Transverse Single Spin Asymmetries in Hard Processes

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Work done with
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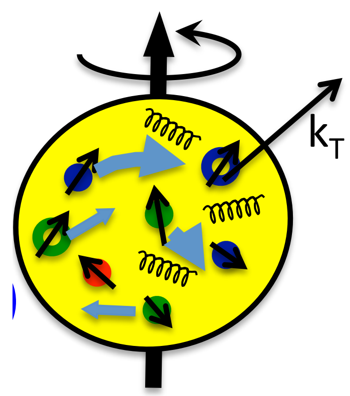
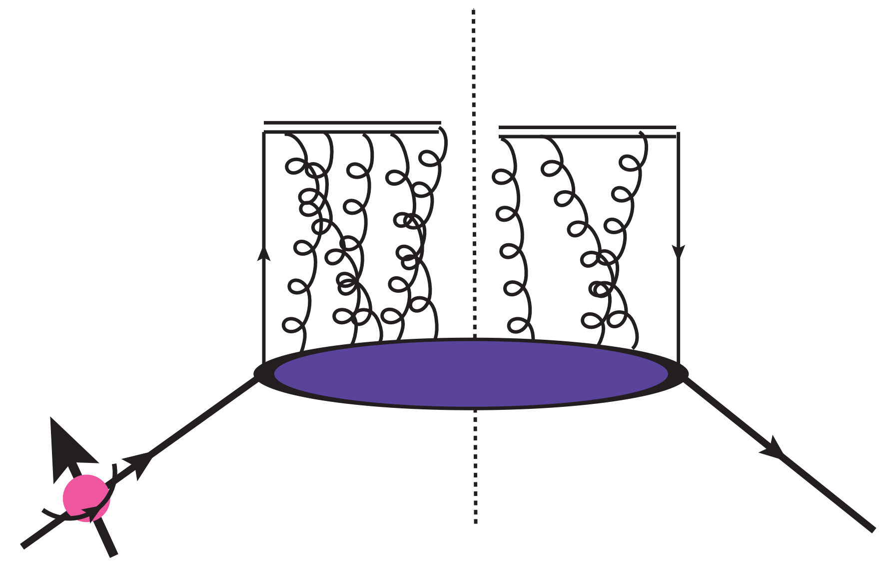
Overview

- We present the results of a combined TMD and twist-3 formalism analysis of single spin asymmetries in SIDIS, e^+e^- annihilation into hadron pairs, and proton-proton scattering to explore what effect evolution has on predicting A_N in $pp \to \pi X$
- Short review of TSSAs theory & experiment
- Can we explain data from RHIC on inclusive meson production in *pp* scattering from Twist -3 & Twist -2 description of TSSAs?what we know
- Summary Challenges-way forward

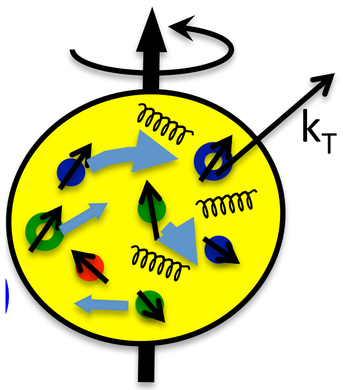
Intro Remarks





- TSSAs are a central observables/tool to extract essential information to unfold “3-dimensional” partonic description sub-structure of the nucleon
- Study through semi-inclusive and inclusive scattering process: @ JLAB-6&12, RHIC, HERMES, COMPASS, Fermi Lab-DY
- Impact for future EIC
- **See RHIC Cold QCD Plan, arXiv:1602.03922**

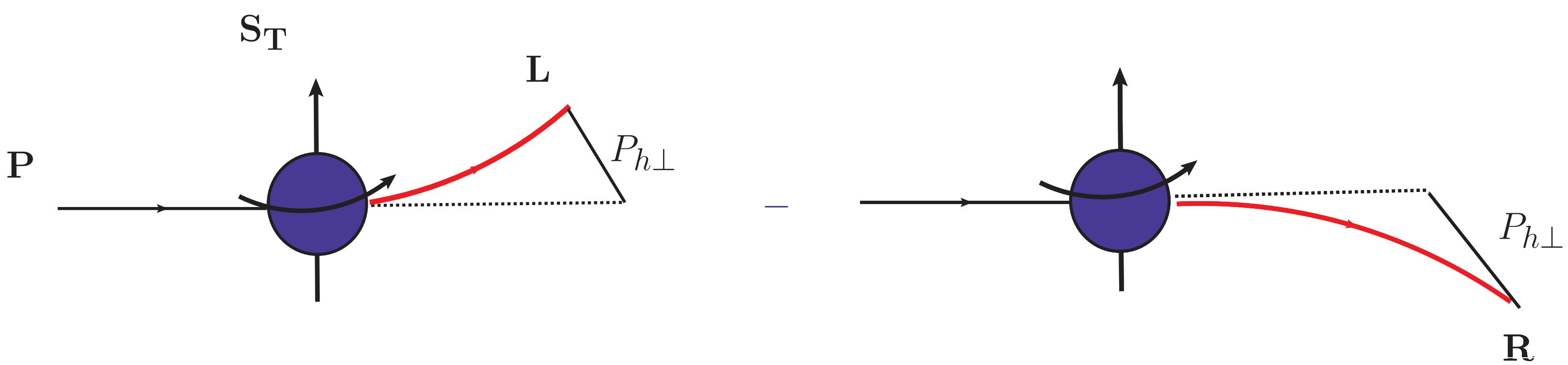
Transverse single spin asymmetries

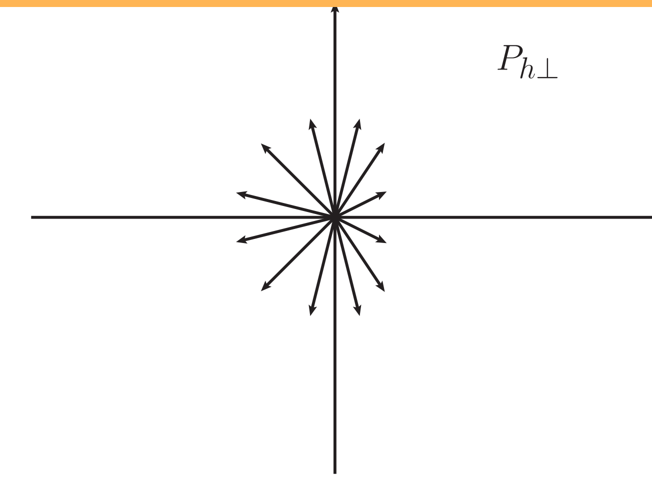


Observables that provide a window to study the 3-dim. momentum structure of the nucleon

- Process: semi-inclusive processes (*SIDIS, e^+e^- , DY*)
  - Information encoded in TMD PDFs-intrinsic properties of the nucleon
  - TMDs contain intrinsic information on spin orbit correlations
  - Information on spin & momentum correlations in CS
- Process: single-inclusive meson production in proton-proton scattering
  e.g. $pp \to \pi X$
  - Information encoded in quark-gluon-quark correlation functions
  - Some attempts to test the relation between TSSAs in these processes

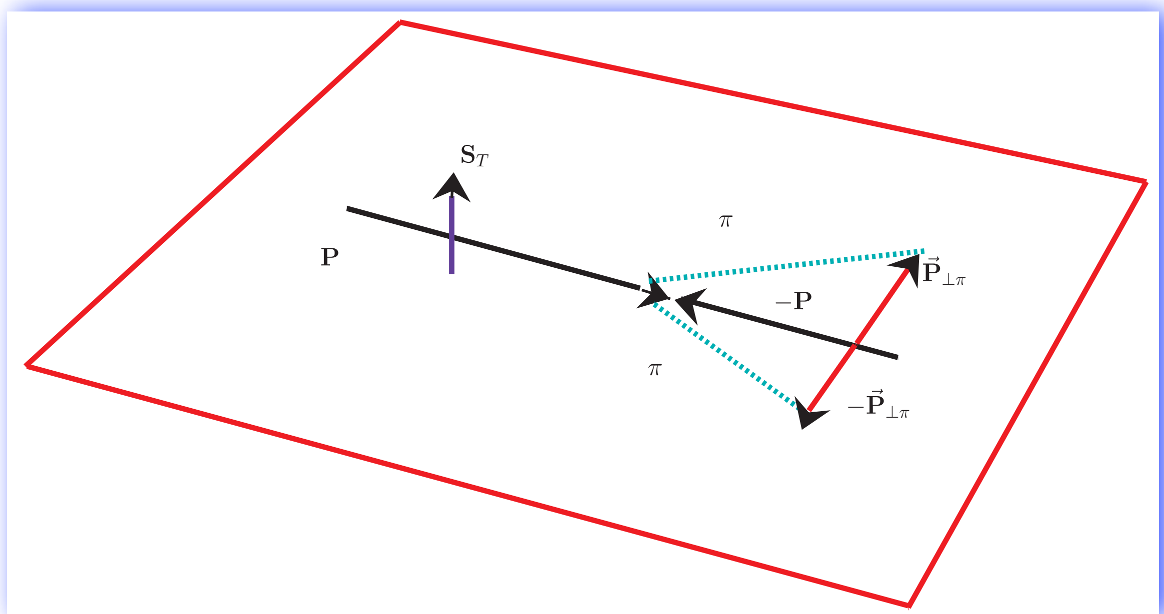
What is transverse single spin asymmetry TSSAs





$\sigma^{\downarrow}(x, P_{\perp}) = \sigma^{\uparrow}(x, -P_{\perp})$ Rotational Invariance *"Left-Right" Asymmetry*

$$A_N = \frac{\sigma^{\uparrow}(x, P_{\perp}) - \sigma^{\uparrow}(x, -P_{\perp})}{\sigma^{\uparrow}(x, P_{\perp}) + \sigma^{\uparrow}(x, -P_{\perp})} \equiv \Delta\sigma$$



QCD is Parity Conserving TSSAs Scattering plane transverse to spin

Naively "T-odd"

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_{\perp}) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

need some mechanism dynamics .... "function"

Spin orbit

TMD factorization: Process Dependence of Sivers and Universality of transverse momentum dependent Sivers function and Universality for Collins Function

T-odd TMDs provide info on color phase structure of the nucleon

$$f_{1T_{(\mathrm{sidis})}}^{\perp}(x, k_{\perp}) = -f_{1T_{(DY)}}^{\perp}(x, k_{\perp})$$

Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman NPB 03,

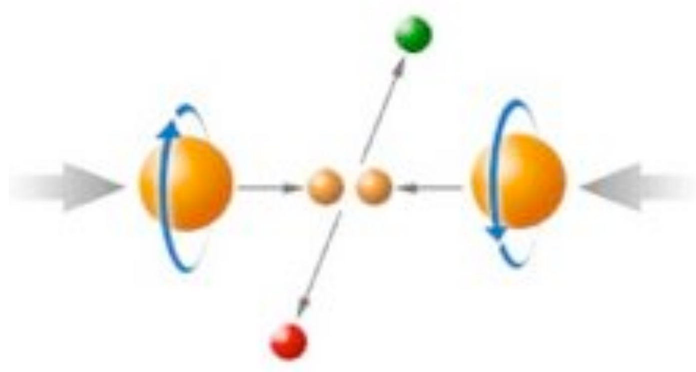
$$H_{1_{(\mathrm{sidis})}}^{\perp}(x, k_{\perp}) = H_{1_{(e^{+}e^{-})}}^{\perp}(x, k_{\perp})$$

Metz PLB 2002, Collins, Metz PRL 2004; L.Gamberg, A. Mukherjee, P. Mulders PRD 2008,2011; F.Yuan PRL & PRD 2008; A. Metz, S. Meissner PRL 2009, Boer, Kang, Vogelsang, Yuan-predictions on Lambda polarization in SIDIS

Motivation Use Universality of Collins and Study Process dependence of Sivers connection between SIDIS, Drell-Yan, e^+e^- *to study 3-D structure*

RHIC , JLAB 12, Belle, BaBar in conjunction with Drell-Yan exp.
Fermi LAB DY, AnDY, Compass, JPARC, NICA -JINR, & EIC

Remarks on TSSAs



- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs
- From theory view notoriously challenging from partonic picture twist-3 power suppressed hard scale (vs. SIDIS, Drell Yan & e^+e^-)

Report to the NSAC sub-committee on performance measures

8/11/2008

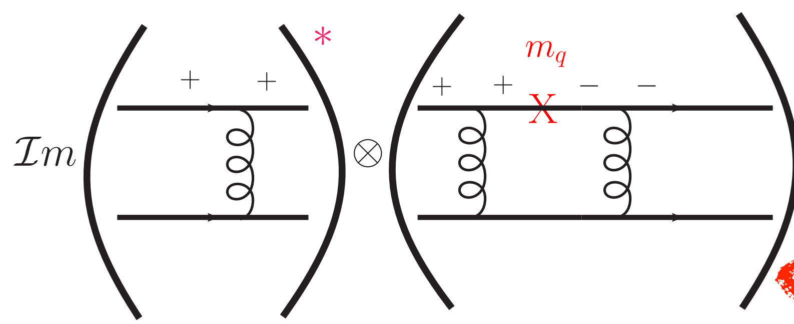
2015	HP13 (new)	Test unique QCD predictions for relations between single-transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering

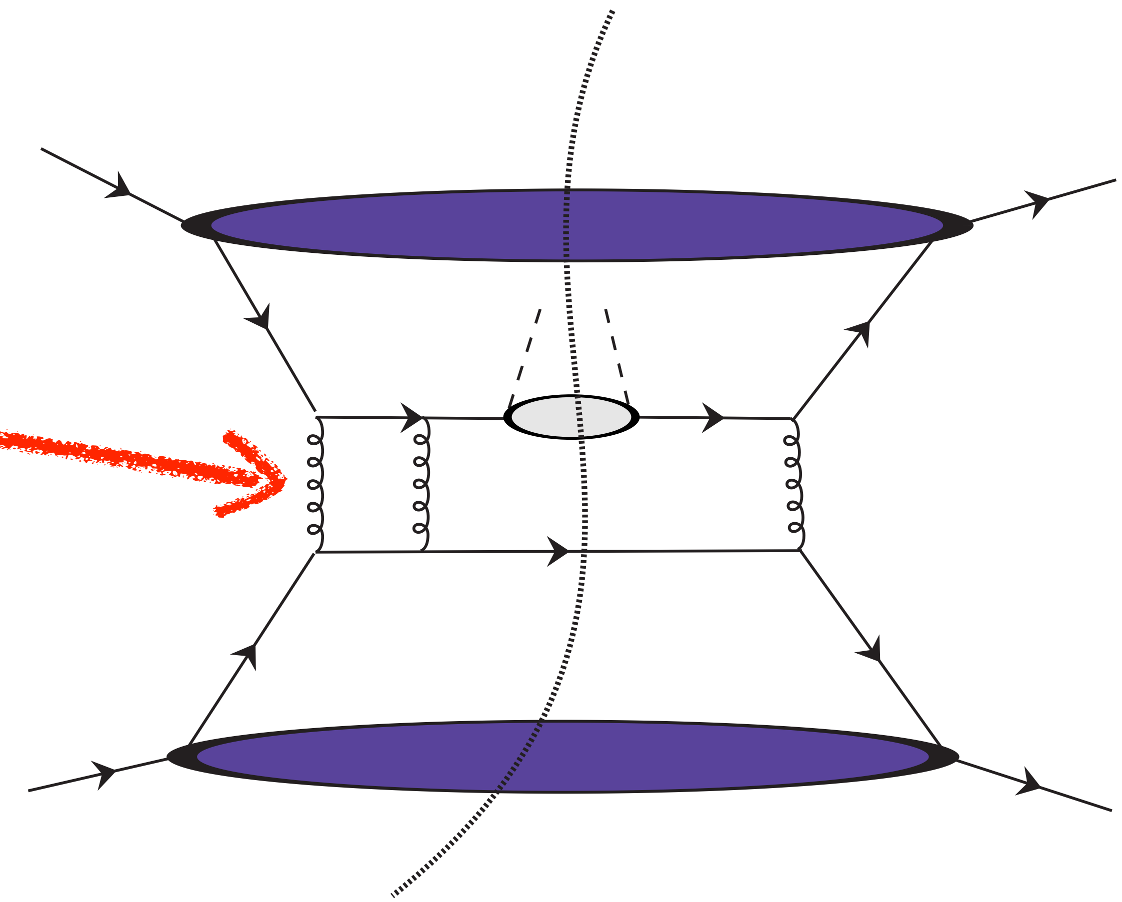
New Milestone HP13 reflects the intense activity and theoretical breakthroughs of recent years in understanding the parton distribution functions accessed in spin asymmetries for hard-scattering reactions involving a transversely polarized proton. This leads to new experimental opportunities to test all our concepts for analyzing hard scattering with perturbative QCD. New Milestone

Reaction Mechanism for TSSAs

Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$





$$\Delta\hat{\sigma} = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$

0) Interference of helicity flip and non-flip amps

1) Relative color phase require higher order correction α_s

2) QCD interactions conserve helicity up to correction
requires breaking of chiral $\mathcal{O}\left(m_q/E_q\right)$

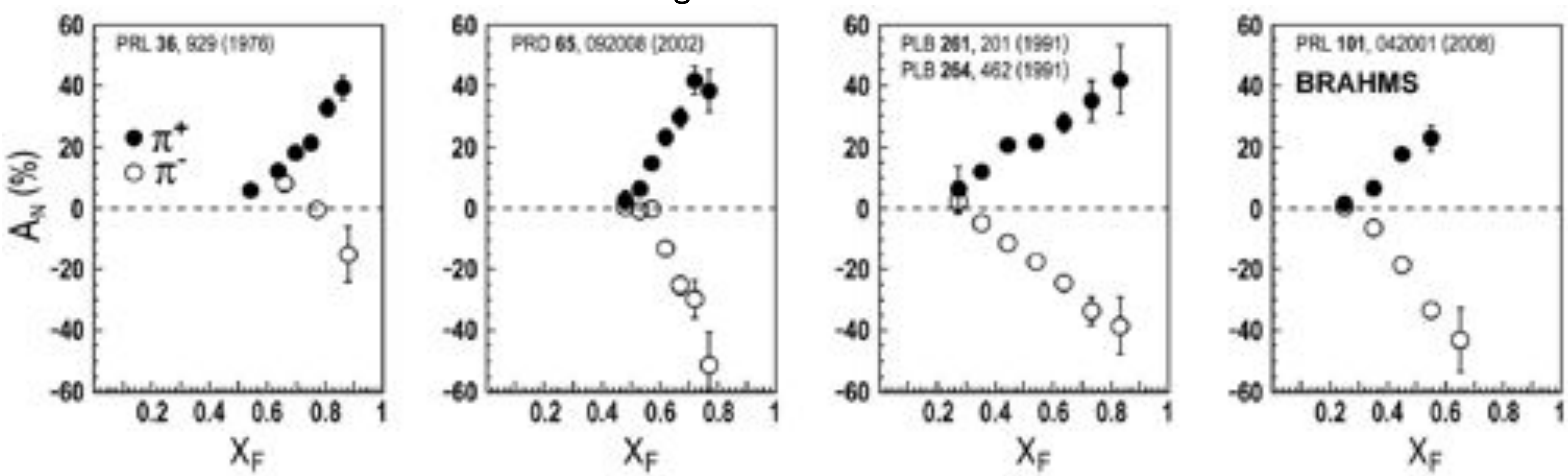
3) Thus, Twist three and trivial in chiral limit

$$\Delta\sigma \propto \frac{m_q}{E}\alpha_s \to 0 \quad \text{chiral limit}$$

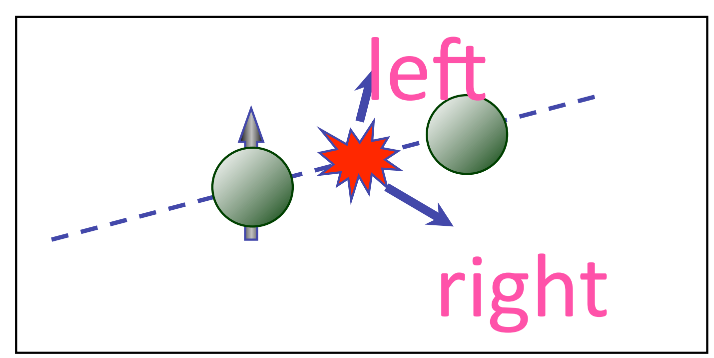
Parton Mdl.Theory striking contrast TSSAs in Inclusive Reactions

Transverse Single-Spin Asymmetries: From Low to High Energies!

p_{beam}=12 Gev/c	p_{beam}=22 Gev/c	p_{beam}=200 Gev/c	
ANL	BNL	FNAL	RHIC
√s=4.9 GeV	√s=6.6 GeV	√s=19.4 GeV	√s=62.4 GeV
Fixed target			Collider



$$x_F = 2p_{long} / \sqrt{s}$$



AGS to RHIC Transverse SSA's at √s = 4.9 -- 500 GeV

BRAHMS PHENIX STAR

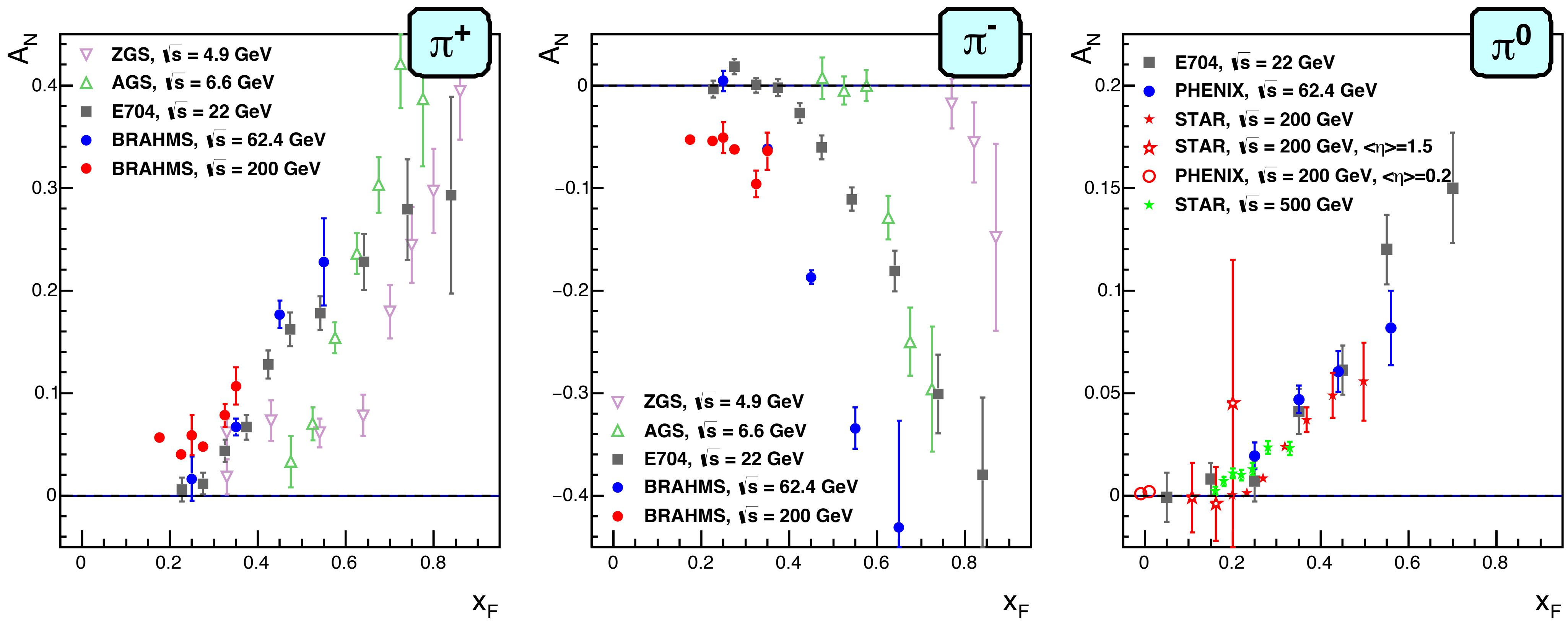
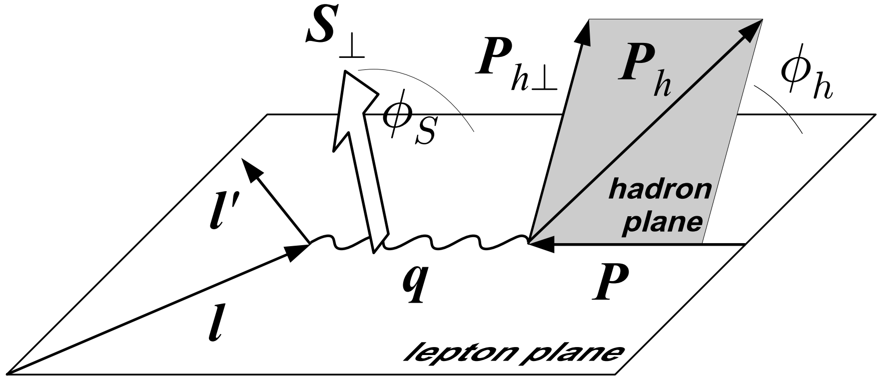
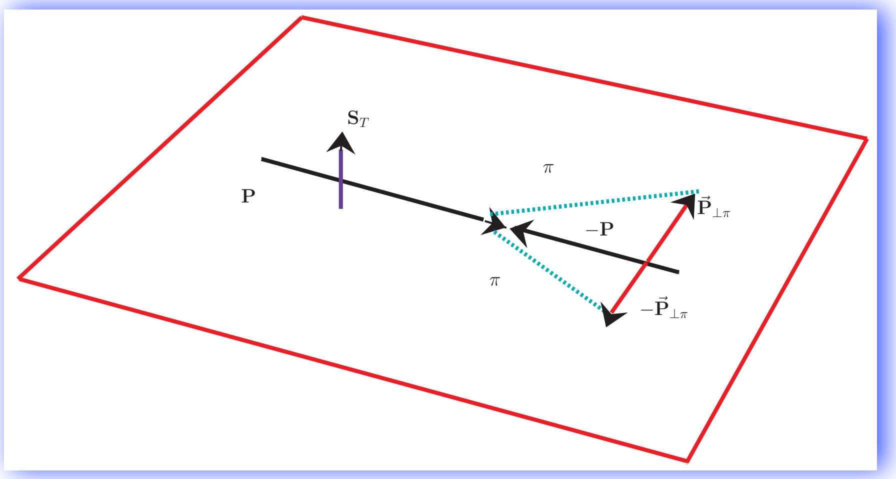


Figure 2-9: Transverse single spin asymmetry measurements for charged and neutral pions at different center-of-mass energies as a function of Feynman-x.

Two methods to generate non trivial TSSA in QCD

- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $P_{h\perp}$ relative to hadronic scale





- $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ two scales-twist 2 TMDs

Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13)

$$\Delta\sigma(P_h, S) \sim \Delta f^{\perp}_{a/A}(x, k_\perp) \otimes D_{h/c}(z, K_\perp) \otimes \hat{\sigma}_{\rm parton}$$

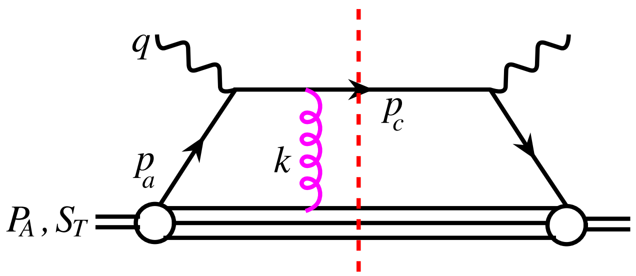
- $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ twist 3 factorization-ETQSs

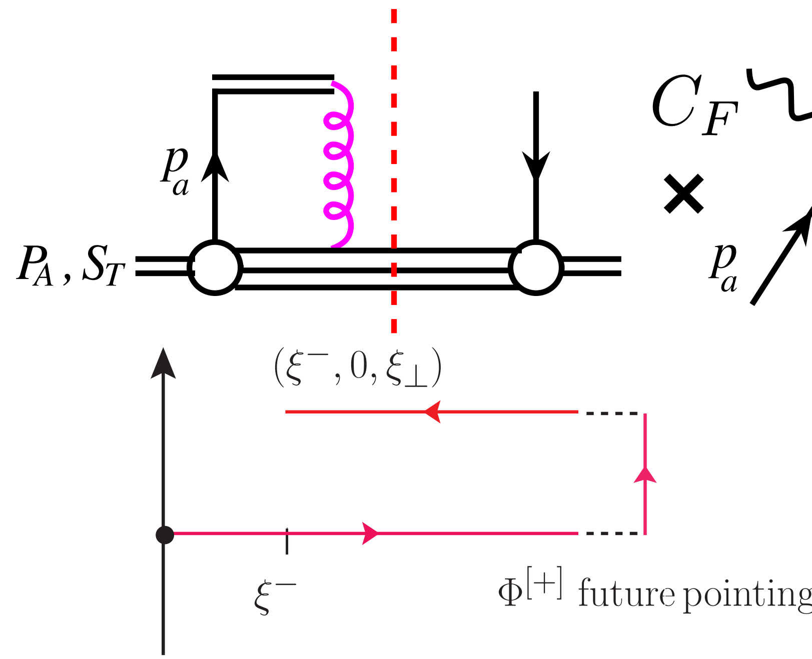
$$\Delta\sigma(P_h, S) \sim \frac{1}{Q}\,\Delta f^{\perp}_{a/A}(x) \otimes f_{b/B}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\rm parton}$$

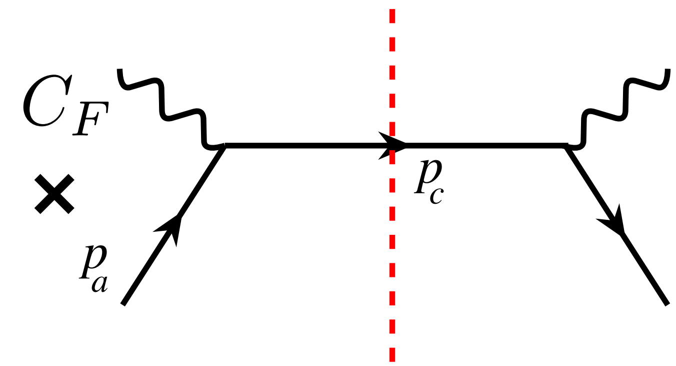
Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang! 2006, Vogelsang and Yuan PRD 2007

QCD phases in gauge link/Wilson line of TMD “T-odd” structure

Final -state interaction in SIDIS

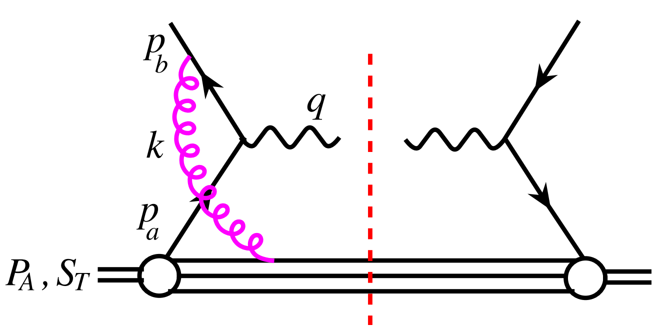


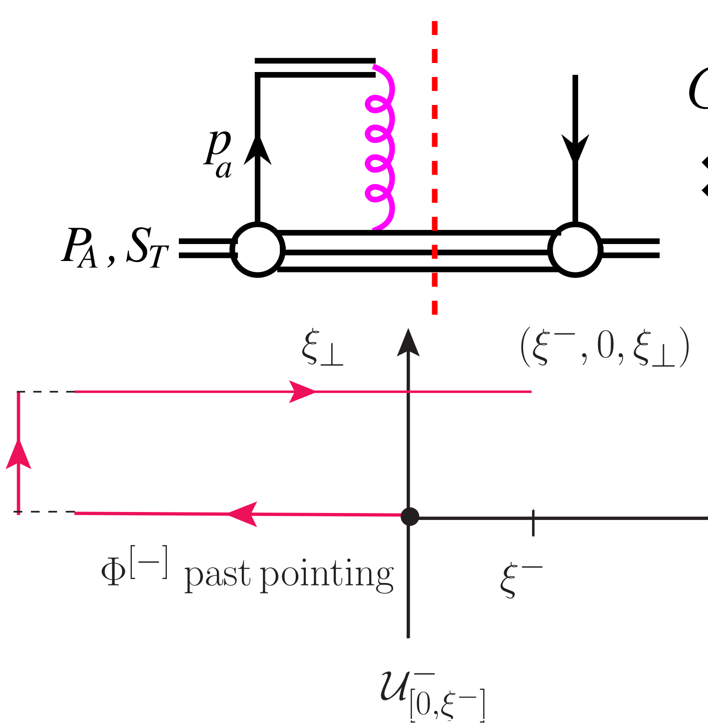


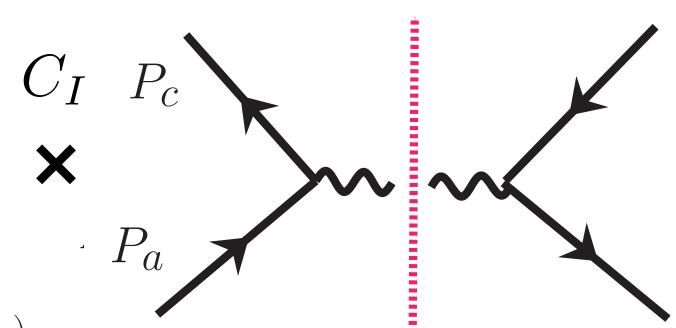


The path of gauge link determined by the hard subprocess in *factorization*

Initial-state interaction in DY



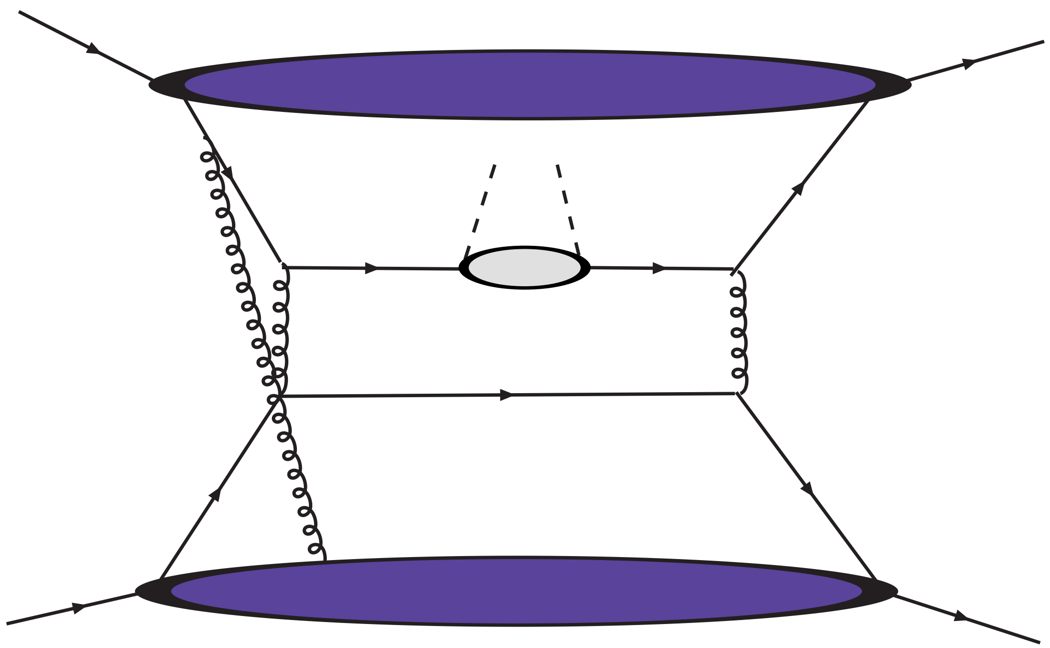


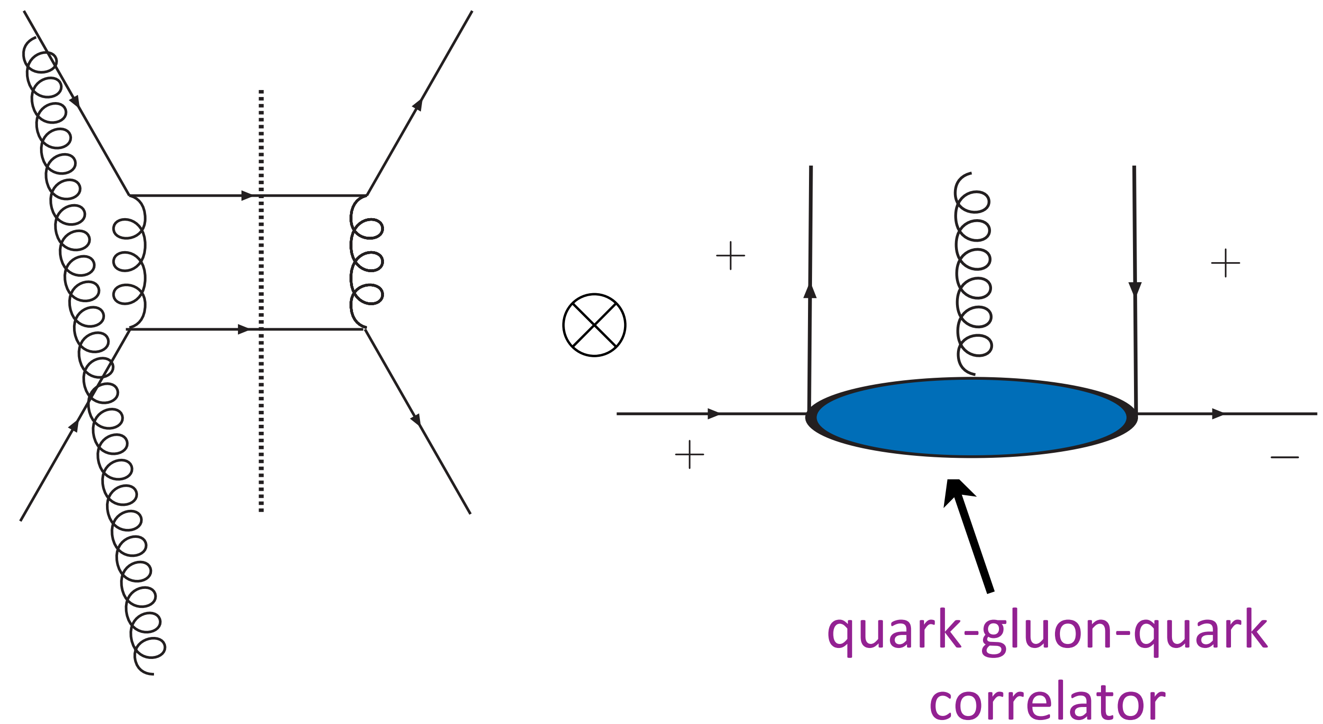


$$C_I = -C_F$$

Twist 3 Factorization ETQS & T-odd Structure

$\Lambda_{\text{QCD}} << P_{h\perp} \sim \sqrt{Q^2}$ one scale Collinear-Twist 3





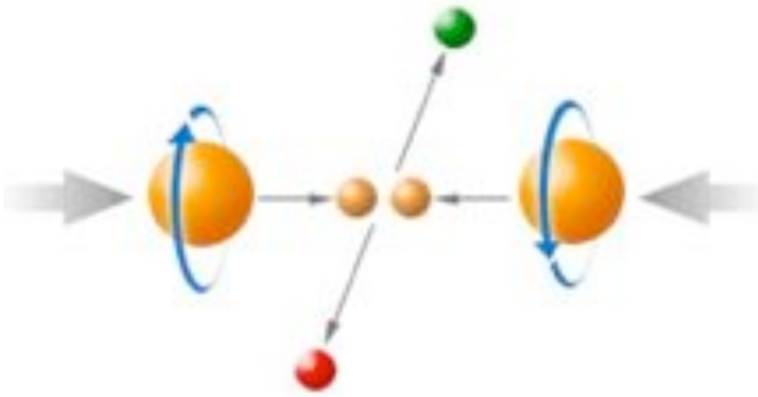
$$\Delta\sigma \sim f_a \otimes T_F \otimes H_{ab\to cd} \otimes D^{q\to h}$$

$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

- Phases from interference two parton three parton scattering amplitudes
  Efremov & Teryaev PLB 1982
- Net asymmetry after integration over parton's transverse momentum
- Twist three suppressed by hard scale but non-trival! $m_q \to M_h$

Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang 2006, Vogelsang and Yuan PRD 2007

Motivation Study Process dependence connection between SIDIS and inclusive processes in part motivated by



- relation btwn twist 2 "TMD" approach and twist 3 ETQS

we study process dependence in inclusive processes

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$ Boer Piljman Mulders NPB 2003

$$= -2M f_{1T}^{\perp(1)}(x) \quad + \text{ “UV” } \ldots$$

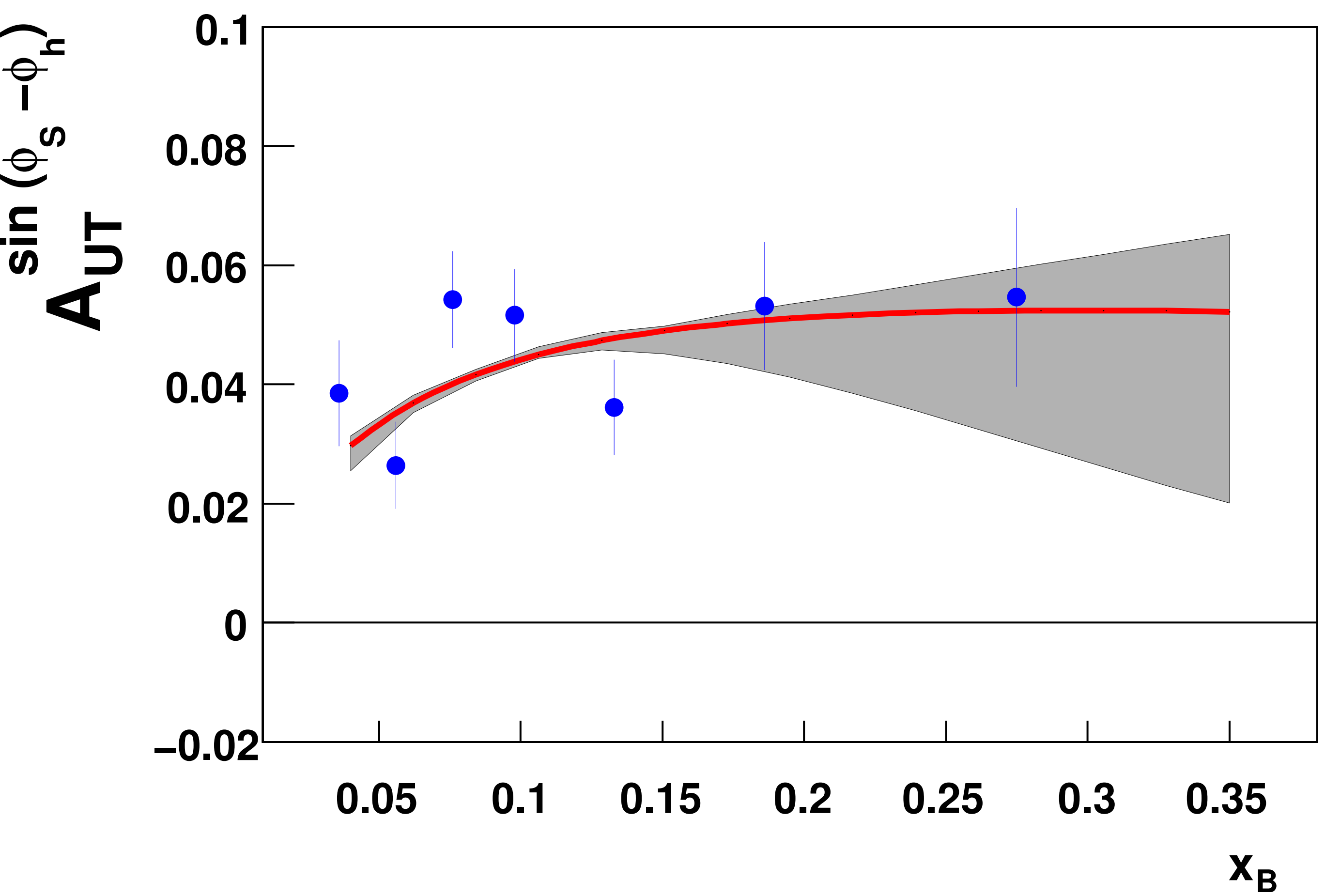
Z. Kang, J.W. Qiu, W. Vogelsang, F. Yuan Phys. Rev D 2011 "compatibility study"
L. Gamberg, Z. Kang, Phys. Lett B696 "compatibility study"
L.Gamberg, Z. Kang, A. Prokudin, Phys. Rev. Lett 2013 "compatibility study"
and others ....

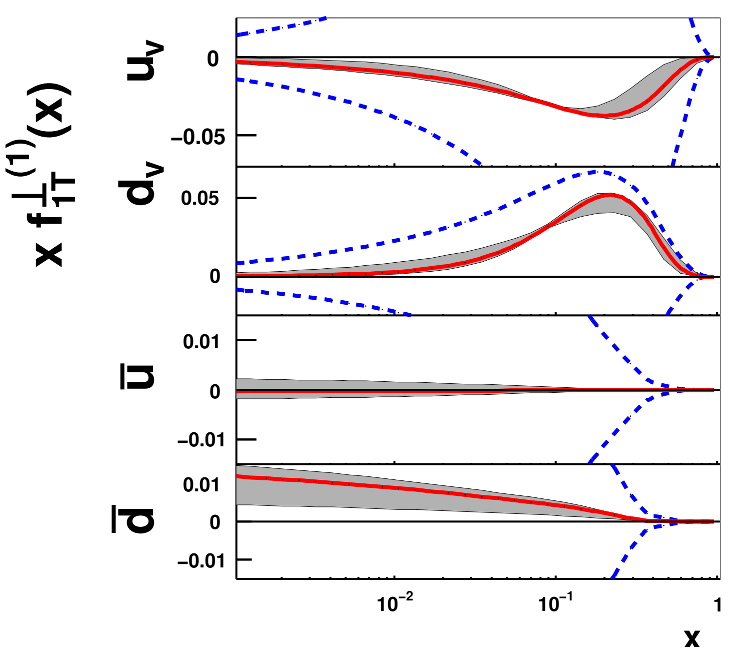
Can use for a Gaussian-like/ parton model

Extract Sivers function from SIDIS data

CGI-GPM

1)Ingredients of Torino Model parametrization but w/ color factors *ie Gauge links*
2)Use GRV98LO for spin average collinear pdf
3)Use DSS for collinear FF
4) Enforce postivitity bound on Sivers and unpol





$$f_{1T}^{\perp(1)}(x) = \int d^2k_\perp \frac{k_\perp^2}{2M} f_{1T}^\perp(x, k_\perp^2)$$

•*Indication on the process-dependence of the Sivers effect*
L. Gamberg, Z. Kang, A. Prokudin, **Phys. Rev. Lett. 110, 232301 (2013)**

Calculate polarized cross section for $P^{\uparrow}P \to Jet\,X$

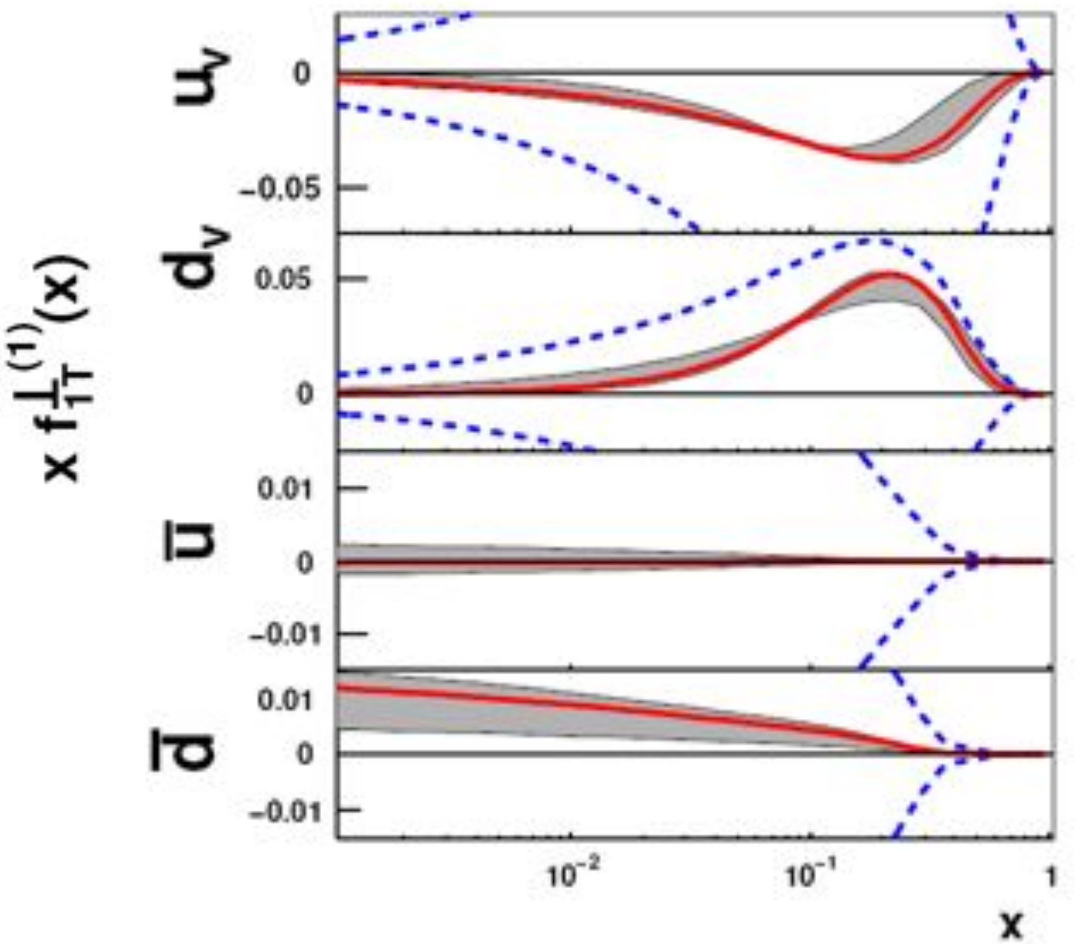
We calculate jet A_N in twist-3:

$$E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} = \epsilon_{\alpha\beta} s_\perp^\alpha P_{J\perp}^\beta \frac{\alpha_s^2}{s} \sum_{a,b} \int \frac{dx}{x} \frac{dx'}{x'} f_{b/B}(x') \times \left[T_{a,F}(x,x) - x\frac{d}{dx} T_{a,F}(x,x) \right] \times \frac{1}{\hat{u}} H_{ab\to c}^{\mathrm{Sivers}}(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u}),$$

Twist-3 TMD relation

Use Sivers that describes SIDIS:

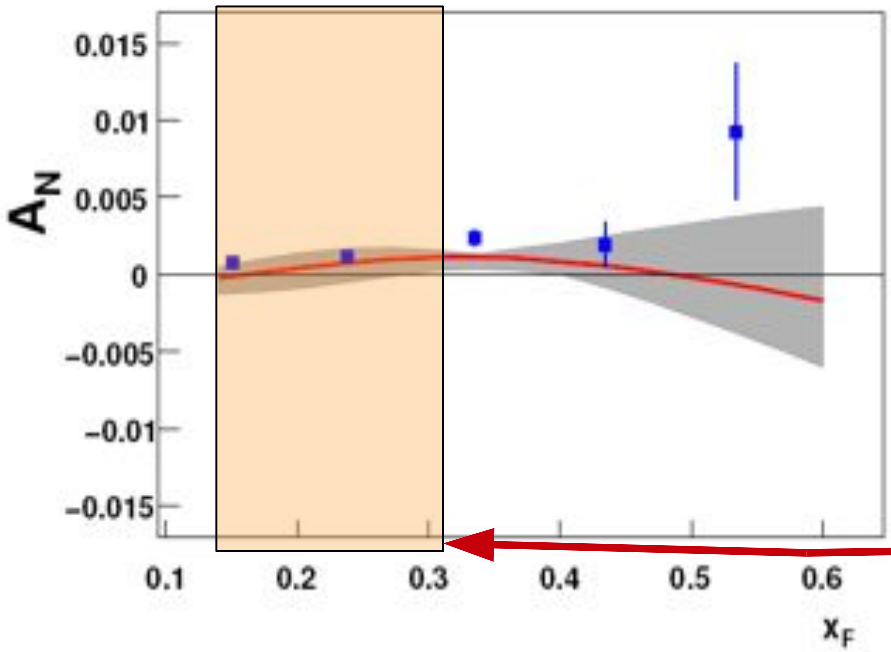
Gamberg, Kang, Prokudin (2013)



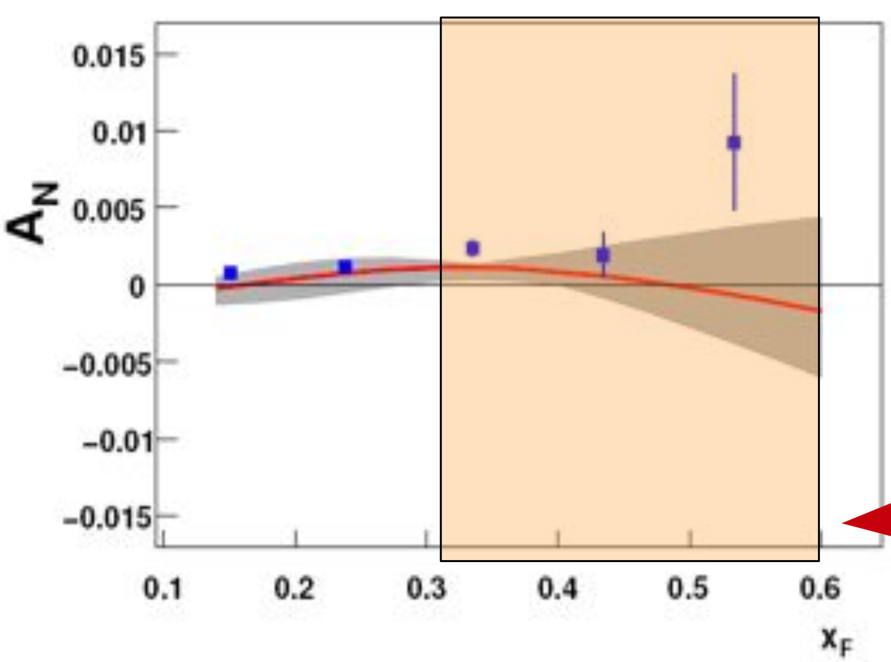
$$A_N = E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} \Big/ E_J \frac{d\sigma}{d^3P_J}$$

Compare with AnDY data:

Gamberg, Kang, Prokudin PRL (2013)



This region corresponds to SIDIS kinematical region: agreement is very encouraging



This region relies on large-x region, future JLab 12 measurement is important

✔ The sign is correct

✔ The size is correct

Result is indication

✔ TMD and twist-3 are compatible

✔ Sivers effect is process dependent

An Indication on the process dependence of Sivers Effect

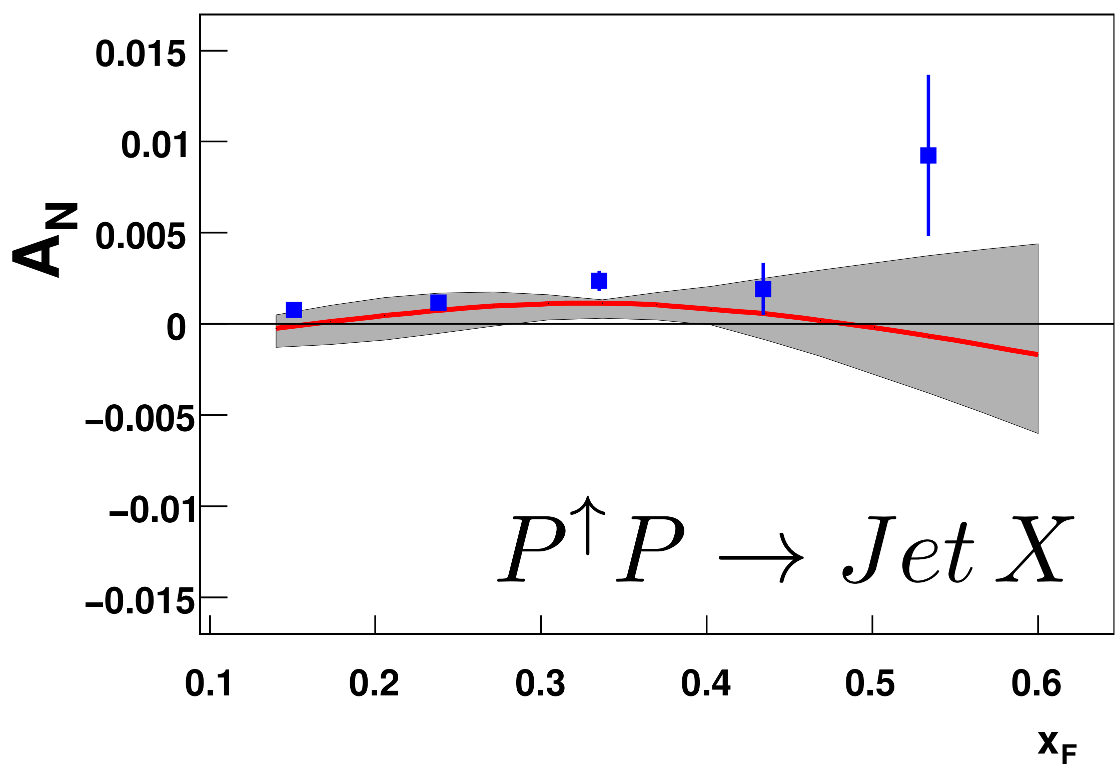
We need Drell Yan Results

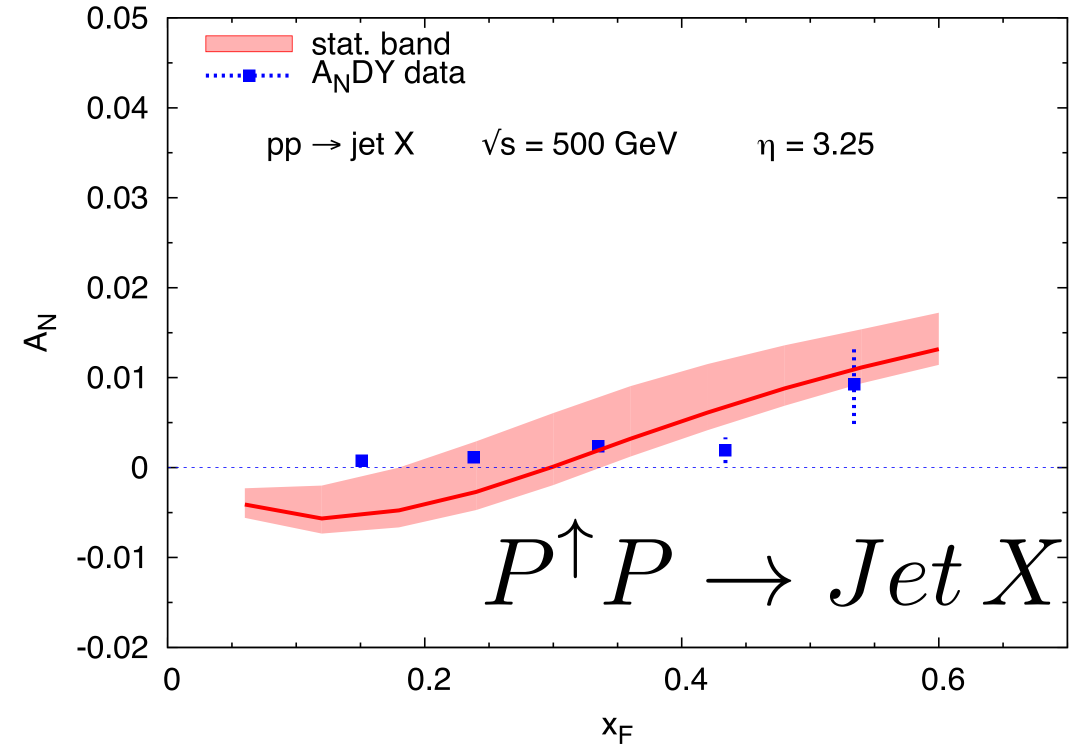
L. Gamberg, Z. Kang, A. Prokudin,
Phys. Rev. Lett. 110 (2013) 232301

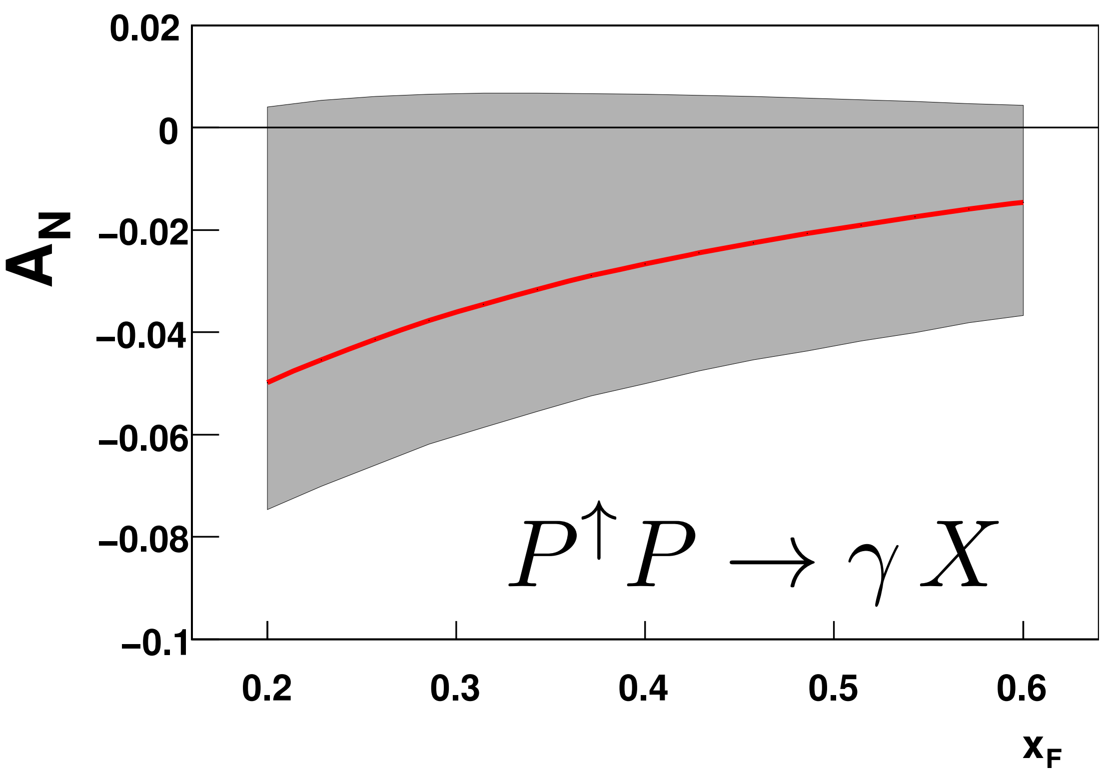
w/ color factors

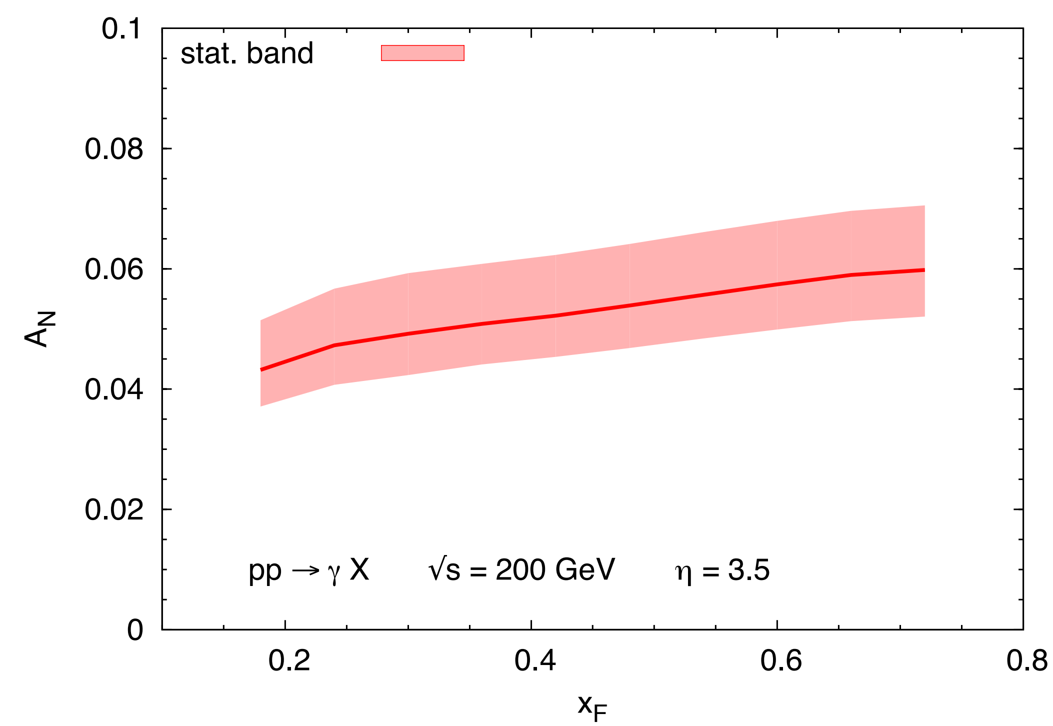
Anselmino et al.
arXiv:1304.7691

w/o color factors









Problem with k_T moments

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

Problem with k_T moments

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

- power counting ... Sivers tail $\qquad f_{1T}^{\perp}(x, k_T) \sim \dfrac{M^2}{\left(k_T^2 + M^2\right)^2}$

  Aybat, Collins, Rogers, Qiu PRD 2012

- "First Moment" diverges but not so if you generalize via Bessel moments **Boer, Gamberg, Musch, Prokuding JHEP 2011**

Remarks

- Use the relation between Bessel Moments of Sivers and Collins function thru *TMD* evolution formalism
- And use TMD evolution in *b*-space to express these TMDs through the OPE
- Fit these moments from SIDIS and e^+e^-
- We use to determine the twist three as input for A_N

★ Intuition from *b*-space interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \\
&\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int \frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+ \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h} \\
&+ S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin 2\phi_h}\Big] \\
&+ S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\Big] \\
&+ |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad + \varepsilon\,\sin(3\phi_h-\phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

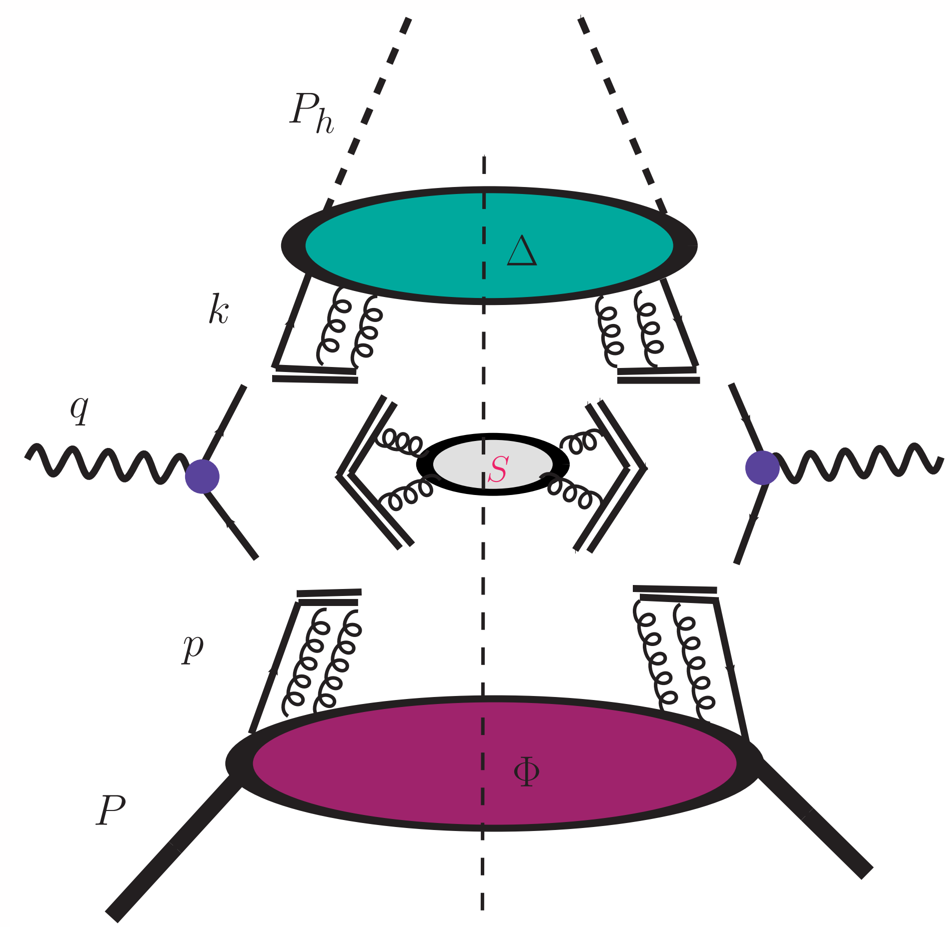
$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}_1]$$

$$\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} = -\mathcal{P}[\tilde{h}_1\tilde{H}_1^{\perp(1)}]$$

Bessel-Weighted Asymmetries in Semi Inclusive Deep Inelastic Scattering,
D. Boer, L. Gamberg, B. Musch, A. Prokudin,
JHEP (2011)

Review of TMD factorization

★ Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13),Echevarria, Idilbi, Scimemi JHEP 2012, Collins Rogers 2015 ....



- TMDs w/Gauge links: color invariant
- *Soft factor w/Gauge links*
- *Hard cross section*

- Divergences at one loop and higher
- Extra parameters needed to regulate light-cone, soft & collinear divergences
- Modifies convolution integral introduction of soft factor

Review of TMD factorization

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu),\mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x,b_T;\mu,\zeta_1)\,\tilde{D}_{H_2/j'}(z,b_T;\mu,\zeta_2)\ +\ Y_{\mathrm{SIDIS}}$$

In full QCD, the auxiliary parameters are exactly arbitrary and this is reflected in the the Collins-Soper (CS) equations for the TMD PDF, and the renormalization group (RG) equations

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Collins, Gamberg, Prokudin, Sato, Wang

Elements of TMD Fact. Cross section

- Y term serves to correct expression for structure function when $P_T \sim Q$

- Evolution kernel contains both perturbative and non-perturbative content arising from TMD factorization $\longleftrightarrow$ evolution

- This structure is based upon earlier CS 81 & CSS 85 formalism & new treatment of soft factor and CSS equations.
  Also see Collins & Rogers PRD 2015

Unpack the Transverse Polarized Target Structure Functions

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,q_T,Q) = -H_{SIDIS}(Q,\mu=Q)\sum_a e_q^2 \int_{k_\perp,p_\perp} f_{1T}^{\perp}(x_B,k_\perp^2;Q)\frac{\hat{P}_{h\perp}\cdot k_\perp}{M} D_1(z_h,p_\perp^2;Q)$$

$$\int_{k_\perp,p_\perp} \equiv \int d^2k_\perp d^2p_\perp \delta^2\left(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}\right)$$

Fourier Bessel Moments of "Sivers Structure Function"

$$\begin{aligned}
F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,q_T,Q) &= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/z,b)\tilde{\mathcal{F}}_{UT}(x,z,b,Q^2) \\
&= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/z,b)\left(\frac{2}{M^2}\frac{\partial}{\partial b^2}\right) f_{1T}^{\perp}(x_B,b;Q)D_1(z_h,b;Q) \\
&= -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b^2 J_1(P_{h\perp}/z,b) f_{1T}^{\perp(1)}(x_B,b;Q)D_1(z_h,b;Q)
\end{aligned}$$

Boer, Gamberg, Musch, Prokudin JHEP 2011 Bessel Moments
Also Aybat, Collins, Qiu, Rogers PRD 2012

$$\tilde{f}_{1T}^{\perp(1)}(x,b;Q) \equiv \frac{2}{M^2}\frac{\partial}{\partial b^2}\tilde{f}_{1T}^{\perp}(x,b;Q)$$

$$\tilde{f}_{1T}^{\perp(1)}(x,b;Q) = \frac{2\pi}{M^2}\frac{1}{b}\int_0^\infty dk_\perp k_\perp^2 J_1(k_\perp b)\ \ f_{1T}^{\perp}(x,k_\perp;Q)$$

Sivers moment in b-space

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\tilde{D}_1]$$

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,b,Q) = H_{UT}(Q;\mu)\sum_q \tilde{f}_{1T\,i/P}^{q(1)}(x,b;Q)\tilde{D}_H^q(z,b;Q)$$

TMDs are defined at a scale Q

Evolution is performed in Fourier space

Over short transverse distance scales, $1/b$ is hard scale, and the b dependence of TMDs can be calculated in perturbation theory

TMD Evolution of unpolarized Structure Functions

Fourier Bessel Transform

$$F_{UU}(Q;P_{h\perp}) = -H_{SIDIS}(Q,\mu)\sum_a e_q^2 \int \frac{db}{2\pi} b\, J_0(P_{h\perp}/z, b)\tilde{f}_1(x_B, b; Q)\tilde{D}_1(z_h, b; Q)$$

$$\tilde{\mathcal{F}}_{UU}(x,z,b,Q) = H_{UU}(Q;\mu)\sum_q e_q^2 \tilde{f}_1^q(x,b;Q)\tilde{D}_1^q(z,b;Q)$$

JCC formalism express evolution of TMDS
OPE in terms of collinear pdfs

Elements of TMD Evolution

I.) Over short transverse distance scales, $1/b$ is hard scale, and the b dependence of TMDs is calculated in perturbation theory

$$\tilde{f}_1(x_B, b; Q) = \tilde{f}_1(x_B, b; \mu_b) e^{-S_{\rm pert}(b,Q)}, \qquad b << 1/\Lambda_{\rm QCD}$$

Standard CSS formalism, evolution starts from

$$S_{\rm pert}(b,Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu})\ln\frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})\right] \qquad \mu_b = c/b,\ c = 2e^{-\gamma_E}$$

$$A = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n A^{(n)} \qquad B = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n B^{(n)}$$

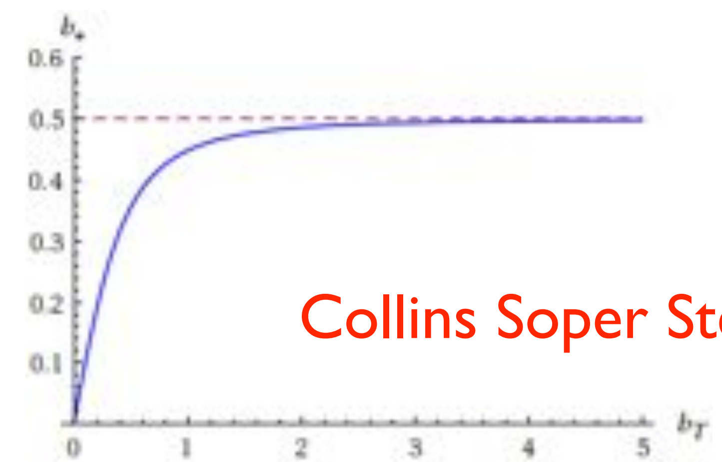
Perturbative contribution CSS NPB 85 JCC 2011

II.) However Fourier transform space involves non-perturbative b region where perturbation theory breaks down

$$f_1(x, k_\perp; Q) = \int_0^\infty \frac{db}{2\pi} b J_0(k_\perp b) \tilde{f}_1(x, b; Q)$$

Non perturbative region treated with b^* prescription

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$



Collins Soper Sterman NPB 85

$$\tilde{f}_1(x, b; Q) = \tilde{f}_1(x, b_*; c/b_*) e^{-\frac{1}{2} S_{pert}(Q, b_*) - \frac{1}{2} S_{NP}^{\text{sivers}}(Q, b)}$$

III.) With $1/b$ as hard scale, the b dependence of TMDs is calculated in perturbation theory and related to their collinear parton distribution (PDFs), fragmentation functions (FFs), or multiparton correlation functions ,, … OPE.

$$\tilde{f}_1(x,b;Q) = \tilde{f}_1(x,b_*;c/b_*)e^{-\frac{1}{2}S_{pert}(Q,b_*)-\frac{1}{2}S_{NP}^{f_1}(Q,b)}$$

$$f_1^i(x,b;Q) = C_{q\leftarrow i}^{f_1} \otimes f_1^i(x,\mu_{b_*})e^{\frac{1}{2}S_{pert}(Q,b_*)-S_{NP}^{f_1}(Q,b)}$$

$$C_{q\leftarrow i} \otimes f_1^i(x_B,\mu_b) \equiv \sum_i \int_{x_B}^1 \frac{dx}{x} C_{q\leftarrow i}\left(\frac{x_B}{x},\mu_b\right) f_1^i(x,\mu_b)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$ Wilson coefficient

Summary TMD Evolution of Structure Functions

$$
\begin{aligned}
\tilde{\mathcal{F}}_{UU}(x,z,b,Q^2) &= H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b,\mu,\zeta_F)\tilde{D}_1^q(z_h,b,\mu,\zeta_D) \\
&= H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b_*,\mu,\zeta_F)\tilde{D}_1^q(z_h,b_*,\mu,\zeta_D)e^{-S_{\mathrm{pert}}(b_*,Q)-S_{UU}^{NP}(b,Q)} \\
&= H_{UU}(Q,\mu=Q)\sum_q e_q^2\, C_{q\leftarrow i}^{\mathrm{SIDIS}} \otimes \tilde{f}_1^i(x,\mu_b)\hat{C}_{j\leftarrow q}^{\mathrm{SIDIS}} \otimes \tilde{D}_{h/j}^q(x,\mu_b)e^{-S_{\mathrm{pert}}(b_*,Q)-S_{UU}^{NP}(b,Q)}
\end{aligned}
$$

JCC formalism can express evolution of TMDS
OPE in terms of collinear pdfs

...and what about Sivers Evolution ??

$$\frac{k_\perp}{M^2} f_{1T}^{\perp}(x, k_\perp; Q) = \int_0^\infty \frac{db}{2\pi} b^2 J_1(k_\perp b) \tilde{f}_{1T}^{\perp(1)}(x, b; Q)$$

With TMD Evolution with, b_* & OPE

$$\tilde{f}_{1T}^{\perp(1)}(x, b; Q) = \Delta \tilde{C}_{i\leftarrow q}^{Sivers} \otimes \tilde{f}_{1T}^{\perp(1)}(x, \mu_b) e^{-\frac{1}{2} S_{pert}(Q, b_*) - \frac{1}{2} S_{NP}^{sivers}}$$

see also Kang, Xaio, Yuan PRL 2011
Aybat, Collins, Qiu, Rogers PRD 2012
Echevarria, Idilbi, Kang, Vitev PRD 2014

Transverse Polarized Target Structure Functions Collins Contribution

$$F_{UT}^{\sin(\phi_h+\phi_s)}(Q;P_{h\perp}) = -H_{\text{SIDIS}}(Q,\mu=Q)\sum_q e_q^2 \int_{k_\perp,p_\perp} h_1(x_B,k_\perp;Q)\frac{\hat{P}_{h\perp}\cdot p_\perp}{M_h} H_{1h/q}^{\perp}(z_h,p_\perp^2;Q)$$

$$\int_{k_\perp,p_\perp} \equiv \int d^2k_\perp d^2p_\perp \delta^2\left(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}\right)$$

Fourier Bessel Transform

$$\begin{aligned}
F_{UT}^{\sin(\phi_h+\phi_s)}(Q;P_{h\perp}) &= -H_{\text{SIDIS}}(Q,\mu=Q)\sum_a e_q^2 \int \frac{db}{2\pi} b^2\, J_1(P_{h\perp}/z,b) h_1(x_B,b;Q)\left(\frac{2}{M_h}\frac{\partial}{\partial b^2}\right) H_1^{\perp}(z_h,b;Q) \\
&= -H_{\text{SIDIS}}(Q,\mu=Q)\sum_a e_q^2 \int \frac{db}{2\pi} b^2\, J_1(P_{h\perp}/z,b) h_1(x_B,b;Q) H_1^{\perp(1)}(z_h,b;Q)
\end{aligned}$$

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

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JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

Kang-Prokudin-Sun-Yuan PRD 2016

$$\frac{p_\perp}{zM_h} H_{1h/q}^{\perp}(z, p_\perp^2; Q) = \frac{1}{z^2} \int_0^\infty \frac{db\, b^2}{(2\pi)} J_1(p_\perp b/z) \delta C_{i \leftarrow q}^{\text{collins}} \otimes H_{1h/i}^{\perp(1)}(z, \mu_b) e^{\frac{1}{2} S_{\text{pert}}(Q, b_*) - S_{\text{NP}}^{\text{collins}}(Q, b)}$$

b-space OPE

$$H_{1h/q}^{\perp(1)}(z, b; Q) \equiv \frac{1}{z^2} \frac{b^2}{2\pi} \delta C_{i \leftarrow q}^{\text{collins}} \otimes H_{1h/i}^{\perp(1)}(z, \mu_b) e^{\frac{1}{2} S_{\text{pert}}(Q, b_*) - S_{\text{NP}}^{\text{collins}}(Q, b)}$$

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

$$h_1^q(x, k_\perp^2; Q) = \int_0^\infty \frac{db\, b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

b-space OPE

$$h_1^q(x, b; Q) = \frac{b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

Coefficient functions *b-space OPE*

Kang, Prokudin, Sun, Yuan PRD 2016

$$C^{(\mathrm{SIDIS})}_{q\leftarrow q'}(x,\mu_b) = \delta_{q'q}\left[\delta(1-x)+\frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-x)-2C_F\delta(1-x)\right)\right],$$

$$C^{(\mathrm{SIDIS})}_{q\leftarrow g}(x,\mu_b) = \frac{\alpha_s}{\pi}T_R x(1-x), \qquad \delta C_{q\leftarrow q'}(x,\mu_b) = \delta_{q'q}[\delta(1-x)+\mathcal{O}(\alpha_s^2)],$$

$$\hat{C}^{(\mathrm{SIDIS})}_{q'\leftarrow q}(z,\mu_b) = \delta_{q'q}\left[\delta(1-z)+\frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-z)-2C_F\delta(1-z)+P_{q\leftarrow q}(z)\ln z\right)\right],$$

$$\hat{C}^{(\mathrm{SIDIS})}_{g\leftarrow q}(z,\mu_b) = \frac{\alpha_s}{\pi}\left(\frac{C_F}{2}z+P_{g\leftarrow q}(z)\ln z\right)$$

$$\delta\hat{C}^{(\mathrm{SIDIS})}_{q'\leftarrow q}(z,\mu_b) = \delta_{q'q}\left[\delta(1-z)+\frac{\alpha_s}{\pi}(\hat{P}^c_{q\leftarrow q}(z)\ln z)\right]$$

For twist 3 keep only homogeneous or diagonal terms in splitting kernal, Kang plb 2011 and for Sivers, Sun Yuan PRD 2013

NLL' extraction from the data $A^{(1,2)}$ $B^{(1)}$ $C^{(1)}$

Parametrizations:

Transversity $$h_1^q(x,Q_0) \propto N_q^h x^{a_q}(1-x)^{b_q}\frac{1}{2}\left(f_1(x,Q_0)+g_1(x,Q_0)\right)$$

Favoured and unfavoured Collins FF

$$\hat{H}_{fav}^{(3)}(z,Q_0) = N_u^c z^{\alpha_u}(1-z)^{\beta_u} D_{\pi^+/u}(z,Q_0)$$

$$\hat{H}_{unf}^{(3)}(z,Q_0) = N_d^c z^{\alpha_d}(1-z)^{\beta_d} D_{\pi^+/d}(z,Q_0)$$

Total 13 parameters: $N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_d, \beta_u, g_c$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

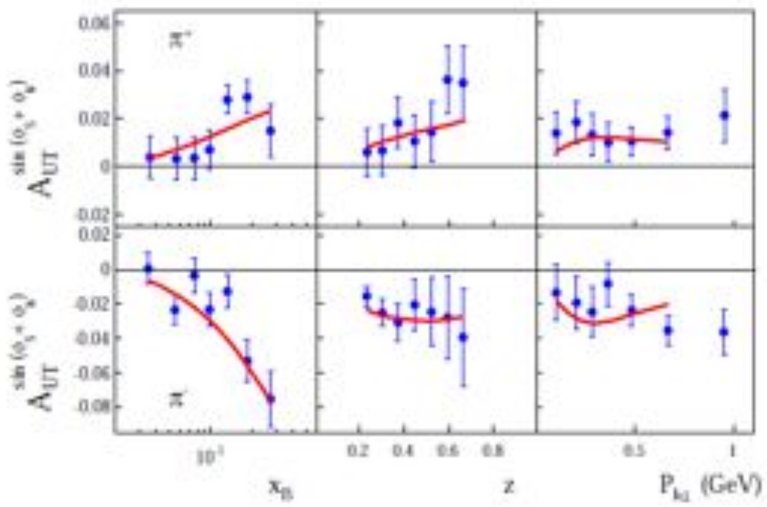
e+e- data used: BELLE, BABAR including PT dependence – 122 points

$$\chi^2/\mathrm{d.o.f.} \simeq .88$$

details in Kang, Prokudin, Sun, Yuan PRD 2016

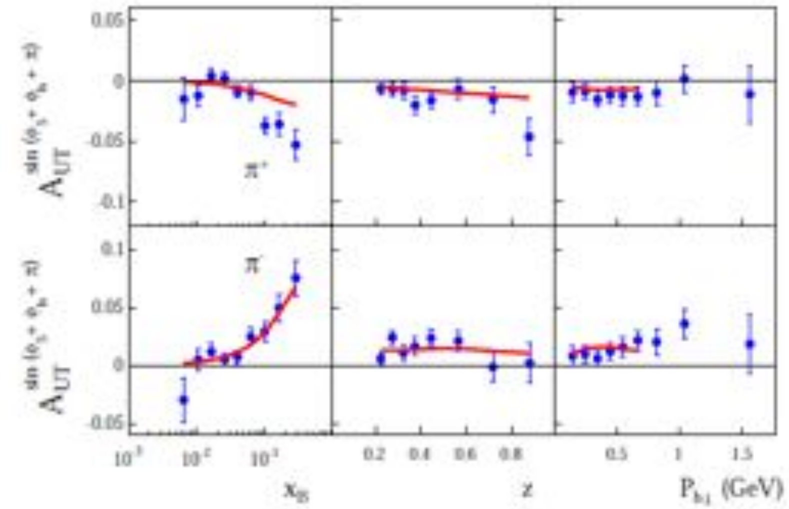
$$\ell P \to \pi^{\pm} X$$

HERMES



$1 \lesssim \langle Q^2 \rangle \lesssim \ 6 \ \mathrm{GeV}^2$

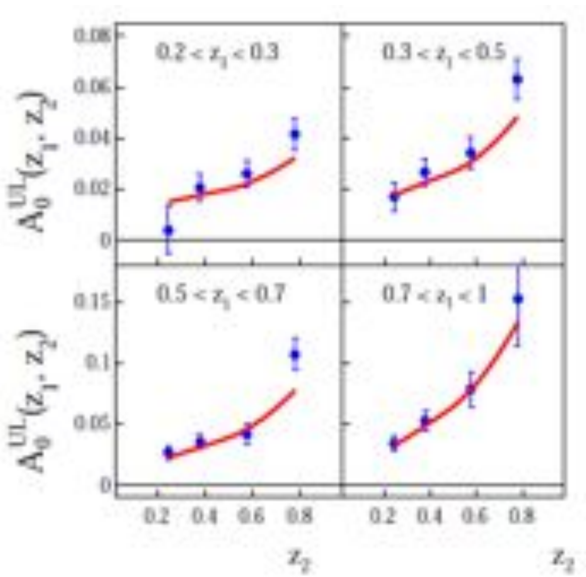
COMPASS



$1 \lesssim \langle Q^2 \rangle \lesssim \ 21 \ \mathrm{GeV}^2$

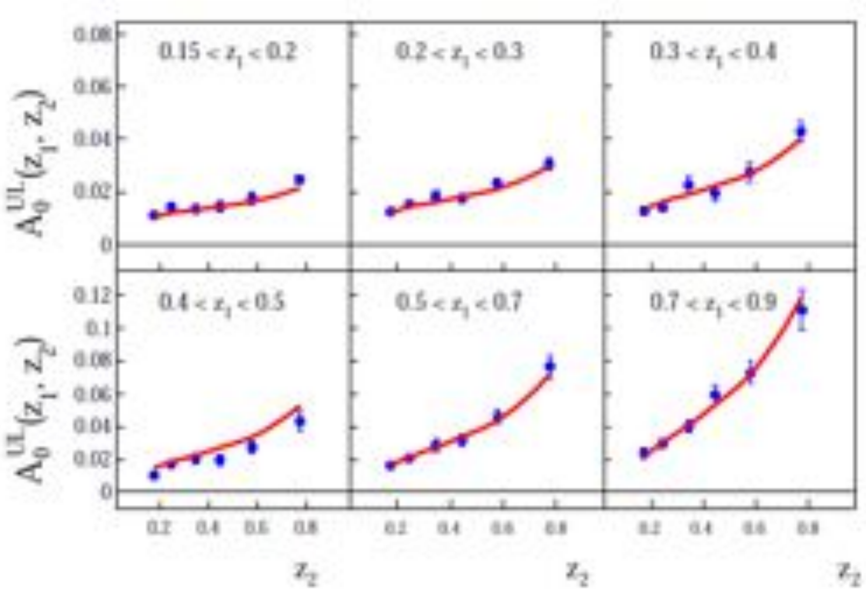
$$e^+ e^- \to \pi \pi X$$

BELLE



$Q^2 = 110 \ \mathrm{GeV}^2$

BABAR



$Q^2 = 110 \ \mathrm{GeV}^2$

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

$$\frac{p_\perp}{zM_h} H_{1h/q}^{\perp}(z, p_\perp^2; Q) = \frac{1}{z^2}\int_0^\infty \frac{db\, b^2}{(2\pi)} J_1(p_\perp b/z)\, \delta C_{i\leftarrow q}^{\text{collins}} \otimes H_{1h/i}^{\perp(1)}(z, \mu_b) e^{\frac{1}{2}S_{\text{pert}}(Q,b_*) - S_{\text{NP}}^{\text{collins}}(Q,b)}$$

b-space OPE

$$H_{1h/q}^{\perp(1)}(z, b; Q) \equiv \frac{1}{z^2}\frac{b^2}{2\pi}\delta C_{i\leftarrow q}^{\text{collins}} \otimes H_{1h/i}^{\perp(1)}(z, \mu_b) e^{\frac{1}{2}S_{\text{pert}}(Q,b_*) - S_{\text{NP}}^{\text{collins}}(Q,b)}$$

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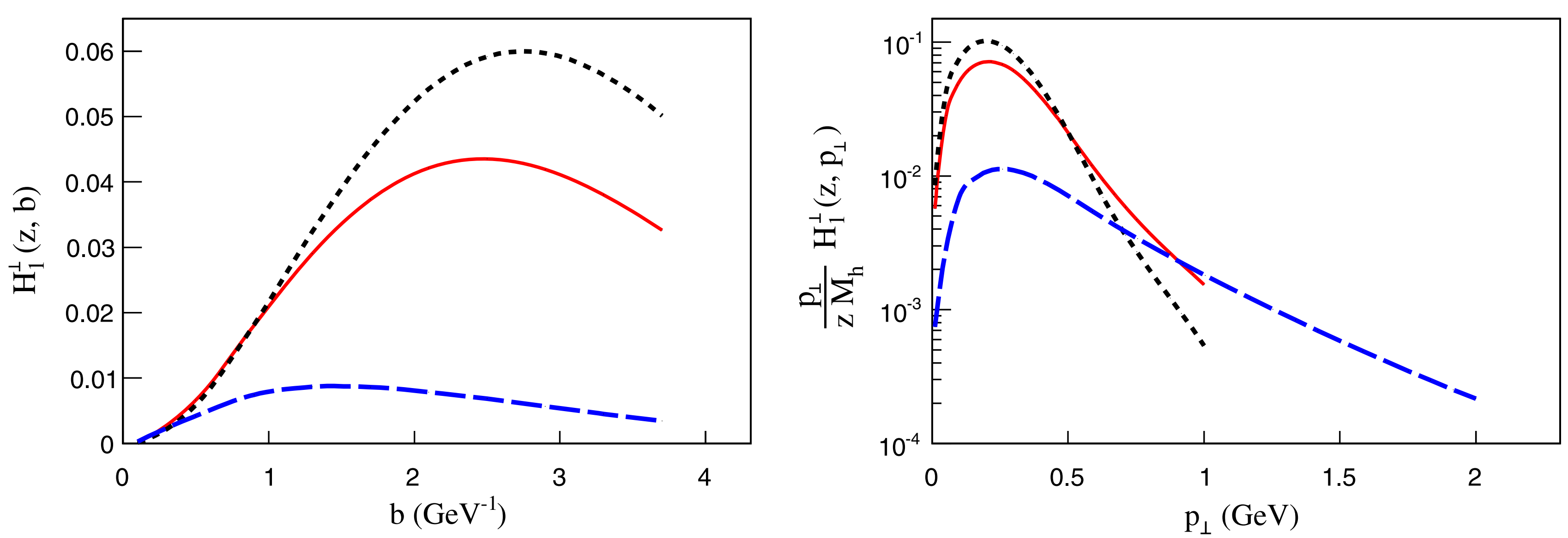

FIG. 11. Collins FF $u \to \pi^+$ as a function of b (a) and as a function of $p_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

$$h_1^q(x, k_\perp^2; Q) = \int_0^\infty \frac{db\, b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

b-space OPE

$$h_1^q(x, b; Q) = \frac{b}{2\pi} J_0(k_\perp b)\, \delta C_{q\leftarrow i} \otimes h_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{h_1}(Q, b)}$$

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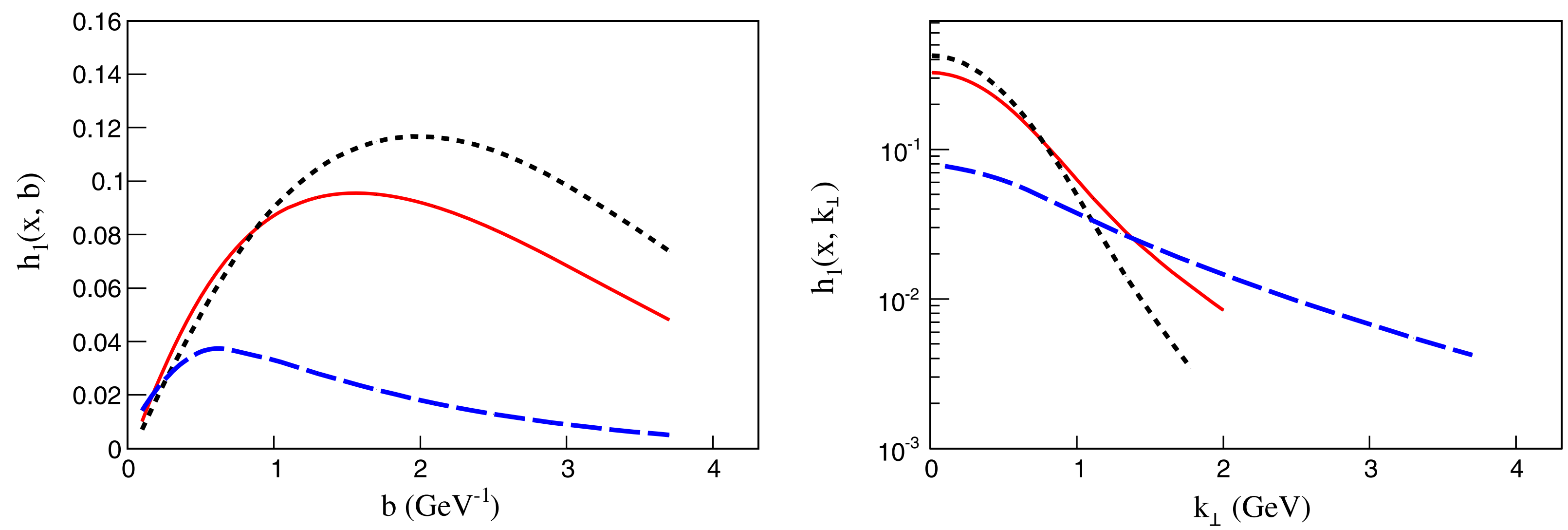

FIG. 9. Transversity u-quark distribution as a function of b (a) and as a function of $k_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

JCC formalism can express evolution of TMDS OPE in terms of collinear pdfs

$$f_1^q(x, k_\perp^2; Q) = \int_0^\infty \frac{db\, b}{2\pi} J_0(k_\perp b)\, C_{q\leftarrow i}^{f_1} \otimes \tilde{f}_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q,b_*) - S_{NP}^{f_1}(Q,b)}$$

b-space OPE

$$f_1^q(x, b; Q) = \frac{b}{2\pi} C_{q\leftarrow i}^{f_1} \otimes \tilde{f}_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q,b_*) - S_{NP}^{f_1}(Q,b)}$$

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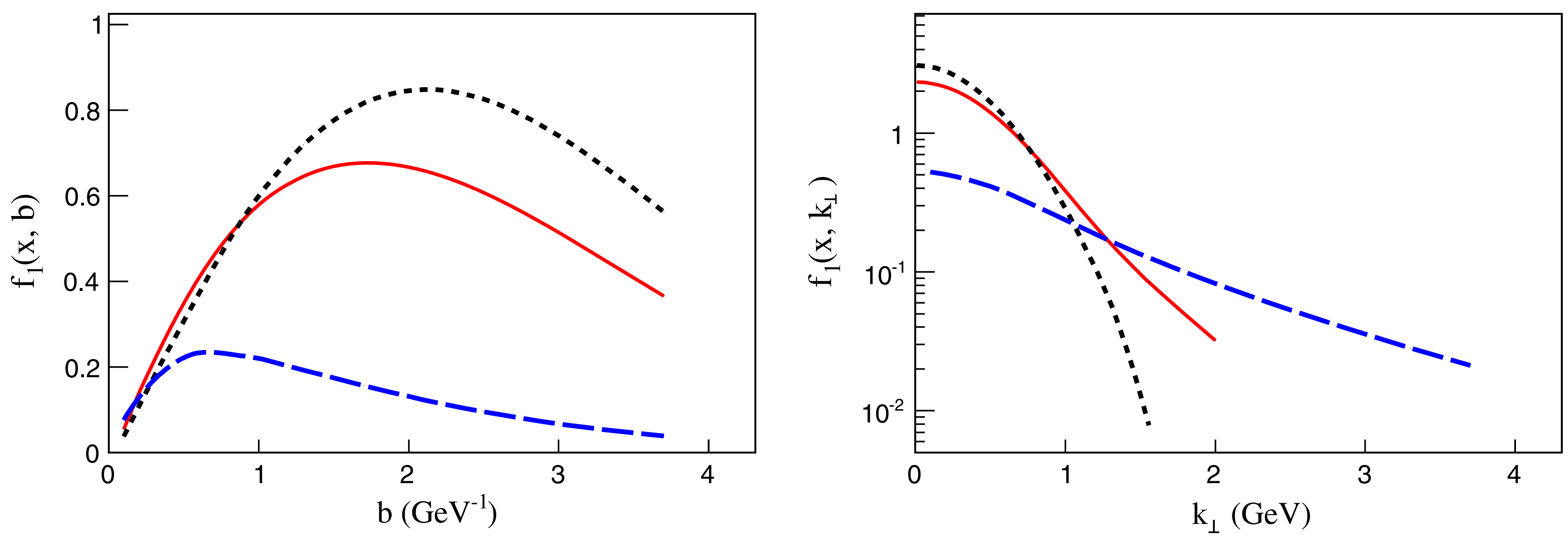

FIG. 8. Unpolarized u-quark distribution as a function of b (a) and as a function of $k_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

TMD Evolution of Structure Functions & TMDs

JCC formalism can express evolution of TMDS
OPE in terms of collinear pdfs

. . .

$$D_{h/q}(z, p_\perp^2; Q) = \frac{1}{z^2}\int_0^\infty \frac{dbb}{(2\pi)} J_0(p_\perp b/z)\hat{C}^{D_1}_{i\leftarrow q} \otimes D_{h/i}(z, \mu_b) e^{-\frac{1}{2}S_{\rm pert}(Q,b_*) - S^{D_1}_{\rm NP}(Q,b)},$$

b-space OPE

$$D_1^q(z, b; Q) \equiv \frac{1}{z^2}\frac{b}{2\pi}\hat{C}^{D_1}_{i\leftarrow q} \otimes D_{h/i}(z, \mu_b) e^{\frac{1}{2}S_{pert}(Q,b_*) - S^{D_1}_{NP}(Q,b)}$$

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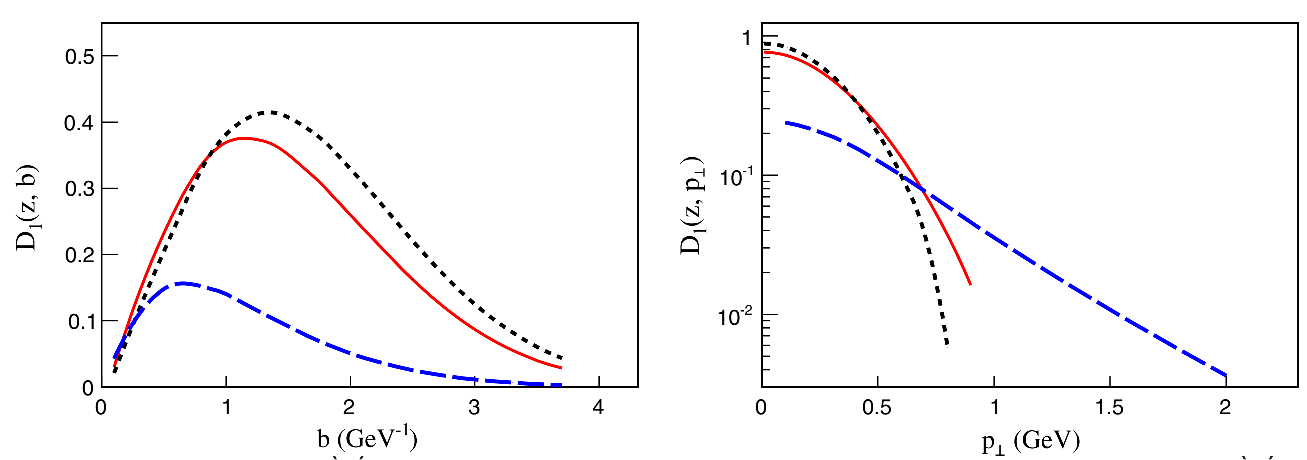

FIG. 10. Unpolarized FF $u \to \pi^+$ as a function of b (a) and as a function of $p_\perp$ (b) at three different scales, $Q^2 = 2.4$ (dotted lines), $Q^2 = 10$ (solid lines), and $Q^2 = 1000$ (dashed lines) GeV^2.

What are evolution effects?

$$e^+e^- \to \pi\pi X$$

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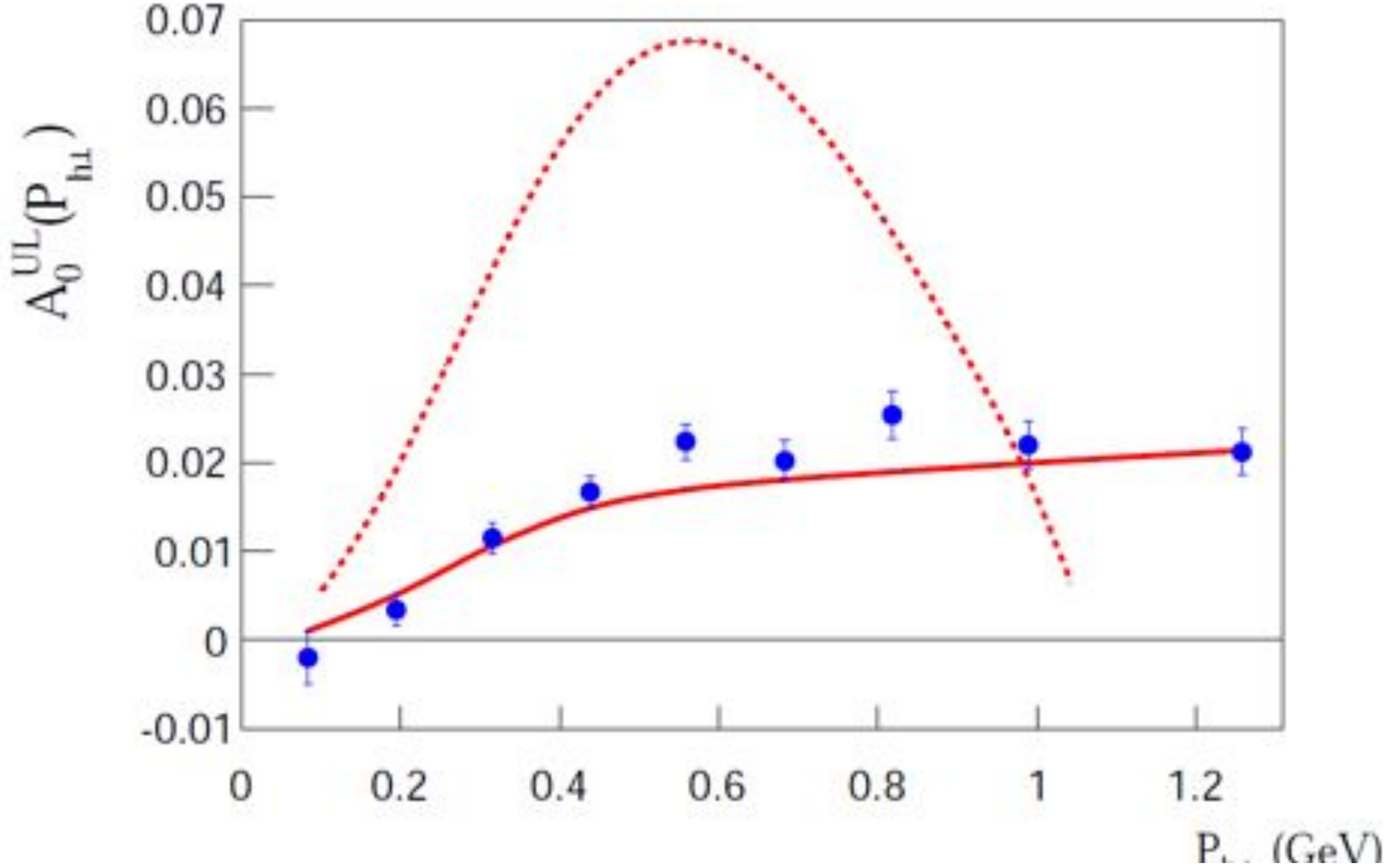
No evolution: $Q^2 = 2.4\ \mathrm{GeV}^2$

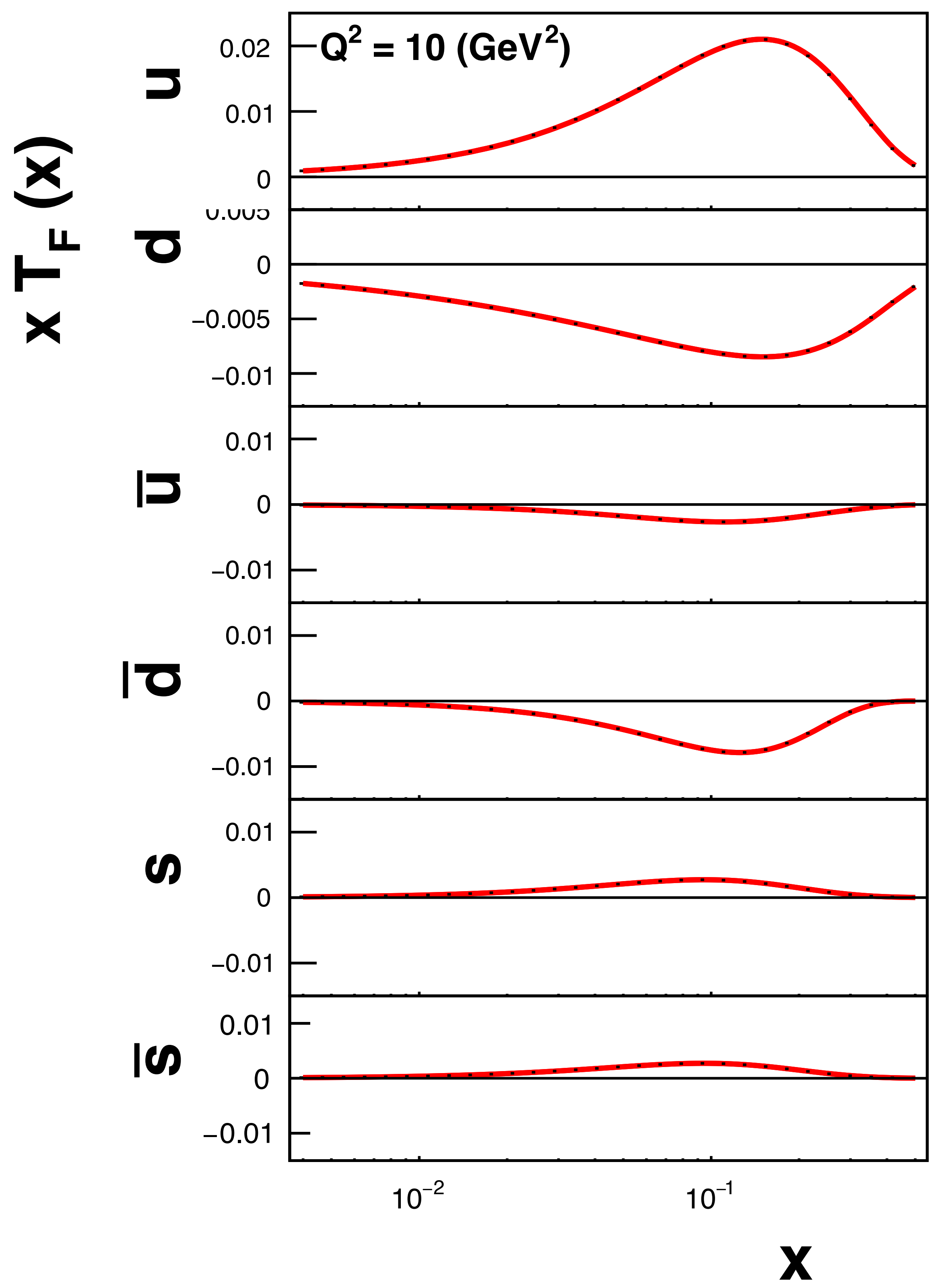
NLL' evolution: $Q^2 = 110\ \mathrm{GeV}^2$



$$Q_1^2/Q_2^2 \simeq 50$$

Asymmetry ratio ~ 3.5

Sivers fit *Preliminary under TMD evolution*



$$\tilde{f}_{1T}^{\perp(1)}(x,b;Q) = \frac{b^2}{(2\pi)}\Delta\tilde{C}_{i\leftarrow q}^{Sivers} \otimes \tilde{f}_{1T}^{\perp(1)}(x,\mu_b)e^{-\frac{1}{2}S_{pert}(Q,b_*)-\frac{1}{2}S_{NP}^{sivers}}$$

Input to A_N

$$E_h \frac{d^3\Delta\sigma(S_\perp)}{d^3P_h}\bigg|_{\text{forward}}$$

$$= \epsilon_{\perp\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{\min}}^{1} \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x)$$

$$\times \int_{z_{\min}}^{1} \frac{dz}{z}\left[-z\frac{\partial}{\partial z}\left(\frac{\hat{H}(z)}{z^2}\right)\right]$$

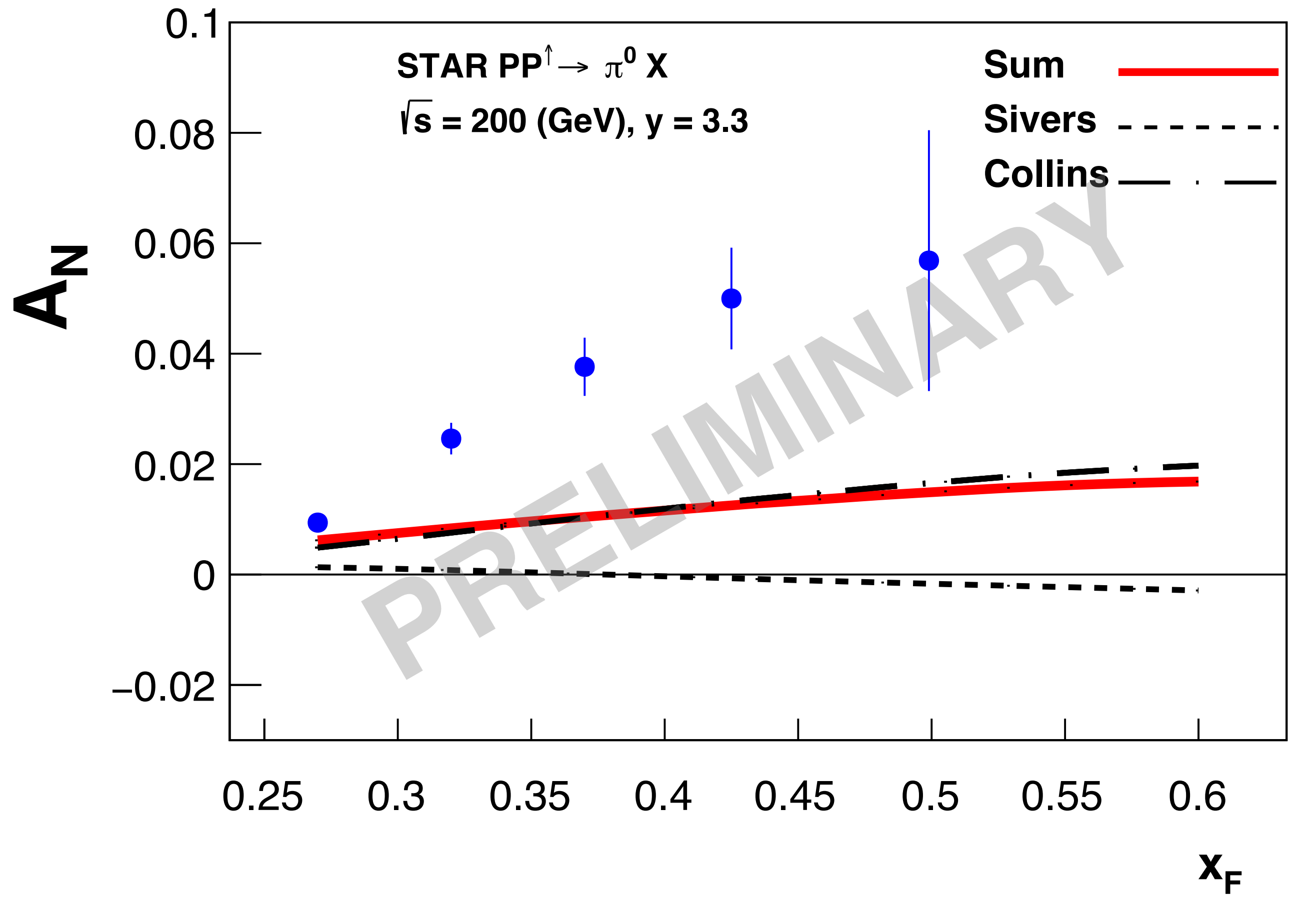
Collins like

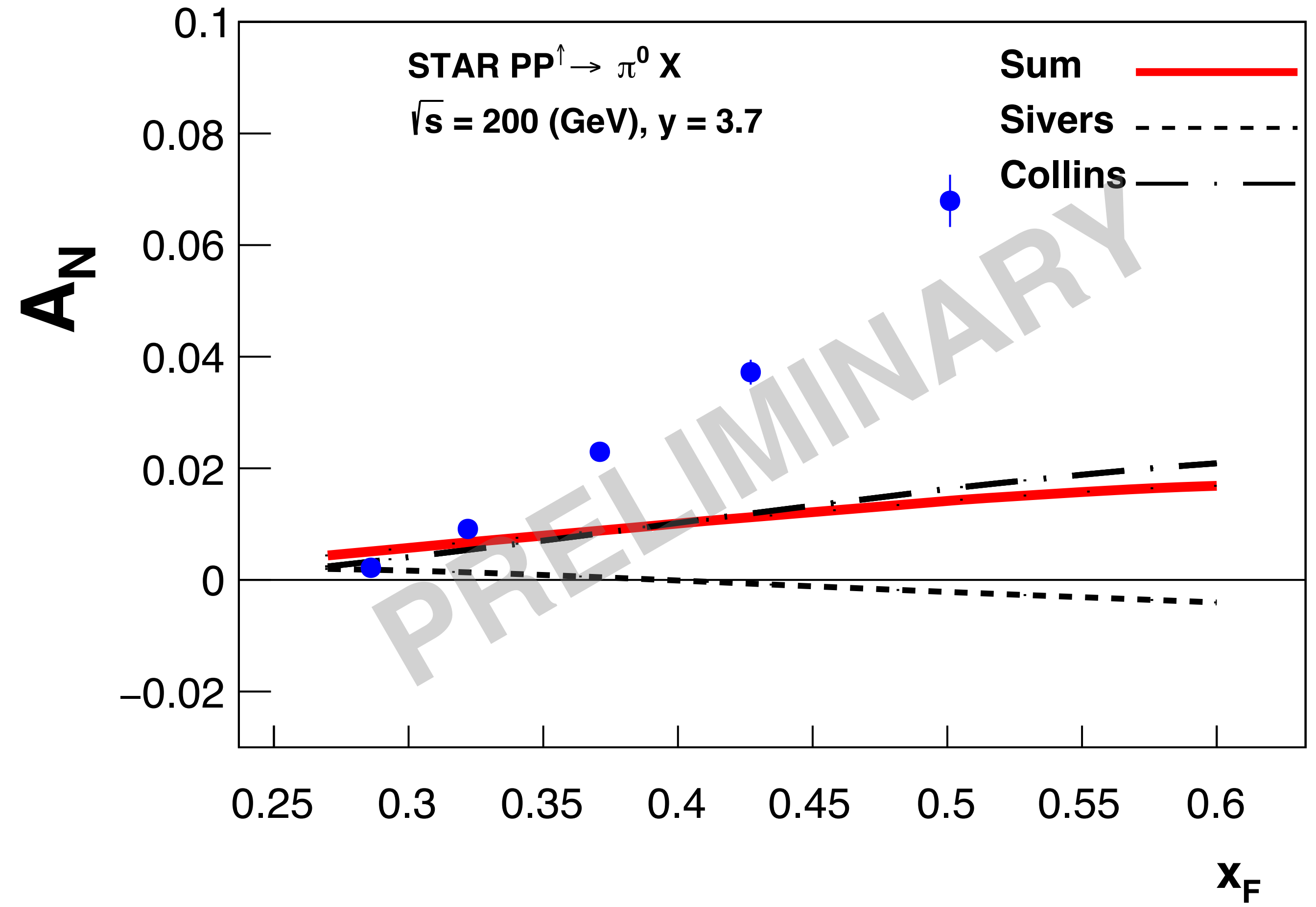
$$\times \frac{1}{x'S + T/z} \frac{1}{-z\hat{u}} H_{ab\to c}(\hat{s}, \hat{t}, \hat{u}).$$

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{b/B}(x') \int \frac{dx}{x} \sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_{h\perp}s_\perp n\bar{n}}}{z\hat{u}}\right)\left[T_{a,F}(x,x) - x\frac{d}{dx}T_{a,F}(x,x)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

Sivers like

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3 P_h} \Big/ E_h \frac{d\sigma}{d^3 P_h} \equiv A_N$$





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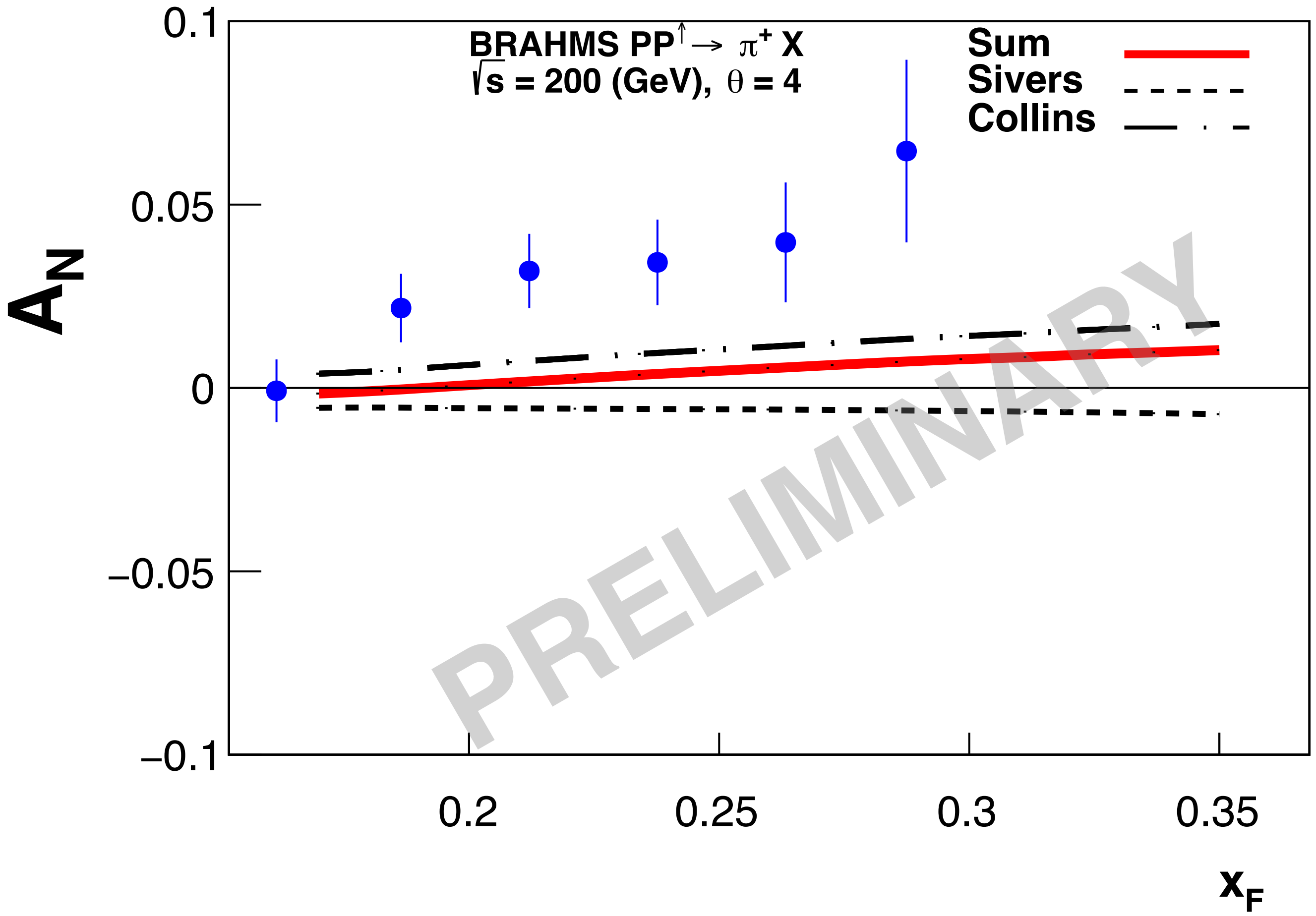

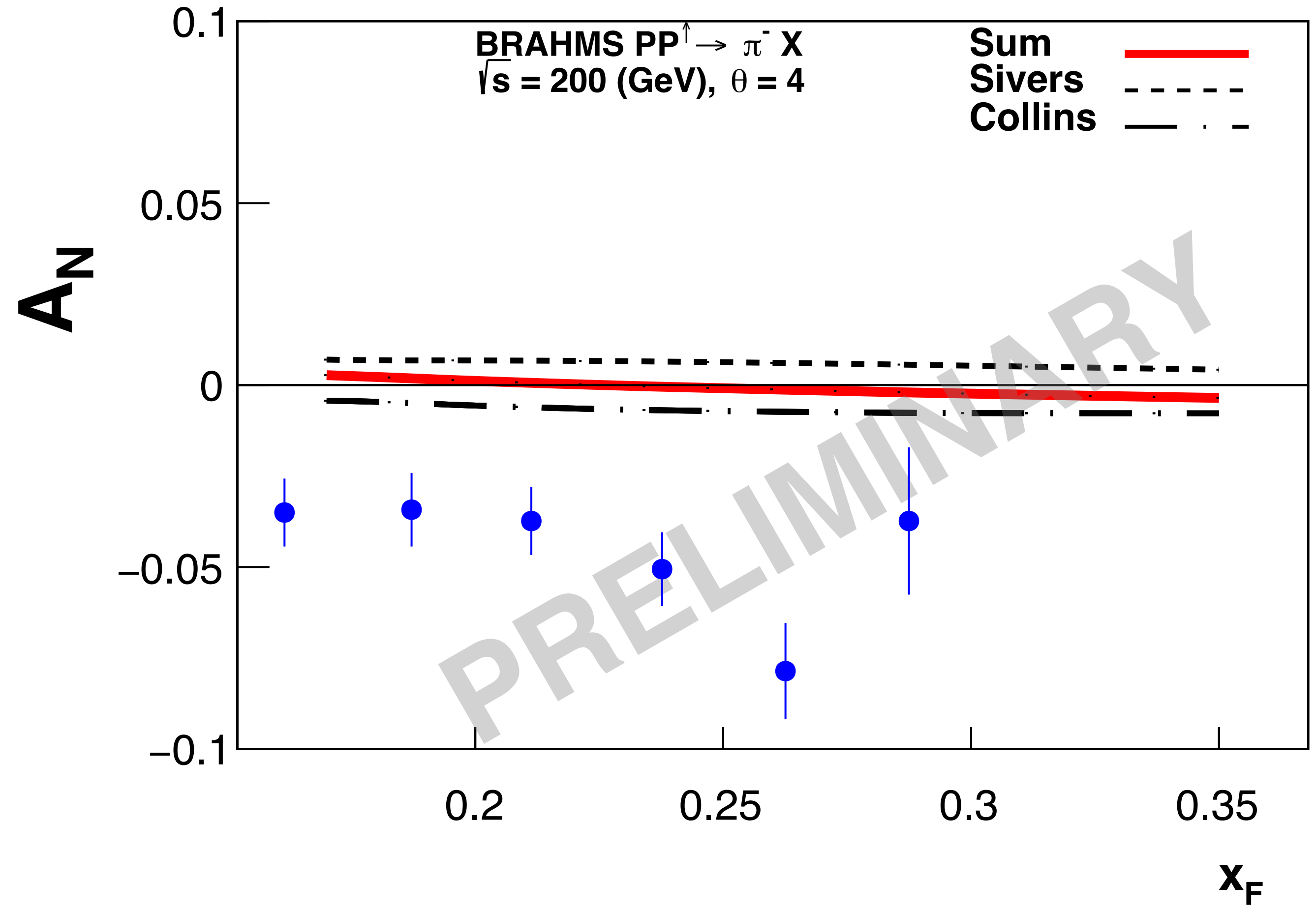

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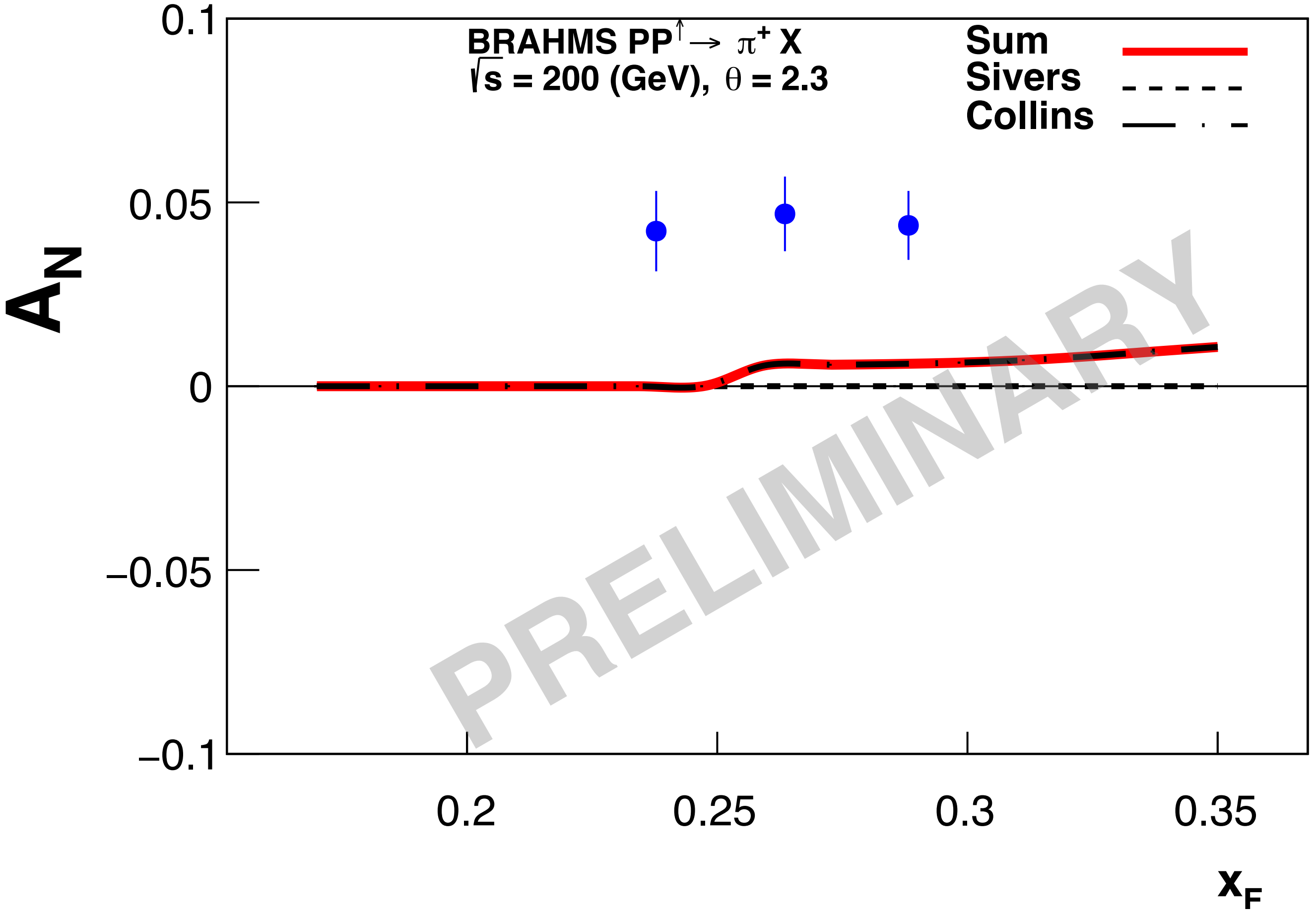
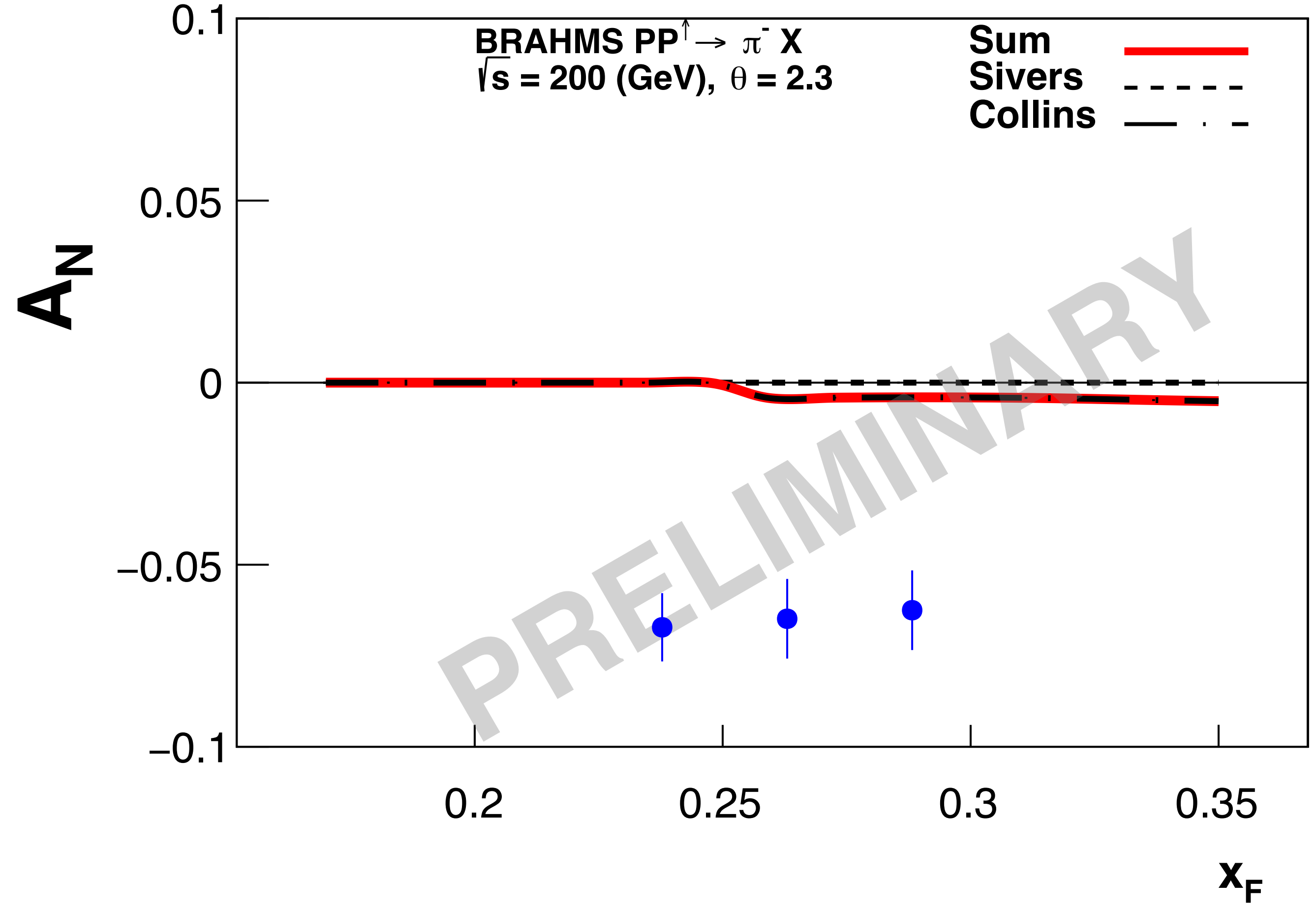




Metz, Pitonyak PLB 2013
Kanazawa, Koike, Metz, Pitonyak PRD 2014

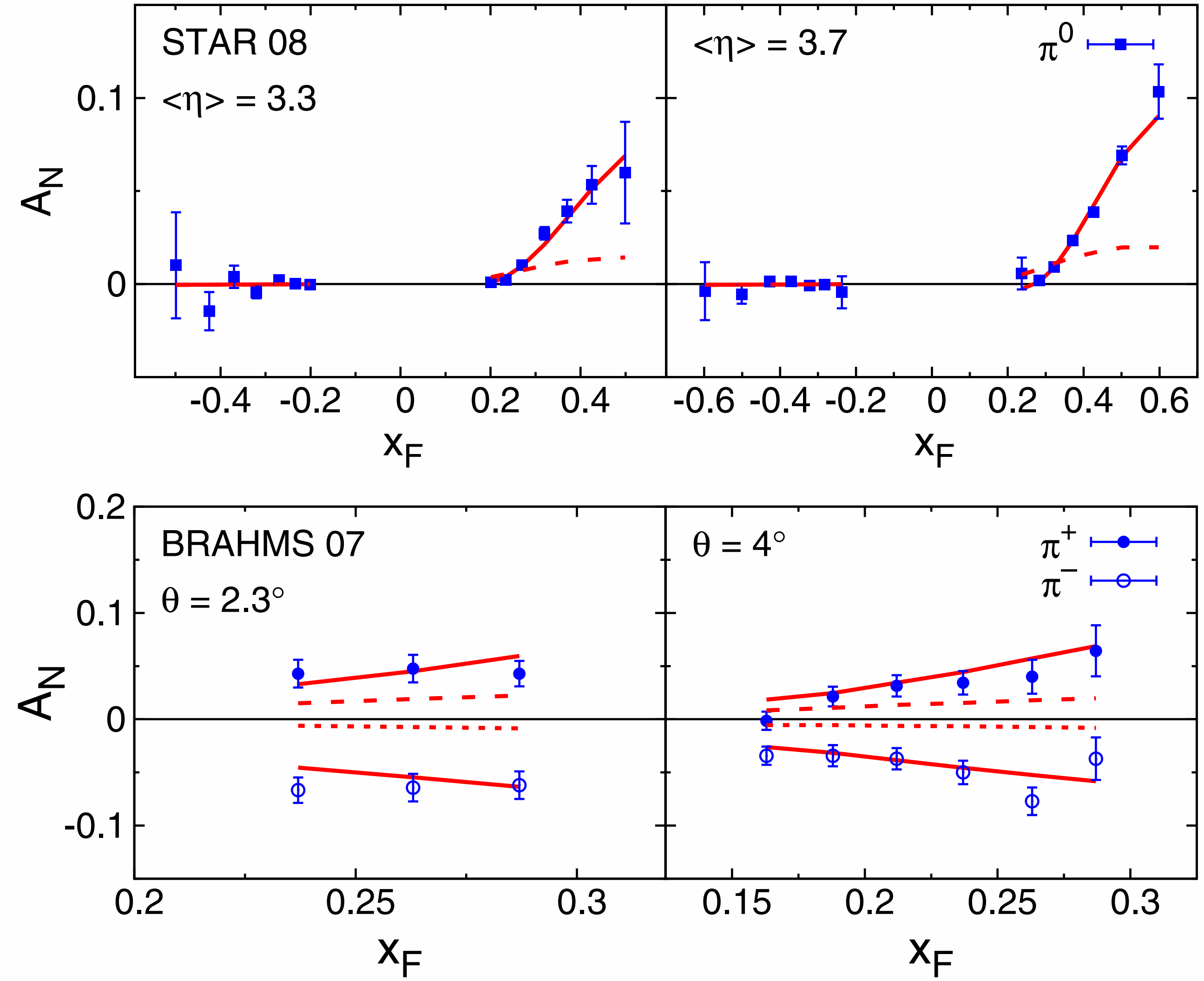


FIG. 1 (color online). Fit results for $A_N^{\pi^0}$ (data from [35–37]) and $A_N^{\pi^\pm}$ (data from [38]) for the SV1 input. The dashed line (dotted line in the case of π^-) means $\hat{H}_{FU}^{\Im}$ switched off.

$$\frac{P_h^0 d\sigma(\vec{S}_\perp)}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\epsilon_{\perp,\alpha\beta}\, S_\perp^\alpha P_{h\perp}^\beta \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3}$$
$$\times \int_{x'_{min}}^1 \frac{dx'}{x'}\frac{1}{x}\frac{1}{x'S + T/z}\frac{1}{-x'\hat{t} - x\hat{u}}\, h_1^a(x)\, f_1^b(x')$$
$$\times \left\{\left[\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right] S_{\hat{H}}^i + \frac{1}{z}H^{C/c}(z)\, S_H^i\right.$$
$$\left. + 2z^2 \int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{C/c,\Im}(z,z_1)\frac{1}{\xi}S_{\hat{H}_{FU}}^i\right\}, \quad (3)$$

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{2M_h^2} H_1^{\perp h/q}(z, z^2\vec{k}_\perp^2)\,.$$

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1)$$

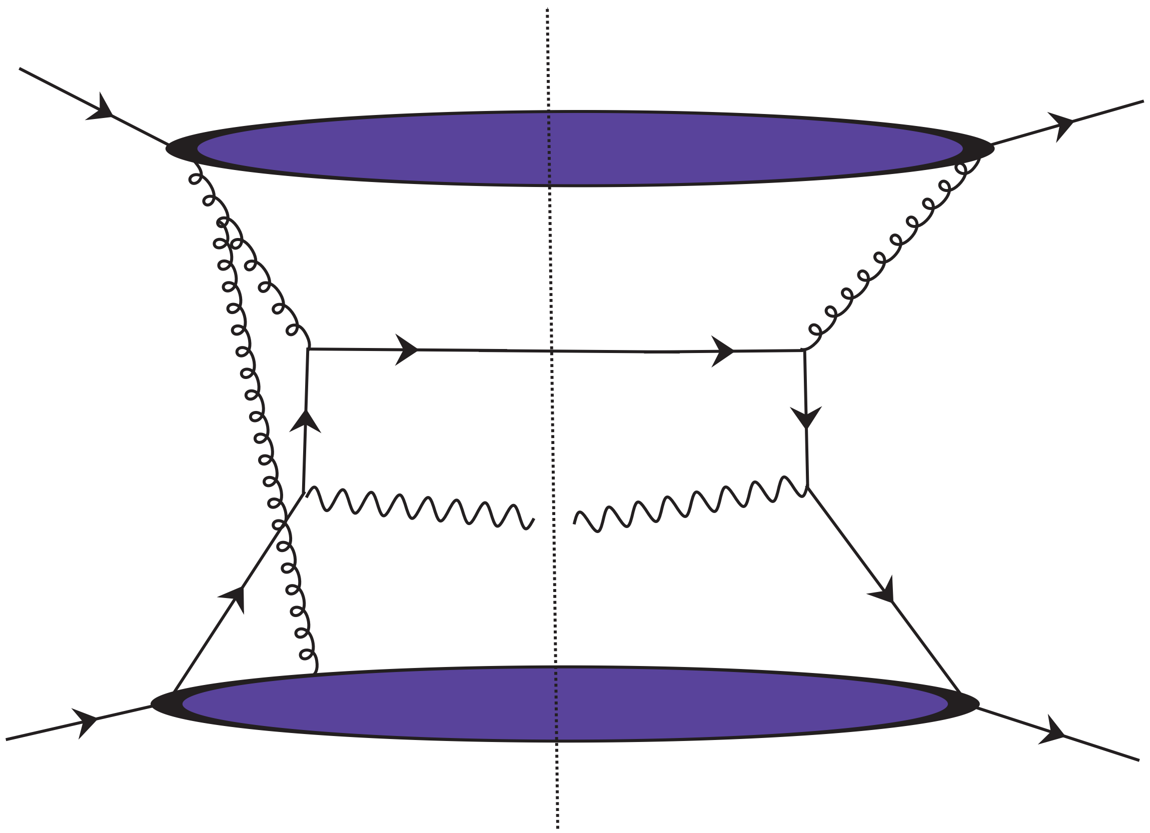
Summary

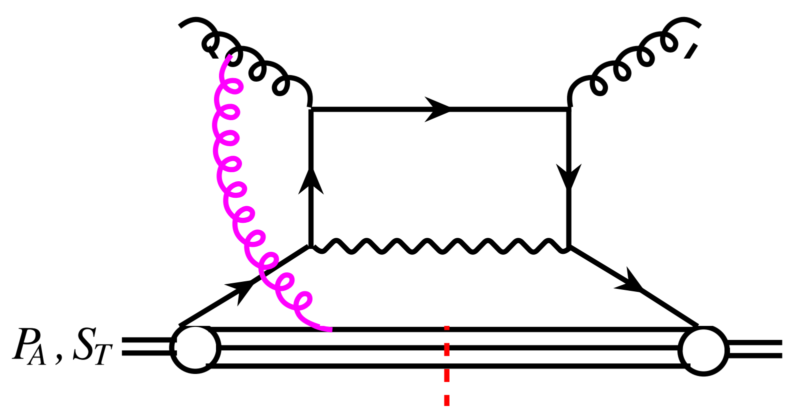
- Many interesting theory issues to address
- Are twist 2 - twist 3 Sivers/Collins interpretation for TSSAs compatible ?
- What is mechanism underlying inclusive meson production?
- QCD Evolution workshop series has and will play essential role in addressing these questions. THANK YOU from an organizers

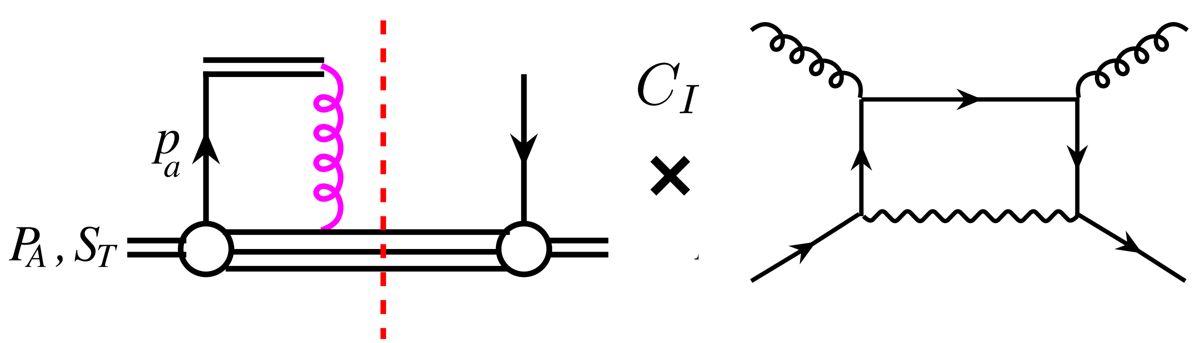
Consider direct Photon

LG & Z. Kang
Phys.Lett. B696 2011

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \hat{\sigma}$$







Color factor dictates process dependence

The FT transform of the e.g. Sivers asympt. reduces to first Bessel moment of Sivers TMD

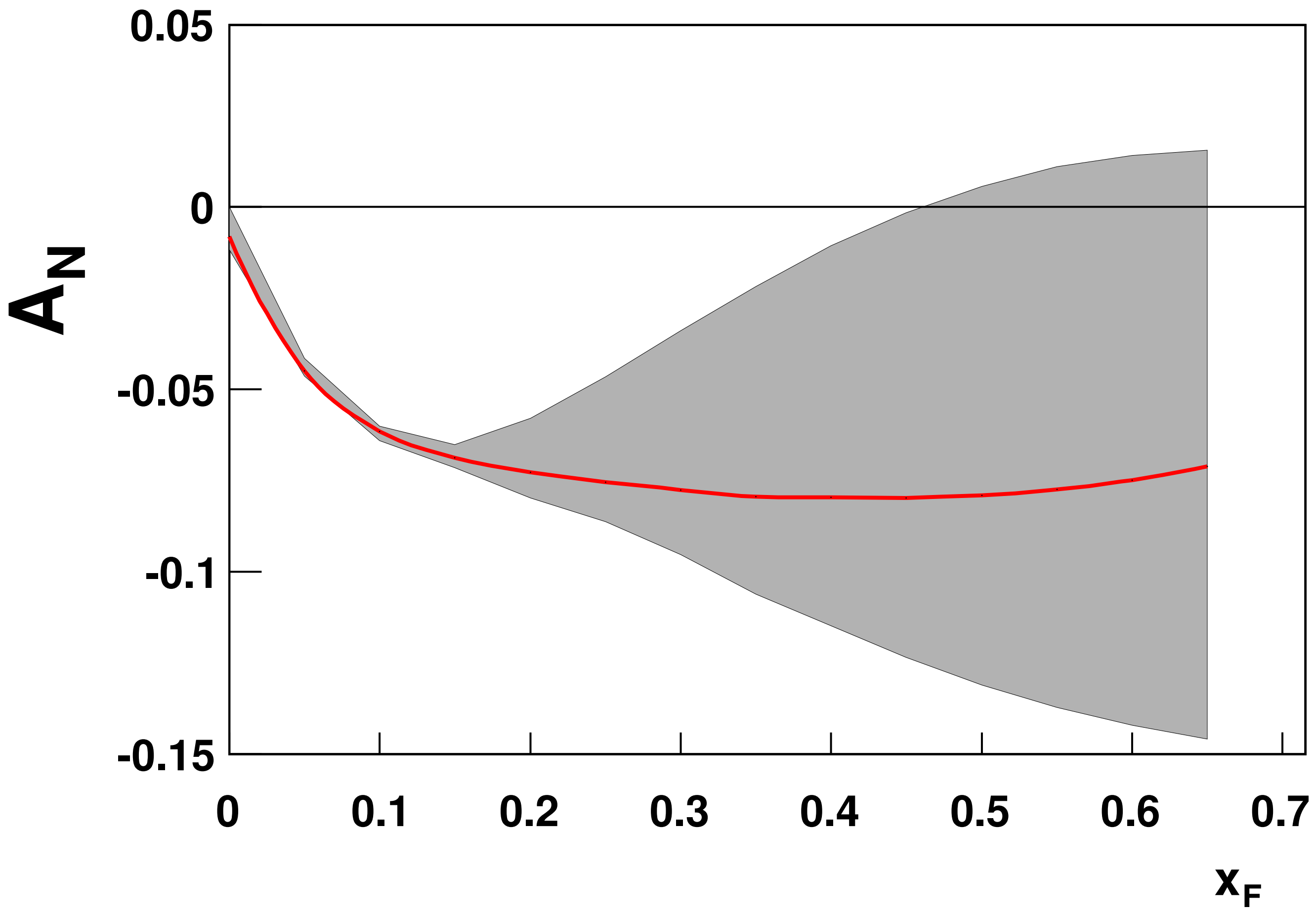
$$\tilde{f}_{1T}^{\perp(1)}(x, b_T) \equiv \frac{2}{M^2} \frac{\partial}{\partial b_T^2} \tilde{f}_{1T}^{\perp}(x, b_T)$$

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T) = \frac{2\pi}{M^2} \frac{1}{b_T} \int_0^\infty dk_\perp k_\perp^2 J_1(k_T\, b_T)\;\; f_{1T}^{\perp}(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, b_T) \approx \frac{2\pi}{M^2} \int_0^\infty dk_T \, \frac{k_T^2}{b_T} \, \frac{k_T\, b_T}{2} f_{1T}^{\perp}(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, 0) = f_{1T}^{\perp(1)}(x)$$

Most important Measurement Drell Yan



Prediction for Sivers asymmetry for DY lepton pair production at COM energy 500 GeV, for the invariant mass 4<Q<8 GeV