2)_+} \right. \\ & + \delta(k^+) \left[ -\frac{1}{\mu^2} \left[ \frac{\ln(k_T^2/\mu^2)}{k_T^2/\mu^2} \right]_+ + \frac{1}{\mu^2} \frac{1}{(k_T^2/\mu^2)_+} \ln \frac{\nu}{\mu} - \frac{\pi^2}{12} \delta(k_T^2) \right] \left. \right\} \end{aligned}$$



# Consistency relations

- Consistency between SCET<sub>I</sub> and SCET+

$$B(Q\mathcal{T}_0, \vec{p}_T) = B(\vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \\ \times \left[ 1 + \mathcal{O}\left(\frac{p_T^2}{Q\mathcal{T}_0}\right) \right]$$



- Consistency between SCET<sub>II</sub> and SCET+

$$S(\mathcal{T}_0, \vec{p}_T) = S_+(\mathcal{T}_0, \vec{p}_T) \otimes S_+(\mathcal{T}_0, \vec{p}_T) \otimes S(\mathcal{T}_0) \left[ 1 + \mathcal{O}\left(\frac{\mathcal{T}_0^2}{p_T^2}\right) \right]$$

- Verified for anomalous dimensions and NLO ingredients

# Scales choices and matching

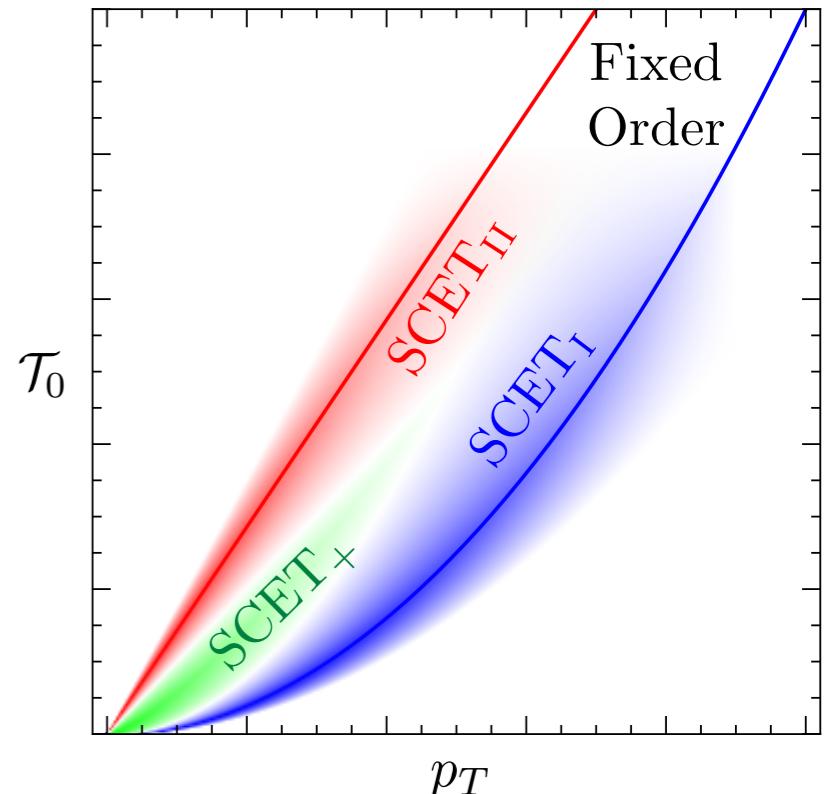
- Natural scales of the ingredients are

$$\mu_H = Q ,$$

$$\mu_B = p_T , \quad \nu_B = Q ,$$

$$\mu_{S_+} = p_T , \quad \nu_{S_+} = p_T^2 / \mathcal{T}_0 ,$$

$$\mu_S = \mathcal{T}_0$$



- Scales / resummation smoothly connect all regimes
- SCET+ sums all logarithms but has more power corrections
- Starting at NNLL/NLO there are corrections from the SCET<sub>I</sub>, SCET<sub>II</sub> and fixed-order boundaries that must be included

### 3. $p_T$ and threshold resummation

Based on arXiv:1605.02740 (Lustermans, WW, Zeune)

# Transverse momentum and threshold resummation

---

- Start from  $p_T$  or threshold factorization

$$1. \quad \frac{d\sigma}{d\vec{p}_T} = H(Q)B(\vec{p}_T, \textcolor{red}{z}) \otimes B(\vec{p}_T, \textcolor{red}{z}) \otimes S(\vec{p}_T) \quad 1 - z \sim 1$$

$$3. \quad \frac{d\sigma}{d\vec{p}_T} = H(Q)f(\textcolor{red}{z}) \otimes f(\textcolor{red}{z}) \otimes S(\vec{p}_T, \textcolor{red}{z}) \quad p_T \sim (1 - z)Q$$

where the threshold parameter is  $1 - \textcolor{red}{z} = 1 - \frac{Q^2}{\hat{s}}$

# Transverse momentum and threshold resummation

---

- Start from  $p_T$  or threshold factorization

$$1. \quad \frac{d\sigma}{d\vec{p}_T} = H(Q)B(\vec{p}_T, z) \otimes B(\vec{p}_T, z) \otimes S(\vec{p}_T) \quad 1 - z \sim 1$$

$$2. \quad \frac{d\sigma}{d\vec{p}_T} = H(Q)f(z) \otimes f(z) \otimes S_+(\vec{p}_T, z) \otimes S_+(\vec{p}_T, z) \otimes S(\vec{p}_T)$$

$$3. \quad \frac{d\sigma}{d\vec{p}_T} = H(Q)f(z) \otimes f(z) \otimes S(\vec{p}_T, z) \quad p_T \sim (1 - z)Q$$

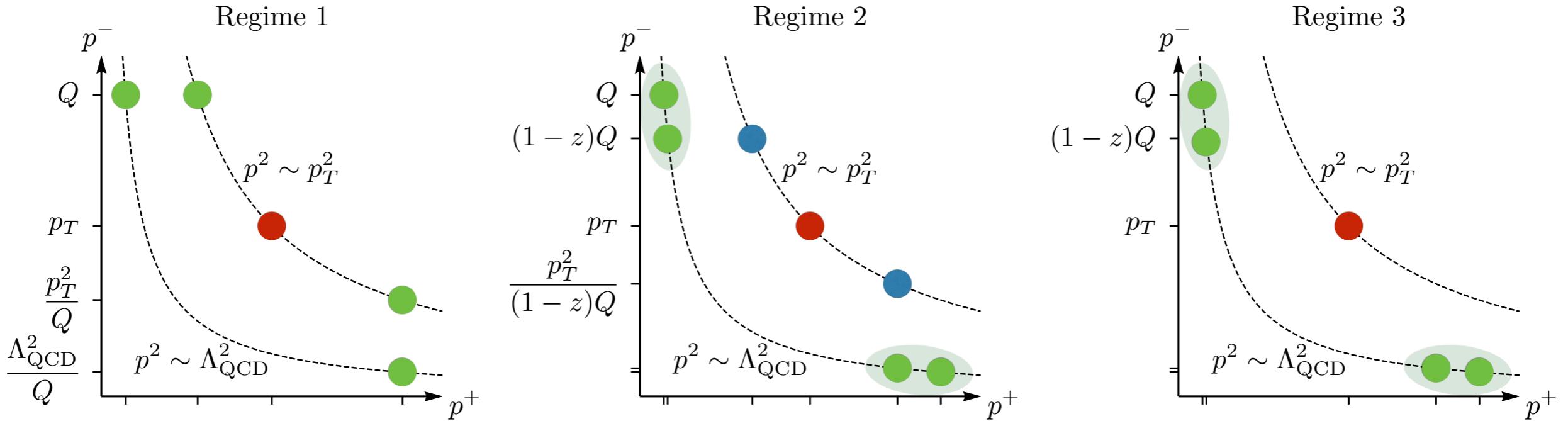
where the threshold parameter is  $1 - z = 1 - \frac{Q^2}{\hat{s}}$

- Intermediate regime 2 described by SCET+ with

$$p_T/Q \ll 1 - z \ll 1$$

# Modes

---



| Regime:              | 1: $1 \sim 1 - z \gg p_T/Q$  | 2: $1 \gg 1 - z \gg p_T/Q$   | 3: $1 \gg 1 - z \sim p_T/Q$  |
|----------------------|--|--|--|
| $n$ -collinear       | $(\Lambda_{\text{QCD}}^2/Q, Q, \Lambda_{\text{QCD}})$<br>$(p_T^2/Q, Q, p_T)$ | $(\Lambda_{\text{QCD}}^2/Q, Q, \Lambda_{\text{QCD}})$<br>$(\Lambda_{\text{QCD}}^2/[(1-z)Q], (1-z)Q, \Lambda_{\text{QCD}})$ | $(\Lambda_{\text{QCD}}^2/Q, Q, \Lambda_{\text{QCD}})$<br>$(\Lambda_{\text{QCD}}^2/[(1-z)Q], (1-z)Q, \Lambda_{\text{QCD}})$ |
| $\bar{n}$ -collinear | $(Q, \Lambda_{\text{QCD}}^2/Q, \Lambda_{\text{QCD}})$<br>$(Q, p_T^2/Q, p_T)$ | $(Q, \Lambda_{\text{QCD}}^2/Q, \Lambda_{\text{QCD}})$<br>$((1-z)Q, \Lambda_{\text{QCD}}^2/[(1-z)Q], \Lambda_{\text{QCD}})$ | $(Q, \Lambda_{\text{QCD}}^2/Q, \Lambda_{\text{QCD}})$<br>$((1-z)Q, \Lambda_{\text{QCD}}^2/[(1-z)Q], \Lambda_{\text{QCD}})$ |
| $n$ -csoft           |  | $(p_T^2/[(1-z)Q], (1-z)Q, p_T)$  |  |
| $\bar{n}$ -csoft     |  | $((1-z)Q, p_T^2/[(1-z)Q], p_T)$  |  |
| soft                 | $(p_T, p_T, p_T)$  | $(p_T, p_T, p_T)$  | $(p_T, p_T, p_T)$  |

# Collinear-soft function

---

- Measures other light-cone component than before,  $p^+ \rightarrow p^-$

$$S_+(p^-, \vec{p}_T) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_{\bar{n}}] \delta(p^- - \hat{p}^-) \delta(\vec{p}_T - \hat{\vec{p}}_T) T [V_n^\dagger X_n] | 0 \rangle$$

- The  $p^+ \leftrightarrow p^-$  symmetry broken by rapidity regulator or due to double counting with soft radiation

$$S_+(p^+, \vec{p}_T) = [S_+(p^-, \vec{p}_T) \text{ with } p^- \rightarrow p^+] \otimes S(\vec{p}_T)$$

# Collinear-soft function and consistency relations

---

- Measures other light-cone component than before,  $p^+ \rightarrow p^-$

$$S_+(p^-, \vec{p}_T) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_{\bar{n}}] \delta(p^- - \hat{p}^-) \delta(\vec{p}_T - \hat{\vec{p}}_T) T [V_n^\dagger X_n] | 0 \rangle$$

- The  $p^+ \leftrightarrow p^-$  symmetry broken by rapidity regulator or due to double counting with soft radiation

$$S_+(p^+, \vec{p}_T) = [S_+(p^-, \vec{p}_T) \text{ with } p^- \rightarrow p^+] \otimes S(\vec{p}_T)$$

- Consistency relations

$$B(\vec{p}_T, z) = S_+(\vec{p}_T, z) \otimes f(z) [1 + \mathcal{O}(1 - z)]$$

$$S(\vec{p}_T, z) = S_+(\vec{p}_T, z) \otimes S_+(\vec{p}_T, z) \otimes S(\vec{p}_T) \left[ 1 + \mathcal{O}\left(\frac{p_T^2}{(1-z)^2 Q^2}\right) \right]$$

## SCET<sub>II</sub> and SCET+

---

$$\begin{aligned} B_i(\vec{p}_T, \textcolor{red}{z}) &= \sum_j I_{ij}(\vec{p}_T, \textcolor{red}{z}) \otimes f_j(\textcolor{red}{z}) \\ &= S_{i+}(\vec{p}_T, \textcolor{red}{z}) \otimes f_i(\textcolor{red}{z}) [1 + \mathcal{O}(1 - z)] \end{aligned}$$

Dictates structure of coefficients  $I_{ij}$  in the threshold limit:

- Logarithms of  $p_T$  and  $z$  predicted by RG evolution

$$\mu_{S_+} = p_T, \quad \nu_{S_+} = (1 - z)Q,$$

Proves conjecture by Echevarria, Scimemi, Vladimirov  
[arXiv:1604.07869]

## SCET<sub>II</sub> and SCET+

---

$$\begin{aligned} B_i(\vec{p}_T, \textcolor{red}{z}) &= \sum_j I_{ij}(\vec{p}_T, \textcolor{red}{z}) \otimes f_j(\textcolor{red}{z}) \\ &= S_{i+}(\vec{p}_T, \textcolor{red}{z}) \otimes f_i(\textcolor{red}{z}) [1 + \mathcal{O}(1 - z)] \end{aligned}$$

Dictates structure of coefficients  $I_{ij}$  in the threshold limit:

- Logarithms of  $p_T$  and  $z$  predicted by RG evolution

$$\mu_{S_+} = p_T, \quad \nu_{S_+} = (1 - z)Q,$$

Proves conjecture by Echevarria, Scimemi, Vladimirov  
[arXiv:1604.07869]

- Can obtain SCET+ from SCET<sub>II</sub>:  $\nu_B = Q \rightarrow (1 - z)Q = \nu_{S_+}$
- Corrections from SCET<sub>I</sub> and fixed-order start at NNLL

# Conclusions

---

- Collisions involve many scales  
→ additional collinear-soft modes = SCET+
- SCET+ enables the simultaneous resummation of
  - Jet resolution parameters (this talk)
  - Jet kinematic logarithms [Bauer, Tackmann, Walsh, Zuberi; Larkoski, Neill, Moult; Pietrulewicz, Tackmann, WW]
  - Jet radius logarithms [Chien, Hornig, Lee; Kolodrubetz, Pietrulewicz, Stewart, Tackmann, WW]
  - Non-global logarithms [Larkoski, Moult, Neill; Becher, Neubert, Rothen, Shao]

Stay tuned!