Evolution of sum rules and positivity constraints

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Main Topics

- Kinetic form of evolution
- *X.Artru , M.Elchikh , J.-M.Richard , J.Soffer, O.V.Teryaev.* ***Phys.Rept. 470 (2009) 1-92 ; arXiv:0802.0164 [hep-ph]***
- SR and positivity for DGLAP
- Energy-momentum tensor and twist-3 Sum Rules
- TMDs and positivity

Kinetic form of evolution

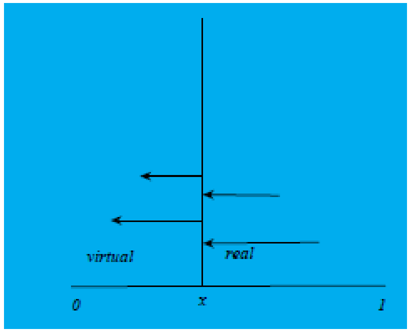
- Gain-loss equation (Collins, Qiu'89)

$$\frac{dq(x)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 dy\,\frac{q(y)}{y}P_+(x/y) \qquad P_+(z) = P(z) - \delta(1-z)\int_0^1 P(y)\,dy$$

$$\frac{dq(x)}{dt} = \frac{\alpha_s}{2\pi}\left[\int_x^1 dy\,\frac{q(y)}{y}P(x/y) - q(x)\int_0^1 P(z)dz\right]$$

- Extra change of variables $z = y/x$

$$\frac{dq(x)}{dt} = \frac{\alpha_s}{2\pi}\left[\int_x^1 dy\,\frac{q(y)}{y}P(x/y) - \int_0^x dy\,\frac{q(x)}{x}P(y/x)\right]$$



- Master equation

$$\frac{dq(x,t)}{dt} = \int dy\,[w(y\to x)q(y,t) - w(x\to y)q(x,t)]$$

$$w(y\to x) = \frac{\alpha_s}{2\pi}P(x/y)\frac{\theta(y-x)}{y}$$

Positivity preservation

- at LO as $P(z) > 0$ $(z<1)$
- Positivity may be violated only by loss term, but it is decreasing when function is approaching zero
- Decreasing exponents never turns to zero; to approach shore with zero velocity boat needs infinite time (V.I. Arnold)
- At NLO – scheme dependent – may be used to select the scheme

Preservation of convexity

- Kramers Moyal expansion

$$\frac{\partial q(x)}{\partial t} = -\frac{1}{x}\exp\left[\frac{\partial}{\partial \ln(1/x)}\frac{\mathrm{d}}{\mathrm{d}n}\right] x\, q(x)\gamma(n)\Big|_{n=1}$$

- Commutes with log derivative – curvatture preserved by evolution
- Keeping lowest terms – diffusion+drift approximation $\frac{\partial q(x)}{\partial t} = \frac{1}{x}\left(v\frac{\partial(xq(x))}{\partial \ln(1/x)} + D\frac{\partial^2(xq(x))}{[\partial \ln(1/x)]^2}\right)$
- with $v = \frac{\alpha_s}{2\pi}(5/4 - \pi^{2/3})$, and $D = \frac{\alpha_s}{4\pi}(-9/8 + 2\zeta(3))$ (NSLO)

Spin dependent case

- Use original form of DGLAP for (positive) distributions with definite helicity $\frac{\mathrm{d}q_{\pm}(x)}{\mathrm{d}t} = \frac{\alpha_s}{2\pi}\left[P_{+\pm}(x/y) \otimes q_{+}(y) + P_{+\mp}(x/y) \otimes q_{-}(y)\right]$. $P_{+\pm}(z) = (P(z) \pm \Delta P(z))/2$
- Required :positivity of helicity kernels

$$|\Delta P(z)| \leq P(z), \quad z < 1$$

- Loss term in the diagonal kernel P_{++} only!

Singlet case

- Coupled kinetic equations (‘reaction - diffusion system”)

$$\frac{\mathrm{d}q_\pm(x)}{\mathrm{d}t} = \frac{\alpha_s}{2\pi}\left[P^{qq}_{+\pm}(x/y)\otimes q_+(y) + P^{qq}_{+\mp}(x/y)\otimes q_-(y)\right]$$
$$+ P^{qG}_{+\pm}(x/y)\otimes G_+(y) + P^{qG}_{+\mp}(x/y)\otimes G_-(y)\,,$$
$$\frac{\mathrm{d}G_\pm(x)}{\mathrm{d}t} = \frac{\alpha_s}{2\pi}\left[P^{Gq}_{+\pm}(x/y)\otimes q_+(y) + P^{Gq}_{+\mp}(x/y)\otimes q_-(y)\right.$$
$$\left. + P^{GG}_{+\pm}(x/y)\otimes G_+(y) + P^{GG}_{+\mp}(x/y)\otimes G_-(y)\right]$$

- Positivity $|\Delta P^{ij}(z)| \le P^{ij}(z), \quad z<1; \quad i,j=q,G$
- Loss terms only in the diagonal kernels

Transversity: Soffer bound stability

- Positive quantities (of mixed chirality)

$$Q_\pm(x) = q_+(x) \pm h_1(x)$$

- Kinetic equations

$$\frac{\mathrm{d}Q_\pm(x)}{\mathrm{d}t} = \frac{\alpha_s}{2\pi}(P^Q_{+\pm}(x/y) \otimes Q_+(y) + P^Q_{+\mp}(x/y) \otimes Q_-(y))$$

$$P^Q_{++}(z) \equiv \frac{P^{(0)}_{qq}(z) + P^{(0)}_h(z)}{2} = \frac{C_F}{2}\left[\frac{(1+z)^2}{(1-z)_+} + 3\delta(1-z)\right]$$

$$P^Q_{+-}(z) \equiv \frac{P^{(0)}_{qq}(z) - P^{(0)}_h(z)}{2} = \frac{C_F}{2}(1-z).$$

- Kernels: positive (LO) for z < 1, loss terms diagonal

"Scale arrow"

- Evolution is kinetic when going to UV and "antikinetic" in the IR
- Positivity preserved when going to UV
- Evolving backwards: small deviations in the UV explode in the IR
- "Turbulence" in the confinement region
- Soffer bound (for d) may be saturated at low scale only: otherwise backward evolution would violate it (M.Radici talk)

Sum rules

- Current (~"particles" number) conservation

$$\int_0^1 \mathrm{d}x \frac{\mathrm{d}q(x)}{\mathrm{d}t} = \int_0^1 \int_0^1 \mathrm{d}x\mathrm{d}y \, [w(y \to x)q(y) - w(x \to y)q(x)] = 0$$

- Singlet case – EMT conservation: 2 sorts of particles (numbers ~ xq(x), xG(x)) mutually transforming one to another ("Mass conservation in reaction-diffusion system")

Energy-momentum tensor related sum rules

- PDF's – momentum SR, first indication for gluons
- GPDs – Ji's SRs
- Follow from momentum and angular momentum conservation
- Evolution of angular momenta – the same as momenta – may be obtained in the kinetic way (OT'98) if $\int_0^1 dx x \Delta P_{Gq}(x) = \frac{1}{2} \int_0^1 dx x P_{Gq}(x)$

Gluonic poles and Energy-Momentum tensor

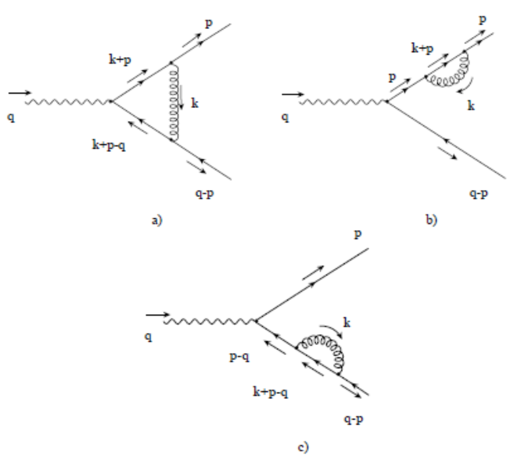
- Consider twist 3 (= relevant moment of Sivers function being infinite tower of twists) gluonic pole
- EMT forward matrix element $< P | T_{\mu\nu} | P >= 2 P_\mu P_\nu$,no spin-dependent structure $P_\mu \varepsilon_{\nu S P n}$ (similar to B~E with q -> n) :

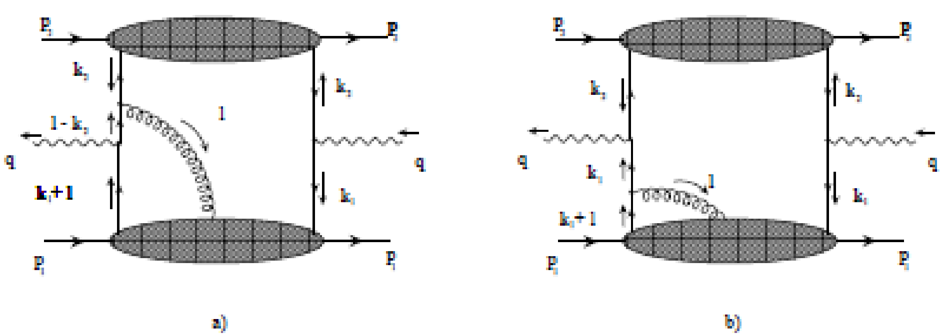
$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2} = 0$$

- Naively: Valid identically due to symmetry properties implied by T-invariance)
- However: such pole in physical processes should get imaginary part due to EMGI and related contour gauge(DY process: Anikin, OT, PLB2010,2015; EPJ2015) – analog of the choice of Wilson line.

Electromagnetic Gauge Invariance in DY process

- Extra diagram – factor 2 in transverse (TM integrated) asymmetry
- Follows also from EM GI
- May be studied at COMPASS, NICA 1408.3959





Pole prescription and Burkardt SR (OT'14)

- Pole prescription provides ("T-odd") symmetric part!

$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$

- SR: $\sum \int dx T(x,x) = 0$
- Burkardt SR+Boer-Mulders-Pijlman relation
- Pole prescription – way to account for dynamics
- Cf with analogous SR (Schafer,OT'01) for Collins functions where only TM conservation was necessary

Validity for separate parton species

- Can it be valid separately for each quark flavour (and gluons) : nodes (Boer, Prokudin)?
- Valid if structures $P_\mu \varepsilon_{\nu SPn}$ forbidden for TOTAL conserved EMT do not appear for each flavour
- Structure contains (besides S) gauge vector n: GI (and/or rotational invariant) separation of EMTs – forbidden: SR valid separately!

Direct test of Twist-3 sum rule evolution (J. Zhou'15 - talk)

- Multiplicative evolution:

$$\frac{\partial \sum_{q,\bar{q},g}[T_F^q(\mu^2) + T_F^{\bar{q}}(\mu^2) + T_G^{(+)}(\mu^2)]}{\partial \ln \mu^2}$$
$$= -\frac{\alpha_s}{2\pi} C_A \sum_{q,\bar{q},g} [T_F^q(\mu^2) + T_F^{\bar{q}}(\mu^2) + T_G^{(+)}(\mu^2)].$$

- Satisfied IF valid at some scale
- R.h.s. from "extra terms" (now confirmed) of Braun, Manashov and Pirnay

Comparing Burkardt and Burkhardt-Cottingham SRs

- Twist 3 contribution to BC SR also evolve multiplicatively in large N_C limit (Ali, Braun, Hiller; Braun, Korchemsky, Manashov)
- BC is related to rotational invariance
- Momentum SR -> BSR: transition from longitudinal to transverse
- BSR ~ JiSR (OT'06) where ANGULAR momentum conservation is required

“Spontaneous” conservation of Burkardt SR

- Pure non-Abelian – rotational properties due to gluon self-interaction?

- Straightforward generalization: If valid separately for each flavour and gluons at some points – also remains stable!

Positivity and TMD

- Low-x – BFKL evolution
- Master-type form for UGDF $f(x_B,k_T)$ with longitudinal time $t=\ln(1/x_B)$ and transverse coordinate $x = \ln k_T$

$$\frac{\mathrm{d}f(x,t)}{\mathrm{d}t} = \int \mathrm{d}y \, [w_+(y \to x) f(y,t) - w_-(x \to y) f(x,t)]$$

- Contains exponential growth besides diffusion and drift
- It is possible to separate these effects

BFKL as a master equation

- Redefined function and kernel

$$f_\sigma(x,t) = f(x,t)\sigma(x), \quad \bar{f}_\sigma(x,t) = f_\sigma(x,t)/\langle f_\sigma(t)\rangle \quad \langle f_\sigma(t)\rangle = \int \mathrm{d}x\, f_\sigma(x,t).$$

$$w_+(x,y) \to w_\sigma(x,y) = w_+(x,y)\sigma(x)/\sigma(y).$$

- Master equation for $\bar{f}_\sigma(x,t)$

$$\frac{\mathrm{d}\bar{f}_\sigma(x,t)}{\mathrm{d}t} = \int \mathrm{d}y\,[w_\sigma(y\to x)\bar{f}_\sigma(y) - w_\sigma(x\to y)\bar{f}_\sigma(x)]$$

- $\sigma(x)$-eigenfunction (BFKL: $(k_T)^a$, 0<a<1) of

$$\int \mathrm{d}x\, w_+(y\to x)\sigma(x) = \left[\lambda_\sigma + \int \mathrm{d}x\, w_-(y\to x)\right]\sigma(y)$$

- Growth power ($\lambda_0 = \lambda_\sigma - \frac{v_\sigma^2}{4D_\sigma}$ = 4 ln2)

$$\lambda_\sigma = \int \mathrm{d}x\,[w_\sigma(y\to x) - w_-(y\to x)]$$

Positivity for BFKL and its extensions

- Master form - preserved positivity
- Scale arrow – directed towards low x
- Unitarization: Nonlinear local terms do not affect positivity
- Coordinate space: BK preserve positivity
- But: Fourier transform of positive functions is only positive definite: for any real x and complex z $\quad \sum f(x_i - x_j) z_i z_j^* > 0$

Positivity and TMD factorization

- W term must turn negative to have zero moment – violation of positivity (in physical momentum space!) signals of inapplicability of approximation
- Two scale arrows for CS and scale evolution
- Positivity preserved for
- Possible qualitative description: reversal of transverse scale arrow (change of log sign) at $Q_T \sim Q$ and its restoration by change W -> Y?

Conclusions/Outlook

- Kinetic interpretation of evolution naturally describes positivity and sum rules preservation
- Scale arrows directed towards large p_T and small x (combined by angle arrow?)
- Burkardt SR in twist 3 approach is controlled by energy-momentum conservation + dynamics (pole prescription)
- Spontaneous conservation of Burkardt SR and its generalization for seprate flavours
- TMD evolution: scale arrow reversal at $Q_T \sim Q$?

Can “standard” conservation of BSR be imposed?

- Twist 3 perfectly survives Abelian limit
- “Extra terms” are pure non-Abelian
- Small x (IR) effect: could it be the room for extra subtraction?
- Recall axial anomaly (Carlitz,Collins, Mueller) – correct IR limits crucial
- Pairs of anomalies: V vs A, EMT conservation vs Trace
- Could the subtraction be related to trace anomaly?!

1-st moments - EM, 2-nd - Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \quad = \quad \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_q(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe ainteraction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $$M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$
Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$$

Equivalence principle for moving particles

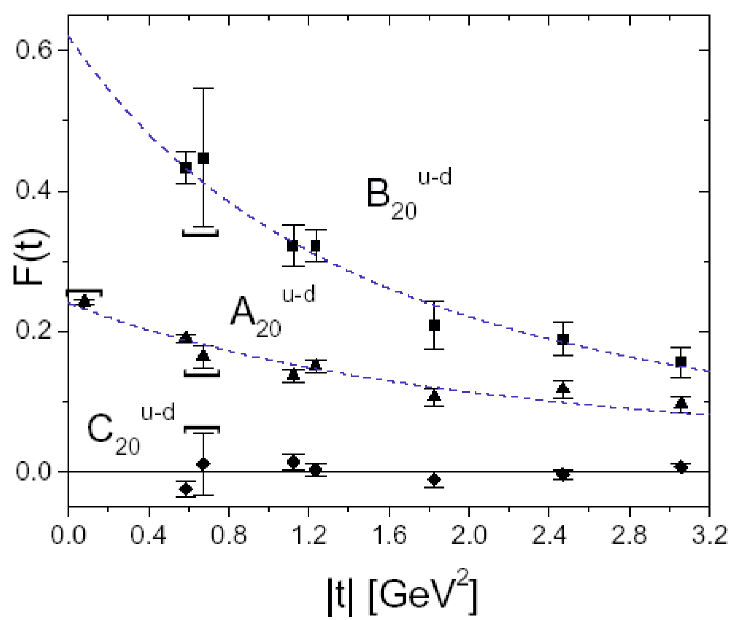
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

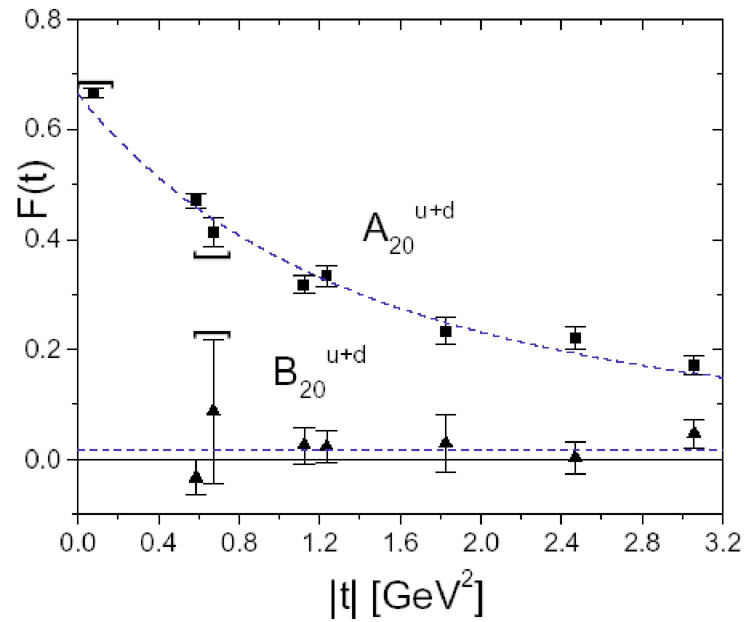
$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \dfrac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, O.T.)

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of EEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by smallness of E (isoscalar AMM)
- Polyakov Vanderhaeghen: dual model with E=0

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible