Gluon TMD in the Loop Space Approach

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Partially based on:

ICh, M. Pieters: *"Gauge-invariant gluon TMD at the LHC: Higgs production"*
in Proceedings of the 7th MPILHC, **DESY-PROC** (2016)-01 188 - 191

ICh: *"Fully gauge-invariant maximally path-dependent gluon TMD: Coordinate representation"*,
J. Phys. Conf. Ser. **678** (2016) 012049

ICh: *"Gauge-invariant gluon TMD and evolution in the coordinate space"*,
`arXiv:1511.00517`

ICh., T. Mertens, F.F. Van der Veken:
"Wilson Lines in Quantum Field Theory", De Gruyter, Berlin (2014)

QCD in the High-Energy Regime

Confinement: fundamental building blocks of QCD – quarks and gluons – do not exist as free particles

Running coupling: the strong coupling α_s changes with the characteristic energy

Asymptotic freedom: at small distance the quarks and gluons are (almost) free particles and the perturbative approach is applicable

Factorization: enables the separation of large- [essentially nonperturbative] and small-distance [perturbative hard scattering matrix elements] contributions

Parton distribution functions [pdfs]: accumulate information about intrinsic structure of hadrons

Parton Distribution Functions

pdfs must be

Gauge-invariant

Universal

Renormalizable

Parton Distribution Functions
Issues

Wilson lines: save gauge invariance; universality questioned; complicate renormalizability

Path-dependence: the structure of the Wilson lines is too complicated; universality may be broken

Factorization scale is arbitrary: transition from one scale to another (different experiments have different characteristic scales) by means of evolution equations

Evolution: DGLAP, BFKL, CCFM... TMD; development of dedicated Monte-Carlo needed [in progress]

Why TMD Factorization?

low-q_T DY: $d\sigma^{Z*}_{\rm DY}(q_T)$ in the range 60 $GeV < M <$ 120 $GeV \rightarrow$ High-q_T (10^2 GeV), the 'peak region' (10 GeV), low-q_T (1 GeV). pQCD convoluted with the collinear pdf $\rightarrow$ $d\sigma^{Z*}_{\rm DY}(q_T)$ diverges at small q_T.

high-energy DIS: rise of the proton structure function at small-x. As parton longitudinal momentum fractions (Bojrken-x) become small, the transverse degrees of freedom becomes increasingly important. The strong corrections at small-x come from multiple radiation of gluons over long intervals in rapidity, in regions not ordered in the gluon transverse momenta $\mathbf{k}_\perp$, and are present in all higher orders of perturbation theory. TMD evolution provides an appropriate framework to resum such corrections.

Definitions of TMD/updfs

Factorization $\rightarrow$ operator definition

updfs via evolution/resummation: DGLAP, BFKL, CCFM

Effective theories: SCET

High-energy/small-x: Balitsky, Kovchegov

$\rightarrow$ Looking for the a unifying approach

Operator structure of TMD

- 'Standard' approach:
  factorization in a convenient gauge $\rightarrow$ gauge-dependent PDF $\rightarrow$ gluon resummation $\rightarrow$ gauge-invariant PDF with Wilson lines, path-dependence as prescribed by the factorisation

- Alternative approach:
  generic gauge-invariant path-dependent object $\rightarrow$ evolution in the coordinate space to fit the factorization scheme $\rightarrow$ operator structure related to a given PDF $\rightarrow$ gauge-invariant PDF with Wilson lines, path-dependence as prescribed by the factorization

Path-Dependent Correlation Functions: Issues

$$\mathcal{F}(k)_\gamma = \mathrm{F.T.}\ \langle h|\ \bar{\Psi}(z)\ \mathcal{W}_\gamma[z,0]\ \Psi(0)\ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**

$$\mathcal{W}_\gamma = \mathcal{P}\ \exp\left[\pm ig \int_0^z d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]_\gamma$$

Issues:

Gauge invariance $\rightarrow$ complicated **structure of the Wilson lines**

Path dependence $\rightarrow$ **universality** is geopardized

Singularities $\rightarrow$ problems with **renormalization**

Factorization $\rightarrow$ **evolution**

Gluon TMD: from Small-x to Large-x

@ [Mulders, Rodrigues (2001); Collins (2011); Dominguez, Marquet, Xiao, Yuan (2011); Balitsky, Tarasov (2014, 2015)]

Small-x

$$\mathbf{G}^{ij}_{\mathrm{small-x}}(x,\mathbf{k}_\perp;P,S) = \int dz^- \int d^2z_\perp \mathrm{e}^{ik_\perp z_\perp}$$

$$\langle h|\, D^i \mathcal{W}_{\mathrm{LC}}(z^-,\mathbf{z}_\perp)\, \mathcal{W}^\dagger{}_{\mathrm{LC}}(z^-,\mathbf{z}_\perp) D^j \mathcal{W}_{\mathrm{LC}}(0^-,\mathbf{0}_\perp)\, \mathcal{W}^\dagger{}_{\mathrm{LC}}(0^-,\mathbf{0}_\perp)\, |h\rangle$$

$$= \int dz^- \int d^2z_\perp \mathrm{e}^{ik_\perp z_\perp}$$

$$\langle h|\, \mathcal{F}^{il}(z^-,\mathbf{z}_\perp) \mathcal{W}^\dagger{}_{\mathrm{LC}}(z)\, \mathcal{W}_{\mathrm{LC}}(0)\, \mathcal{F}^{lj}(0^-,\mathbf{0}_\perp)\, |h\rangle$$

$$= \int dz^- \int d^2z_\perp \mathrm{e}^{ik_\perp z_\perp} \langle h|\, \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\mathrm{LC}}}(z,0)\, \mathcal{F}^{lj}(0)\, |h\rangle$$

Rapidity cutoff: $\ln x$; single-logs $\alpha_s \ln x$; non-linear dynamics, BK Eq.

Gluon TMD: from Small-x to Large-x

@ [Mulders, Rodrigues (2001); Collins (2011); Dominguez, Marquet, Xiao, Yuan (2011); Balitsky, Tarasov (2014, 2015)]

Large/Moderate-x

$$\mathbf{G}^{ij}_{\mathrm{small-x}}(x,\mathbf{k}_\perp;P,S) = \int dz^- \int d^2z_\perp \mathrm{e}^{-ixp^+z^-+ik_\perp z_\perp}$$
$$\langle h|\ \mathcal{F}^{il}(z^-,\mathbf{z}_\perp)\mathcal{W}^\dagger{}_{\mathrm{LC}}(z)\ \mathcal{W}_{\mathrm{LC}}(0)\ \mathcal{F}^{lj}(0^-,\mathbf{0}_\perp)\ |h\rangle$$
$$= \int dz^- \int d^2z_\perp \mathrm{e}^{-ixp^+z^-+ik_\perp z_\perp} \langle h|\ \mathcal{F}^{il}(z)\mathcal{W}_{\gamma_{\mathrm{LC}}}(z,0)\ \mathcal{F}^{lj}(0)\ |h\rangle$$

Rapidity cutoff: $\eta \neq \ln x$; double-logs are possible $\alpha_s \eta \ln x$; linear evolution.

Gluon TMD: from Small-x to Large-x

Two definitions in two regimes—look so similar, but in fact very different:

$$\text{small} = \int dz^- \int d^2 z_\perp \mathrm{e}^{ik_\perp z_\perp} \langle h| \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z,0)\, \mathcal{F}^{lj}(0)\, |h\rangle$$

VS

$$\text{large} = \int dz^- \int d^2 z_\perp \mathrm{e}^{-ixp^+ z^- + ik_\perp z_\perp} \langle h| \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z,0)\, \mathcal{F}^{lj}(0)\, |h\rangle$$

Factorization schemes are different, evolution is different: how to relate?
Connection can be stablished @ [Balitsky, Tarasov (2014, 2015)]
However: the operator structure is the same. Let us start with it and forget (for a while) about the factorization issues.

Quark and Gluon TMD: Generic Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

Highly **gauge-dependent quark correlator** for a hadron h with a momentum P and spin S

$$\mathbf{Q}_{\mathrm{g-d}}(x, \mathbf{k}_\perp; P, S) = \int d^4z \; \mathrm{e}^{-ikz} \; \langle h| \; \bar{\psi}(z)\psi(0) \; |h\rangle$$

Highly **gauge-dependent gluon correlator** for a hadron h with a momentum P and spin S

$$\mathbf{G}^{\mu\nu}_{\mathrm{g-d}}(x, \mathbf{k}_\perp; P, S) = \int d^4z \; \mathrm{e}^{-ikz} \; \langle h| \; \mathcal{A}^\mu(z)\mathcal{A}^\nu(0) \; |h\rangle$$

Fully Unintegrated Gluon PDF: Gauge-Invariant Operator Definition

@ [Mulders, Rodrigues (2001); Collins (2011)]

$$\mathbf{G}^{\mu\nu|\mu'\nu'}(k;P,S) = \int d^4z\ \mathrm{e}^{-ikz}\ \langle h|\ \mathcal{F}^{\mu\nu}(z)\ \mathcal{W}_\gamma\ \mathcal{F}^{\mu'\nu'}(0)\ |h\rangle$$

Wilson line (system of lines) $\mathcal{W}_\gamma$ in the adjoint representation

$$\mathcal{F}_{\mu\nu} = F^a_{\mu\nu} T^a$$

Respects the desirable operator structure

Knows nothing about any factorization scheme: maximally path-dependent, γ is entirely arbitrary

Still difficult to evaluate

Gluon TMD: Several Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

Gluon TMD from the generic **gauge-invariant correlator**

$$\mathbf{G}^{\mu\nu|\mu'\nu'}(k;P,S) =$$

$$\int d^4z\ \mathrm{e}^{-ikz}\ \langle h|\ \mathcal{F}^{\mu\nu}(z)\ \mathcal{W}_\gamma\ \mathcal{F}^{\mu'\nu'}(0)\ |h\rangle$$

$$\rightarrow$$

$$\mathbf{G}^{ij}(x,k_\perp;P,S) \sim \int dk^-\ \mathbf{G}^{+i|+j}(k;P,S) =$$

$$\int dz^- d^2z_\perp\ \mathrm{e}^{-ikz}\ \langle h|\ \mathcal{F}^{+i}(z)\ \mathcal{W}_\gamma\ \mathcal{F}^{+j}(0)\ |h\rangle$$

Loop Space Approach

Try the *opposite direction*: start from a generic object in a loop space

Equations of Motion in the Loop Space

@ [Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt et al. (1981, 1982)]

Wilson loops as the (fundamental) gauge-invariant degrees of freedom:

$$\langle \mathcal{W}_\gamma \rangle = \langle \mathcal{P}_\gamma \exp \oint_\gamma d\zeta_\mu \mathcal{A}^\mu(\zeta) \rangle$$

or

$$\langle \mathcal{W}^n_{\gamma_1,\ldots\gamma_n} \rangle = \langle \mathcal{T} \mathcal{W}_{\gamma_1} \cdots \mathcal{W}_{\gamma_n} \rangle$$

Loop space and differential operators

Area derivative:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\langle\mathcal{W}_\gamma\rangle = \lim_{|\delta\sigma_{\mu\nu}(z)|\to 0}\frac{\langle\mathcal{W}_{\gamma\delta\gamma_x}\rangle - \langle\mathcal{W}_\gamma\rangle}{|\delta\sigma_{\mu\nu}(z)|}$$

Path derivative:

$$\partial_\mu\langle\mathcal{W}_\gamma\rangle = \lim_{|\delta z_\mu|\to 0}\frac{\langle\mathcal{W}_{\delta z_\mu^{-1}\gamma\delta z_\mu}\rangle - \langle\mathcal{W}_\gamma\rangle}{|\delta z_\mu|}$$

Differential operators in the loop space $\to$ evolution of the Wilson loops in the coordinate representation $=$ equations of motion

Loop space and differential operators

The Wilson functionals obey the **Makeenko-Migdal** loop equations:

$$\partial^\nu \frac{\delta}{\delta\sigma_{\mu\nu}(x)} \langle \mathcal{W}^1_\gamma \rangle = \bar{g}^2 \oint_\gamma dz^\mu \, \delta^{(4)}(x-z) \langle \mathcal{W}^2_{\gamma_{xz}\gamma_{zx}} \rangle$$

Stokes-Mandelstam uPDF

Non-Abelian Stokes' theorem

@ [Arefeva (1980) etc.]

$$\mathcal{P}_\gamma \exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right] = \mathcal{P}_\gamma \mathcal{P}_\sigma \exp\left[\int_\sigma d\sigma_{\rho\rho'}(\zeta)\mathcal{F}^{\rho\rho'}(\zeta)\right]$$

Mandelstam formula

@ [Mandelstam (1968)]

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\mathcal{P}_\gamma \exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right] = \mathcal{P}_\gamma \mathcal{F}^{\mu\nu}(x)\exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right]$$

Stokes-Mandelstam uPDF

$$\tilde{\mathbf{G}}^{\mu\nu|\mu'\nu'}(z;P,S) = \frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu'\nu'}(0)}\langle h|\mathcal{W}_{\gamma^{[z,0]}}\ |h\rangle =$$

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu'\nu'}(0)}\sum_X \langle h|\mathcal{W}'_{\gamma^{[z]}}|X\rangle\langle X|\mathcal{W}'_{\gamma^{[0]}}\ |h\rangle$$

Non-Abelian exponentiation

$$\langle W_{\gamma^{[z,0]}}\rangle = \exp\left[\sum a_n W^{(n)}\right]\ ,\ W^{(n)} = \text{hadronic correlators}$$

Gauge invariance, Path dependence, Universality

Evolution in the Coordinate Space: Abelian Case

Abelian exponentiation

$$\langle \mathcal{W}_\gamma \rangle = \langle h | \mathcal{P}_\gamma \, \exp \left[\oint_\gamma \, d\zeta_\rho \mathcal{A}^\rho(\zeta) \right] | h \rangle =$$

$$\exp \left[-\frac{g^2}{2} \oint_\gamma \, d\zeta_\mu \oint_\gamma \, d\zeta'_\nu \, D_{\mu\nu}(\zeta - \zeta') \right]$$

Basic hadronic correlator

$$D_{\mu\nu}(\zeta - \zeta') = \langle A_\mu(\zeta) A_\nu(\zeta') \rangle$$

Evolution in the Coordinate Space: Abelian Case

Parameterization

$$D_{\rho\rho'}(z) =$$
$$g_{\rho\rho'}\ D_1(z,P) + \partial_\rho\partial_{\rho'}\ D_2(z,P) + \{P_\rho\partial_{\rho'}\}\ D_3(z,P) + P_\rho P_{\rho'}\ D_4(z,P)$$

In general, the hadronic correlator contains all necessary information

$$D_{\rho\rho'}(\zeta-\zeta') = \langle P,S|A_\rho(\zeta)A_{\rho'}(\zeta')|P,S\rangle$$

Evolution in the Coordinate Space: Abelian Case

Area derivative

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\langle h|\mathcal{W}_\gamma|h\rangle =$$

$$-\frac{g^2}{2}\left[\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\oint_\gamma d\zeta_\rho \oint_\gamma d\zeta'_{\rho'}\; D_{\rho\rho'}(\zeta-\zeta')\right]\langle h|\mathcal{W}_\gamma|h\rangle$$

Non-vanishing terms after taking the path-derivative ∂_ν

– standard Makeenko-Migdal term

$$\sim \oint_\gamma d\zeta^\nu\; \partial^2\; D_1(z^2,P^2)$$

– hadron momentum-dependent term

$$\sim \oint_\gamma d\zeta^\nu\; (P\partial)^2\; D_4(z^2,P^2)$$

Evolution in the Coordinate Space: Abelian Case

Shape evolution equation

$$\partial^z_\mu \frac{\delta}{\delta\sigma_{\mu\nu}(z)} \langle h|\mathcal{W}_\gamma|h\rangle =$$

$$-\frac{g^2}{2}\left[\oint_\gamma d\zeta^\nu \left(\partial^2 \; D_1(z,P) + (P\partial)^2 \; D_4(z,P)\right)\right] \langle h|\mathcal{W}_\gamma|h\rangle$$

Consistency check: Wilson loops in vacuum

$$\partial^2 D_1(z) = -\delta^{(4)}(z), \;\; D_4 = 0$$

$$\partial^z_\mu \frac{\delta}{\delta\sigma_{\mu\nu}(z)} \langle 0|\mathcal{W}_\gamma|0\rangle = g^2 \oint_\gamma d\zeta^\nu \; \delta^{(4)}(z-\zeta)$$

= Makeenko-Migdal Eq. in the LO.

Some Explicit Results I: A 2D Model

γ is a circle with the radius R in 2D-Euclidean space

Gauge-field vacuum correlator

$$D_{\mu\nu}(z) = \delta_{\mu\nu} D(z) \quad , D(z) = \frac{1}{4\pi} \ln \left[z^2 \Lambda^2 \right]$$

Loop integral

$$\partial_\mu \oint_\gamma d\zeta^\rho D_{\nu\rho}(z-\zeta) = \frac{1}{2} \frac{1}{z^2 + R^2} \delta_{\mu\nu}$$

"Fully unintegrated pdf"

$$\tilde{G}(z) = \frac{\bar{g}^2}{16} F(z,R) \langle W_\gamma \rangle \quad , F(z,R) = \frac{1}{z^2 + R^2}$$

Some Explicit Results I: Evolution from the MM Eq.

The Wilson loop is a function of the area σ_γ:

$$\langle W_\gamma \rangle = W(\sigma_\gamma) = \exp\left[-\frac{1}{2}\bar{g}^2\sigma_\gamma\right]$$

Evolution in the coordinate space:

$$\frac{d}{d\sigma_\gamma} W(\sigma_\gamma) = -\frac{1}{2}\bar{g}^2 \; W(\sigma_\gamma)$$

uPDF

$$\tilde{G}(z) = \frac{\bar{g}^2}{16}\frac{1}{z^2+R^2} \; W(\sigma_\gamma)$$

Some Explicit Results II: A 4D Model

uPDF

$$\tilde{G}(z) = \frac{\bar{g}^2}{16\pi^2}\, F(z,R)\, \langle W_\gamma \rangle$$

$$F(z,R) = \frac{1}{\left(2z^+z^- - \vec{z}_\perp^2 + R^2\right)^2}$$

$\langle W_\gamma \rangle$ is not a simple function of the area

More realistic case: (i) non-Abelian; (ii) $(3+1)D$ Minkowkian; (iii) on the light-cone

Outlook:

A fully unintegrated gluon TMD distribution function can be formulated within fully gauge-invariant, generically path-dependent framework based on the loop space formalism in the coordinate representation. It is not associated with any factorization framework but respects the needed operator structure

This approach goes 'in the opposite direction' to the standard one: one starts with a maximally general object and then extracts a gluon TMD which is adjustable to any specific factorization scheme by means of the geometrical evolution in the coordinate space

The main ingredients of this approach are the hadronic matrix elements of Wilson loops $\langle h| \mathcal{W}_\gamma |h\rangle$.

Non-Abelian exponentiation enables separation of the non-local path-dependence and local UV-divergent contributions and appropriate parameterisation of various gTMD functions**?**