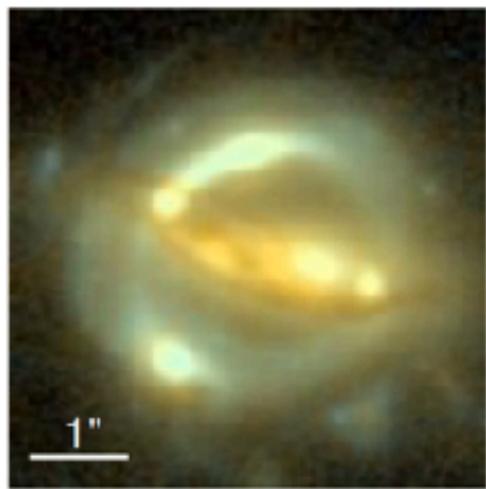


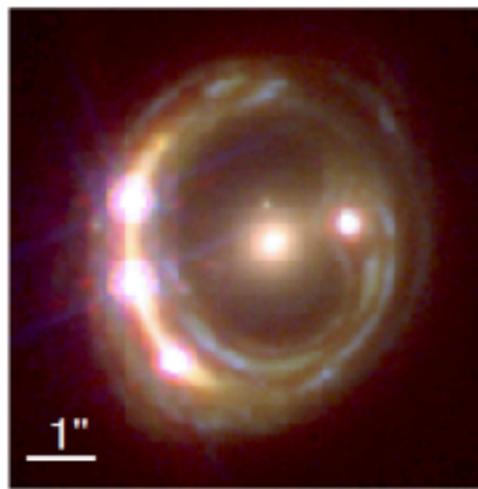
# Time-delay cosmography: *Tensions between the Hubble constant inferred from the early and late Universe*

Léon Koopmans  
(Kapteyn Astronomical Institute)

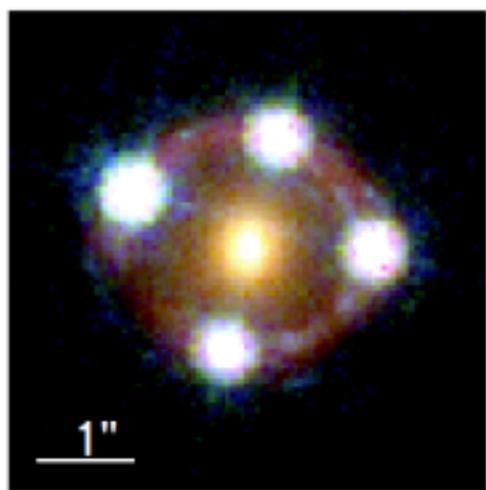
Based on work by the H0LICOW, COSMOGRAIL,  
STRIDES and SHARP collaborations



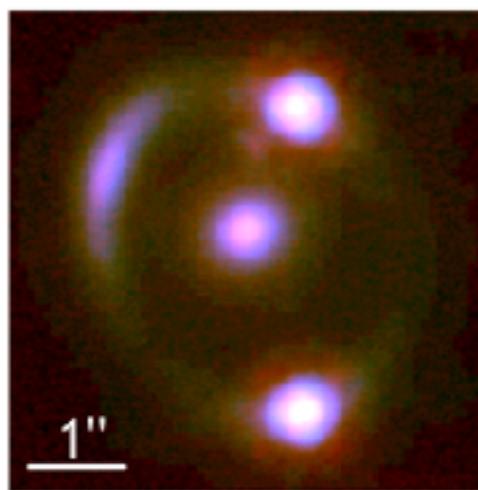
(a) B1608+656



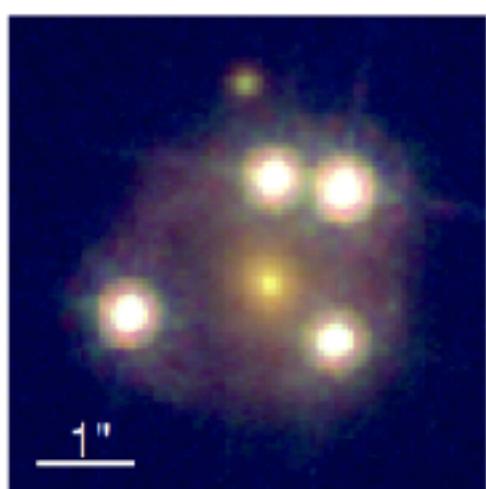
(b) RXJ1131-1231



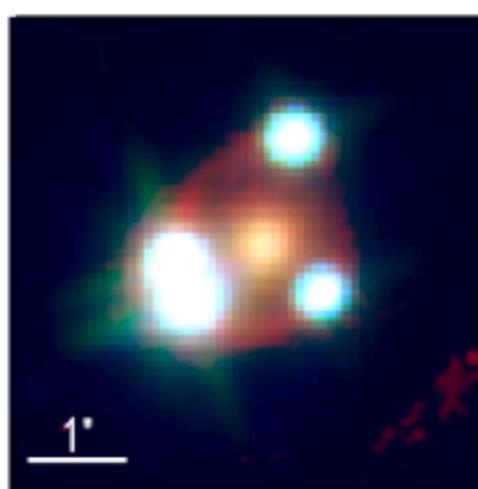
(c) HE 0435-1223



(d) SDSS 1206+4332



(e) WFI2033-4723



(f) PG 1115+080

# Time-delay cosmography: *Tensions between the Hubble constant inferred from the early and late Universe*

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# The “Standard” Cosmological Model

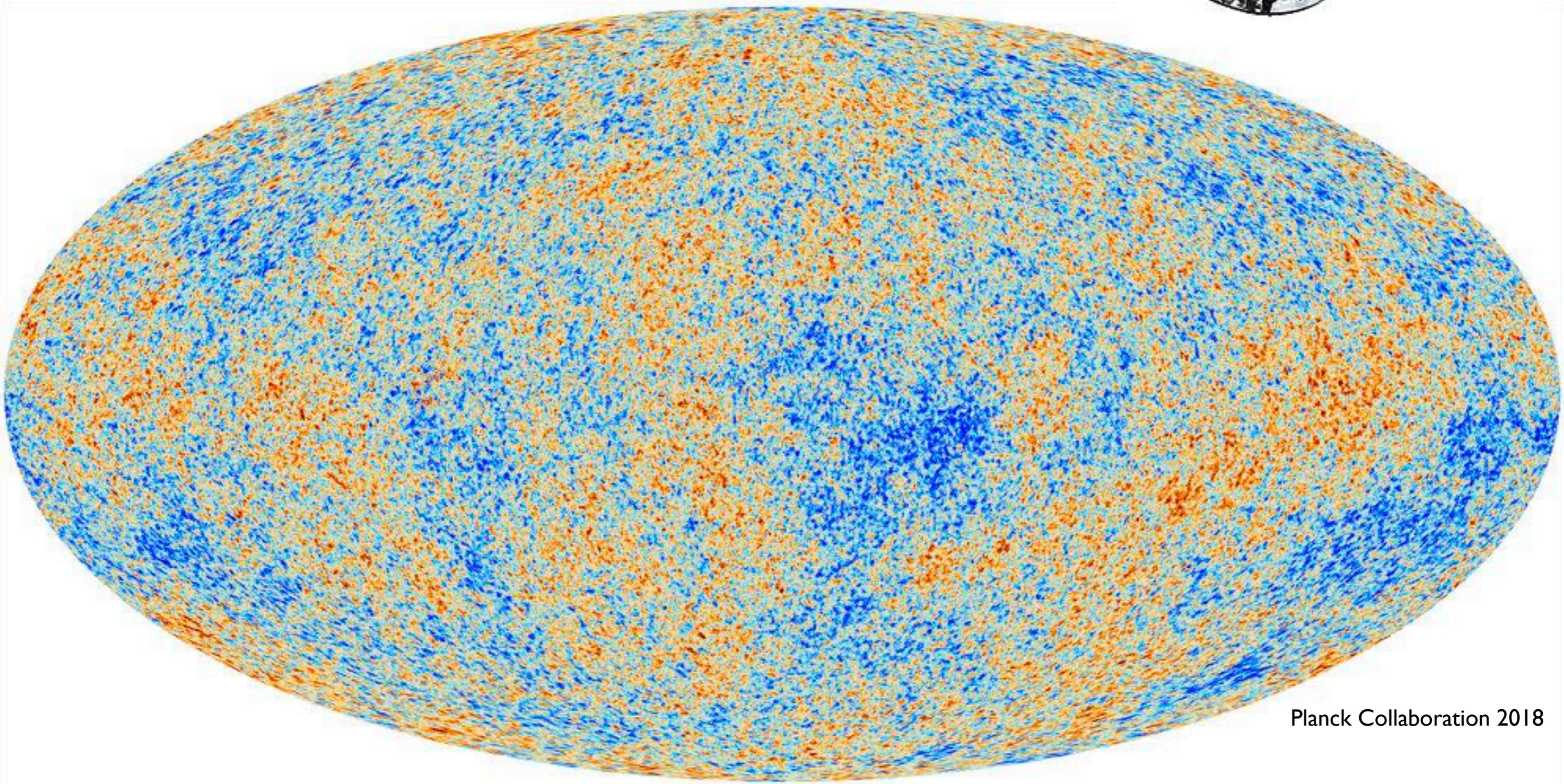
*The standard (flat-) $\Lambda$ CDM cosmological model  
described (until recently) the entire Universe  
on large scales with only 6 free parameters  
(e.g.  $t_0$ ,  $\Omega_{\text{baryons}}$ ,  $\Omega_{\text{DE}}$ ,  $\tau$ ,  $n_s$ ,  $\Delta^2$ ).  
No significant evidence for deviations.*

*However, some cracks are appearing in the model!*

# Expanding and Evolving Universe

Radiation emitted during “[recombination](#)” (a.k.a. CMBR) is about 3000K while emitted and currently seen at T=2.7K.

[Smooth to level of  \$\sim 10^{-5}\$  \( \$\mu\text{k}\$ \)](#)

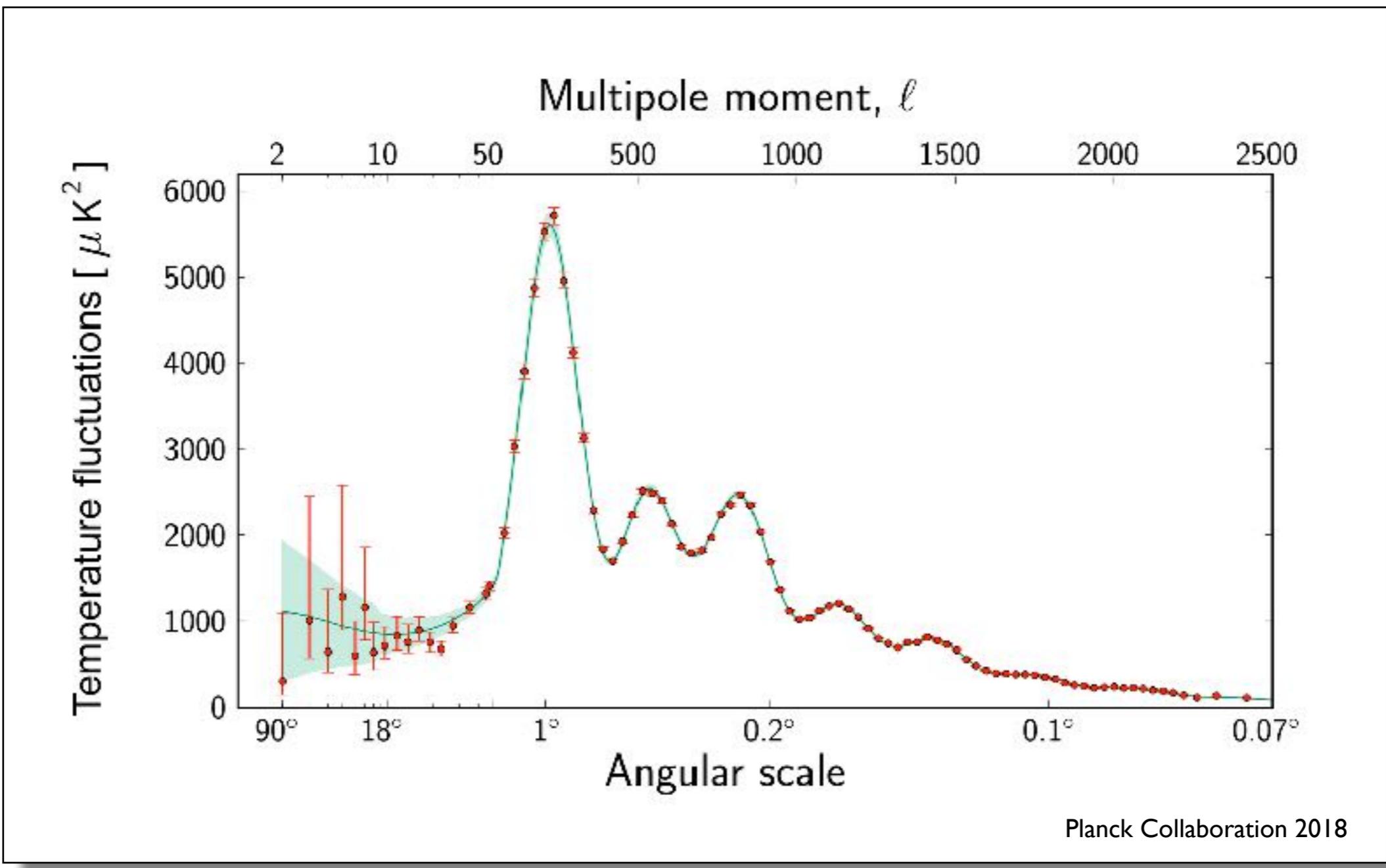


Planck Collaboration 2018

Earliest “[baby picture](#)” of the Universe about 380,000 years after the Big Bang;

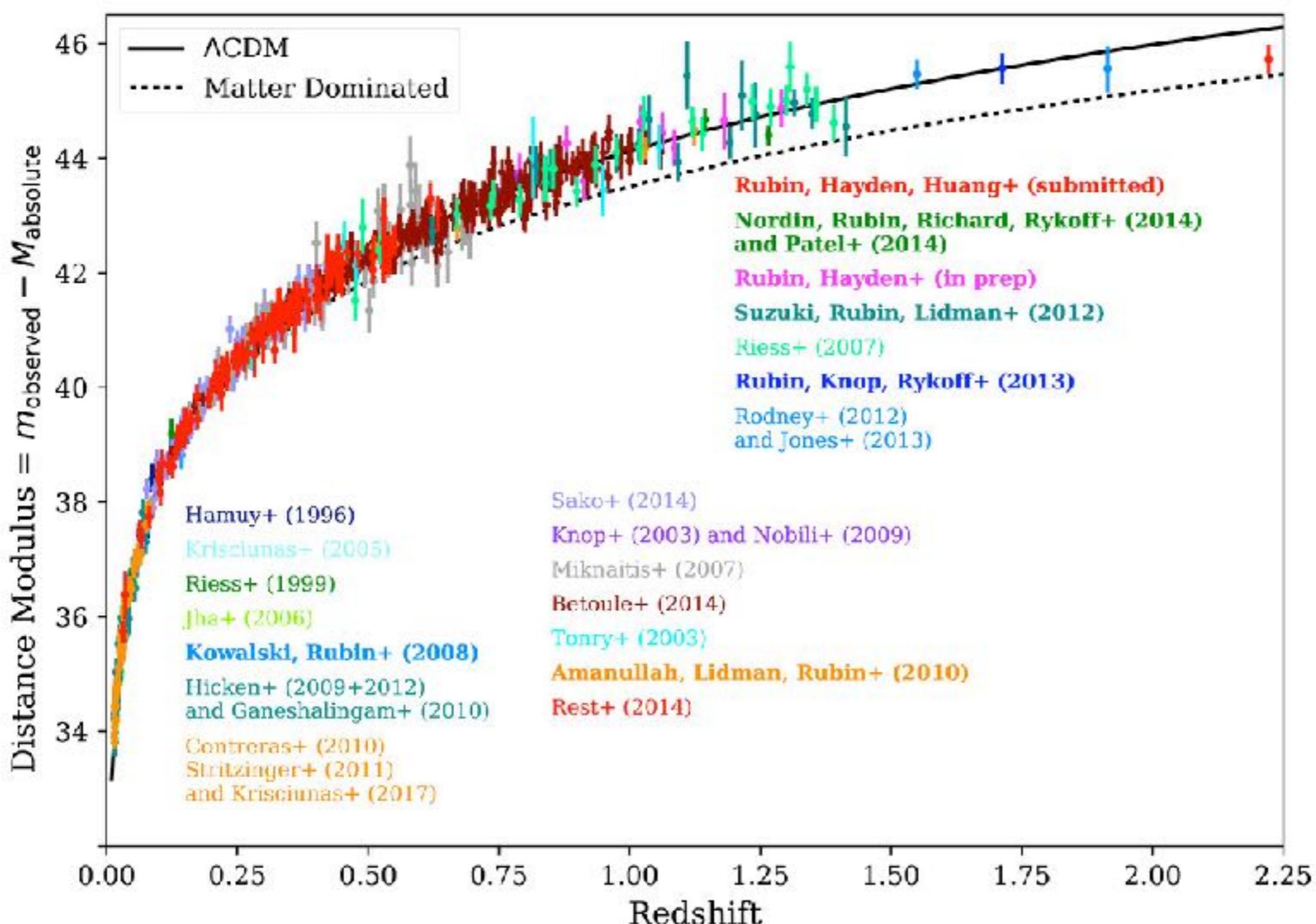
# Expanding and Evolving Universe

The CMB Radiation has small temperature fluctuations, caused by “acoustic oscillations” of the ionised hydrogen in a dark-matter background density field. Its power-spectrum provides the initial conditions for the formation of structure (e.g. galaxies), via gravity.



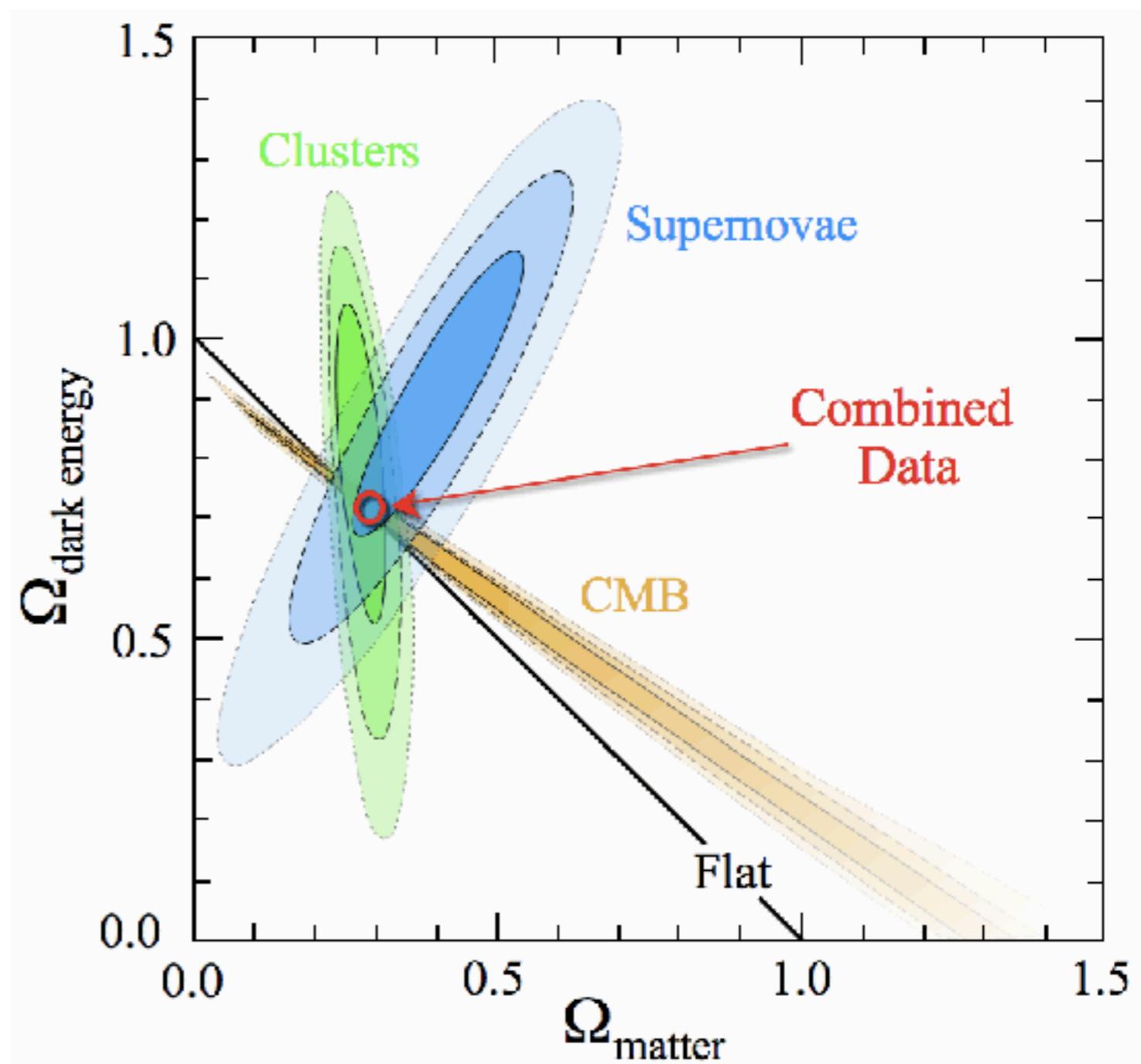
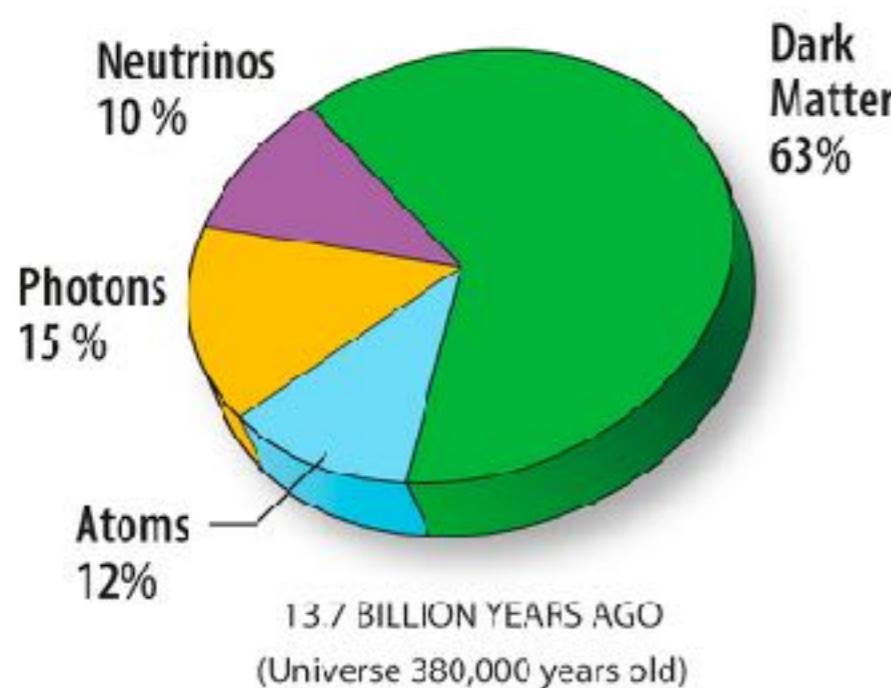
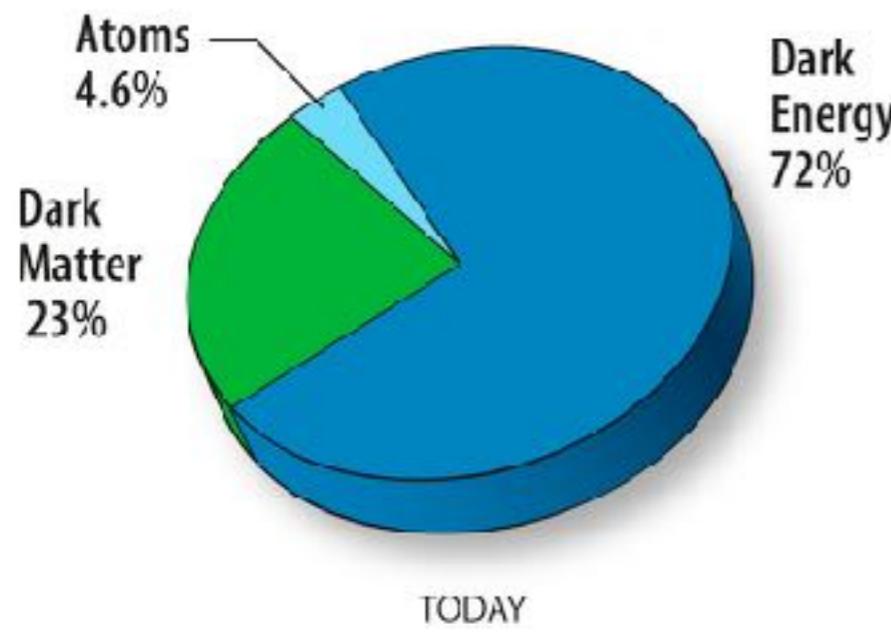
# Expanding and Evolving Universe

Observations of Supernovae show that the Universe is accelerating rather than decelerating: “Dark Energy” or Einstein’s Cosmological Constant?



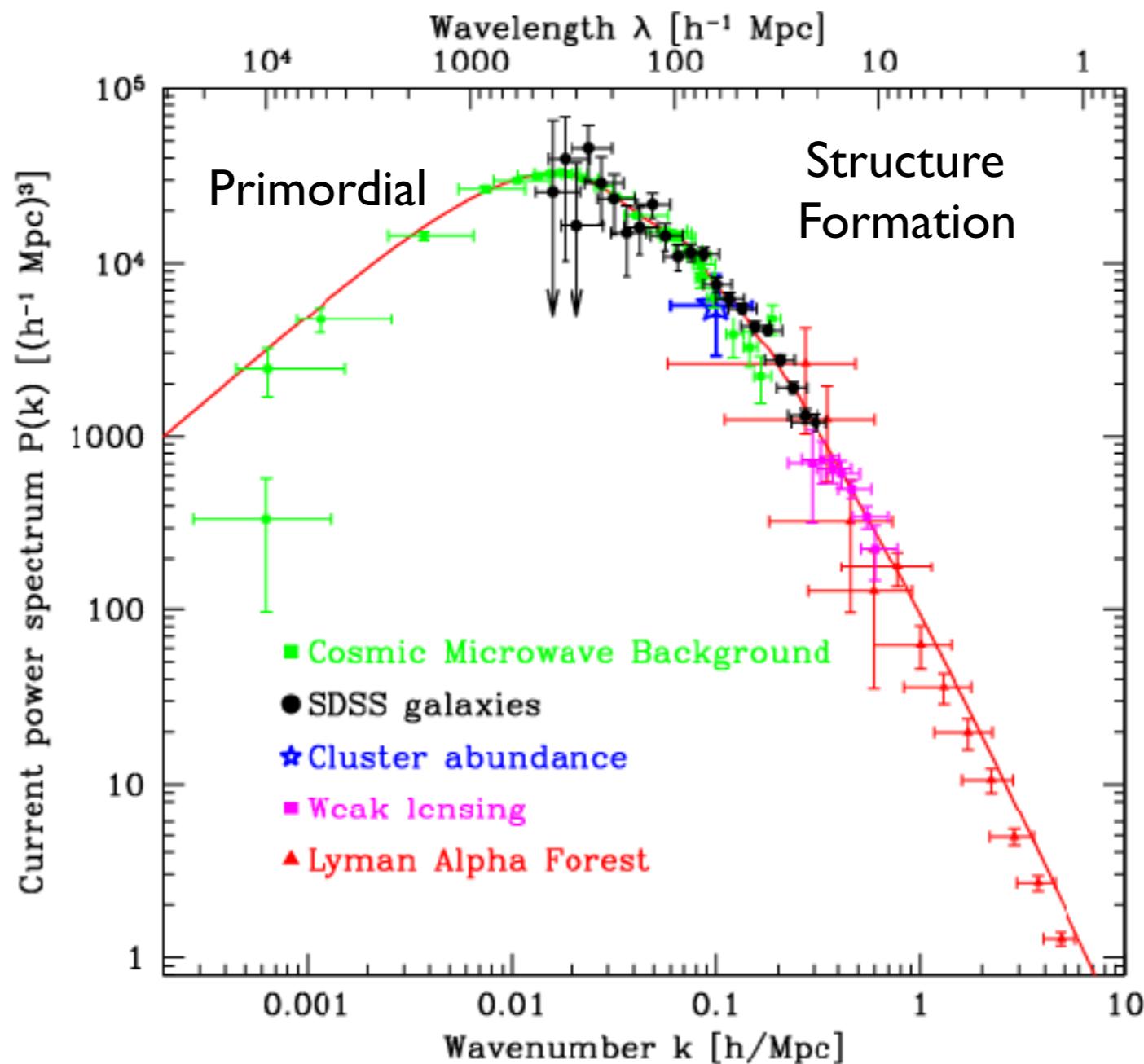
# Expanding and Evolving Universe

Observations of LSS, SuperNovae, Clusters, CMBR, ... have led to a “Standard Cosmological Model”: the Universe contains “Dark Matter” and “Dark Energy” (both unknown energy-density components)



# Expanding and Evolving Universe

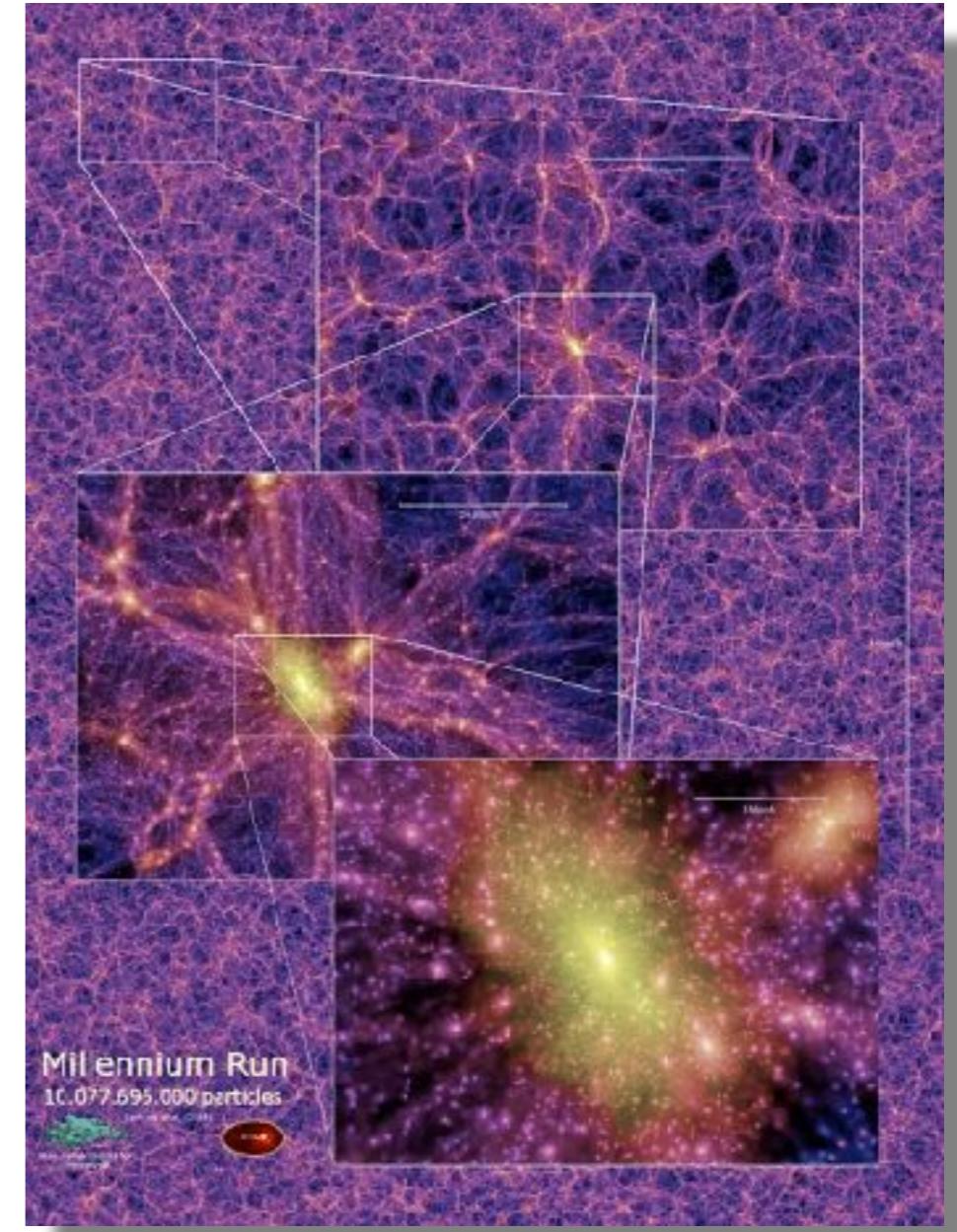
From the CMBR we know the [initial/primordial density fluctuations](#) of the Universe.  
These evolve under the influence of gravity in over and under-dense regions.



# Structure Formation: Dark & Baryonic Matter

Combining these “ingredients” — Cold Dark Matter, baryons with density fluctuations (according to CMB) and gravity — leads to “[large scale structure formation](#)”.

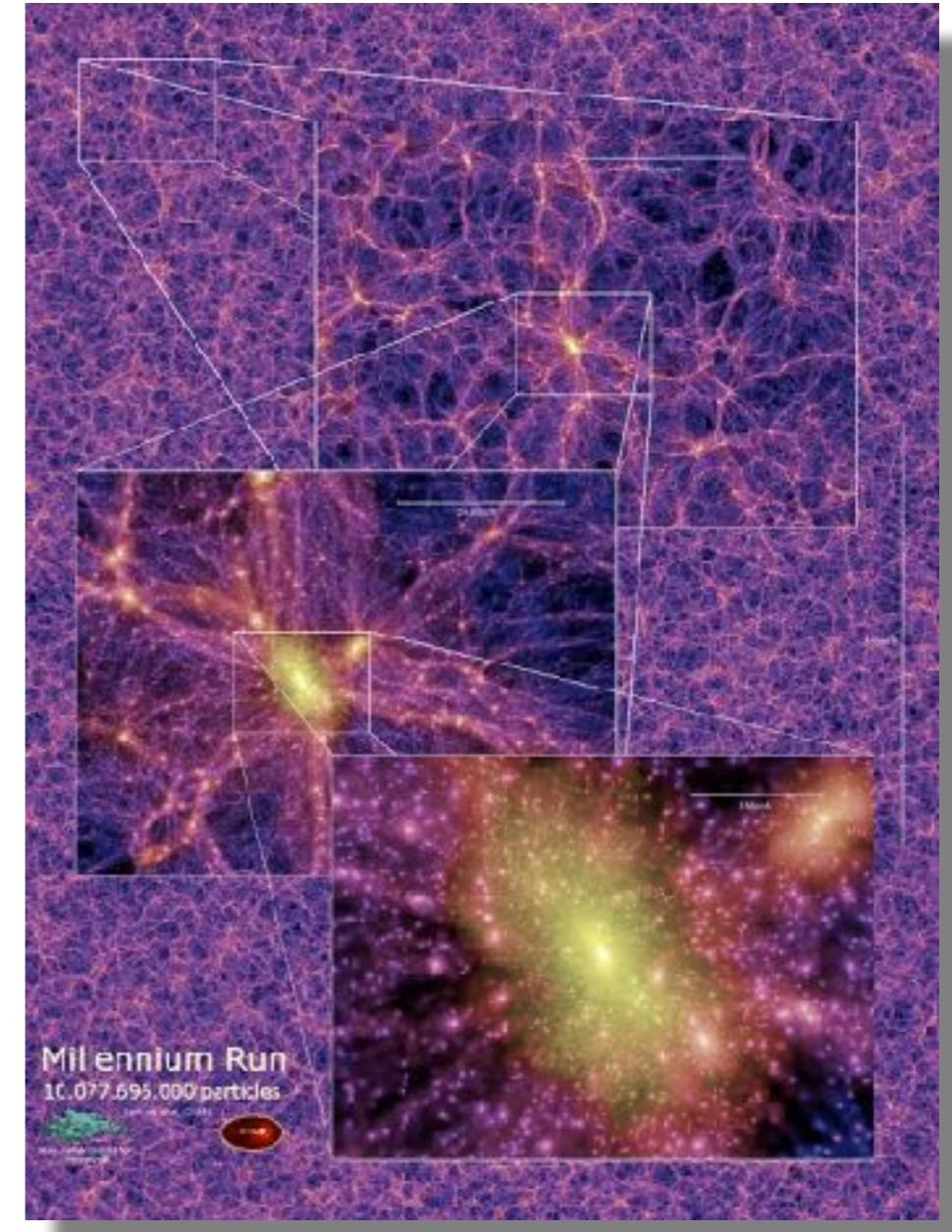
Powerful computers can now accurately simulate the formation of (large scale) structure



# Structure Formation: Dark & Baryonic Matter

Combining these “ingredients” — Cold Dark Matter, baryons with density fluctuations (according to CMB) and gravity — leads to “[large scale structure formation](#)”.

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# The Hubble Constant

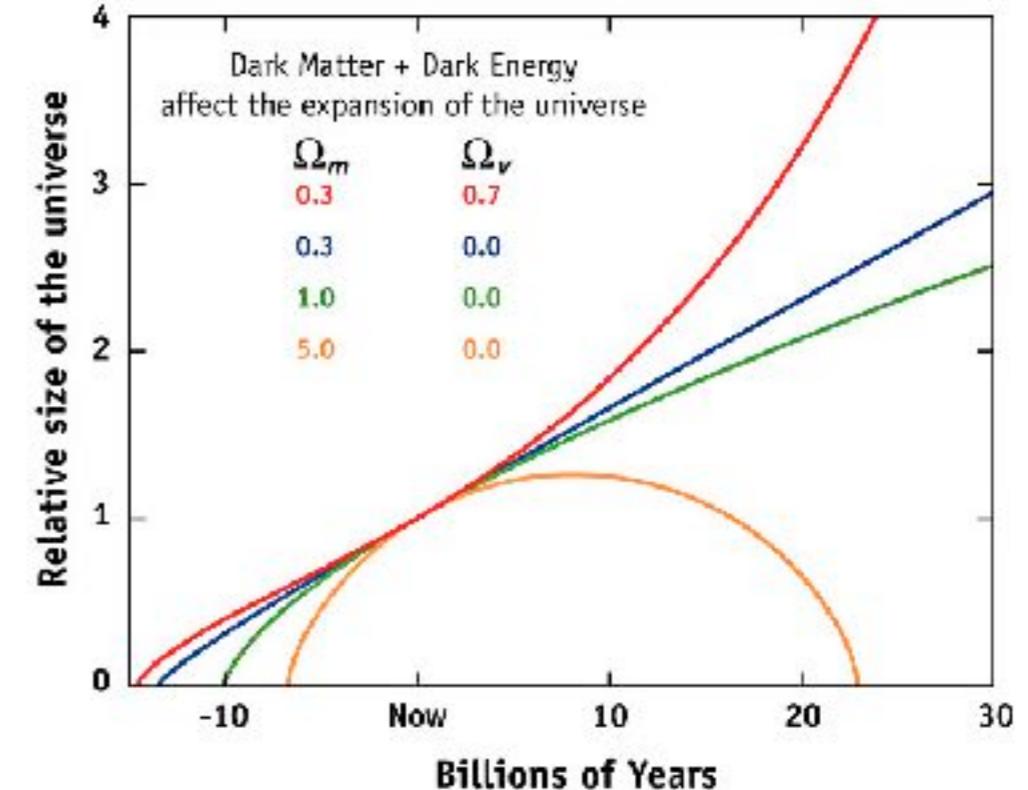
*Given the standard  $\Lambda$ CDM cosmological model  
the expansion history of the Universe is known  
apart from a scale factor, which given by the  
Hubble Constant (see next slides)*

# Why is $H_0$ important? Why do we care?

$H_0 = H(z=0)$  sets all scales in the Universe and  $H(z)$  is related to its expansion history (hence the energy-density content).

It allows us to infer:

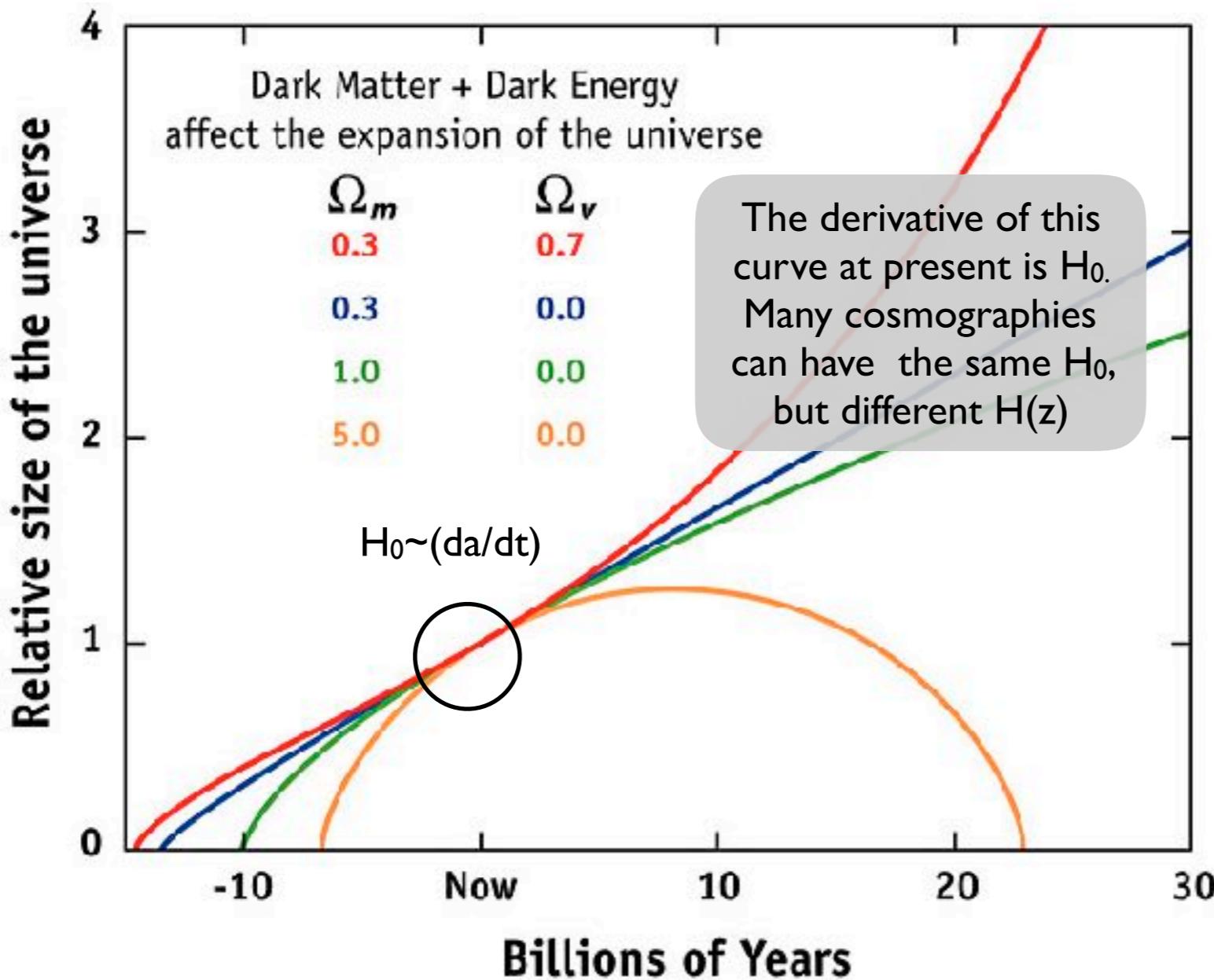
- *The age of the Universe*
- *The physical scales of objects such as galaxies*
- *The energy budget of luminous sources*
- *The masses of e.g. galaxies and clusters*
- *etc.*



By comparing  $H_0$  from various methods and from the early and late Universe also the underlying systematics, or faulty assumption (e.g. the cosmological model) can be tested => *this colloquium*

# The Hubble Constant

$$H^2(a) = H_0^2 \left( \Omega_m a^{-3} + \Omega_X \exp \left[ 3 \int_{\log a}^0 (1+w) d \log a \right] + \Omega_k a^{-2} \right)$$



The Hubble Constant ( $H_0$ ) is the current derivative (slope) of the scale size of the Universe as function of cosmic time.

Inferring it from the early Universe requires knowledge about its expansion history, which assumes we know the energy-density (DM and DE) going in the FLRW metric.

*Currently, the metric is assumed flat- $\Lambda$ CDM. But if it is different,  $H_0$  inferred from the early universe can yield a value of  $H_0$  different from that late universe.*



# Current Status of measuring $H_0$

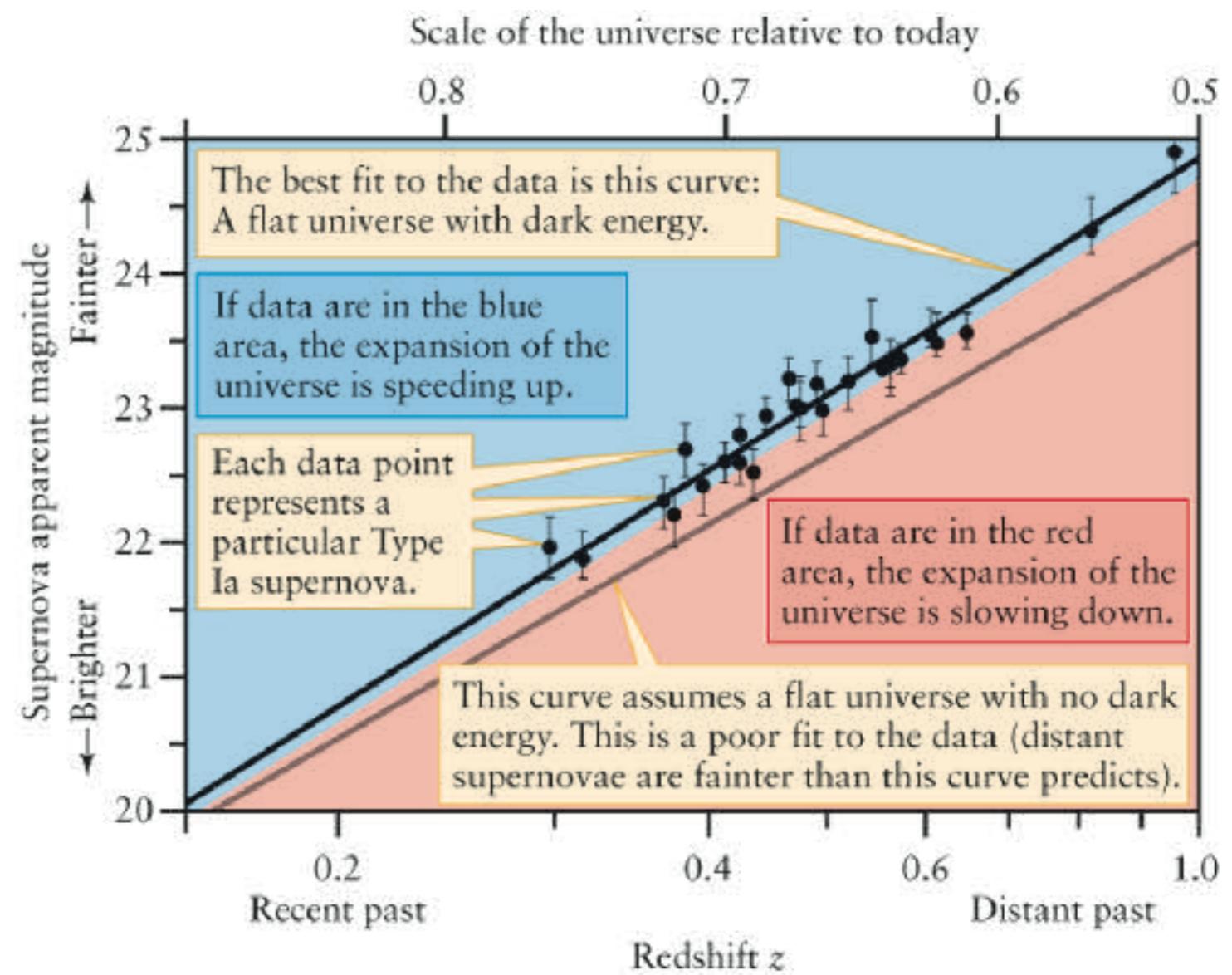
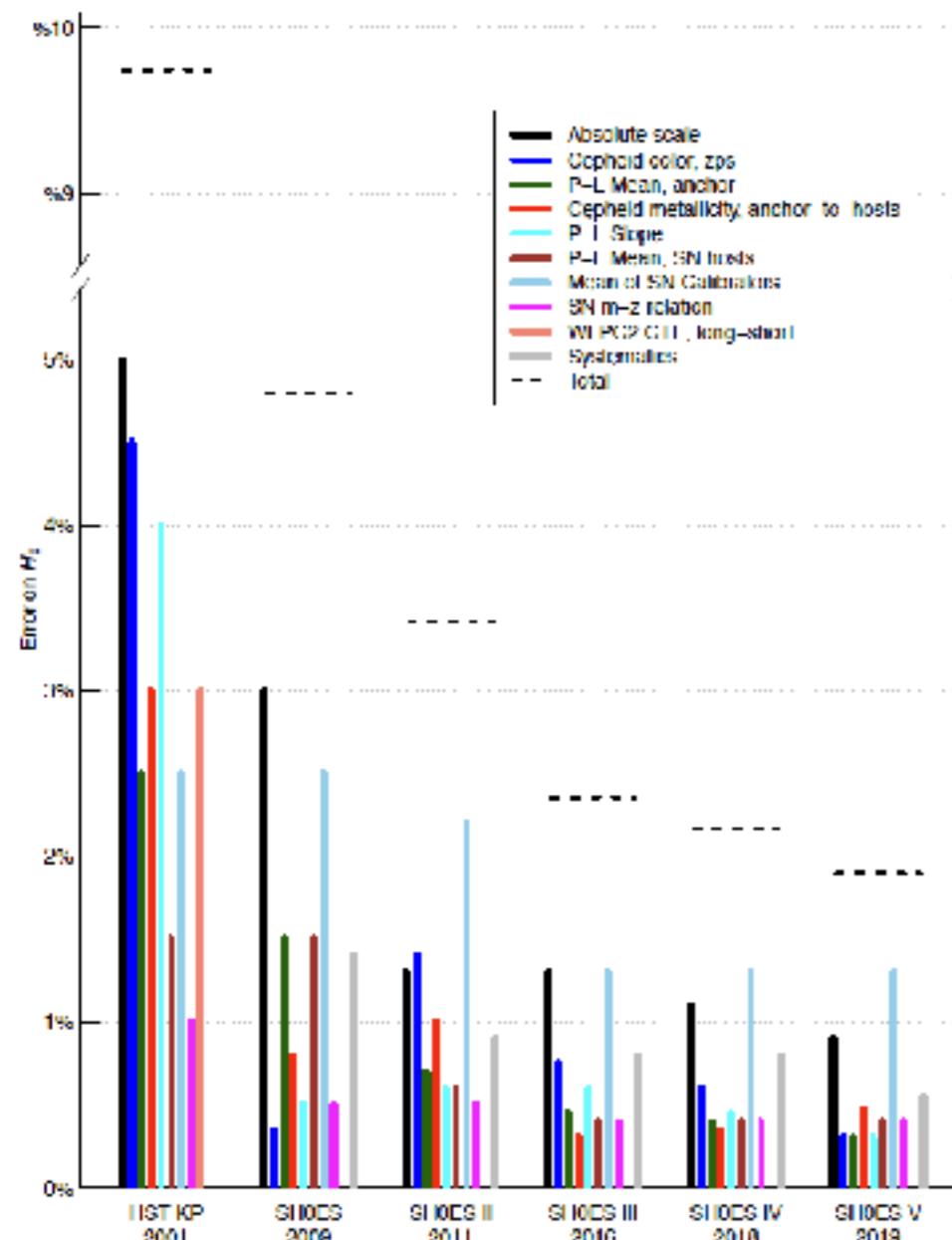
*Over the past two decades the precision  
(accuracy?) of the Hubble constant improved  
from a factor of two to a few-percent error.*

*Not all inferences agree, however...*

# What about this “tension”?

Error budgets from late-universe measurements (e.g. SNe) are shrinking, but values of  $H_0$  in the late universe are not changing for  $\sim 20$  years.

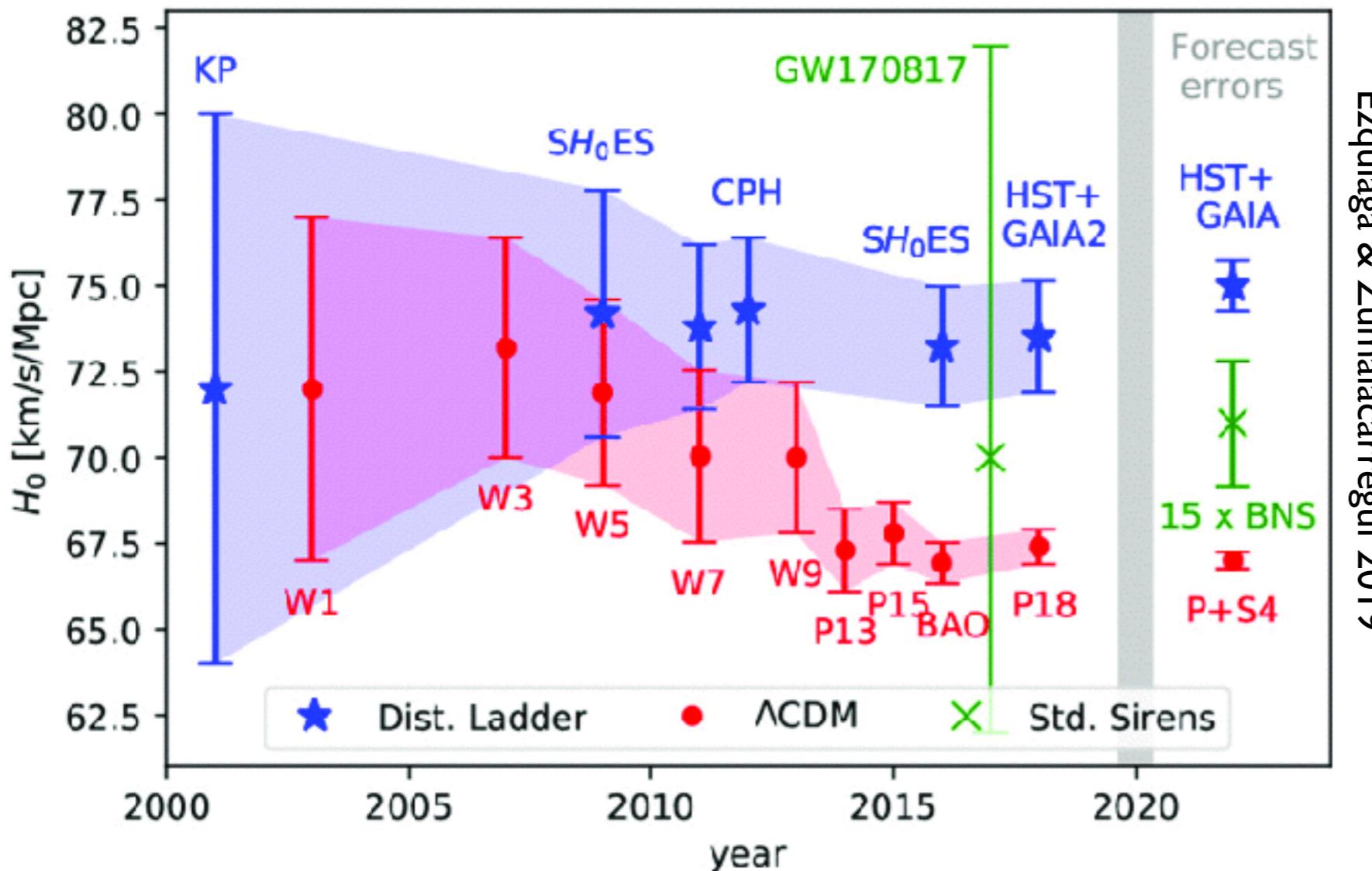
Illustration from SHOES (Supernovae) Riess et al.



# What about this “Hubble tension”?

The late Universe and early Universe measurement (CMB, BAO) have been drifting apart as the data improves and errors get smaller.

In particular the early Universe measurements have decreased since Planck.



Why does Planck find lower value  $H_0$  than the inference from WMAP?

Higher l-mode data,  
better foreground  
removal (e.g MW).



# Early & Late Universe

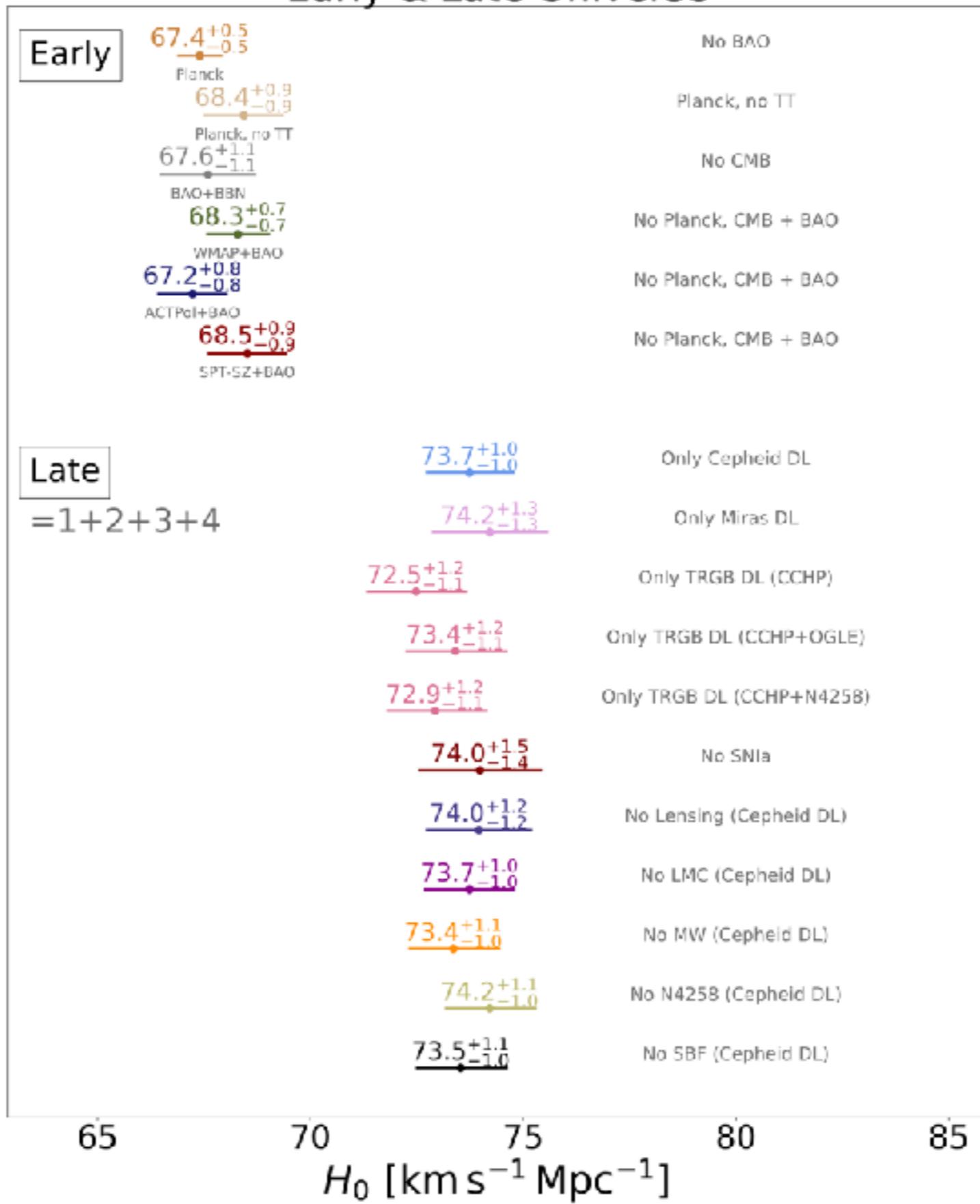


Figure: Adam Amarra

# Early & Late Universe

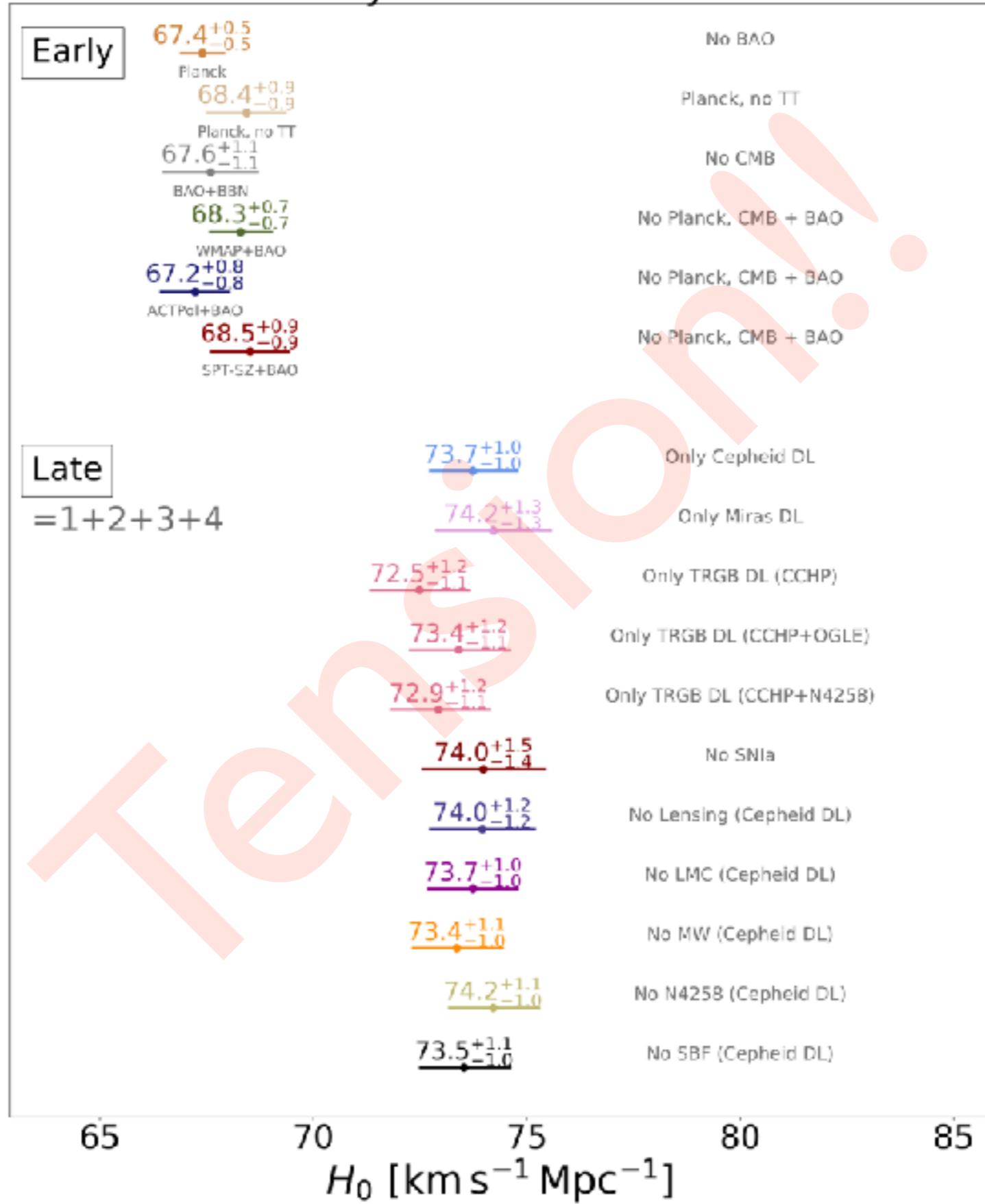
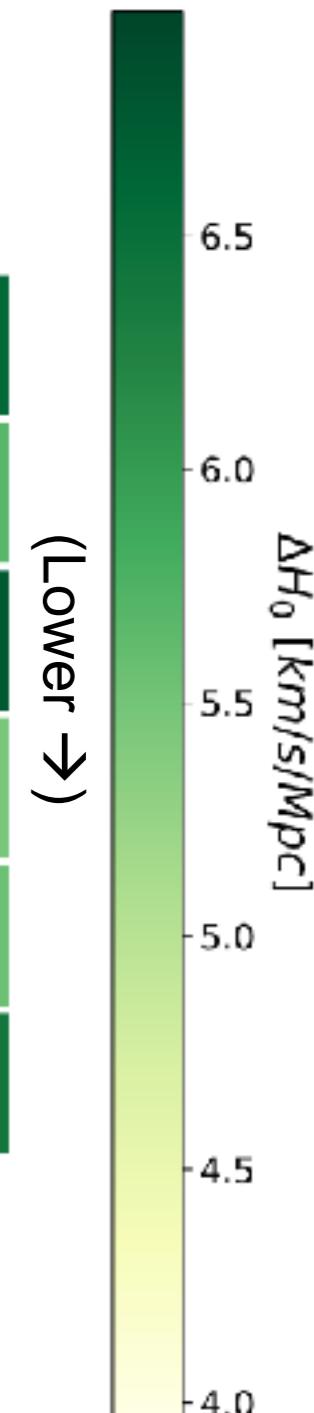


Figure: Adam Amarra

# The tension matrix

E  
A  
R  
L  
Y

$\Delta H_0$  vs Measurement Precision (Lower →)



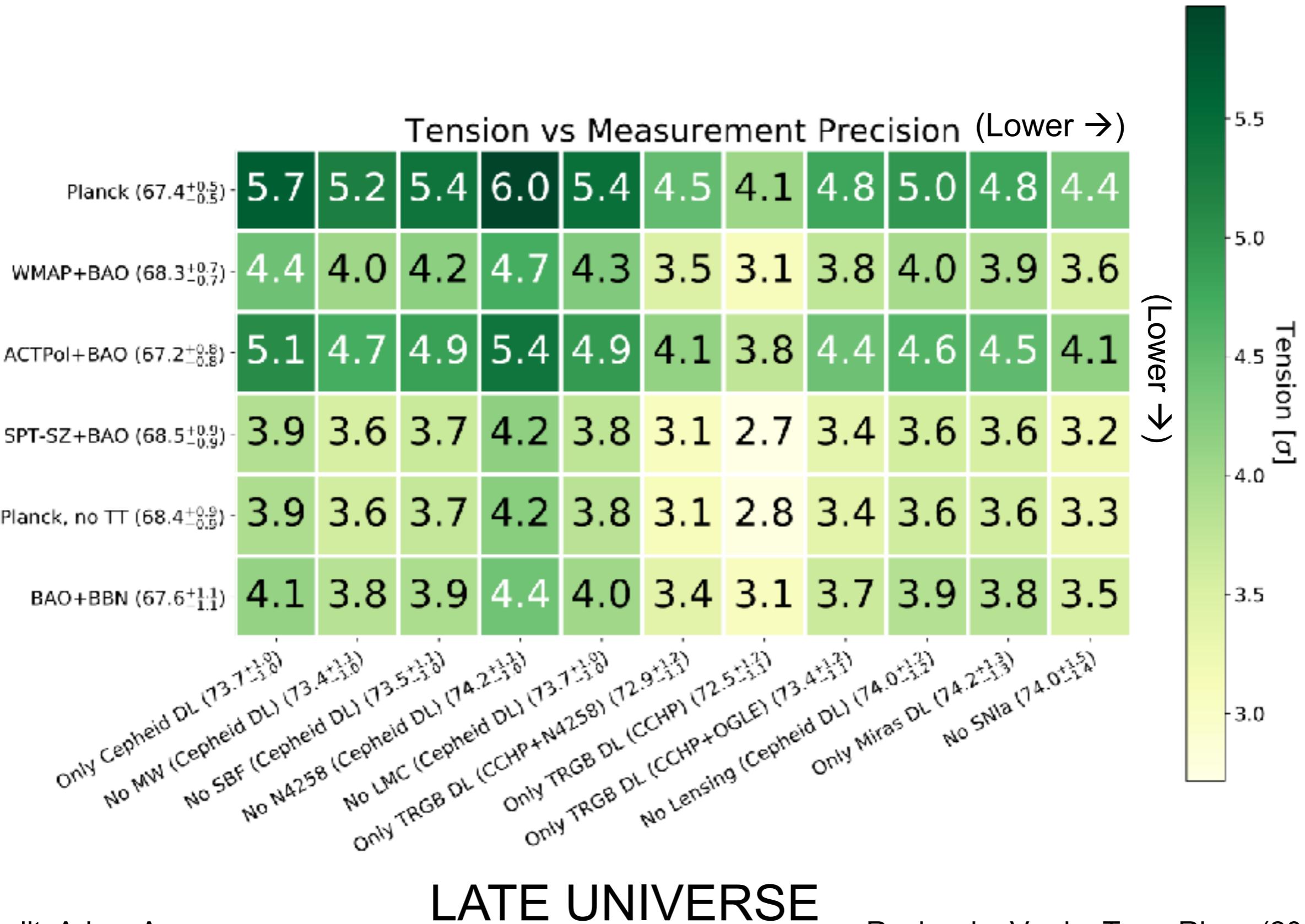
(Lower ↓)

	Only Cepheid DL (73.7 <sup>+1.0</sup> <sub>-1.0</sub> )	No MW (Cepheid DL) (73.4 <sup>+1.3</sup> <sub>-1.0</sub> )	No SBF (Cepheid DL) (73.5 <sup>+1.3</sup> <sub>-1.0</sub> )	No N4258 (Cepheid DL) (74.2 <sup>+1.3</sup> <sub>-1.0</sub> )	No LMC (Cepheid DL) (73.7 <sup>+1.0</sup> <sub>-1.0</sub> )	Only TRGB DL (CCHP+N4258) (72.9 <sup>+1.2</sup> <sub>-1.1</sub> )	Only TRGB DL (CCHP+OGLE) (73.4 <sup>+1.2</sup> <sub>-1.1</sub> )	No Lensing (Cepheid DL) (74.0 <sup>+1.2</sup> <sub>-1.1</sub> )	Only Miras DL (74.2 <sup>+1.3</sup> <sub>-1.3</sub> )	No SNIa (74.0 <sup>+1.5</sup> <sub>-1.4</sub> )	
Planck (67.4 <sup>+0.5</sup> <sub>-0.5</sub> )	6.3	6.0	6.1	6.8	6.3	5.5	5.1	6.0	6.6	6.8	6.6
WMAP+BAO (68.3 <sup>+0.7</sup> <sub>-0.7</sub> )	5.4	5.1	5.2	5.9	5.4	4.6	4.2	5.1	5.7	5.9	5.7
ACTPol+BAO (67.2 <sup>+0.8</sup> <sub>-0.8</sub> )	6.5	6.1	6.3	7.0	6.5	5.7	5.3	6.2	6.7	7.0	6.8
SPT-SZ+BAO (68.5 <sup>+0.9</sup> <sub>-0.9</sub> )	5.2	4.8	5.0	5.7	5.2	4.4	4.0	4.9	5.4	5.7	5.5
Planck, no TT (68.4 <sup>+0.9</sup> <sub>-0.9</sub> )	5.3	4.9	5.1	5.8	5.3	4.5	4.0	5.0	5.5	5.8	5.5
BAO+BBN (67.6 <sup>+1.1</sup> <sub>-1.1</sub> )	6.1	5.8	5.9	6.6	6.1	5.3	4.9	5.8	6.4	6.6	6.4

LATE UNIVERSE

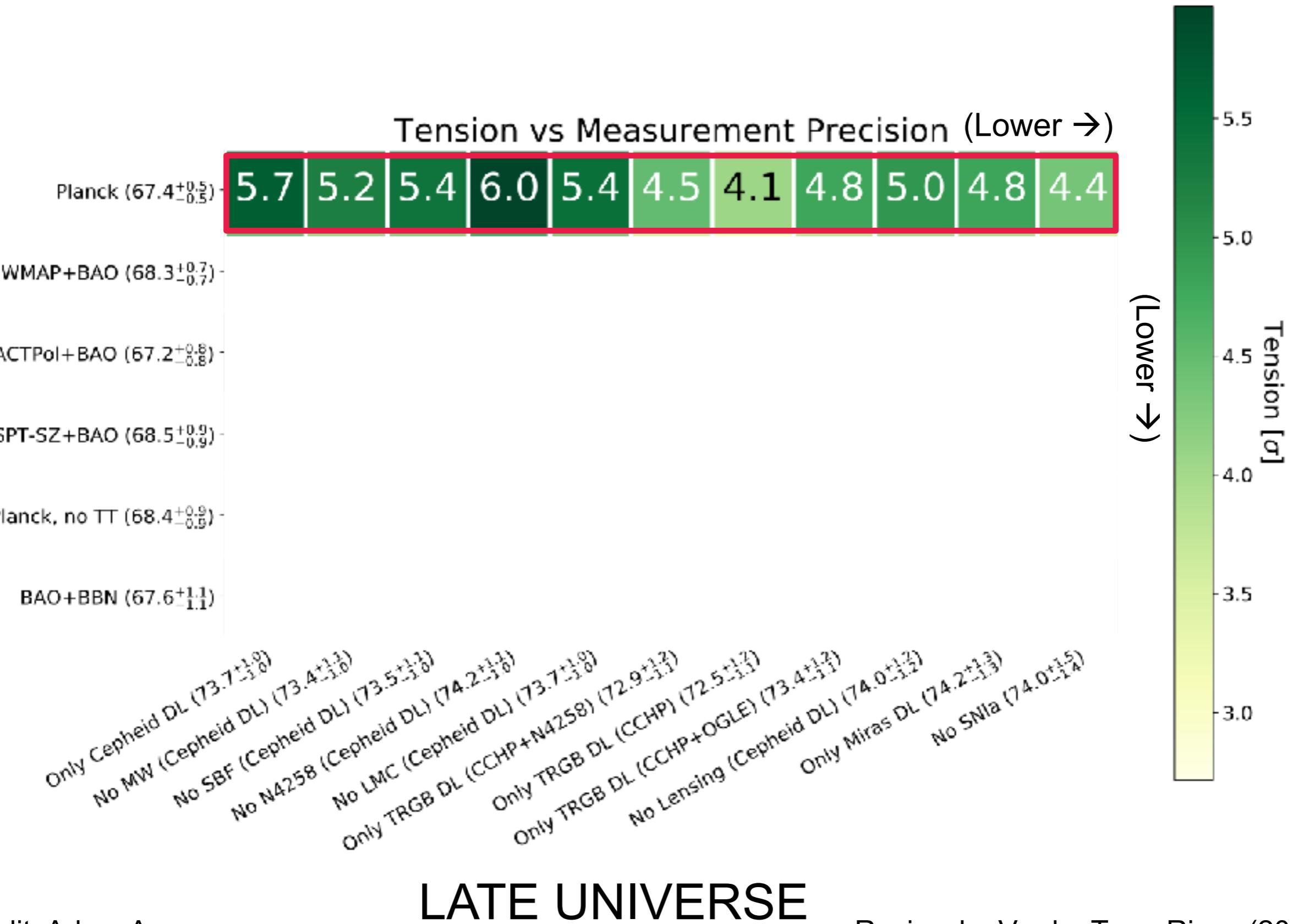
# The tension matrix

E  
A  
R  
L  
Y



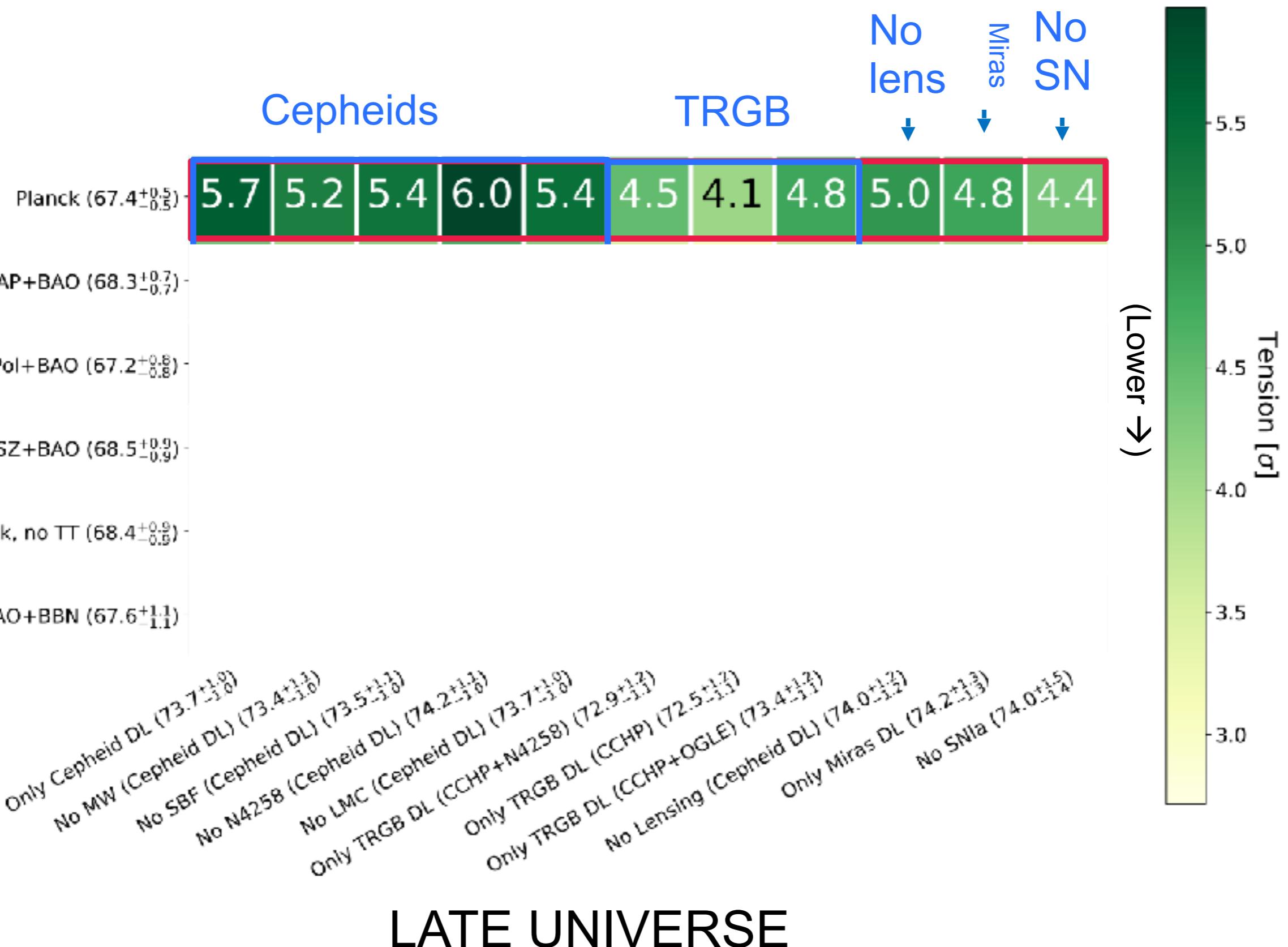
# The tension matrix

E  
A  
R  
L  
Y



# The tension matrix

E  
A  
R  
L  
Y



LATE UNIVERSE



*Are there other, fully independent, methods from CMB+BAO  
and distant ladder methods that can also measure  $H_0$ ?*

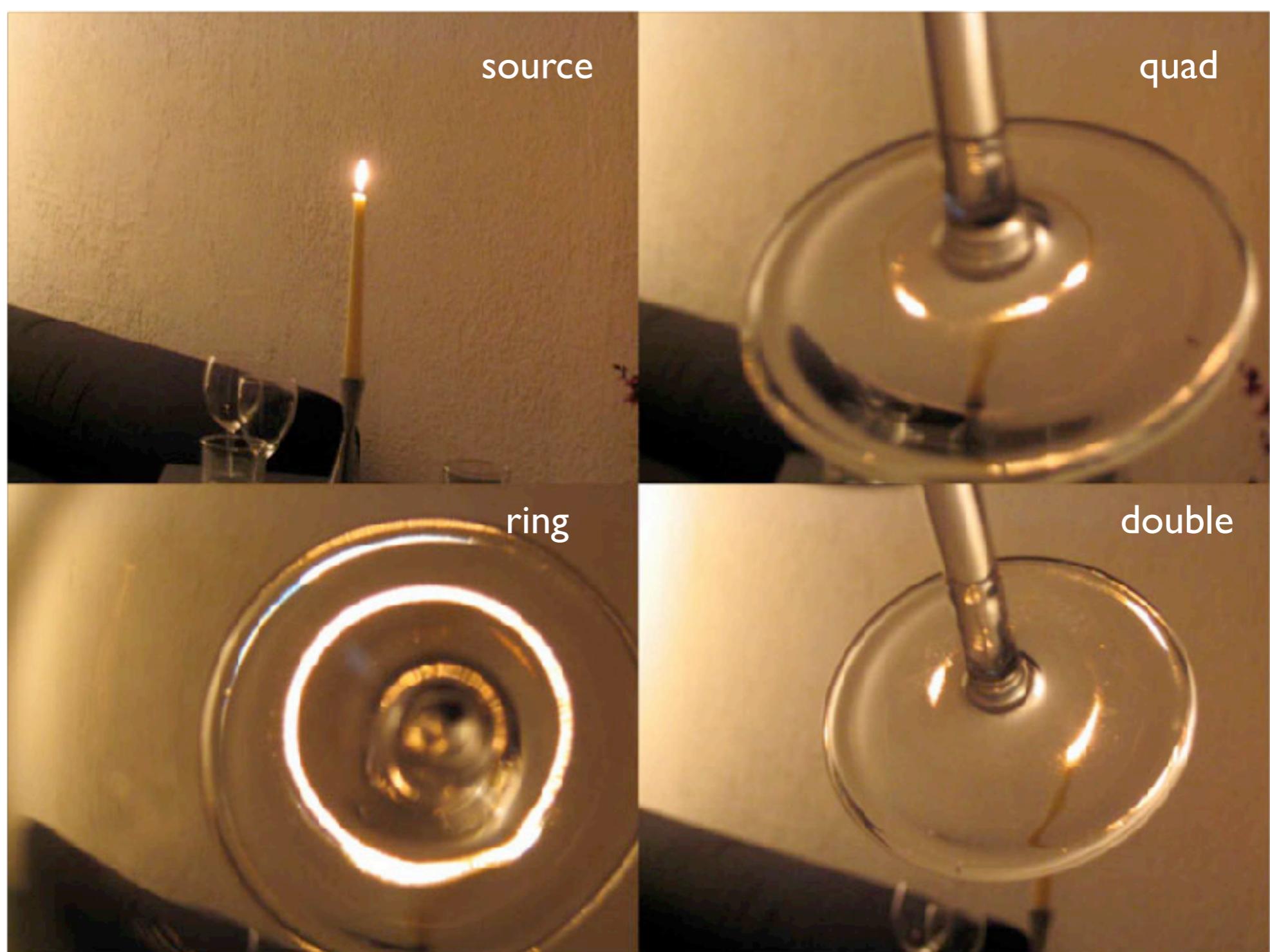
# Strong Gravitational Lensing & Time-delay Cosmography

*Strong lenses are e.g. galaxies at intermediate cosmological distances gravitationally imaging a more distance source in to multiple images.*

*They are powerful probes of the mass-density of galaxies, and can also probe the size of the Universe in one direct step with limited astrophysical complexities.*

# All of you must have seen this before!

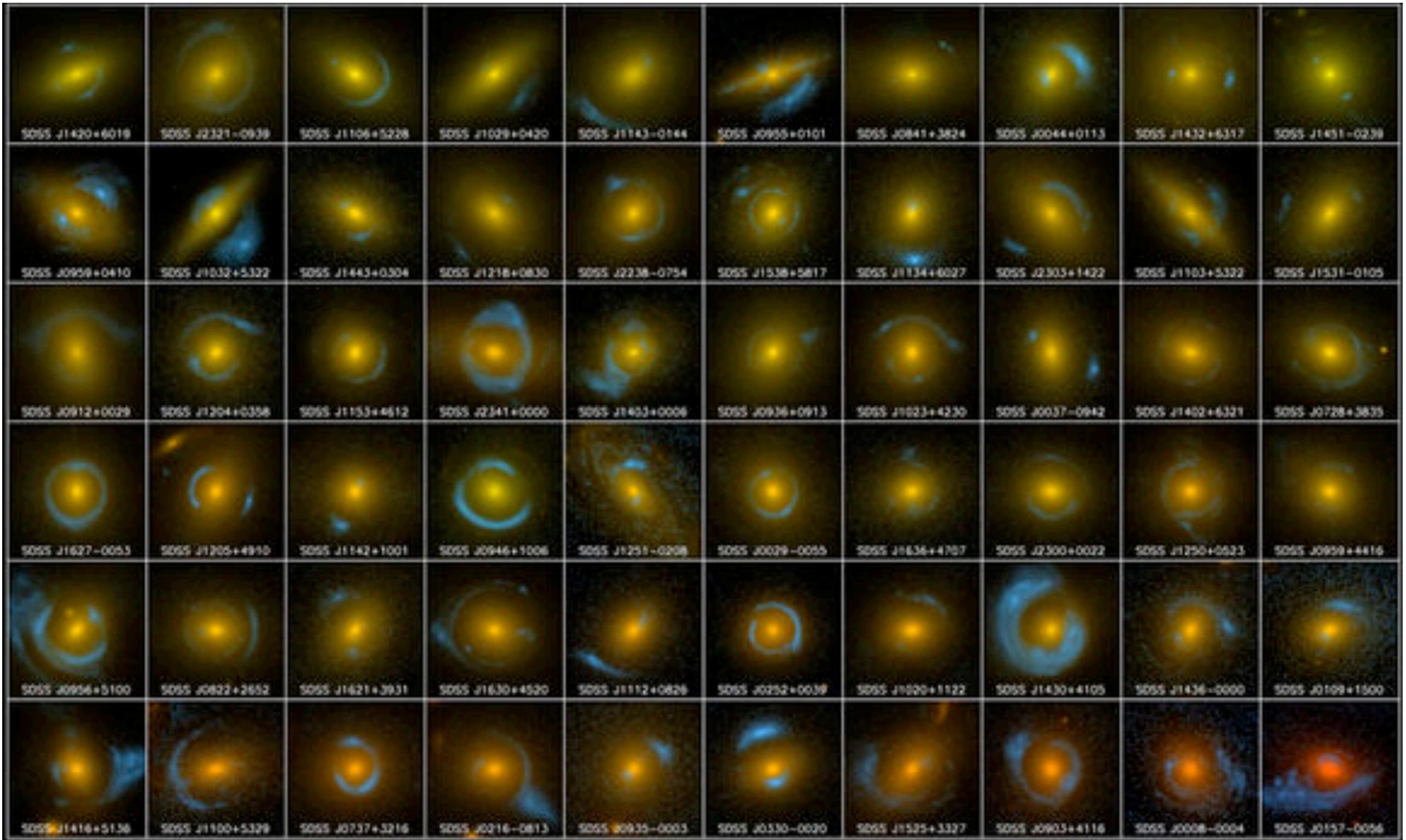
Optical lenses can behave just like gravitational lenses

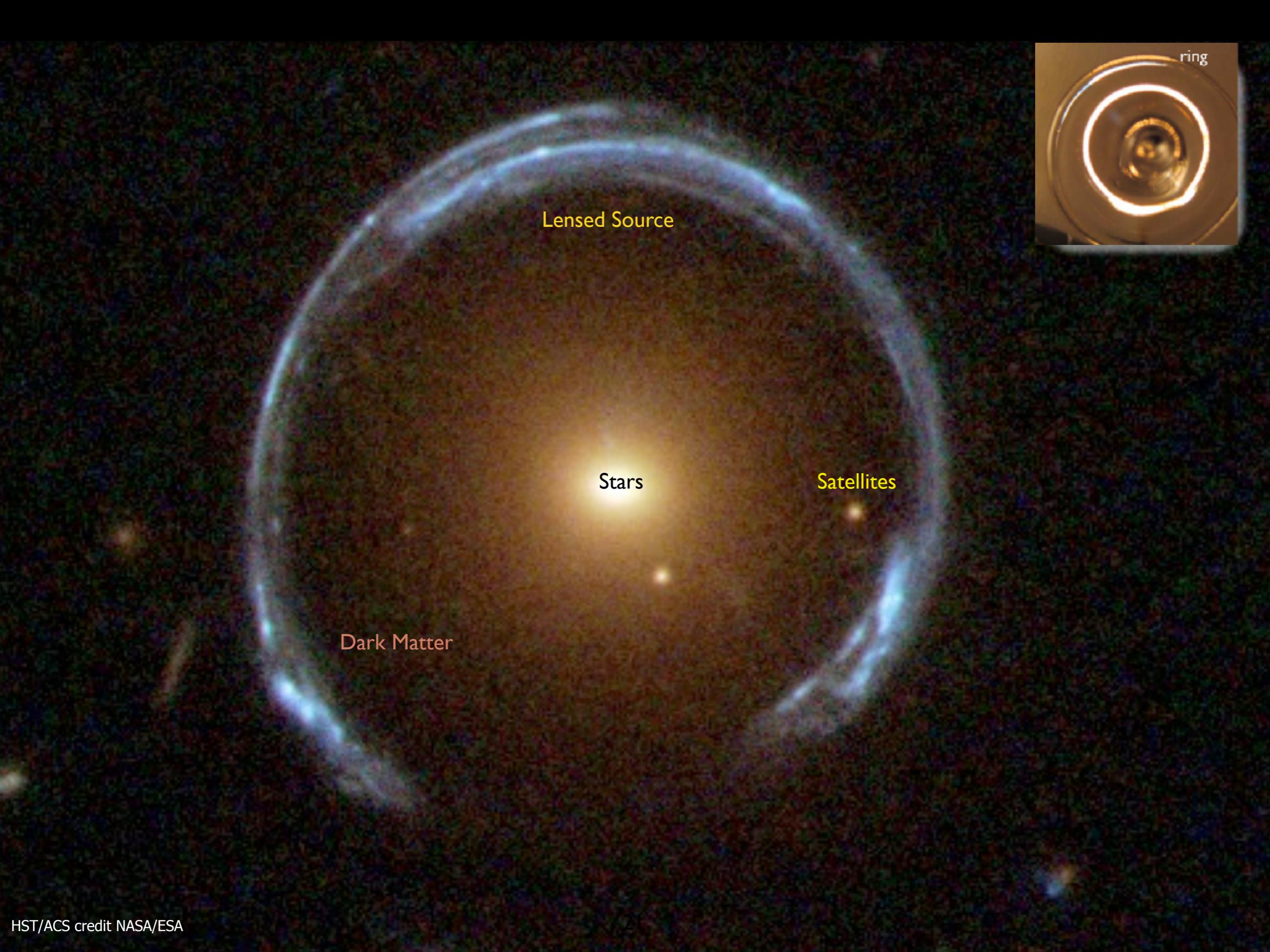


# Spectacular Examples: Clusters



# Examples: Galaxies





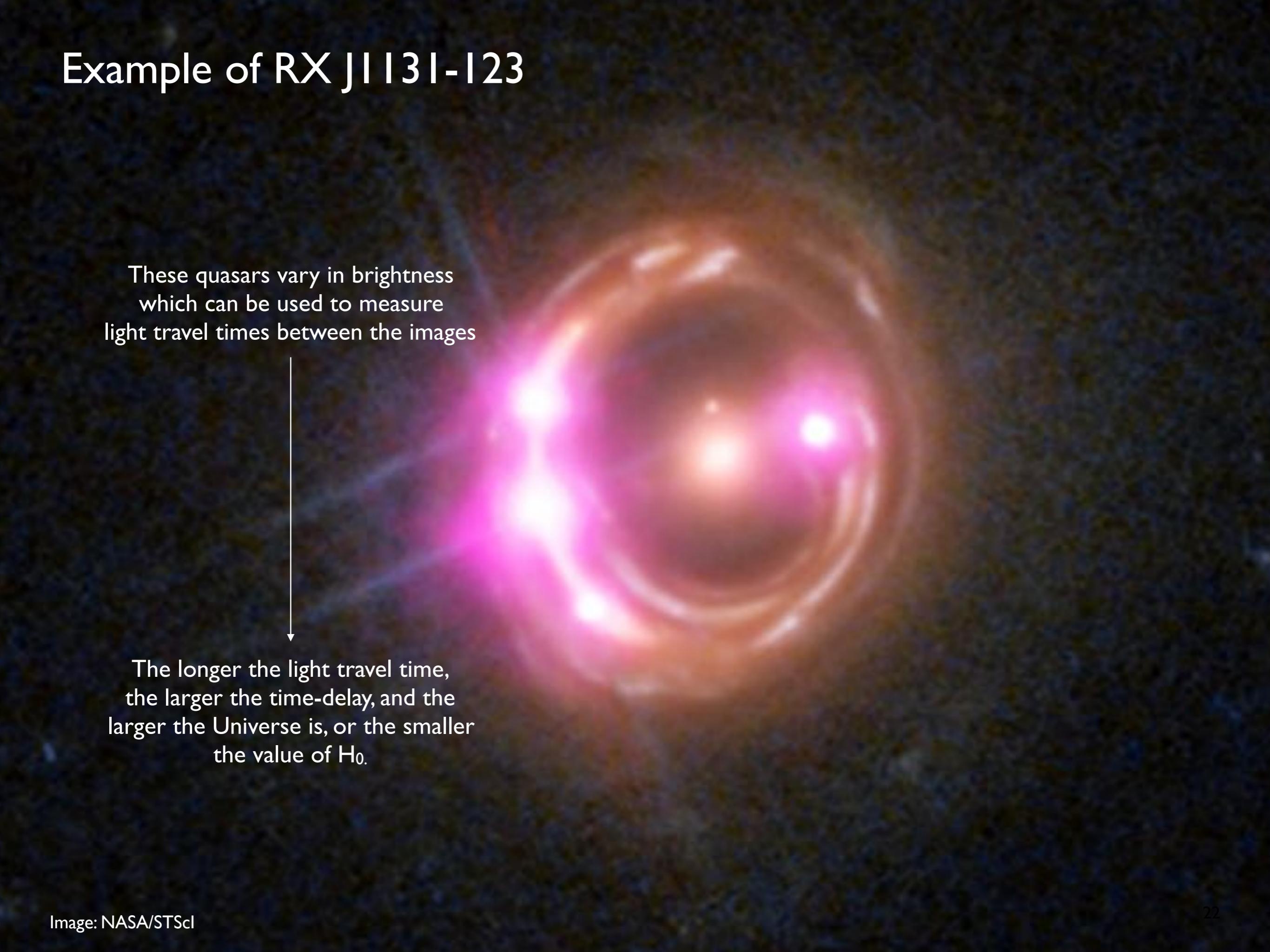
# Example of RX J1131-123

These quasars vary in brightness  
which can be used to measure  
light travel times between the images



The longer the light travel time,  
the larger the time-delay, and the  
larger the Universe is, or the smaller  
the value of  $H_0$ .

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These quasars vary in brightness  
which can be used to measure  
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The longer the light travel time,  
the larger the time-delay, and the  
larger the Universe is, or the smaller  
the value of  $H_0$ .



# Theory on Strong Lensing 101

*Strong lensing is based on Einstein's GR theory, but can be “simplified” in nearly all (weak-field, thin-lens) cases to geometric optics.*

# Gravitational Lensing: General Relativity

The perturbed Minkowski space-time metric reads

$$ds^2 = - \left(1 + 2\frac{\Phi}{c^2}\right) c^2 dt^2 + \left(1 - 2\frac{\Phi}{c^2}\right) (dx^2 + dy^2 + dz^2)$$

The effect on a light-ray can be expressed through an effective refractive index (as in geometric optics; next next slides)

$$n_{ref} = 1 - \frac{2}{c^2} \Phi \geq 1$$

The **deflection angle** integrated along the line-of-sight then becomes

$$\vec{\alpha} = - \int dl \vec{\nabla}_\perp n_{ref} = \frac{2}{c^2} \int dl \vec{\nabla}_\perp \Phi$$

# Gravitational Lensing: Time Delays

This **time-delay** of a signal that travels through a potential is given by

$$dt = \frac{dl}{v}$$

The total time due to the potential will be given, to first order, by

$$\Delta t = \int_{\text{path}} dt \approx \int_{\text{path}} \frac{2}{c^3} |\phi| dl$$

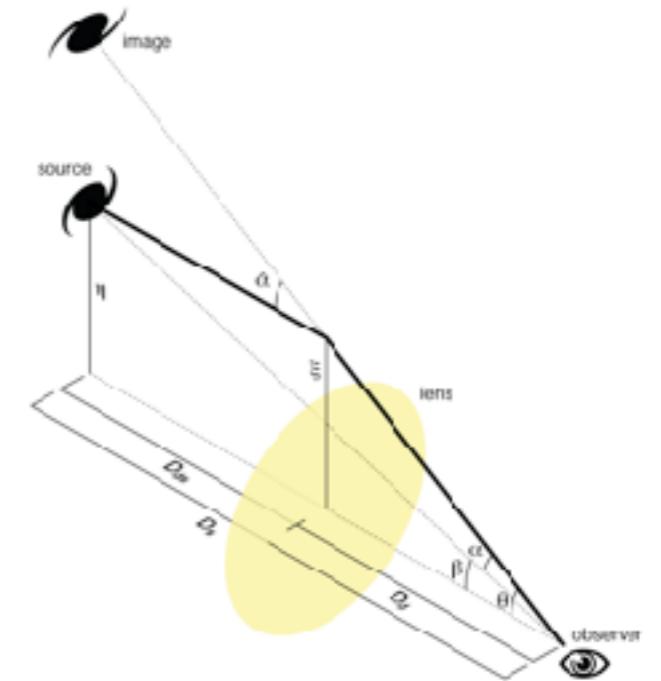
Note that if we know  $|\Phi|$  from say a lens model, and we can measure  $\Delta t$ , we can then derive the path length. This path length turns out to be proportional to  $l/H_0$  and hence we can measure the **Hubble Constant**.

# Gravitational Lensing: Deflection Angles

The second effect of gravitational lensing is the **deflection** of light.

Just as in the case of optics, if the refractive index  $n \neq 1$  a ray of light is deflected along a different (non-straight) path. Similarly this happens in gravitational lensing

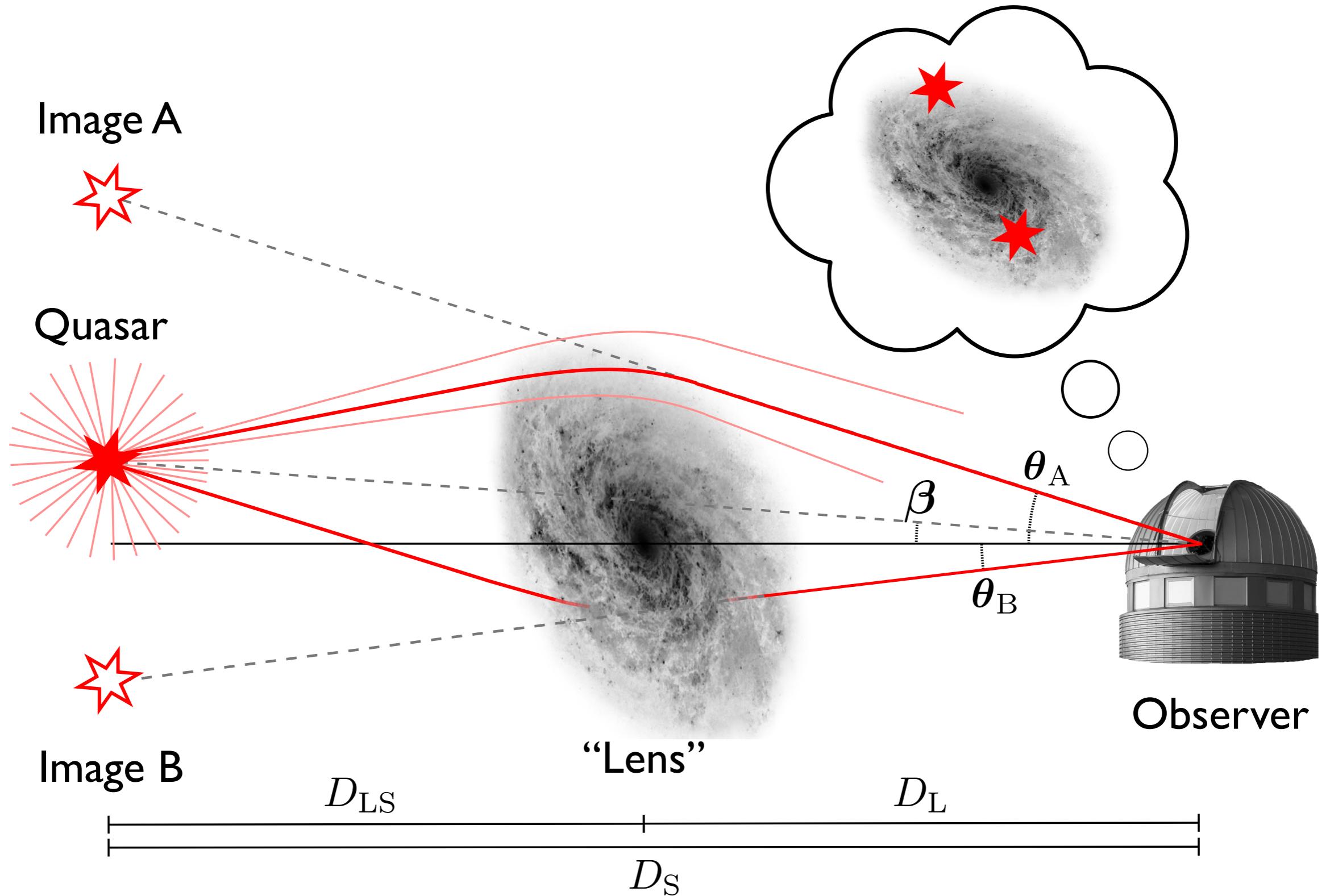
$$\vec{\alpha} = \int_{\text{path}} \vec{\nabla}_{\perp} n dl = \frac{2}{c^2} \int_{\text{path}} \vec{\nabla}_{\perp} \phi dl$$



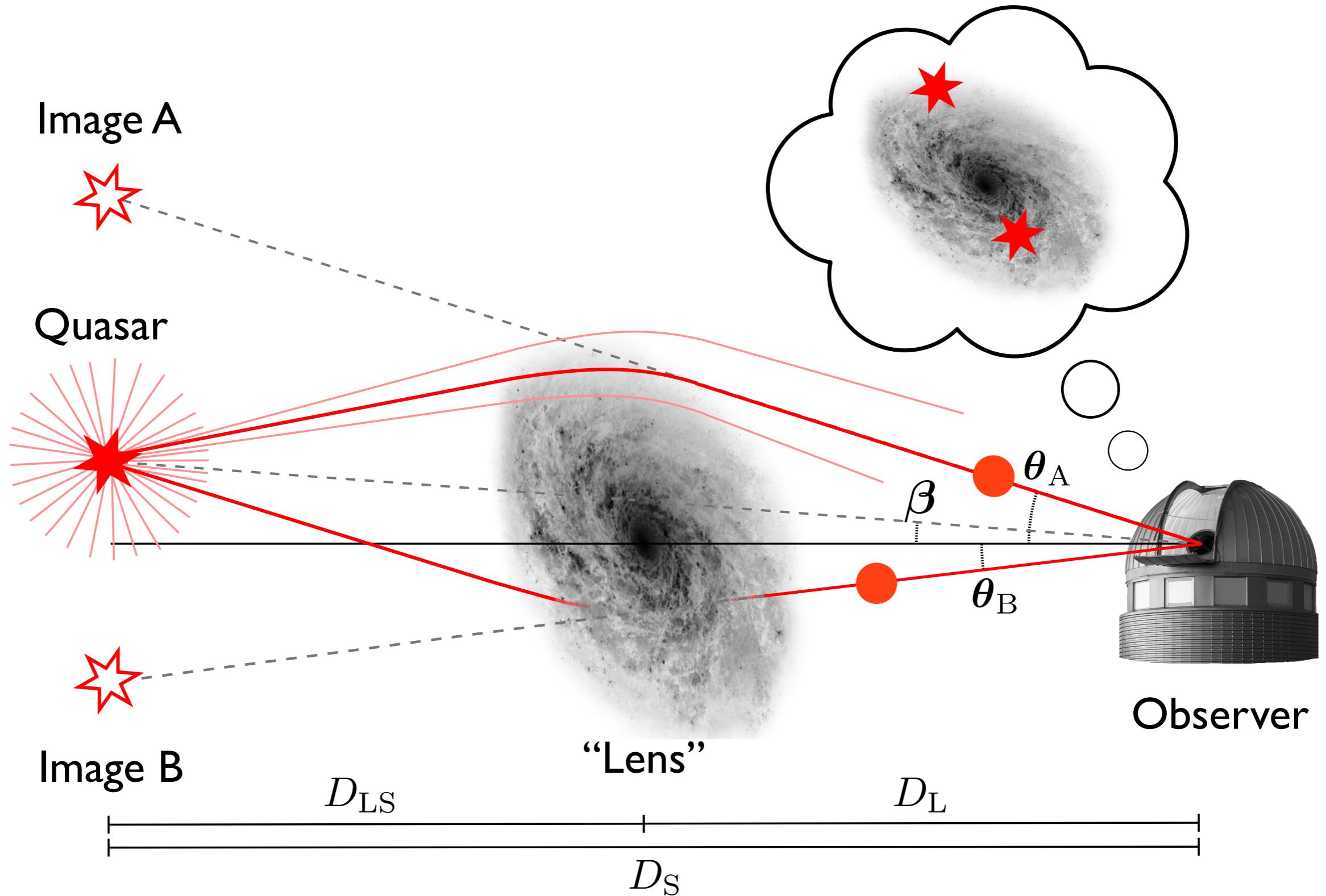
Hence a light ray is deflected by an angle that depends (to first order) on the integral over the gradient of the lens potential perpendicular to the line of sight. This approximation holds because to first order the ray of light travels along a straight path (Born approximation).

NOTE: We can bring both concepts, time-delay and deflection, together in a single frame work: the “Fermat Principle”.

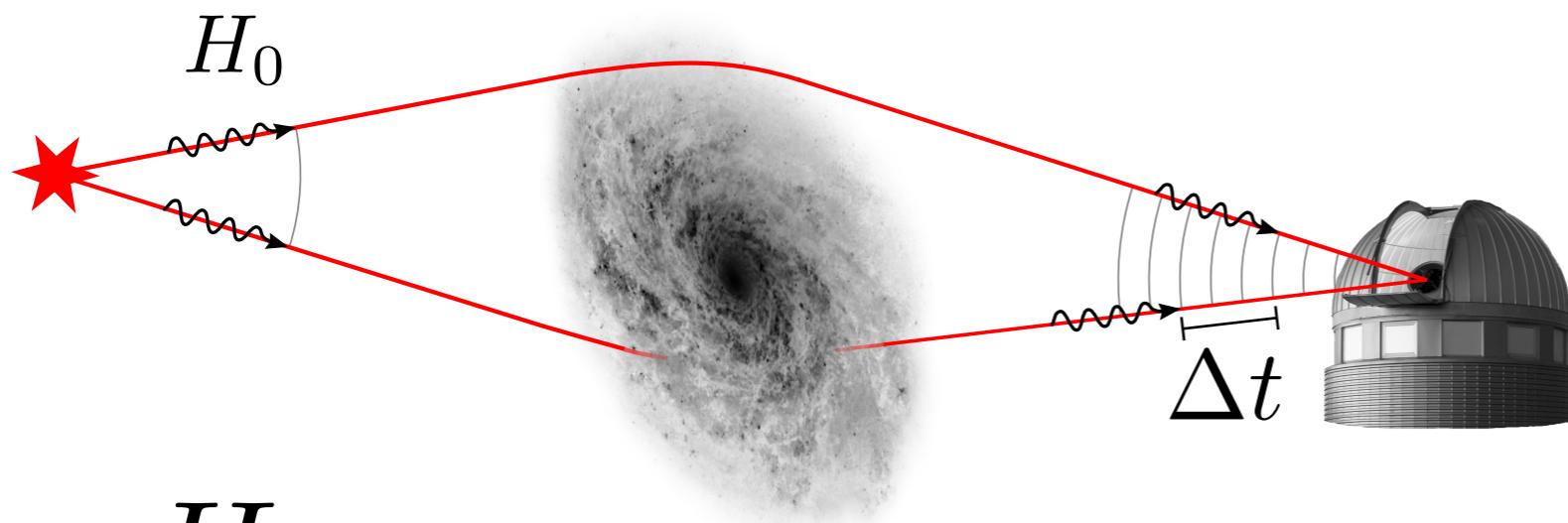
# Time Delays in Strongly Lensed Quasars



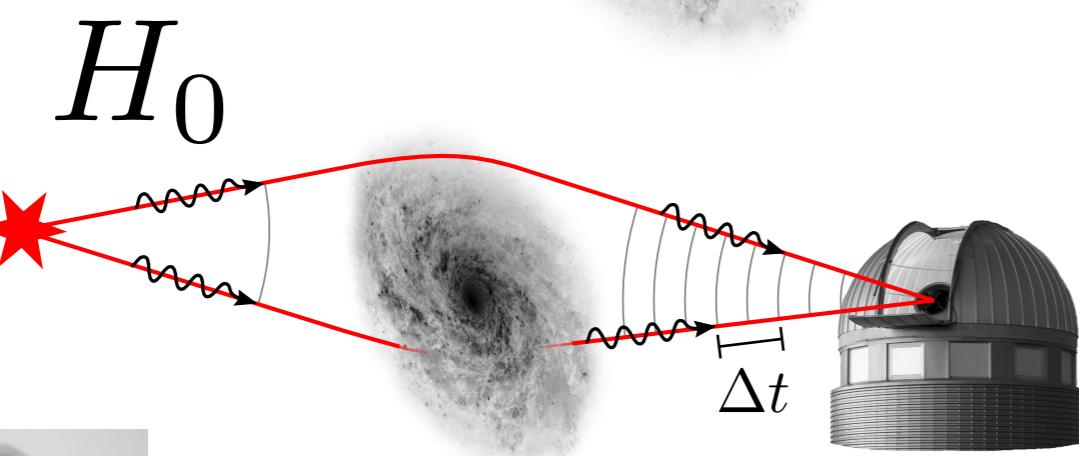
# Time Delays in Strongly Lensed Quasars



# Time Delays Measure the Hubble Constant $H_0$



Lens potential  
at the images



Source position  
(unconstrained)

Astrometry  
of the images

$$\Delta t = \frac{1 + z_L}{c} \underbrace{\frac{D_L D_S}{D_{LS}}}_{\propto 1/H_0} \cdot \Delta \left( \frac{1}{2} |\vec{\theta} - \vec{\beta}|^2 - \psi(\vec{\theta}) \right)$$

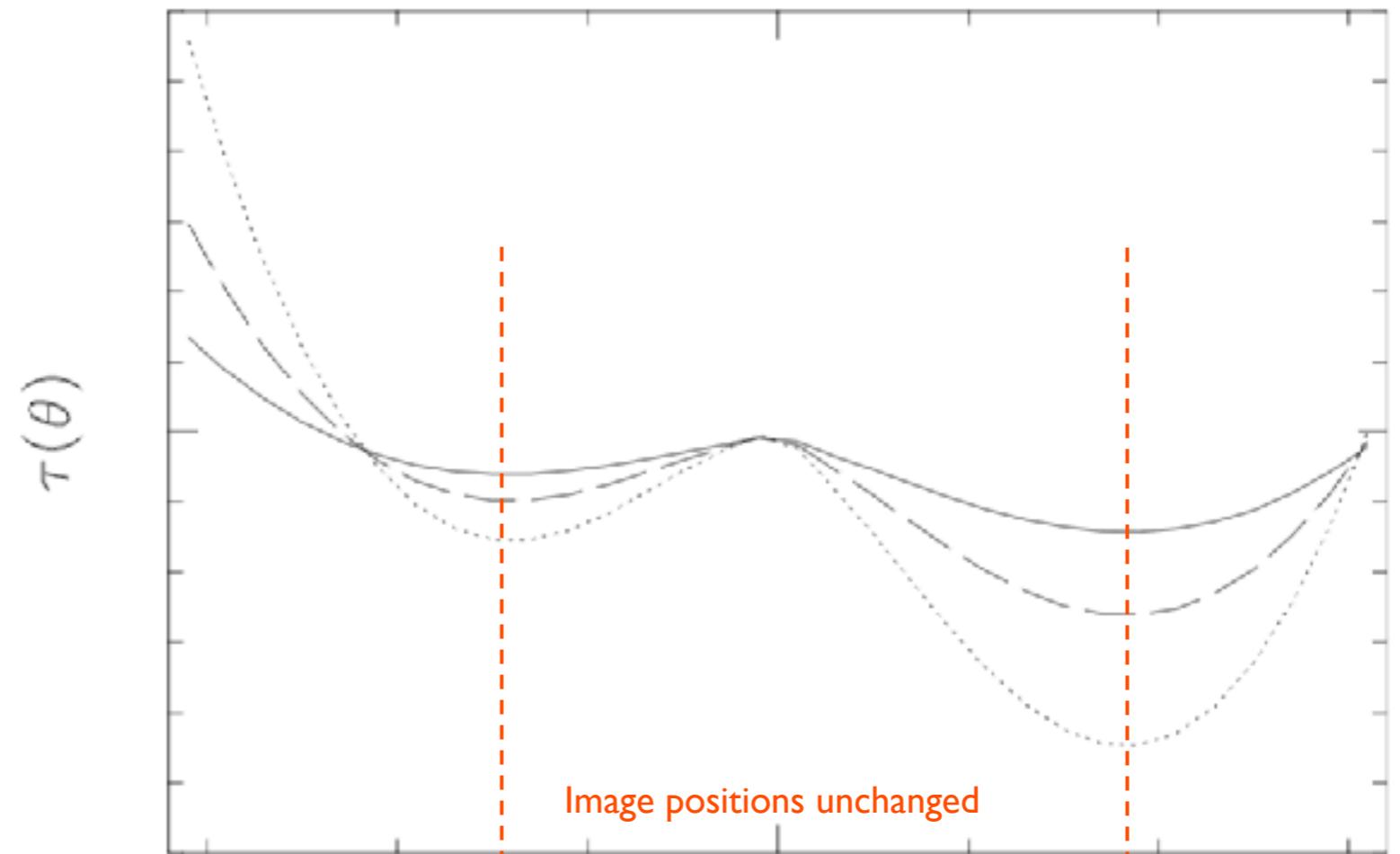


Sjur Refsdal

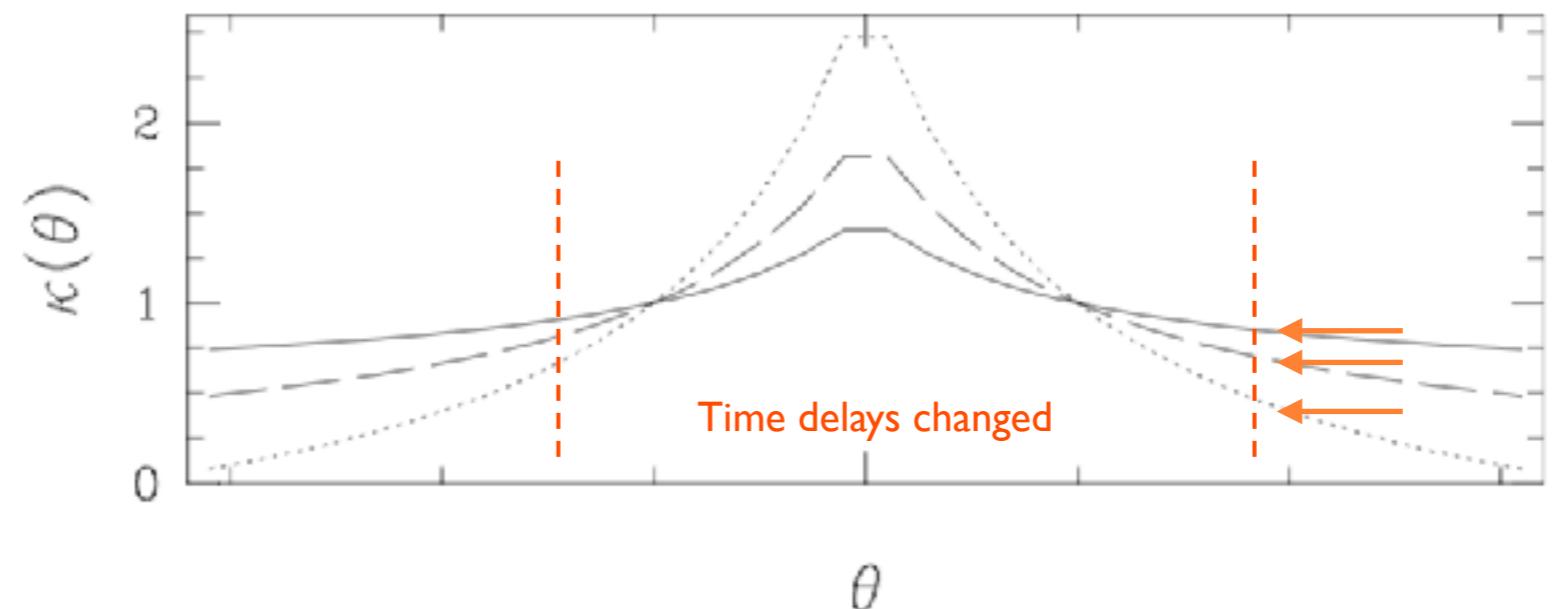
Time delays provide a *single-step* and *independent* constraint on  $H_0$ .

# Impact of the gravitational potential of the lens

Arrival time surfaces  
in the image plane



Normalised mass profile  
 $\kappa$  = projected mass density  
in units of the critical mass



# What's Needed and State of the Art — Blind Analyses!

1. **Time delays** measurements
2. **Mass model** for the lens
3. **Environment** of the lens
4. **Line of Sight** contribution

## PAST

- 15-30% precision
- Simplistic models
- Few constraints
- External shear
- External shear

## PRESENT

- 1-5% precision/accuracy
- Flexible elliptical models
- Deep sharp HST/AO images
- Lens dynamics
- Include nearby companions
- Multi-plane lensing
- Photo and spectro-z
- Galaxy counts
- Cosmological simulations
- Weak lensing



PI: S. Suyu

# H<sub>0</sub> Lenses in CO<sup>S</sup>MOGRAIL'S Wellspring

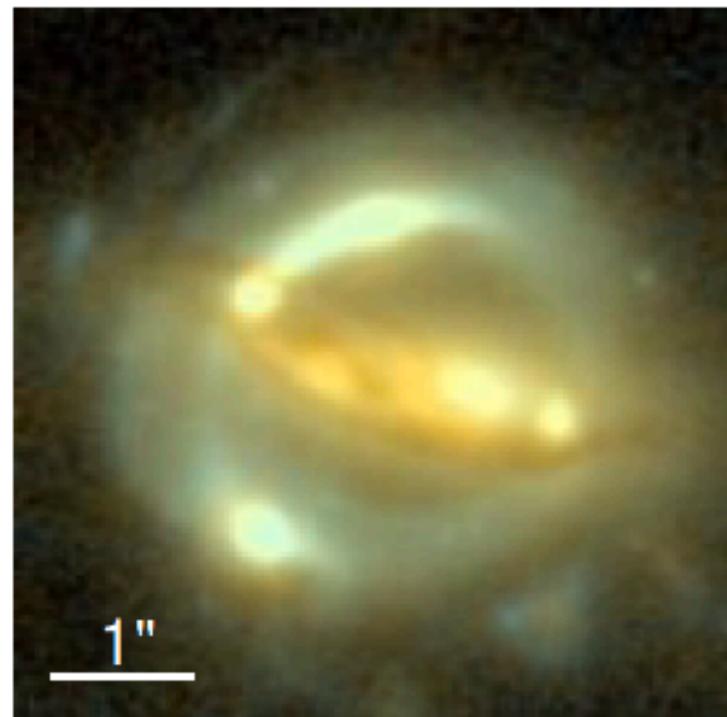
Obtain imaging and  
spectroscopic data.  
Modelling the lenses and  
their environments and  
lines of sight. Infer H<sub>0</sub>.

PI: F. Courbin

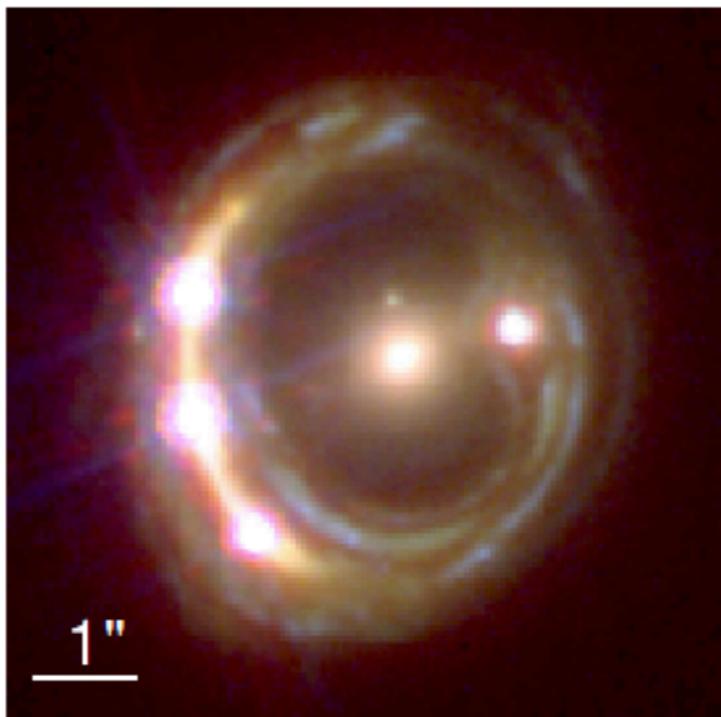
# COSmological MOonitoring GRAVItational Lenses

Measuring the time-  
delays between quasar  
images during decade-  
long campaigns.

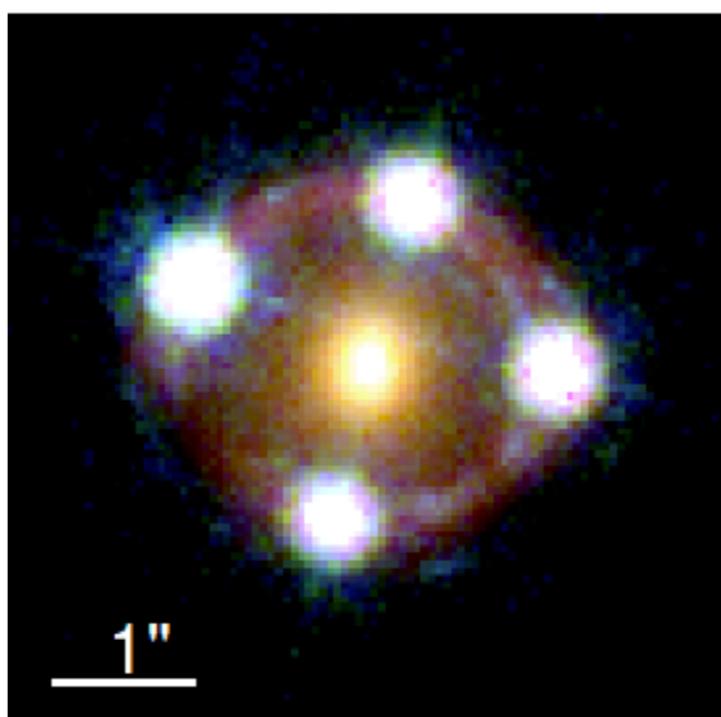
# The current sample: six multiply-imaged quasars



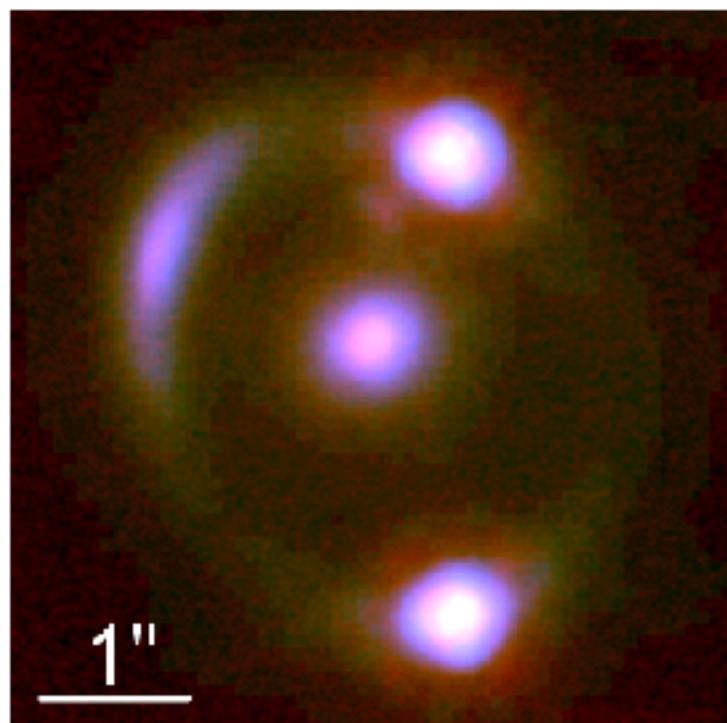
(a) B1608+656



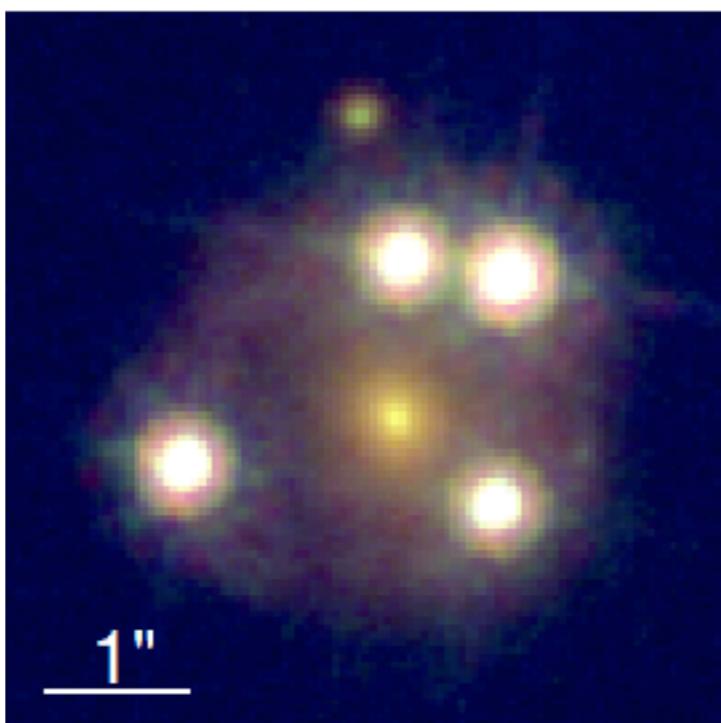
(b) RXJ1131-1231



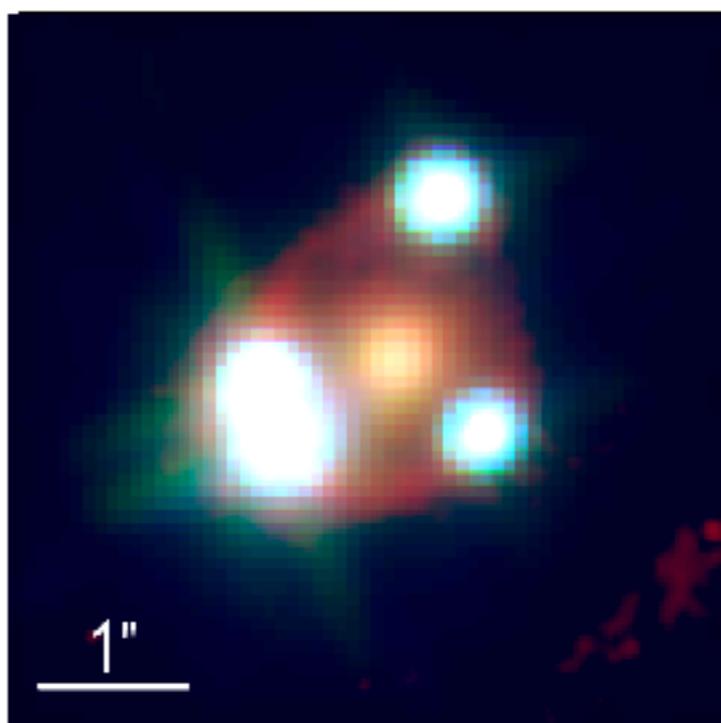
(c) HE 0435-1223



(d) SDSS 1206+4332

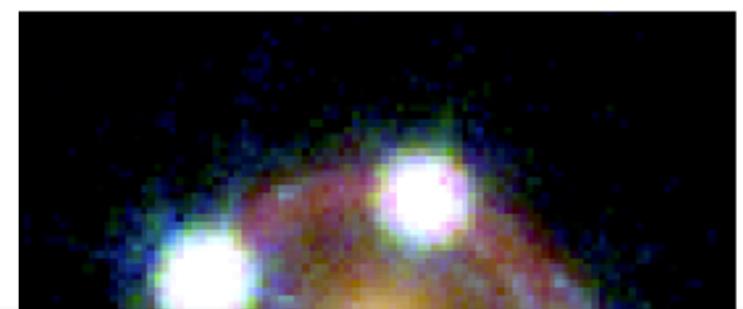


(e) WFI2033-4723

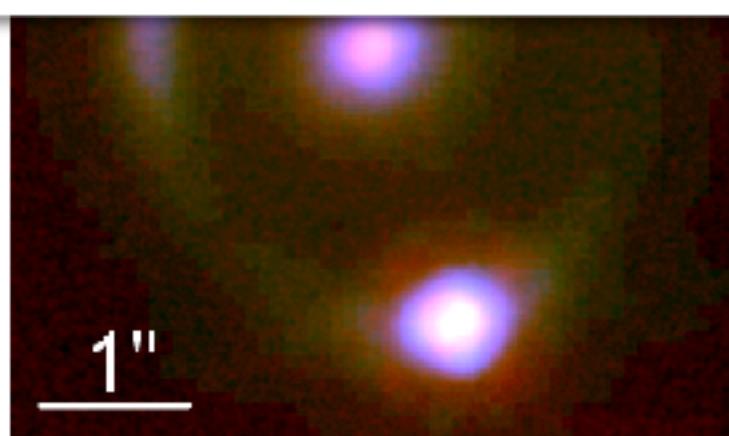


(f) PG 1115+080

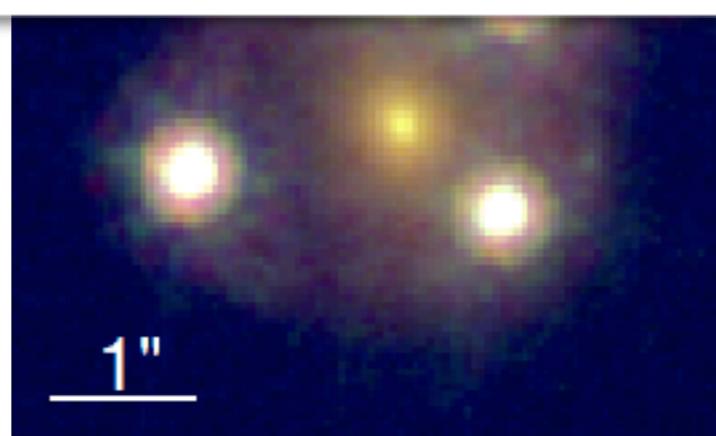
# The current sample: six multiply-imaged quasars



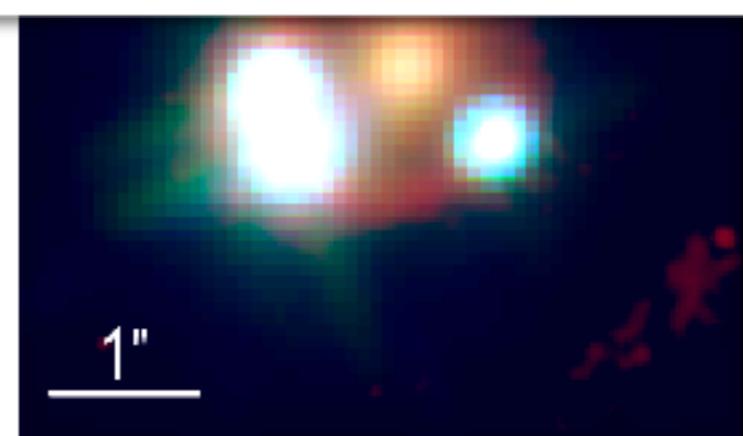
Lens name	$\alpha$ (J2000)	$\delta$ (J2000)	$z_d$	$z_s$	<i>HST / AO</i> data
B1608+656 <sup>a</sup>	16:09:13.96	+65:32:29.0	0.6304 <sup>a</sup>	1.394 <sup>b</sup>	<i>HST</i>
RXJ1131-1231 <sup>c</sup>	11:31:51.6	-12:31:57.0	0.295 <sup>c</sup>	0.654 <sup>d</sup>	<i>HST + AO</i>
HE 0435-1223 <sup>c</sup>	04:38:14.9	-12:17:14.4	0.4546 <sup>f,g</sup>	1.693 <sup>h</sup>	<i>HST + AO</i>
SDSS J206+4332 <sup>i</sup>	12:06:29.65	+43:32:17.6	0.745 <sup>j</sup>	1.789 <sup>i</sup>	<i>HST</i>
WFI2033-4723 <sup>k</sup>	20:33:41.9	-47:23:43.4	0.6575 <sup>l</sup>	1.662 <sup>h</sup>	<i>HST</i>
PG 1115+080 <sup>m</sup>	11:18:16.899	+7:45:58.502	0.311 <sup>n</sup>	1.722 <sup>m</sup>	<i>HST + AO</i>



(d) SDSS J206+4332



(e) WFI2033-4723



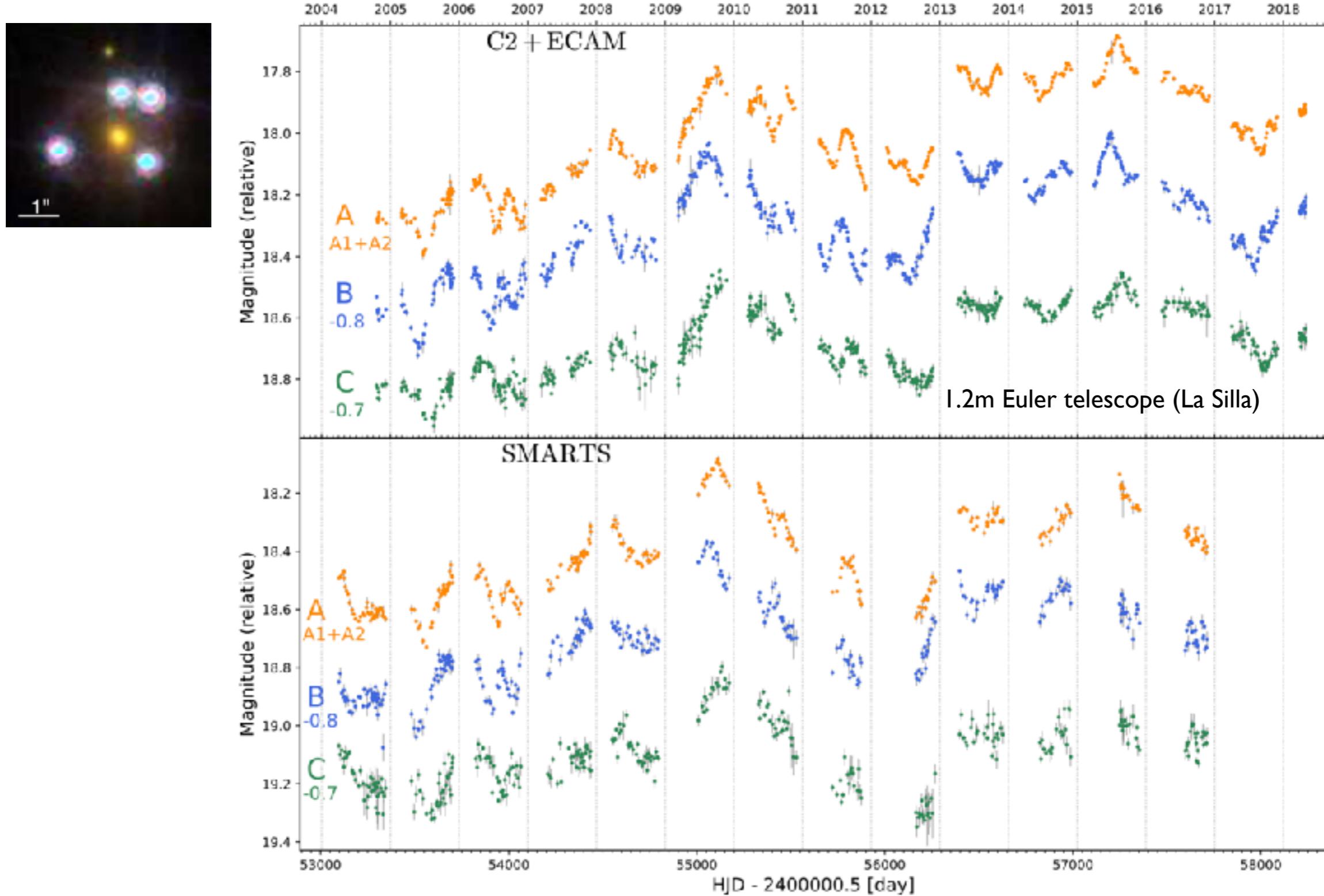
(f) PG 1115+080



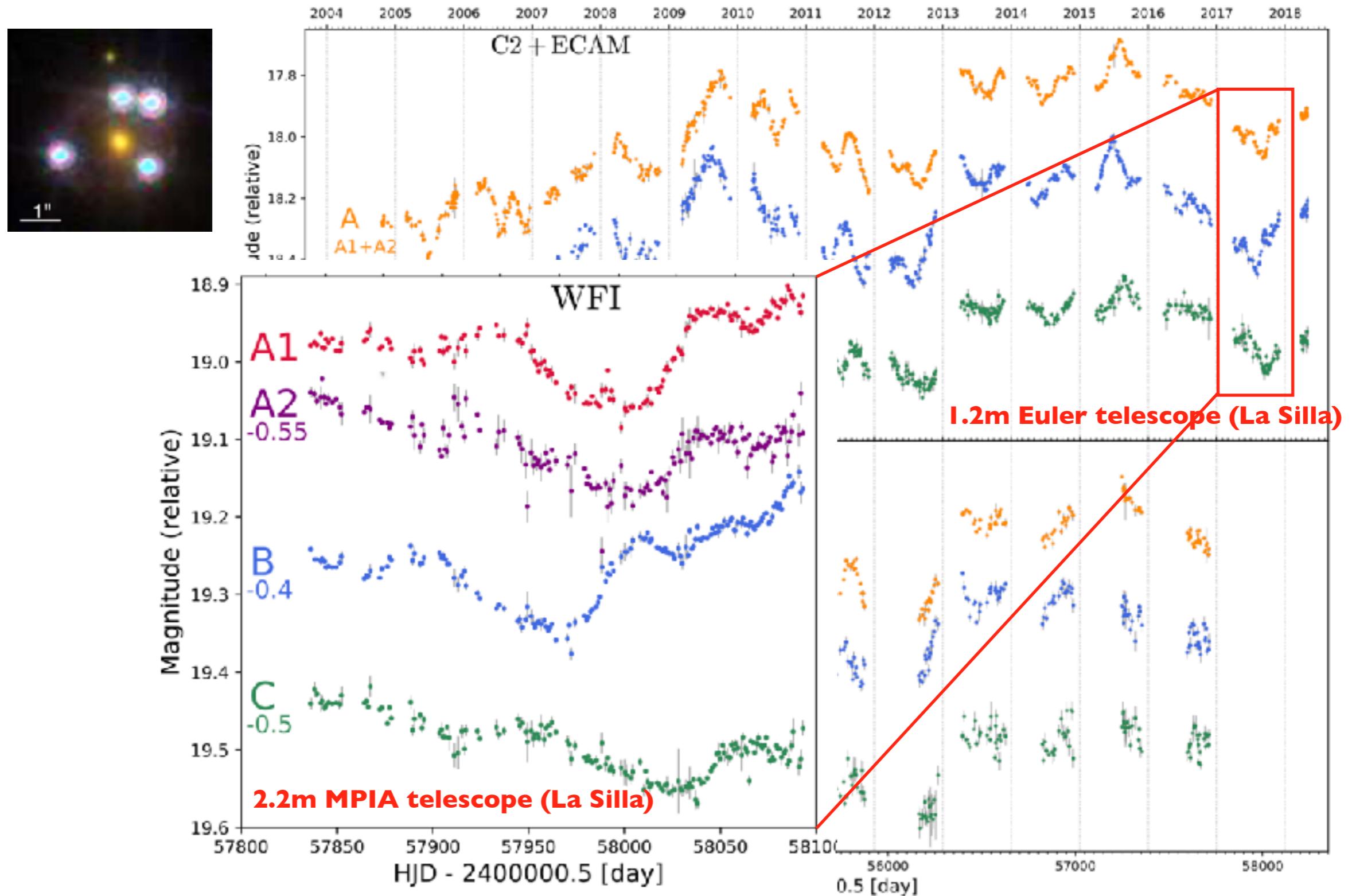
# *Ingredient I:* **Time Delay Measurements**

*Time-delay measurements are hard and require a dedicated team to monitor these lensed quasars with telescopes around the world for periods of a decade or more.*

# COSMOGRAIL: Light Curves of WFI2033-4723



# MPIA 2.2m Light Curves of WFI2033-4723

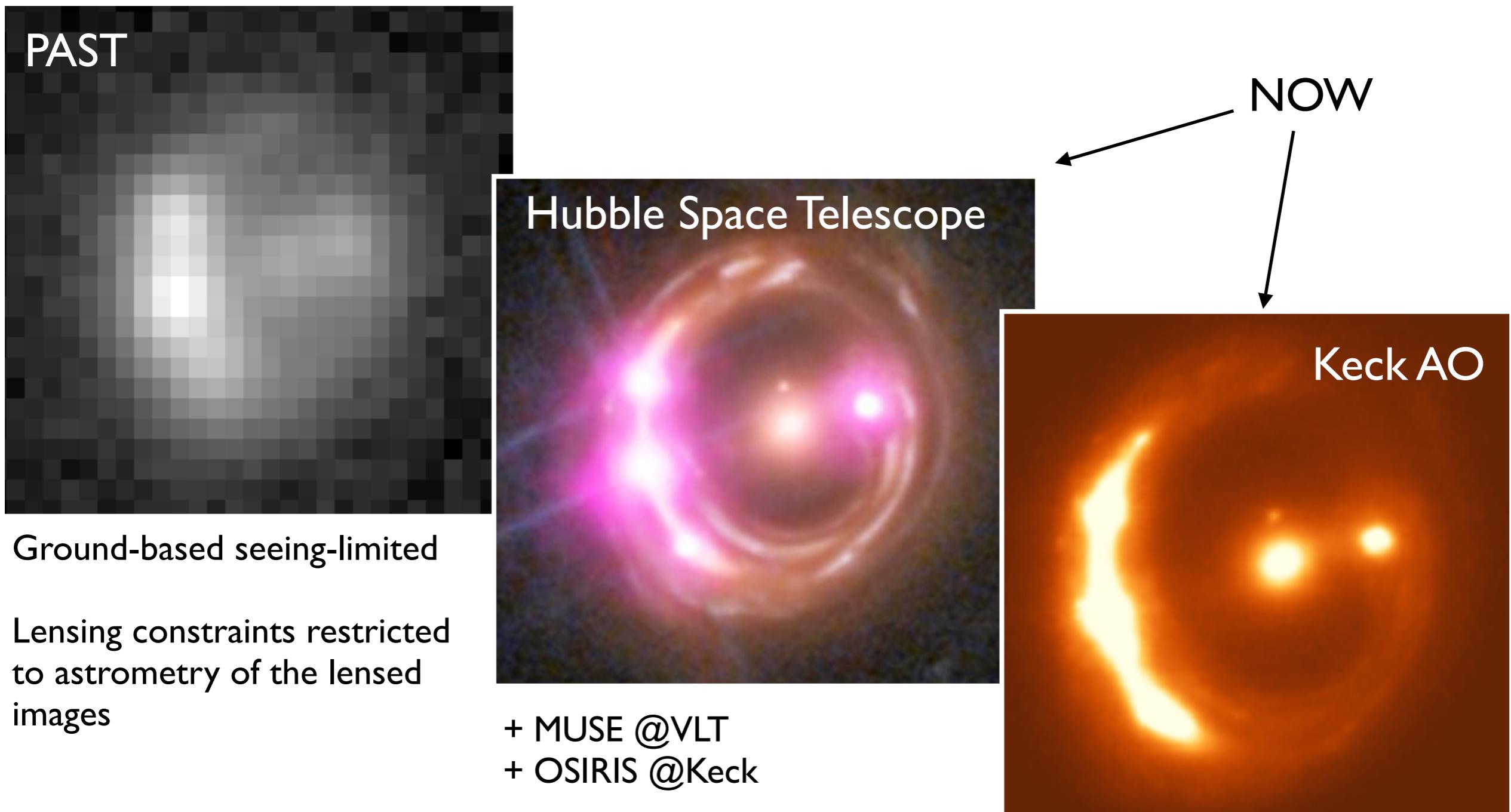




*Ingredients 2&3:*  
**Constraining the mass model of  
the lens and nearby companions**

*The mass model of the lens and nearby companions provide  
information on the deflection angles of the lens galaxy, the true  
source position and the gravitational time-delays.*

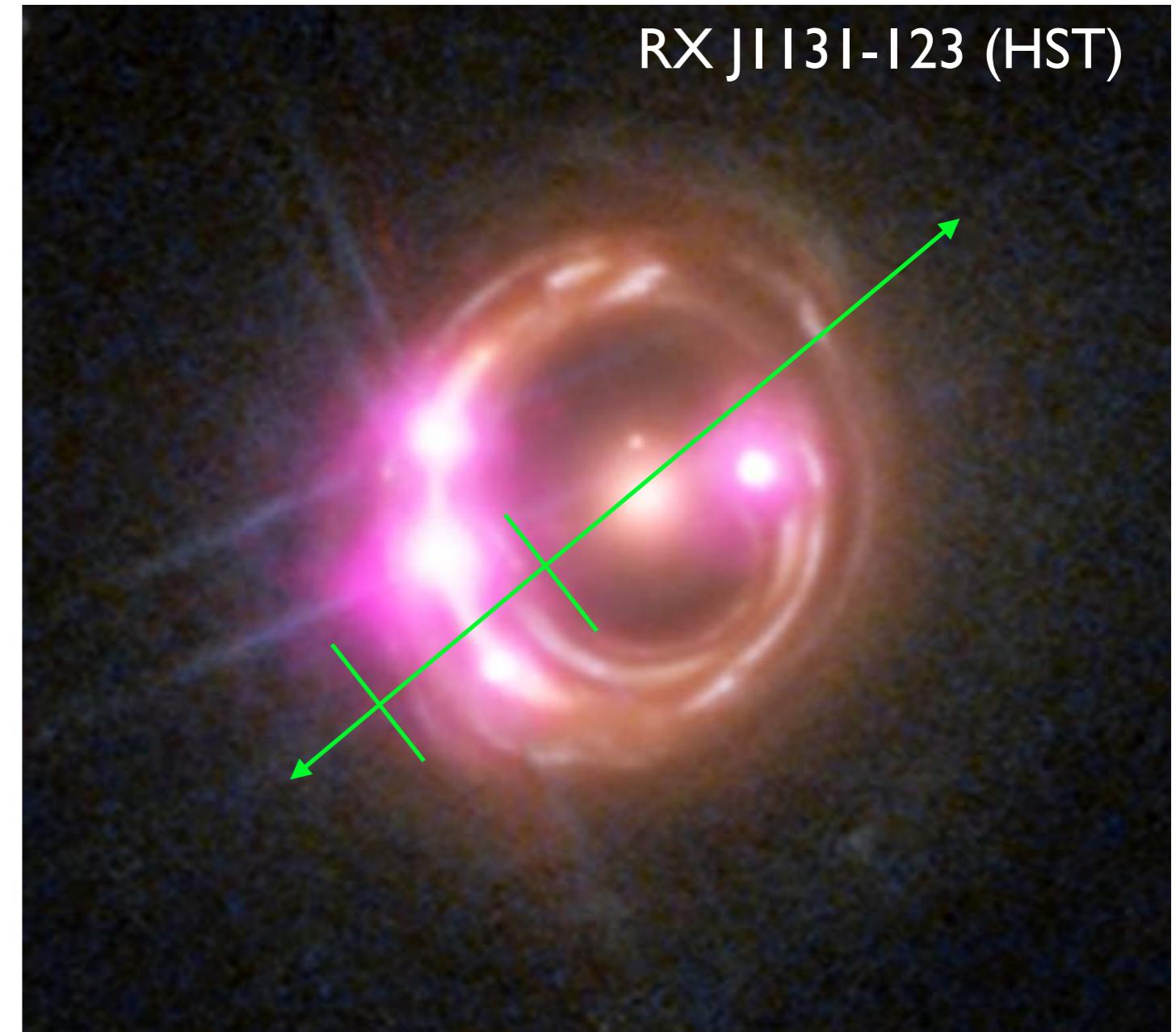
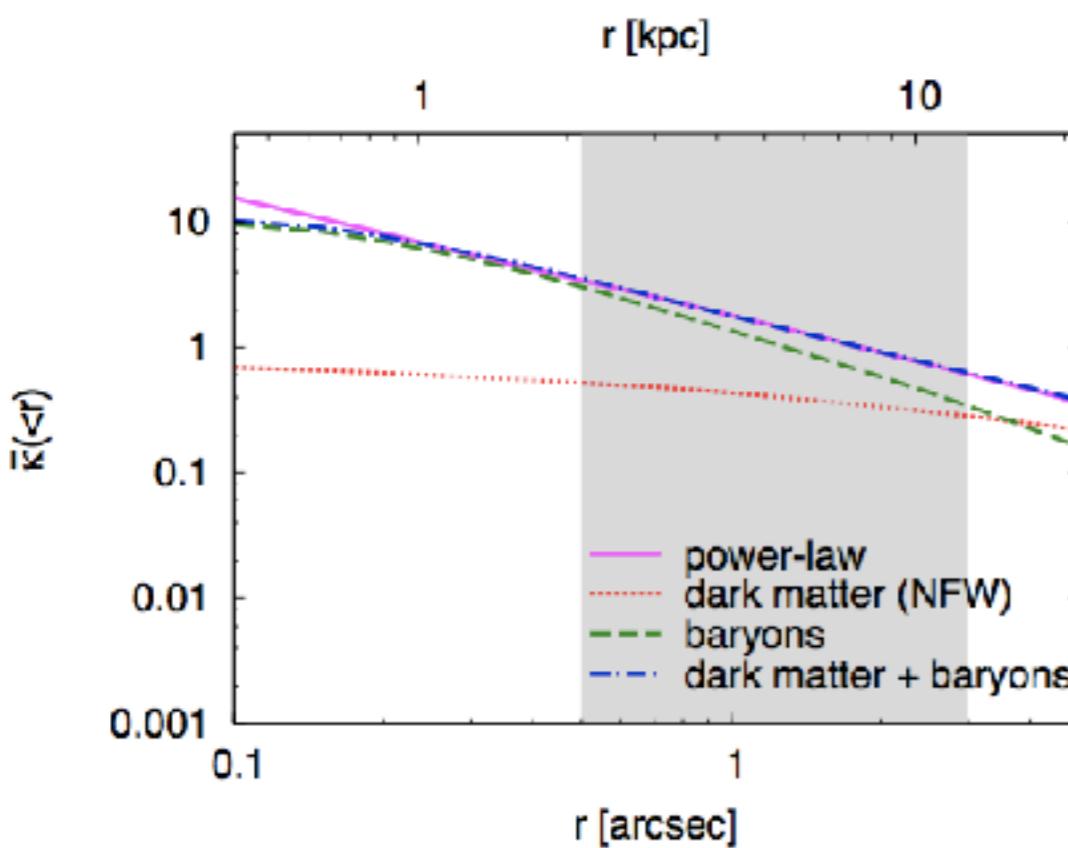
# Imaging Data for RX J1131-123



For detailed analyses of Keck AO imaging see  
Chen et al. (2019, arXiv1907.02533) — SHARP<sub>37</sub>

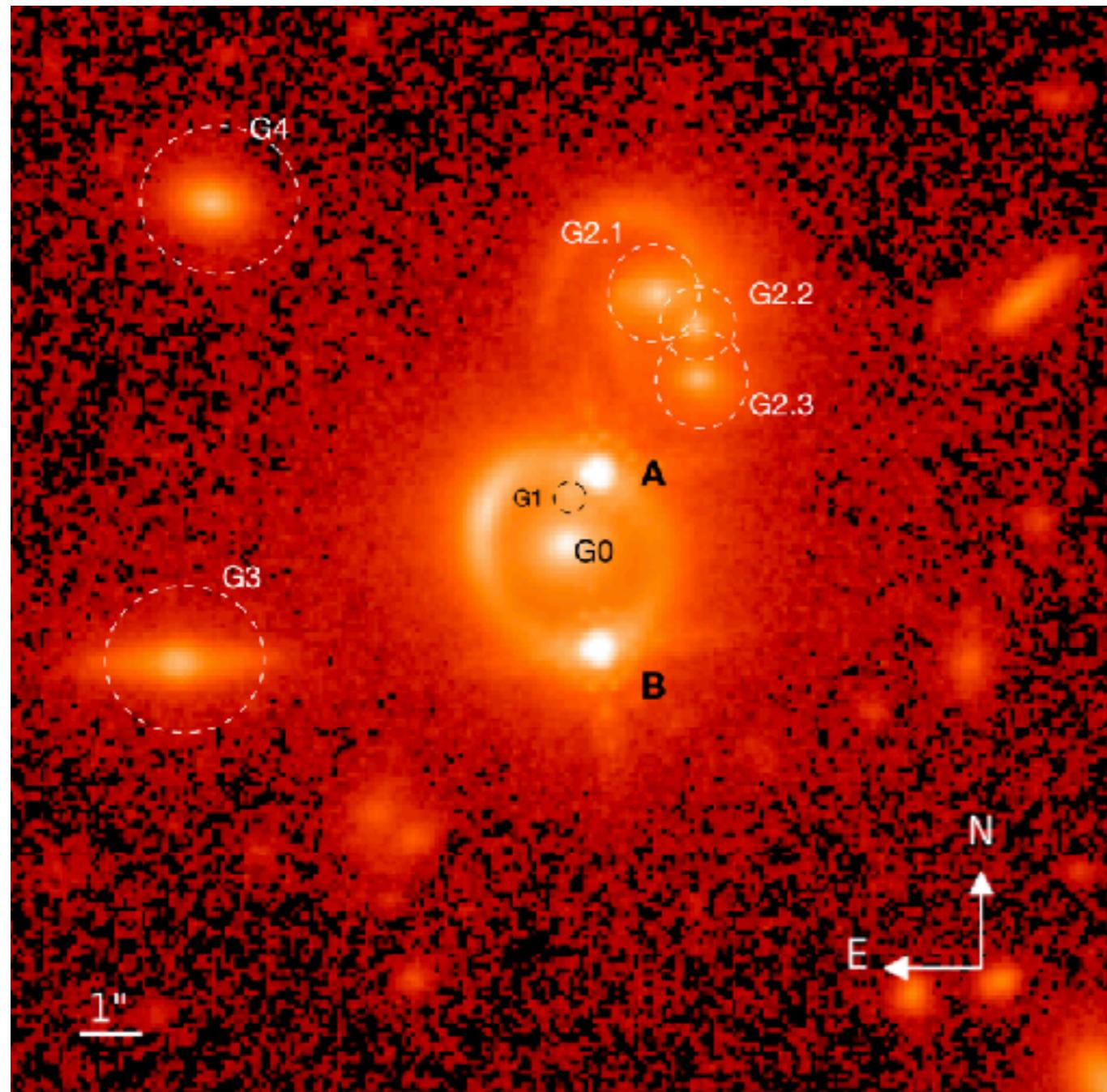
# Constraining models with extended lensed images

Stellar dynamics is included where available to further constrain the lens models



Lensing constraints come from all pixels covered by the Einstein ring formed by the quasar host.

# Constraining models with extended lensed images



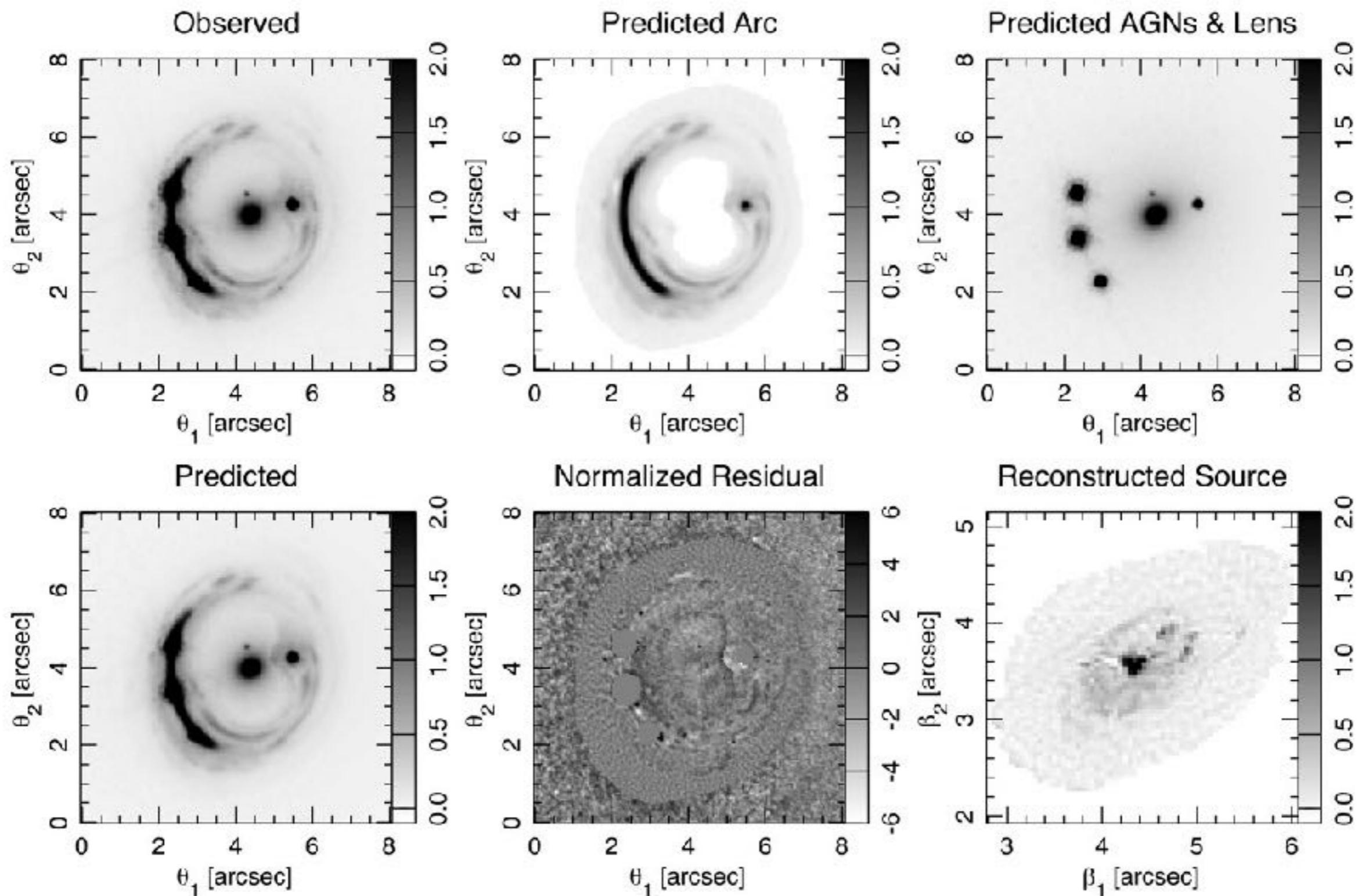
Birrer et al. 2019  
(Paper IX; MNRAS 484, 4726)  
Cosmology results for 4 lenses

## The local environment

The surrounding fields are often complex, since massive lens galaxies often reside in over-dense regions of the Universe.

These nearby galaxies are accounted for in the mass model (e.g. via multi-plane lens modelling).

# Constraining models with extended lensed images





# *Ingredient 4:* **Effects of line of sight**

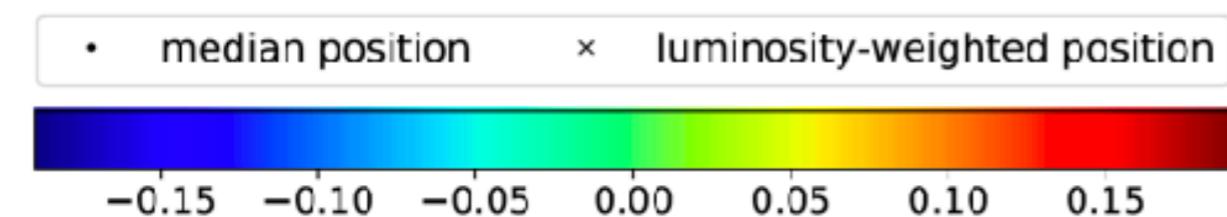
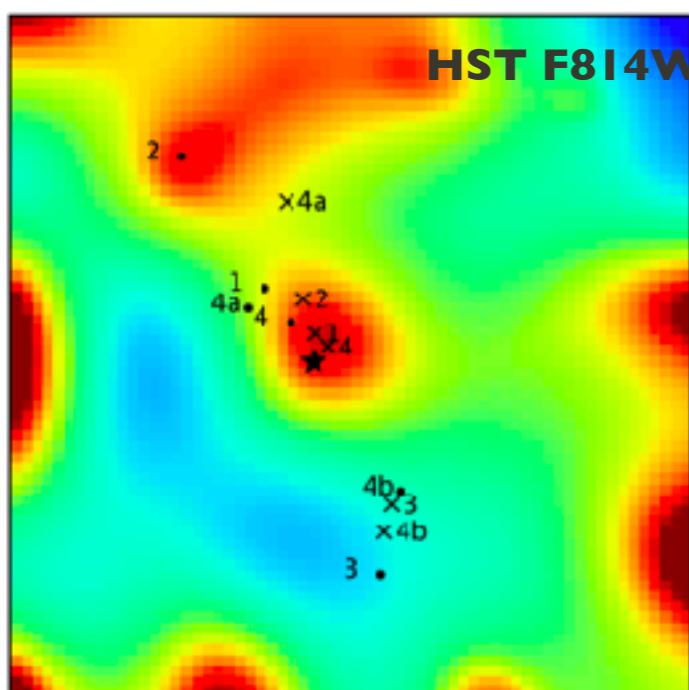
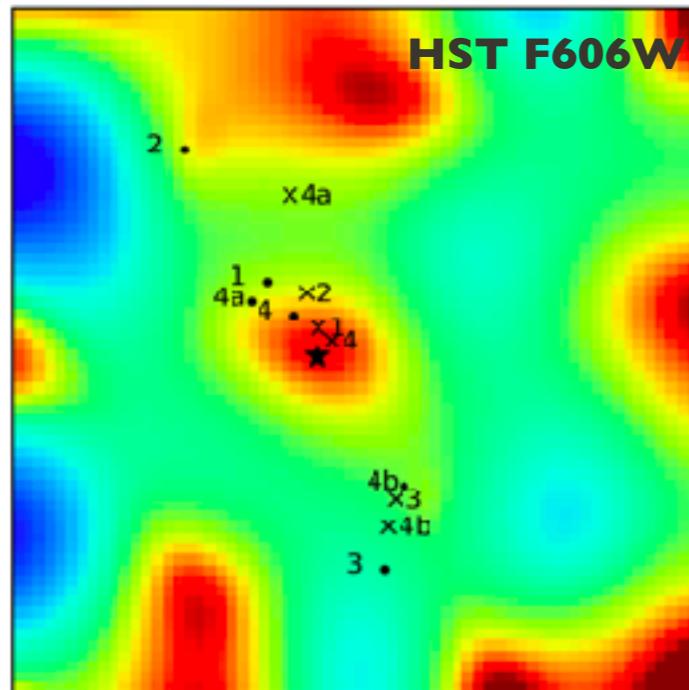
*A lens galaxy is only several 100 kpc in scale, but the distance to the lensed source can be  $\sim 1$  Gpc in scale. A lot of mass is seen along the line of sight which needs to be accounted for.*

# Mass-sheet Degeneracy (Line-of-Sight Contribution)

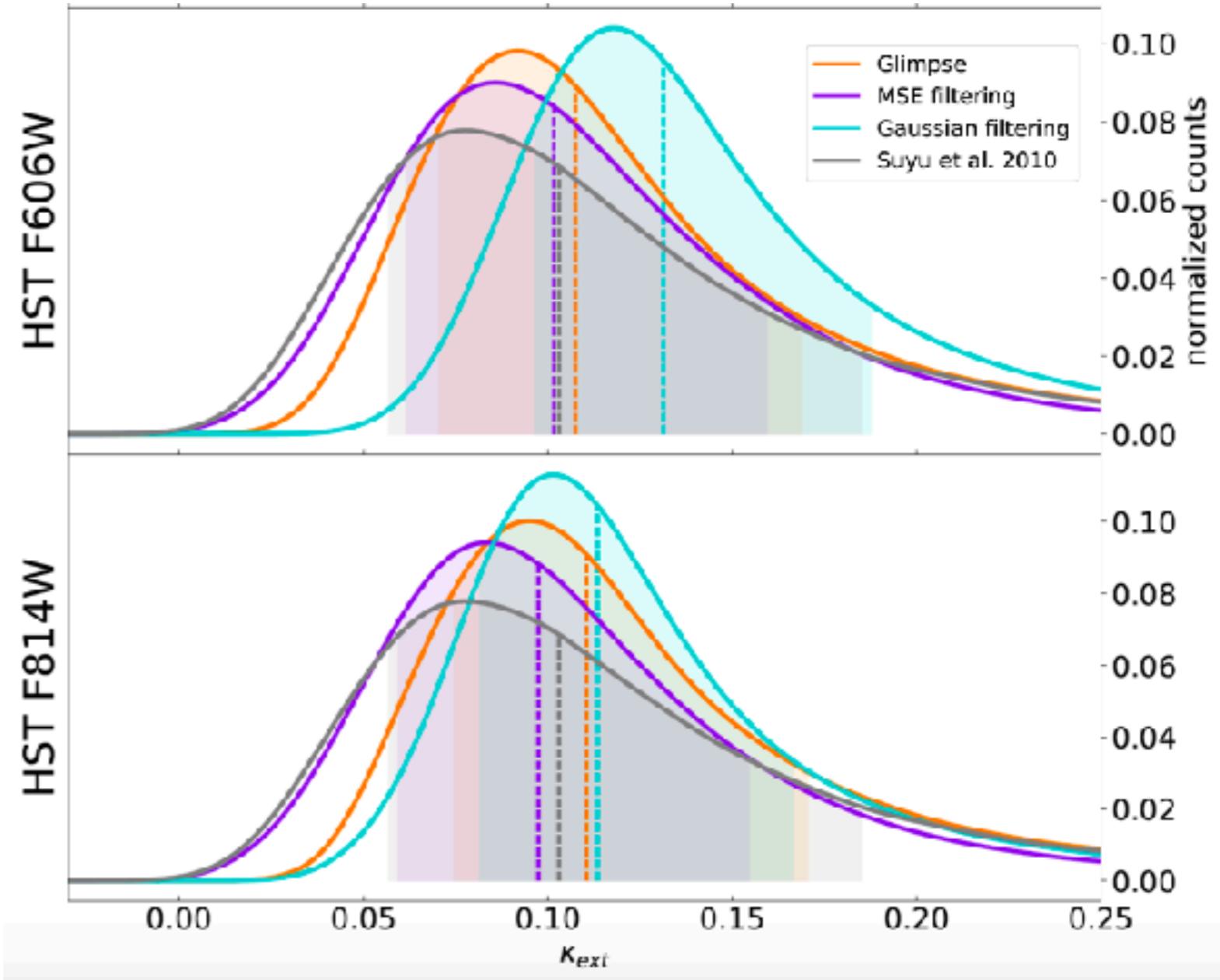
Mass along the line of sight brings extra mass inside the Einstein radius.  
This can be estimated in different ways:

- Using **galaxy counts** in lens fields and compare with the general field (Fassnacht et al. 2006, ApJ, 642, 30)
- Using weighted galaxy counts (Collett et al. 2013, MNRAS 432, 679; Greene et al. 2013, ApJ, 768, 39)
- Calibrating with **cosmological simulations** (e.g. Suyu et al. 2013, ApJ, 766, 70)
- Using **weak lensing maps** (e.g. Tikhonova et al. 2018, MNRAS, 477, 5657)

# LoS contribution: Galaxy Counts and Weak Lensing



Galaxy counts and weak lensing agree !  
Field of B1608: HST and Subaru imaging

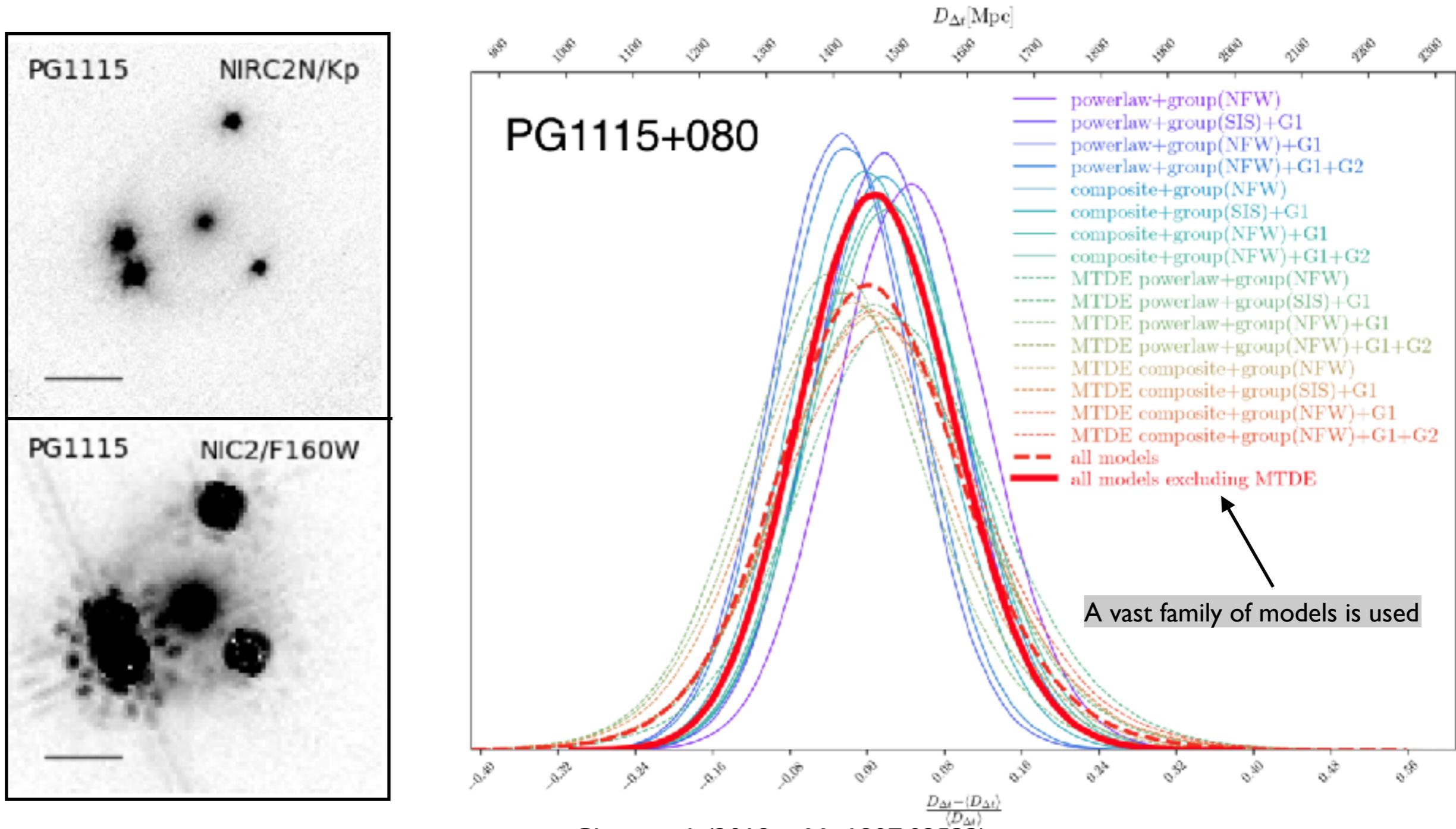


Tikhonova et al. 2019, submitted

# Blind Analysis!

## No-one knows $H_0$ until paper is finished

Results of the unblinding process are published as they are !



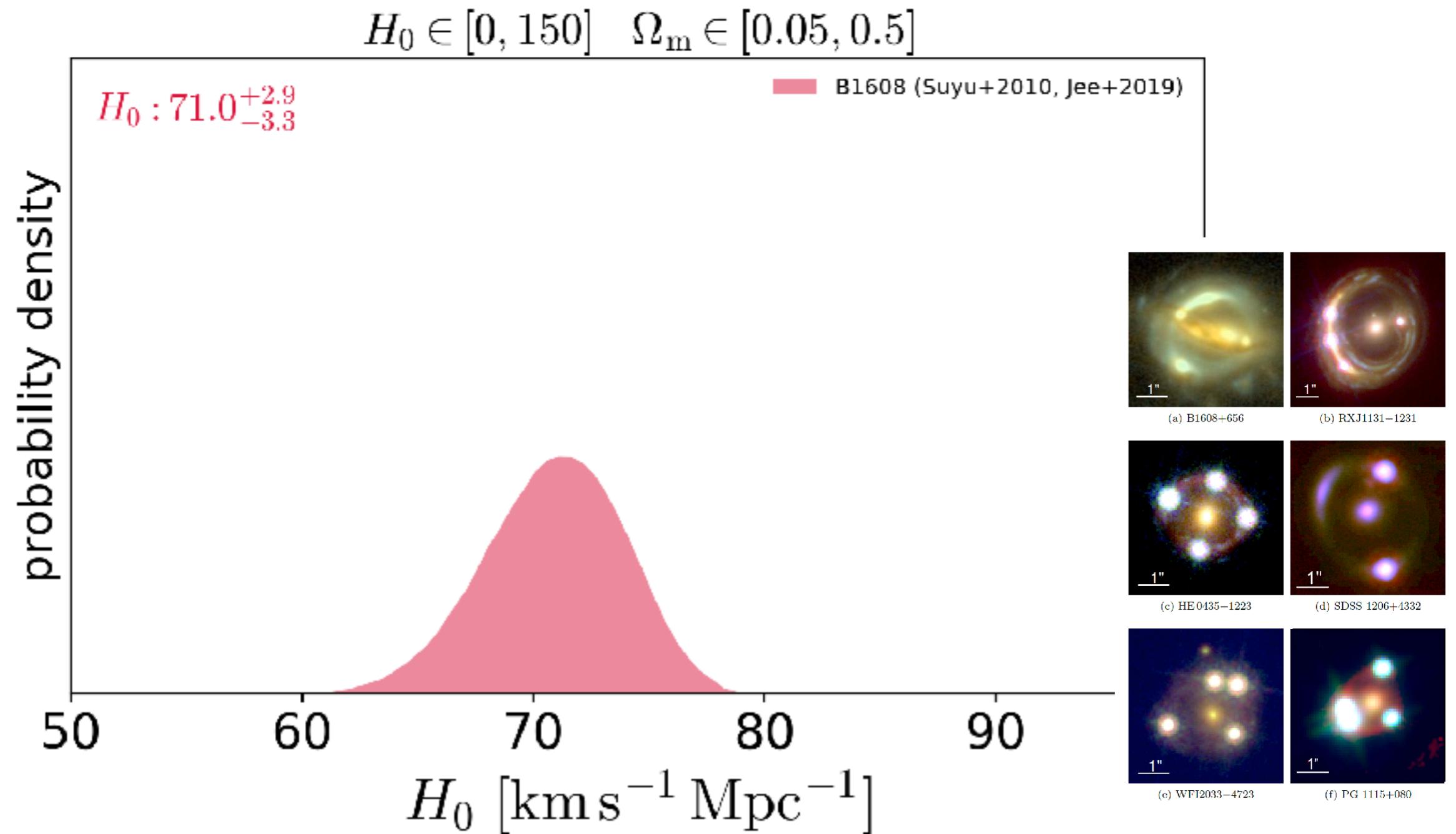


# $\Lambda$ CDM Cosmological Results

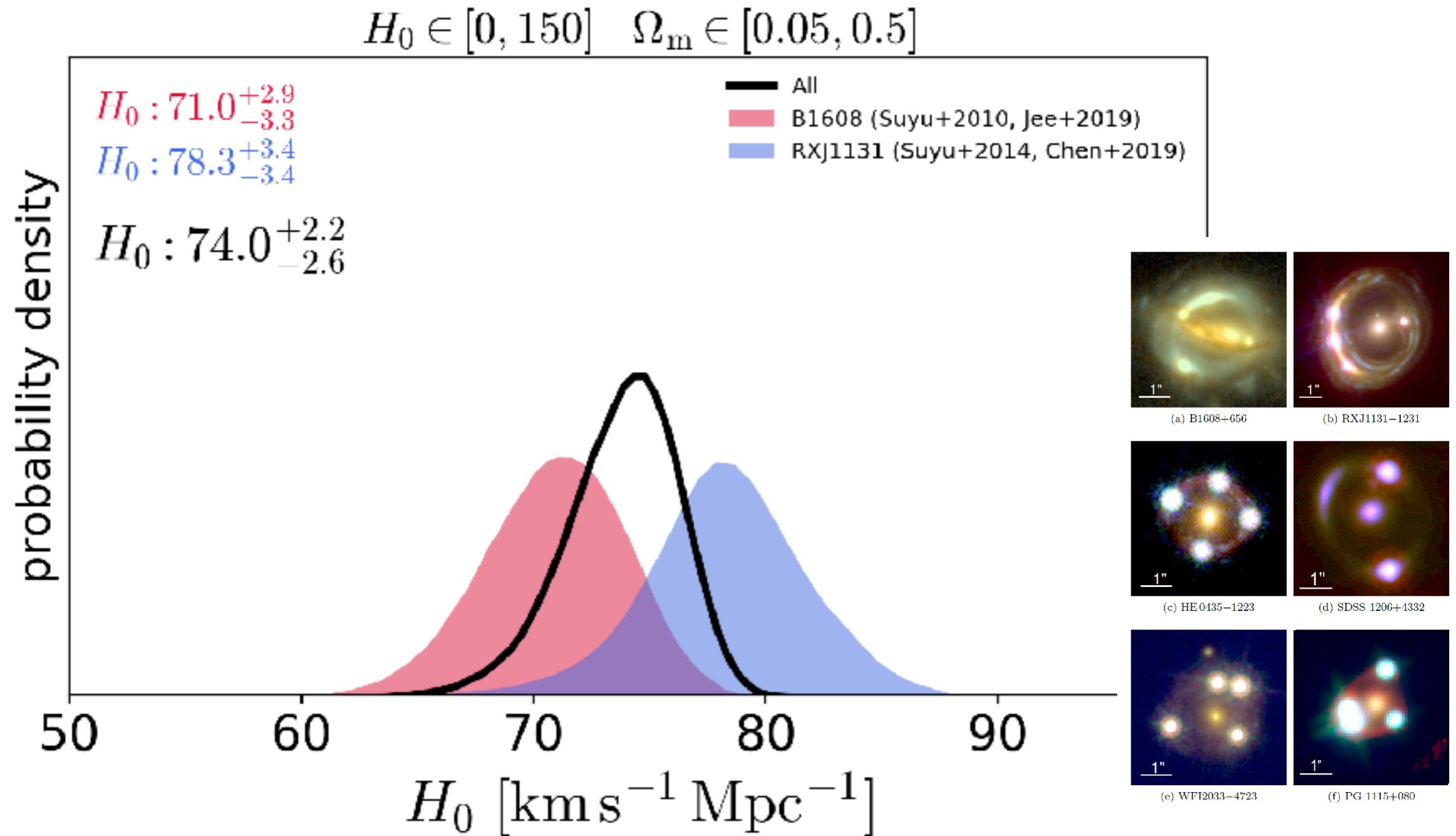
*With the time-delays and mass models in hand, we can now infer the value of  $H_0$  from the model for a given cosmography. For now we assume the standard flat- $\Lambda$ CDM cosmological model.*

*A full MCMC analysis is done over the full model parameters space.*

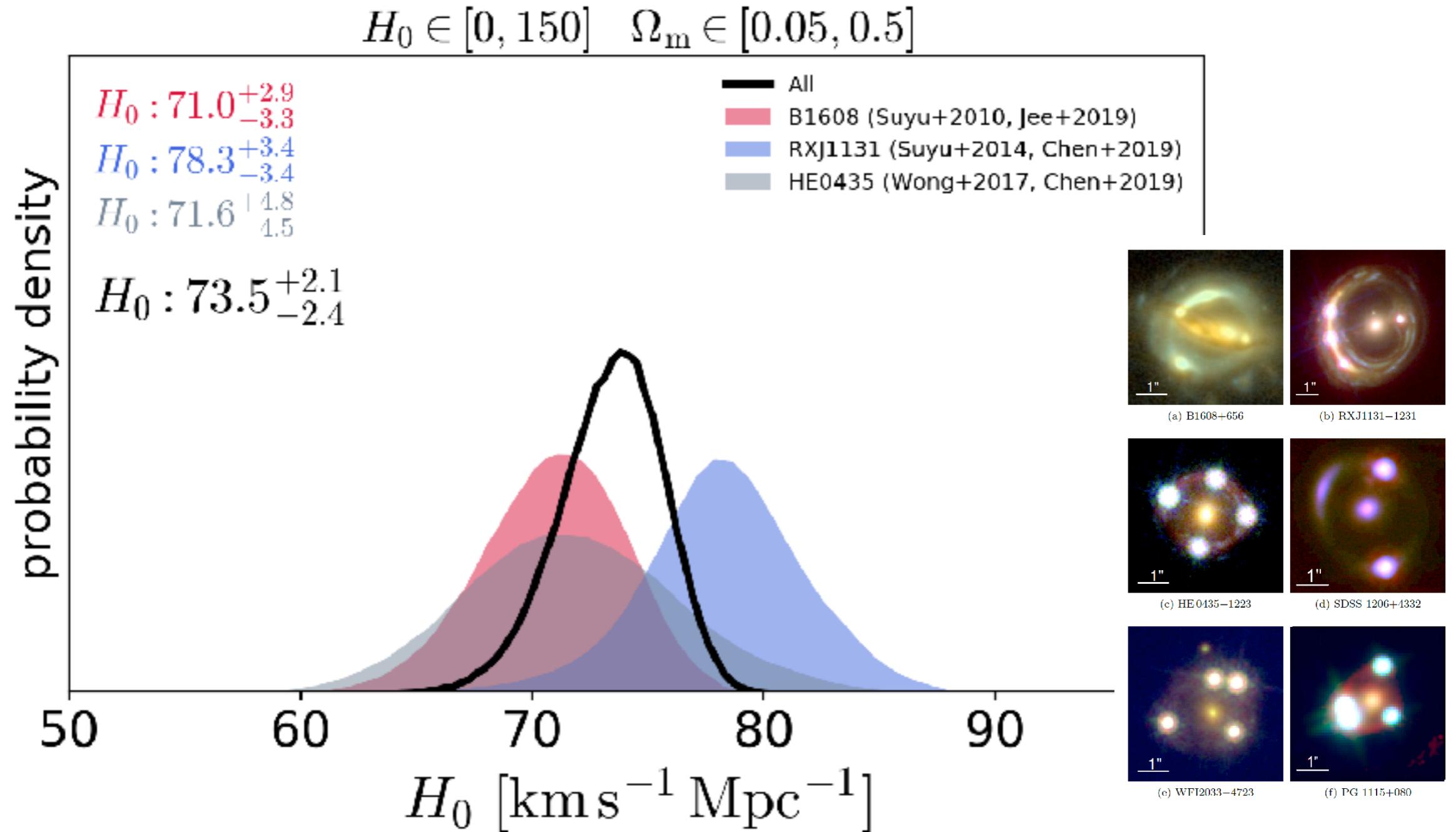
# Cosmology Results for I Lens in flat $\Lambda$ CMD



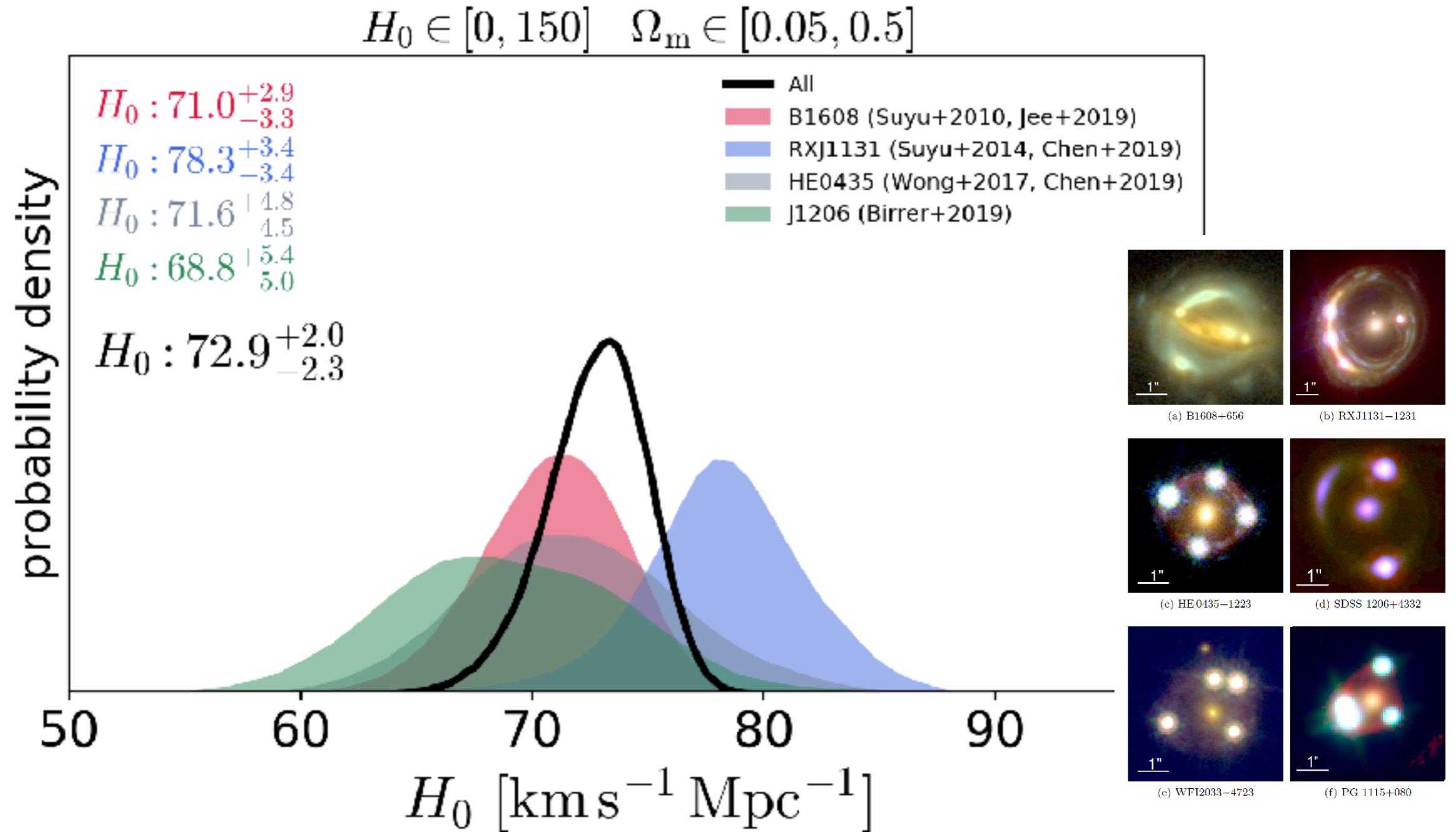
# Cosmology Results for 2 Lens in flat $\Lambda$ CMD



# Cosmology Results for 3 Lens in flat $\Lambda$ CMD

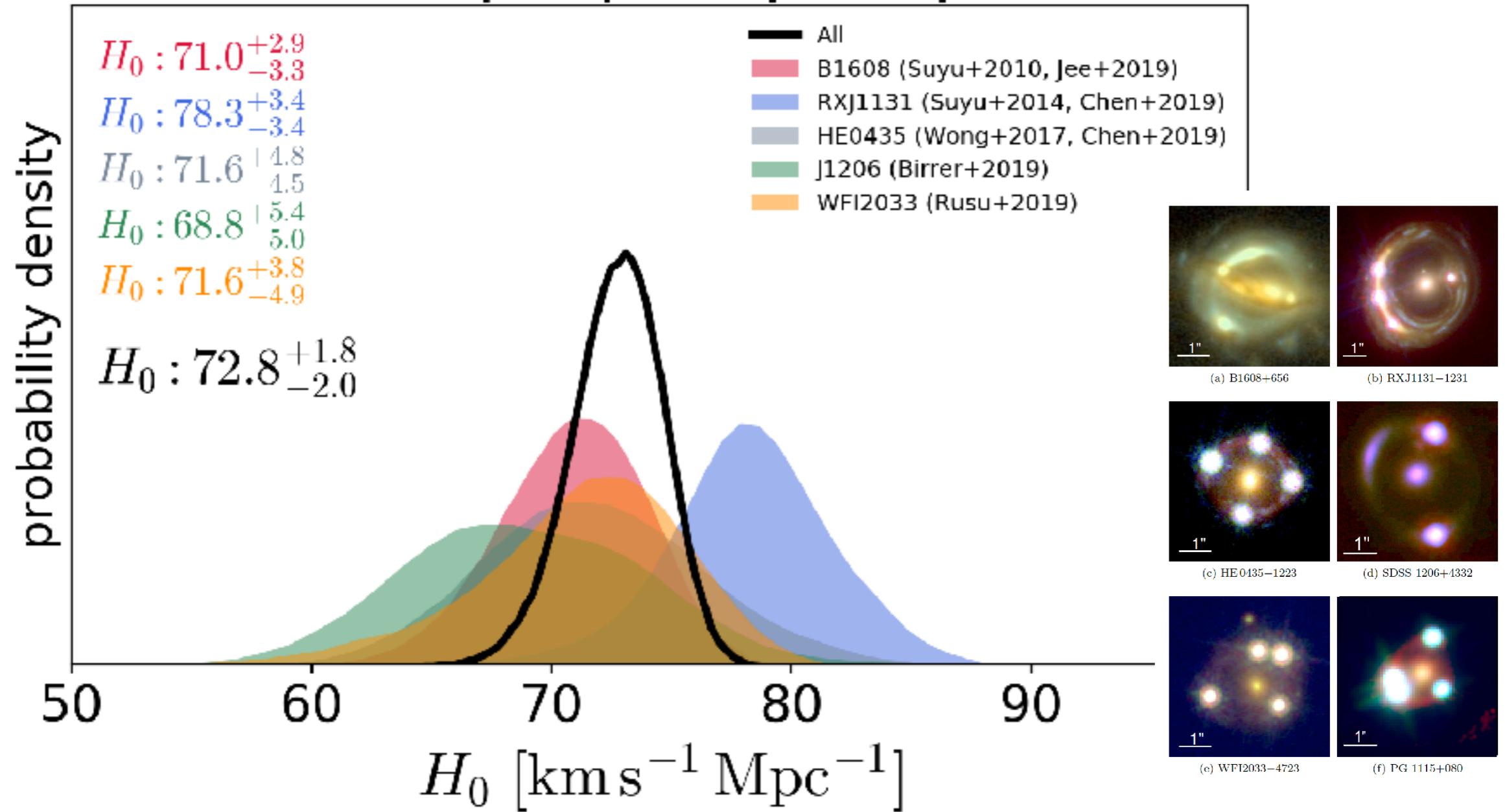


# Cosmology Results for 4 Lens in flat $\Lambda$ CMD



# Cosmology Results for 5 Lens in flat $\Lambda$ CMD

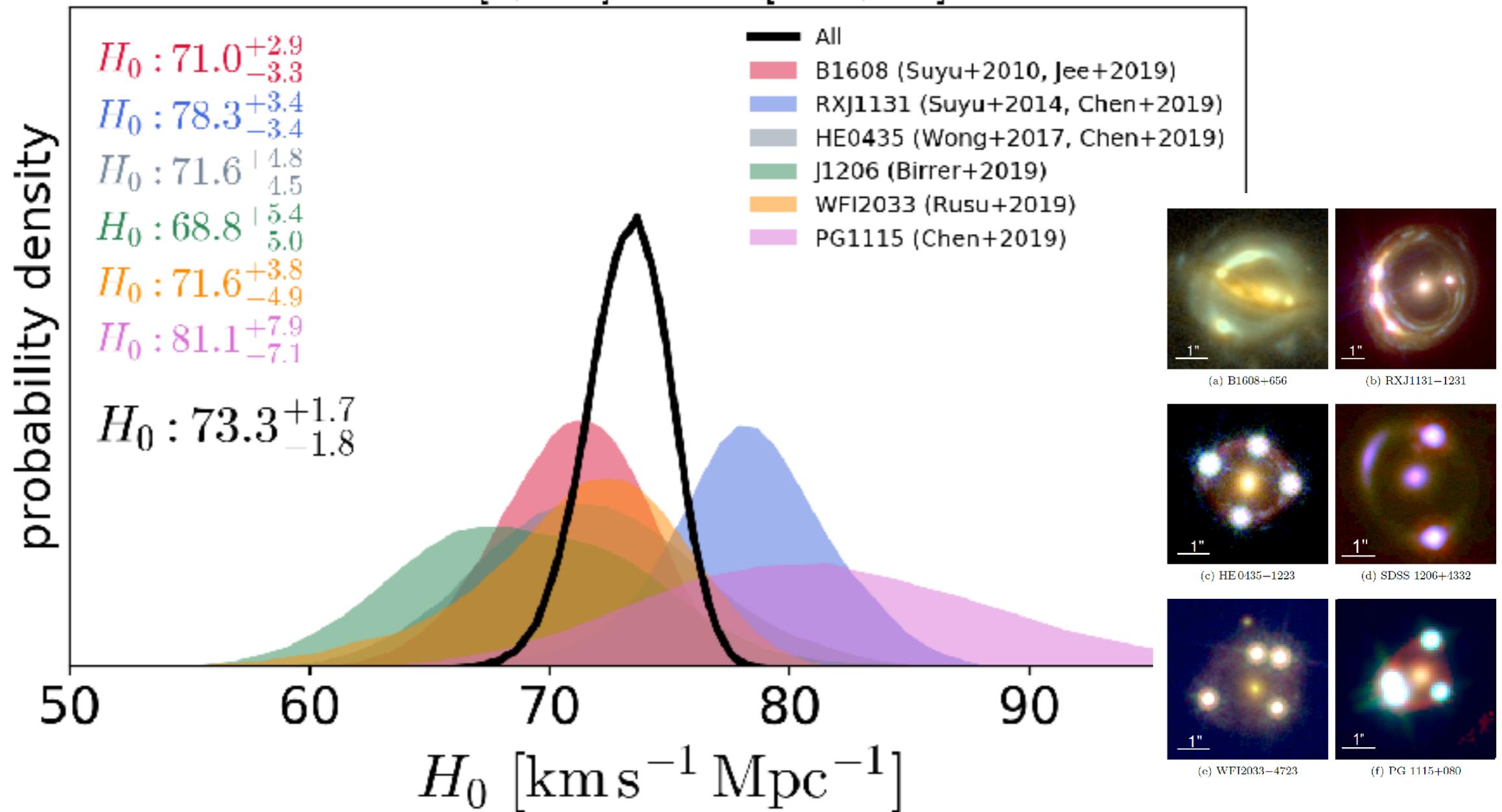
$$H_0 \in [0, 150] \quad \Omega_m \in [0.05, 0.5]$$



# Cosmology Results for 6 Lenses in flat $\Lambda$ CMD

All lenses (except first) was done blindly and values of  $H_0$  are consistent with each other given their error budgets

$$H_0 \in [0, 150] \quad \Omega_m \in [0.05, 0.5]$$



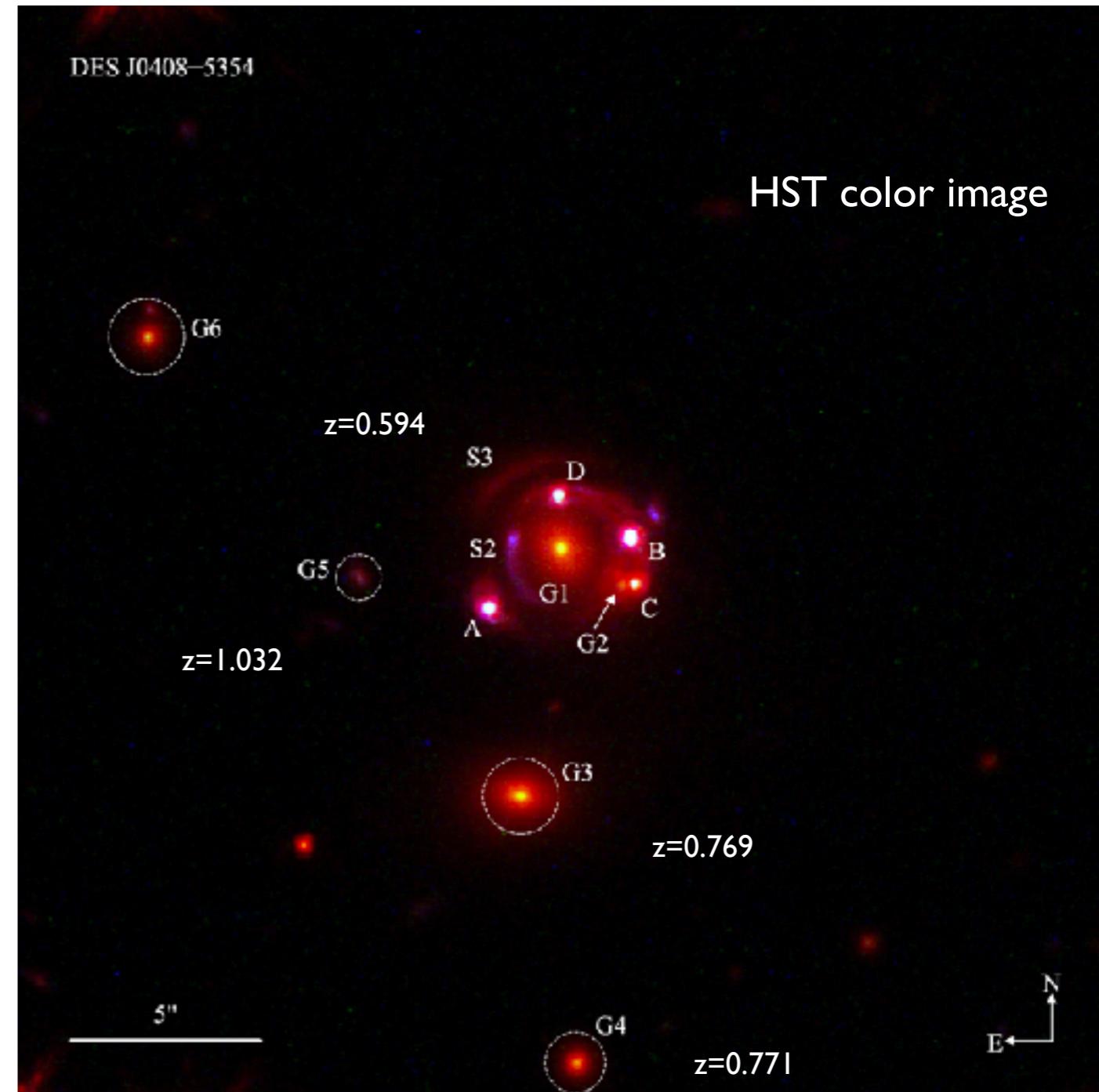
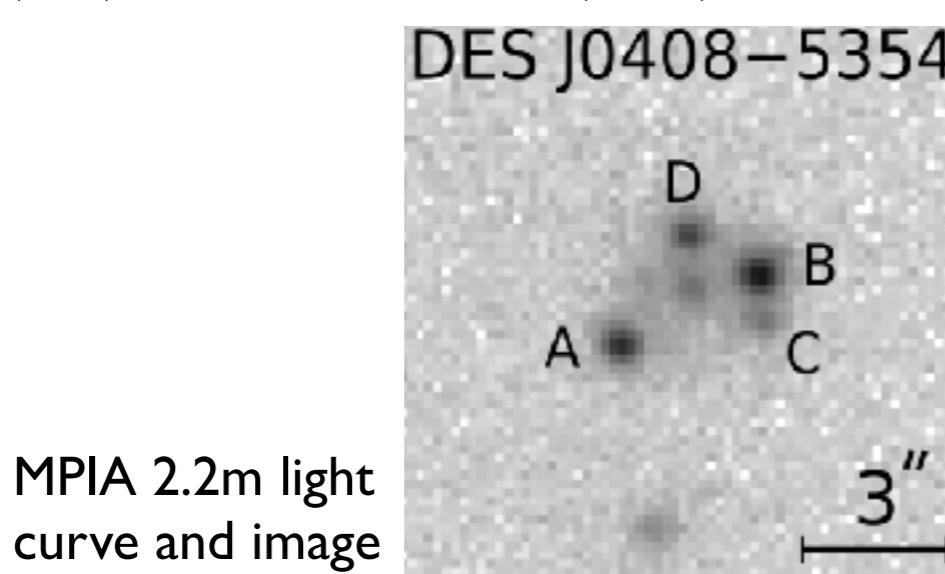
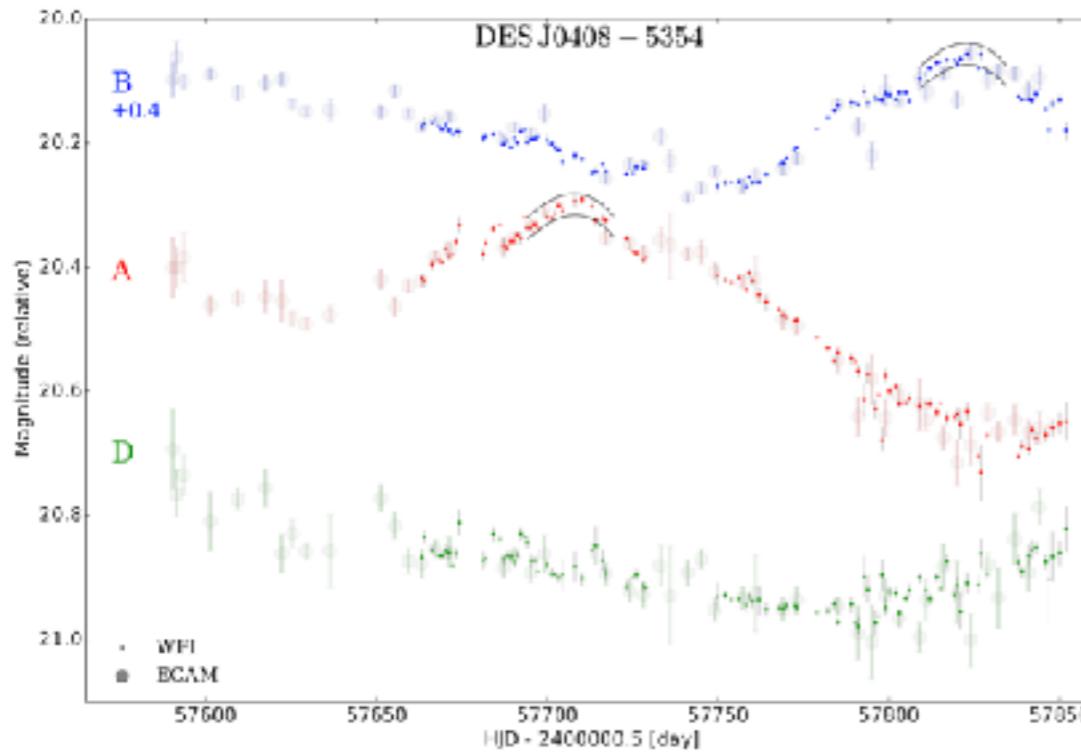


# Latest results: DES J0408-5354

*A seventh lens from the DES collaboration has recently  
been analysed. Fresh off the press...*

# DES J0408-5354: Data

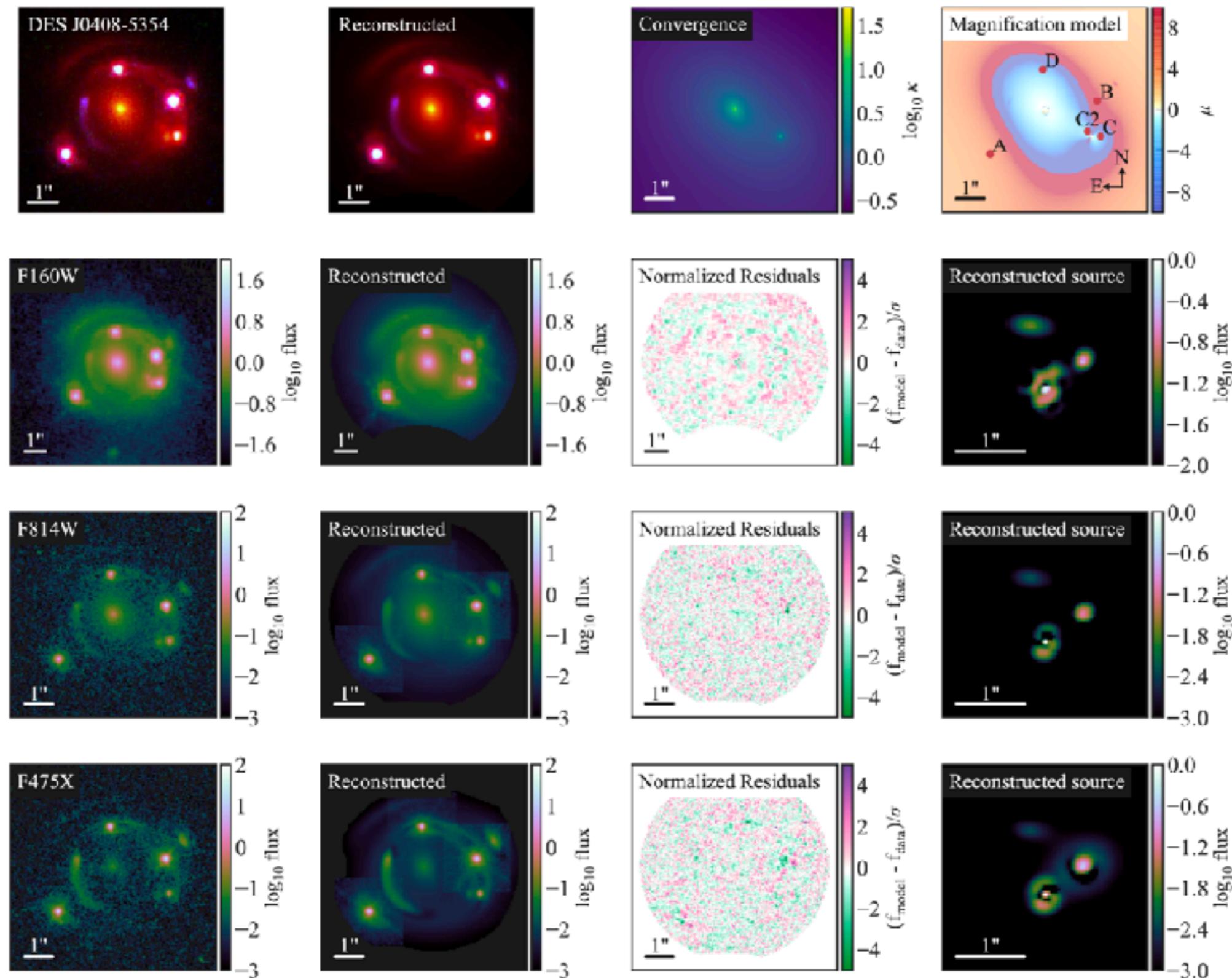
*Time delay measured after  $\sim 1$  year!*



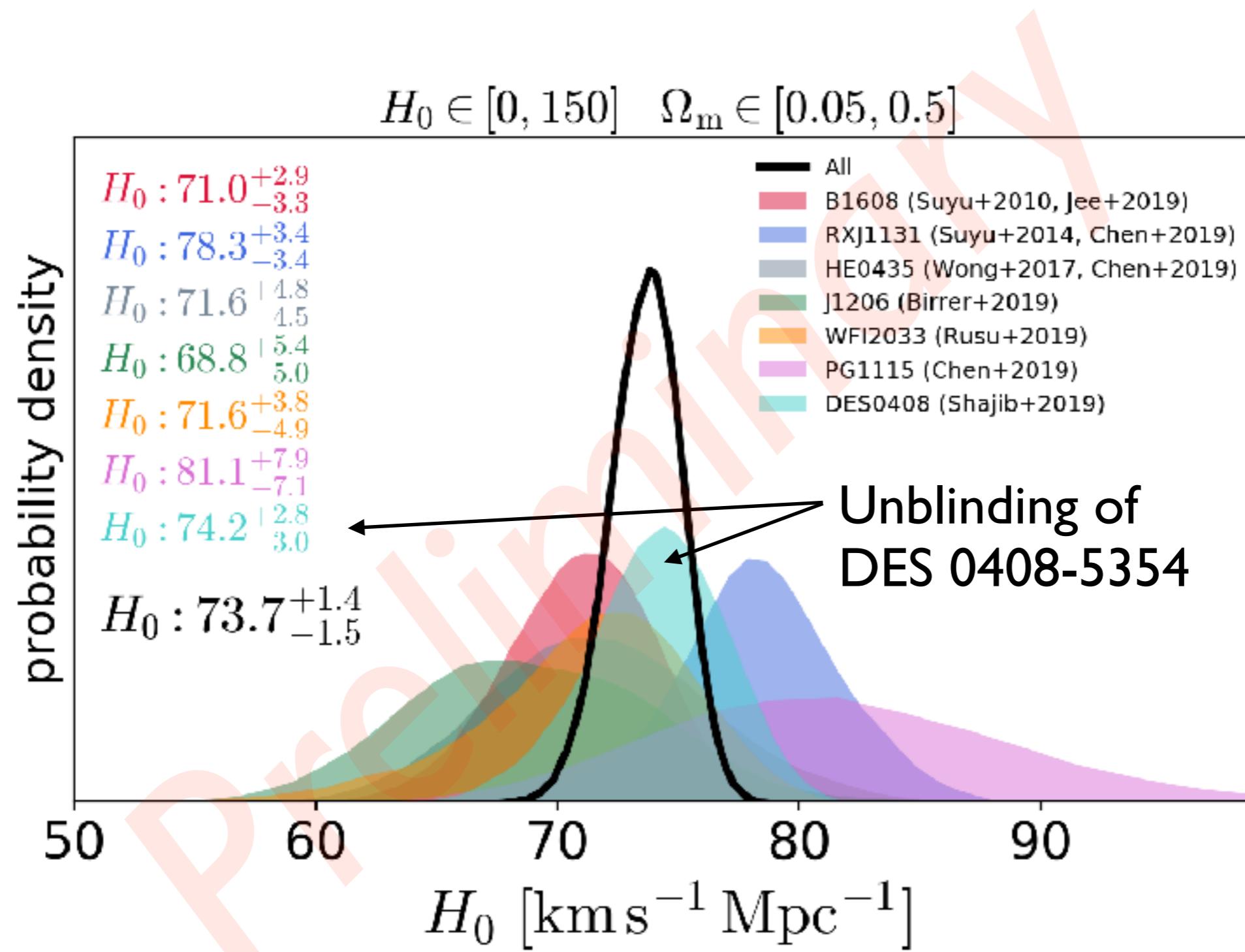
Time delays: Courbin et al. (2018)  
Discovery: Lin et al. 2017

Shajib et al. (2019 arXiv1910.06306)

# DES J0408-5354: Modelling with Lenstronomy



# New Cosmology Results for 7 Lenses in flat $\Lambda$ CMD



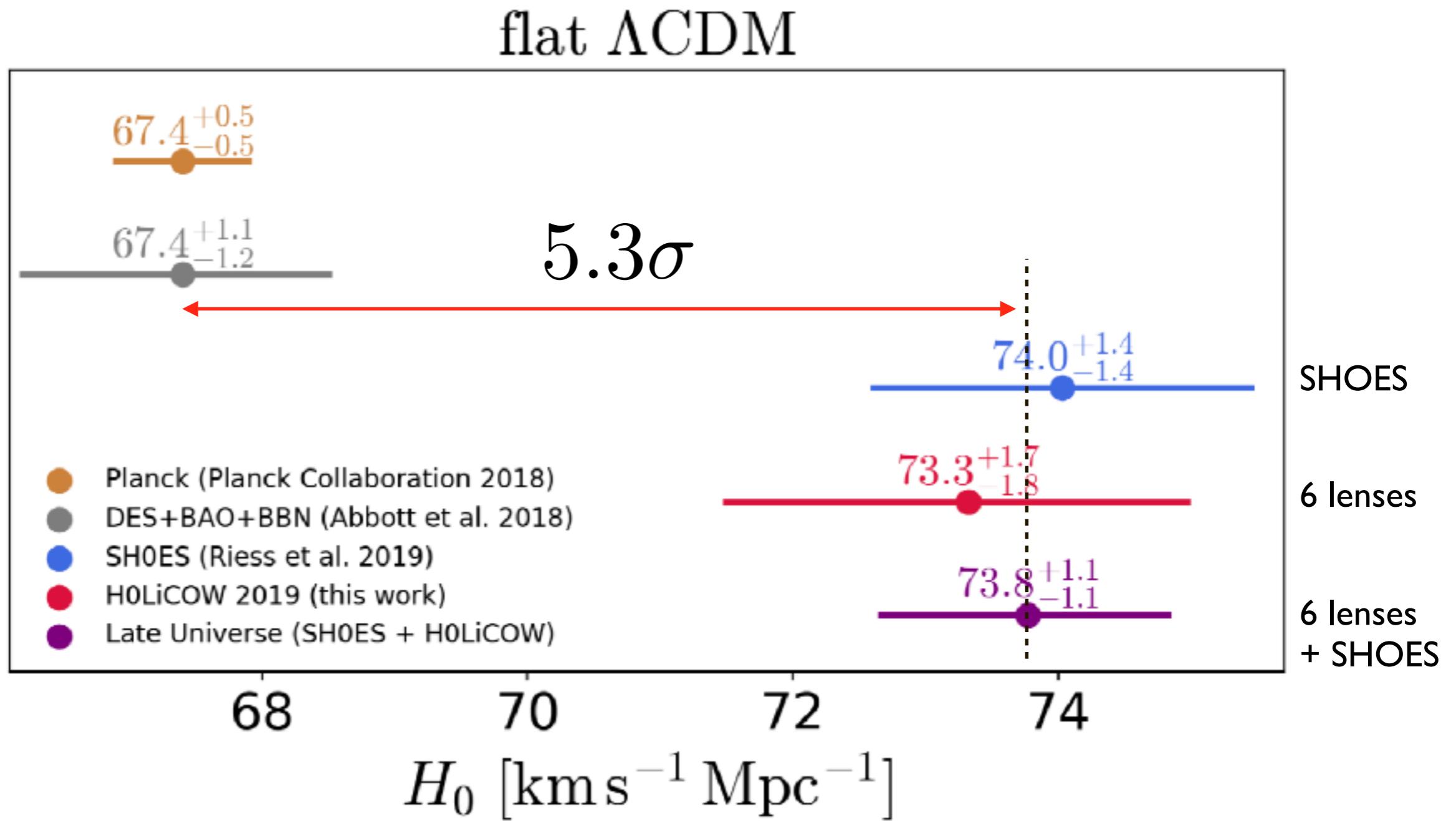
Results again consistent with the other lenses!



# More “tension”?

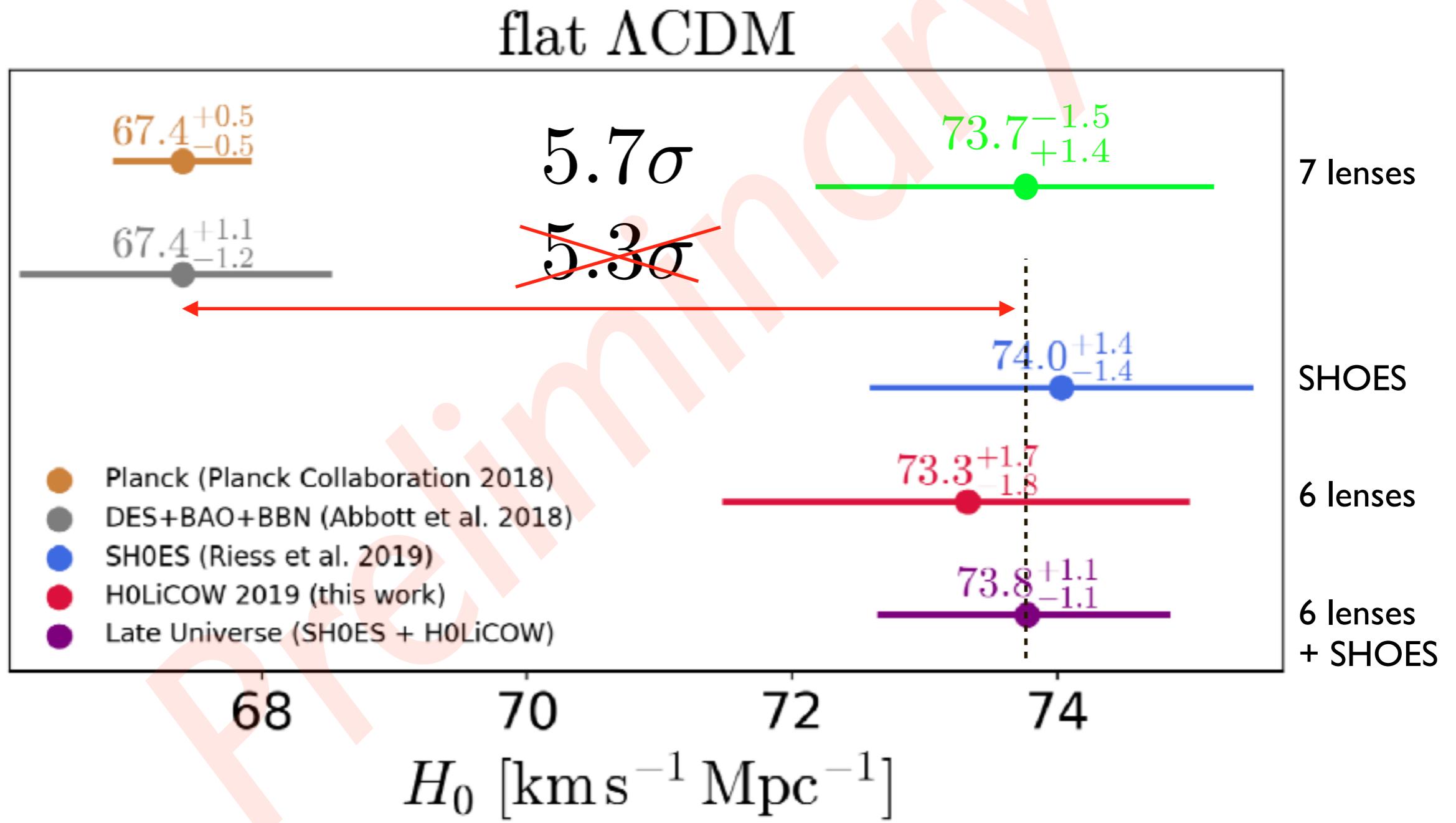
*Lensing is independent from distance-ladder methods, but still a late-Universe measurement. What does it tell us?*

# Tension between values of $H_0$ from lensing and those from early-universe measurements (CMB, BBN, BAO) remains!



H0LiCOW XIII milestone paper by Wong et al. (2019, arXiv1907.04869)

# Tension between values of $H_0$ from lensing and those from early-universe measurements (CMB, BBN, BAO)



Adding DES0408-5354 by Shajib et al. (2019, arXiv1910.06306)

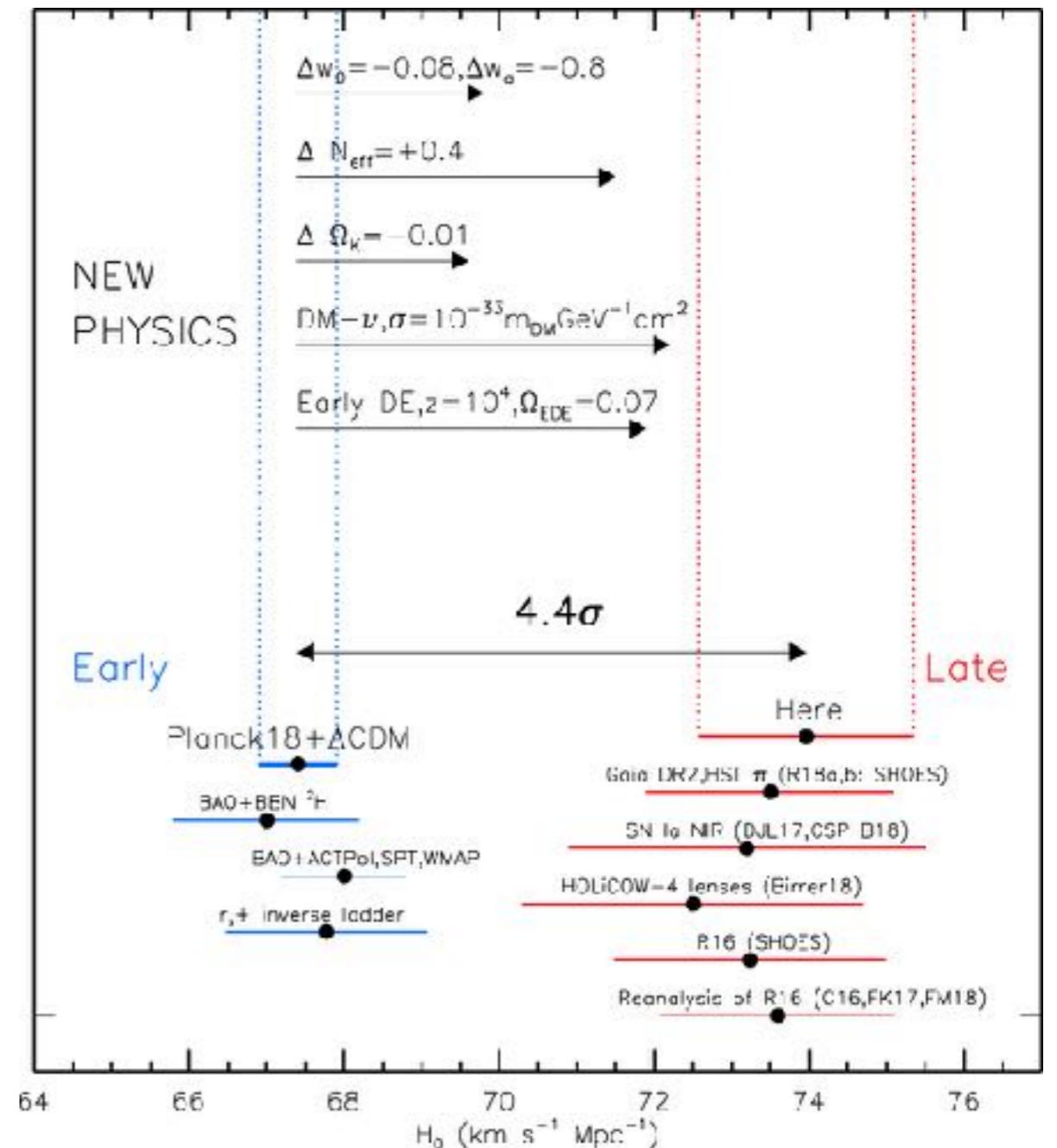


# Can this tension be alleviated?

*What modification to the standard cosmological model is needed?*

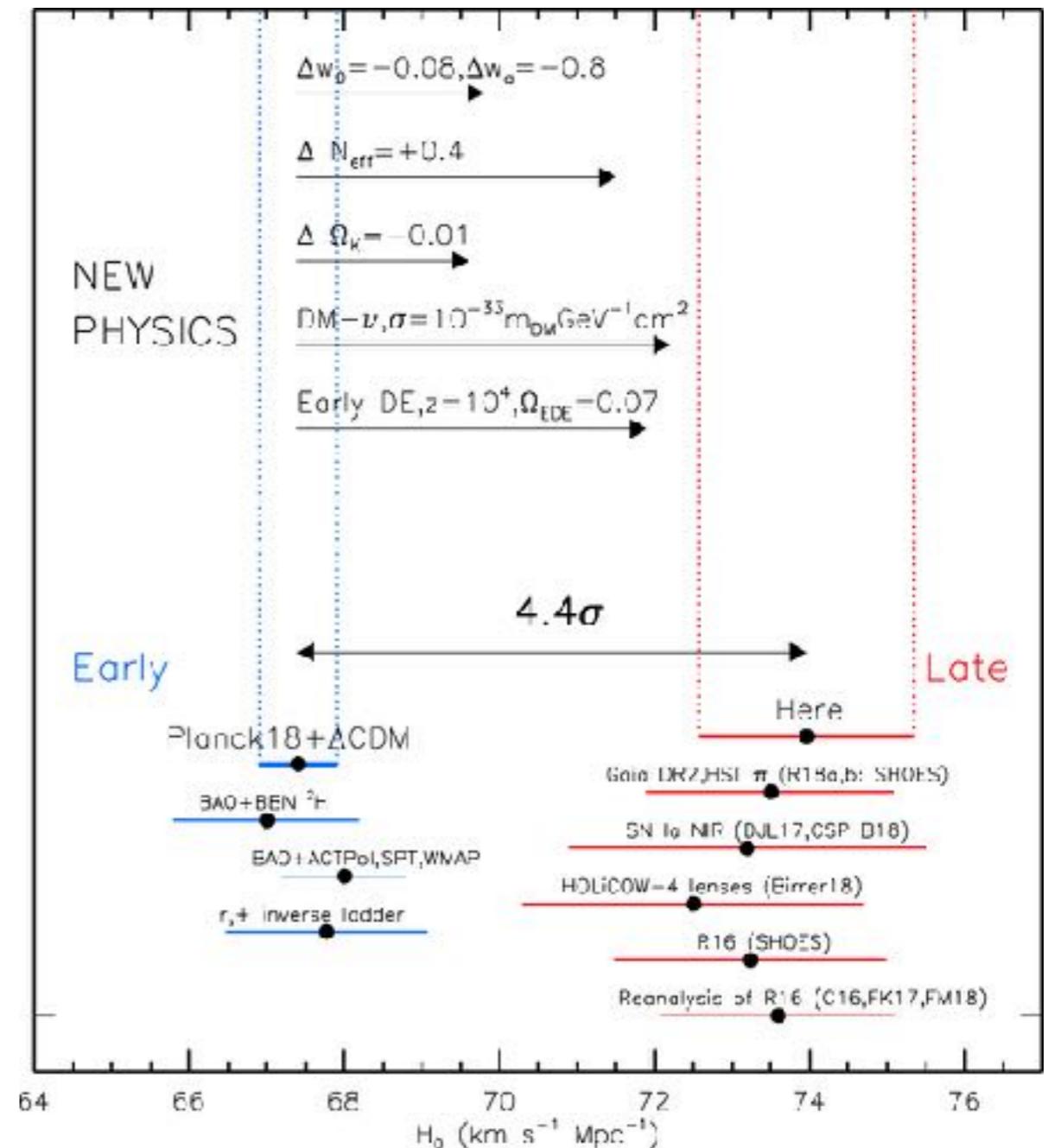
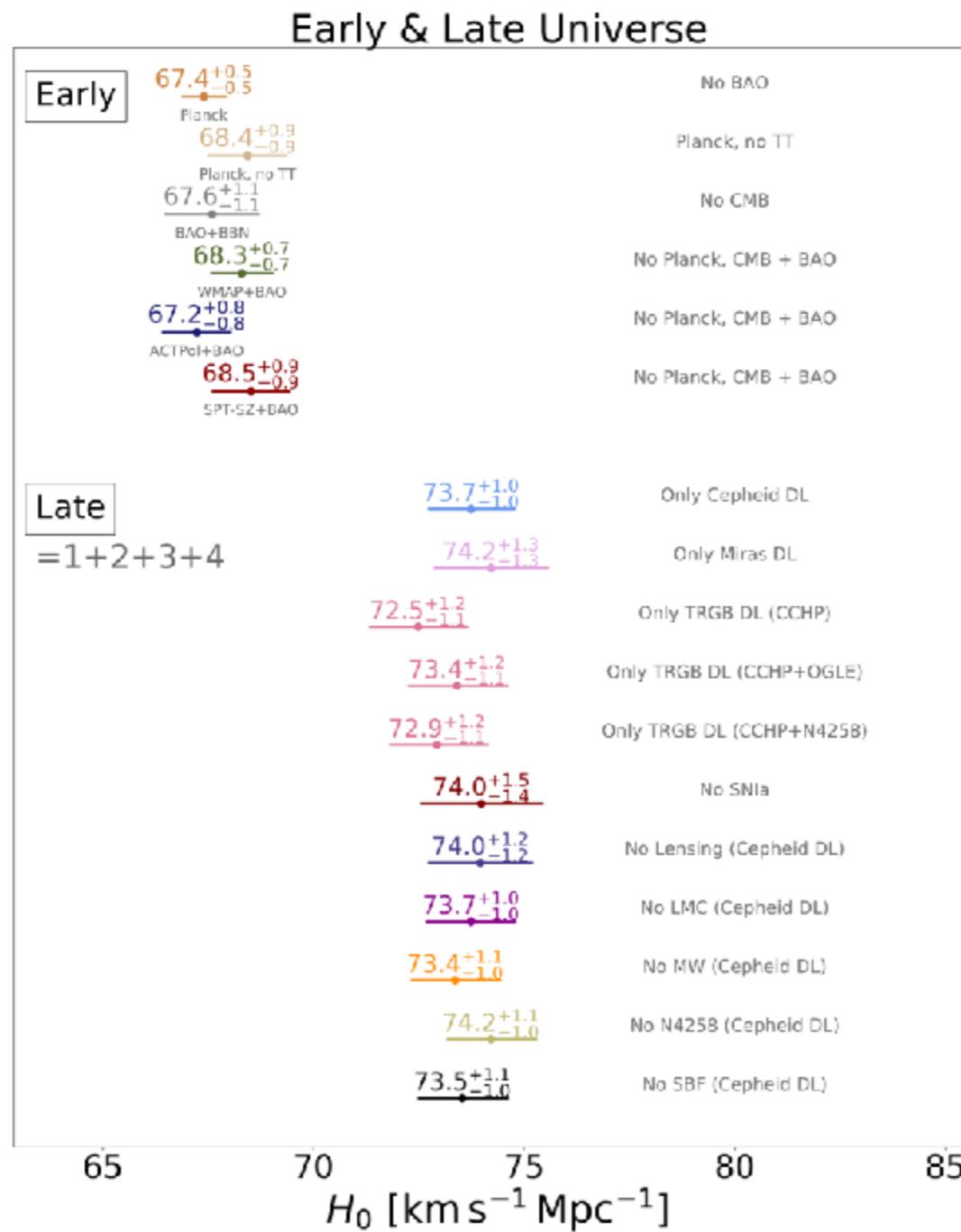
# Alleviating tension needs physics beyond $\Lambda$ CMD

For example: Dark energy with changing equation of state, more relativistic particle species, non-flat universe, DM interactions, early dark energy, ...



# Alleviating tension needs physics beyond $\Lambda$ CMD

For example: Dark energy with changing equation of state, more relativistic particle species, non-flat universe, DM interactions, early dark energy, ...



# Testing many models using our lenses

Model name	Description	Priors
Time-delay cosmography only		
U $\Lambda$ CDM	Flat $\Lambda$ CDM	$H_0$ uniform in [0, 150] km s $^{-1}$ Mpc $^{-1}$ $\Omega_m = 1 - \Omega_\Lambda$ $\Omega_m$ uniform in [0.05, 0.5]
Uo $\Lambda$ CDM	Open $\Lambda$ CDM	$H_0$ uniform in [0, 150] km s $^{-1}$ Mpc $^{-1}$ $\Omega_m$ uniform in [0.05, 5] $\Omega_k$ uniform in [-0.5, 0.5] $\Omega_\Lambda = 1 - \Omega_m - \Omega_k > 0$
U $w$ CDM	Flat $w$ CDM	$H_0$ uniform in [0,150] km s $^{-1}$ Mpc $^{-1}$ $\Omega_m$ uniform in [0.05, 5] $w$ uniform in [-2.5, 0.5] $\Omega_{DE} = 1 - \Omega_m$
U $w_0w_a$ CDM	Flat $w_0w_a$ CDM	$H_0$ uniform in [0,150] km s $^{-1}$ Mpc $^{-1}$ $\Omega_m$ uniform in [0.05, 5] $w_0$ uniform in [-2.5, 0.5] $w_a$ uniform in [-2, 2] $\Omega_{DE} = 1 - \Omega_m$
Time-delay cosmography combined with other probes		
$\Lambda$ CDM	Flat $\Lambda$ CDM	$\Omega_m = 1 - \Omega_\Lambda$ JLA/Pantheon for $\{H_0, \Omega_\Lambda\}$
o $\Lambda$ CDM	Open $\Lambda$ CDM	$\Omega_m = 1 - \Omega_\Lambda - \Omega_k > 0$ Planck (Section 5.3.1) or JLA/Pantheon (Section 5.4) for $\{H_0, \Omega_\Lambda, \Omega_m\}$

$Uw_0w_a\text{CDM}$ Flat  $w_0w_a\text{CDM}$  $H_0$  uniform in  $[0, 150]$  km s $^{-1}$  Mpc $^{-1}$  $\Omega_m$  uniform in  $[0.05, 5]$  $w_0$  uniform in  $[-2.5, 0.5]$  $w_a$  uniform in  $[-2, 2]$ 

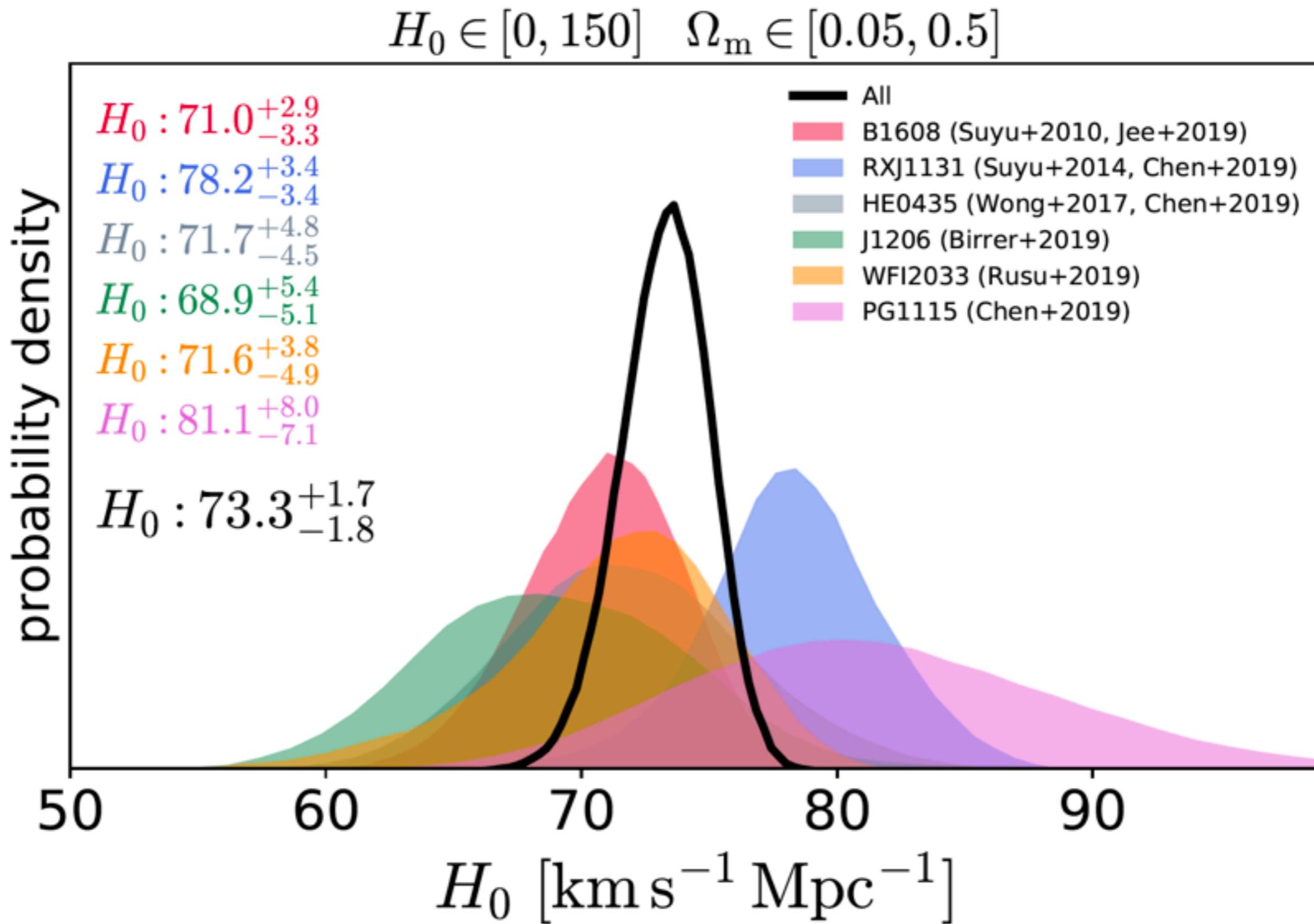
$$\Omega_{\text{DE}} = 1 - \Omega_m$$

# Testing many models using our lenses

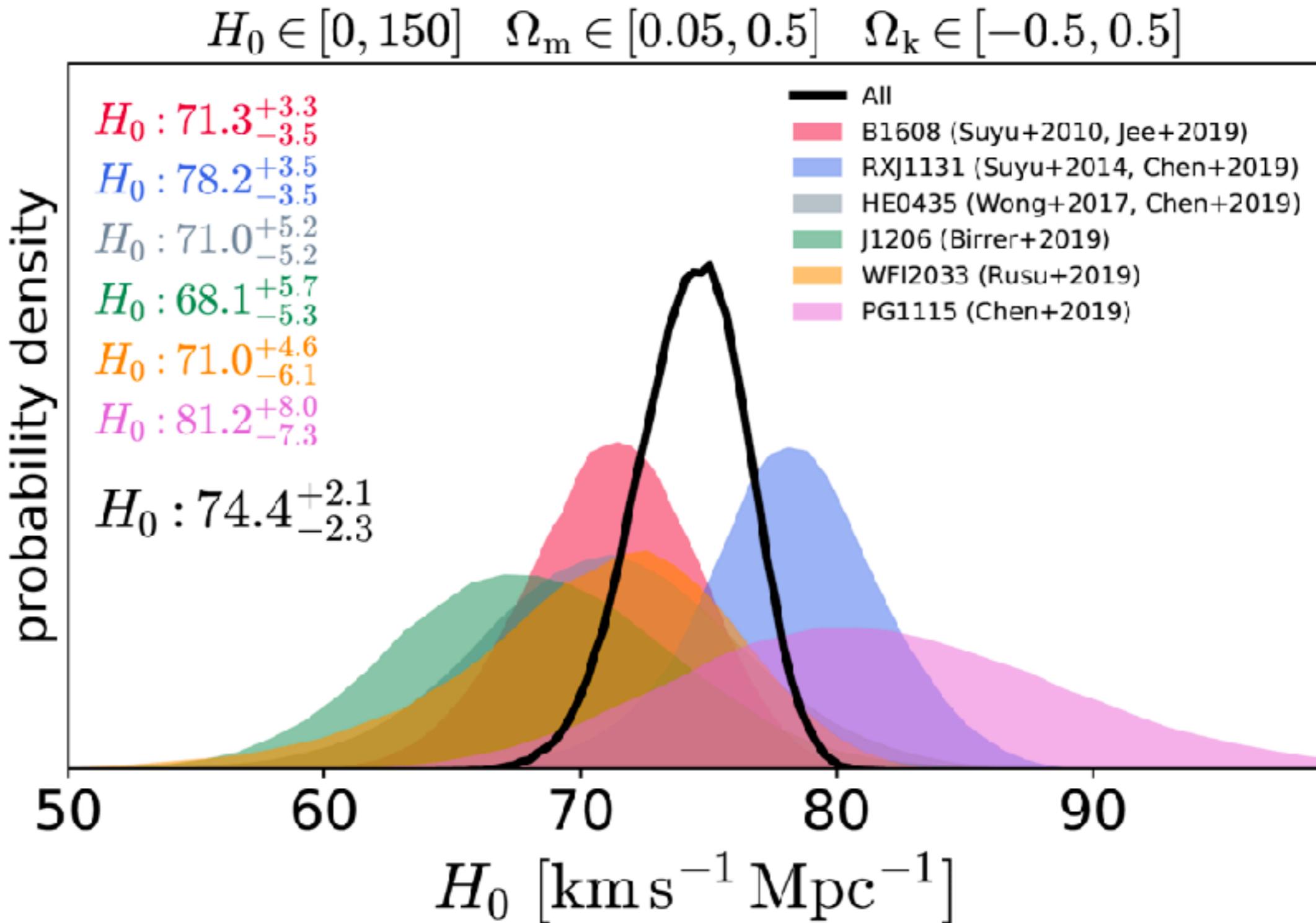
Time-delay cosmography combined with other probes

$\Lambda\text{CDM}$	Flat $\Lambda\text{CDM}$	$\Omega_m = 1 - \Omega_\Lambda$ JLA/Pantheon for $\{H_0, \Omega_\Lambda\}$
$o\Lambda\text{CDM}$	Open $\Lambda\text{CDM}$	$\Omega_m = 1 - \Omega_\Lambda - \Omega_k > 0$ <i>Planck</i> (Section 5.3.1) or JLA/Pantheon (Section 5.4) for $\{H_0, \Omega_\Lambda, \Omega_m\}$
$w\text{CDM}$	Flat $w\text{CDM}$	$\Omega_m = 1 - \Omega_{\text{DE}}$ <i>Planck</i> (Section 5.3.2) or JLA/Pantheon (Section 5.4) for $\{H_0, \Omega_{\text{DE}}, w\}$
$N_{\text{eff}}\Lambda\text{CDM}$	Flat $\Lambda\text{CDM}$ Variable $N_{\text{eff}}$	<i>Planck</i> for $\{H_0, \Omega_\Lambda, N_{\text{eff}}\}$
$m_\nu\Lambda\text{CDM}$	Flat $\Lambda\text{CDM}$ Variable $\sum m_\nu$	<i>Planck</i> for $\{H_0, \Omega_\Lambda, \sum m_\nu\}$
$N_{\text{eff}}m_\nu\Lambda\text{CDM}$	Flat $\Lambda\text{CDM}$ Variable $N_{\text{eff}}$ and $\sum m_\nu$	<i>Planck</i> for $\{H_0, \Omega_\Lambda, N_{\text{eff}}, \sum m_\nu\}$
$w_0w_a\text{CDM}$	Flat $w_0w_a\text{CDM}$	<i>Planck</i> (Section 5.3.4) or JLA/Pantheon (Section 5.4) for $\{H_0, w_0, w_a\}$
$ow\text{CDM}$	Open $w\text{CDM}$	$\Omega_m = 1 - \Omega_{\text{DE}} - \Omega_k > 0$ JLA/Pantheon for $\{H_0, \Omega_k, \Omega_{\text{DE}}, w\}$
$ow_0w_a\text{CDM}$	Open $w_0w_a\text{CDM}$	$\Omega_m = 1 - \Omega_{\text{DE}} - \Omega_k > 0$ JLA/Pantheon for $\{H_0, \Omega_k, \Omega_{\text{DE}}, w_0, w_a\}$

# It's hard to lower $H_0$ from lensing

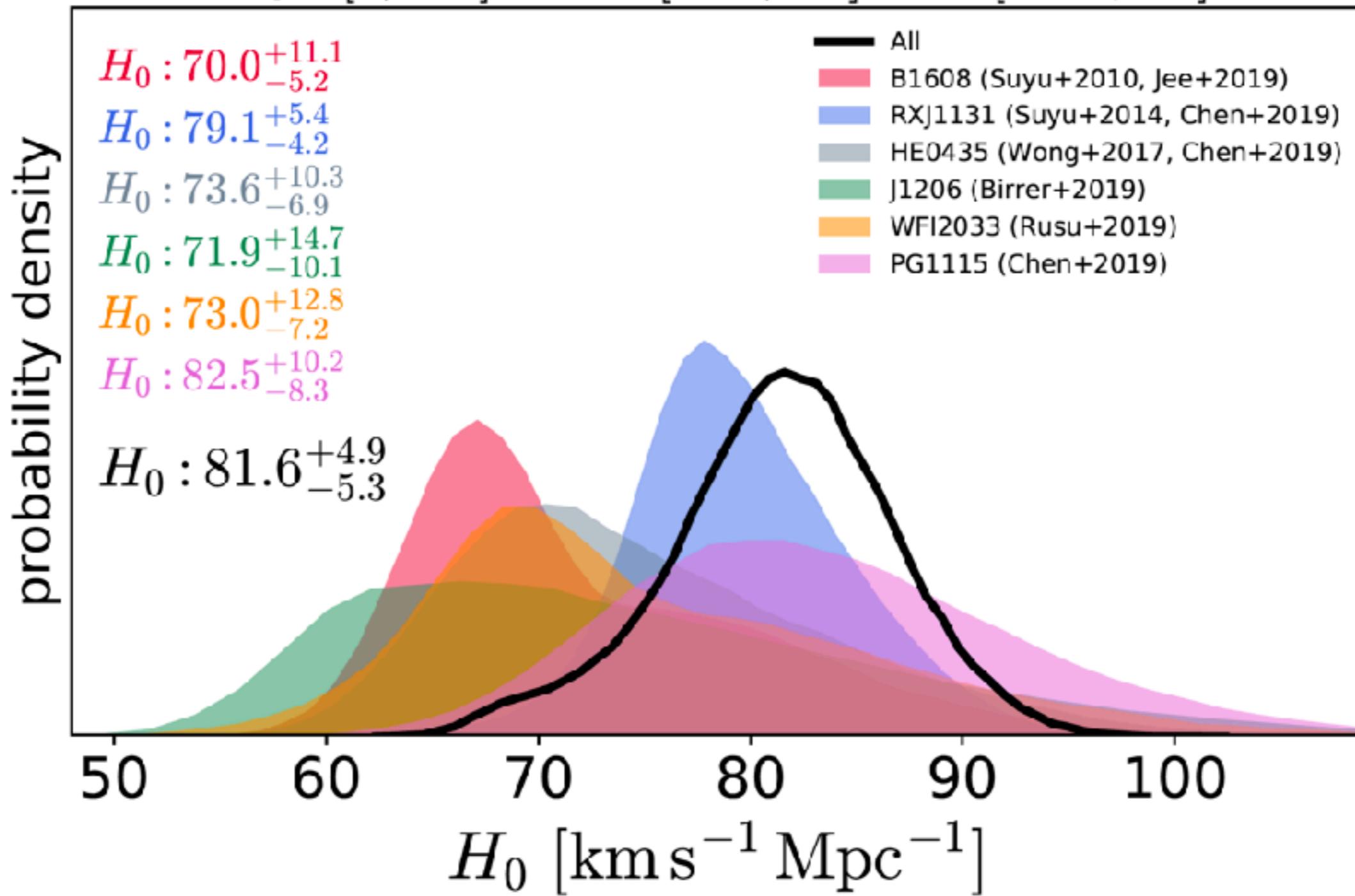


# It's hard to lower $H_0$ from lensing



# It's hard to lower $H_0$ from lensing

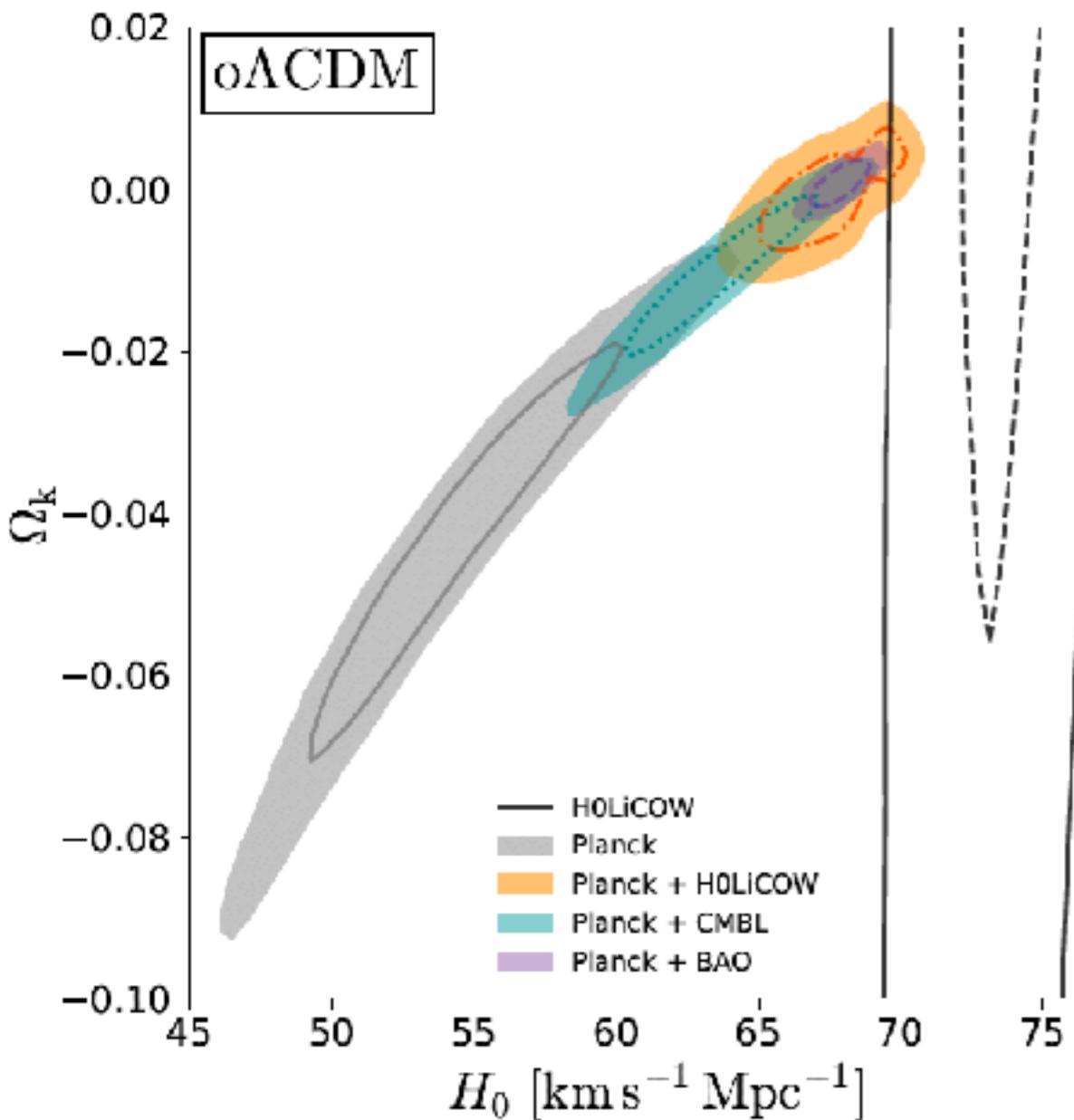
$$H_0 \in [0, 150] \quad \Omega_m \in [0.05, 0.5] \quad w \in [-2.5, 0.5]$$



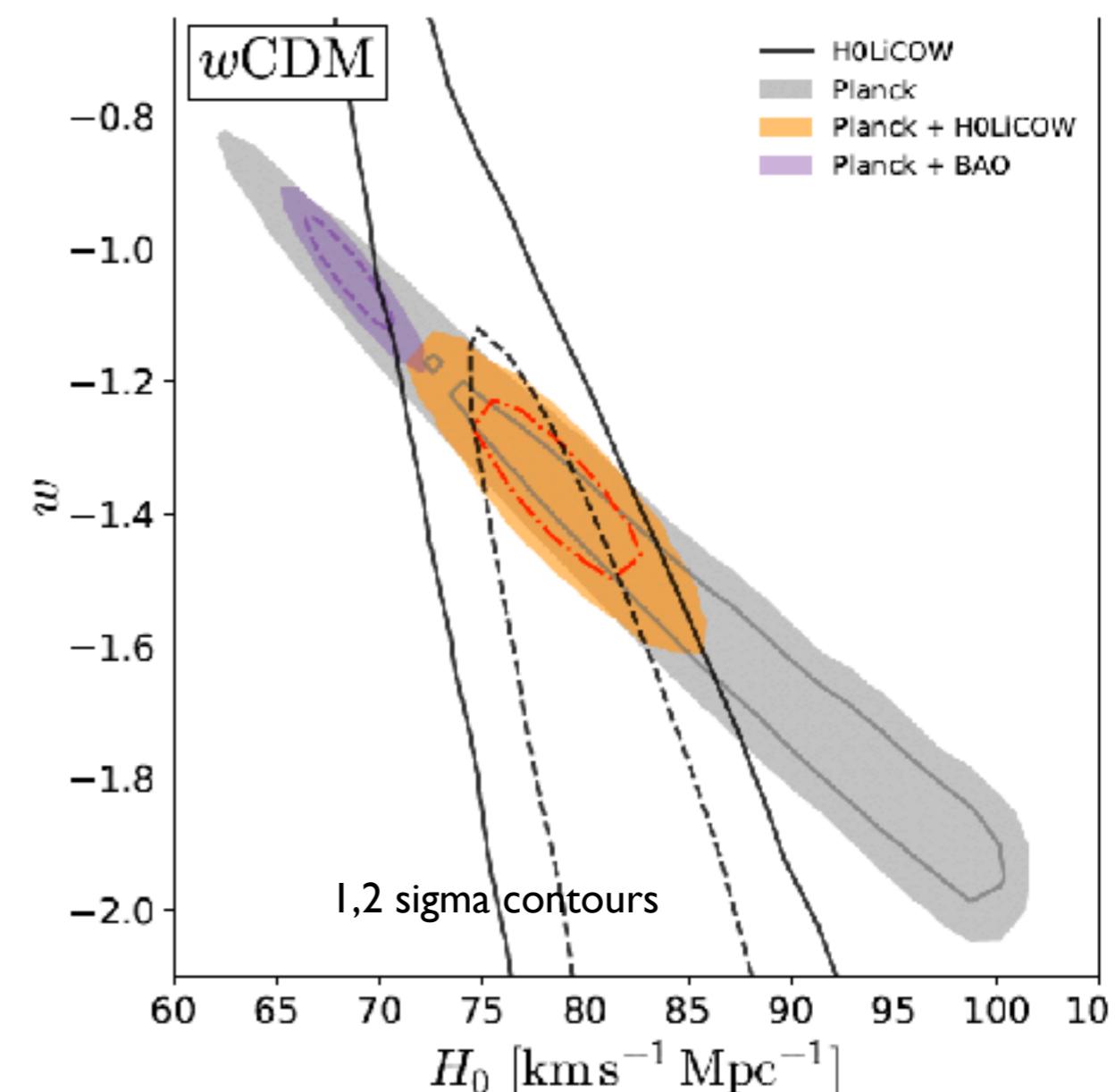
# It's hard to lower $H_0$ from lensing

*Playing with the various energy-density parameters*

Open universe

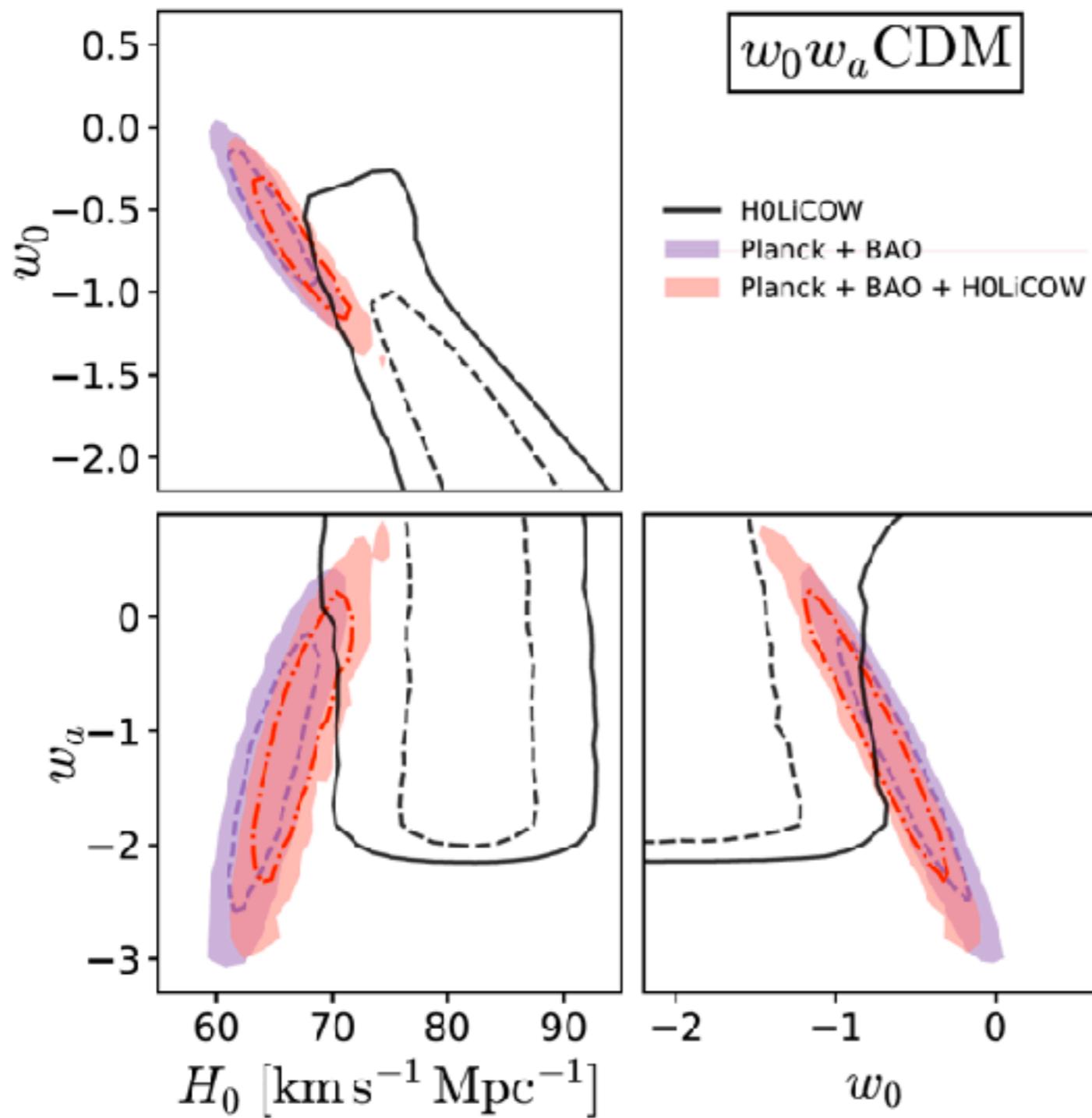


Difference DE equation of state



# It's hard to lower $H_0$ from lensing

*Playing with the various energy-density parameters*



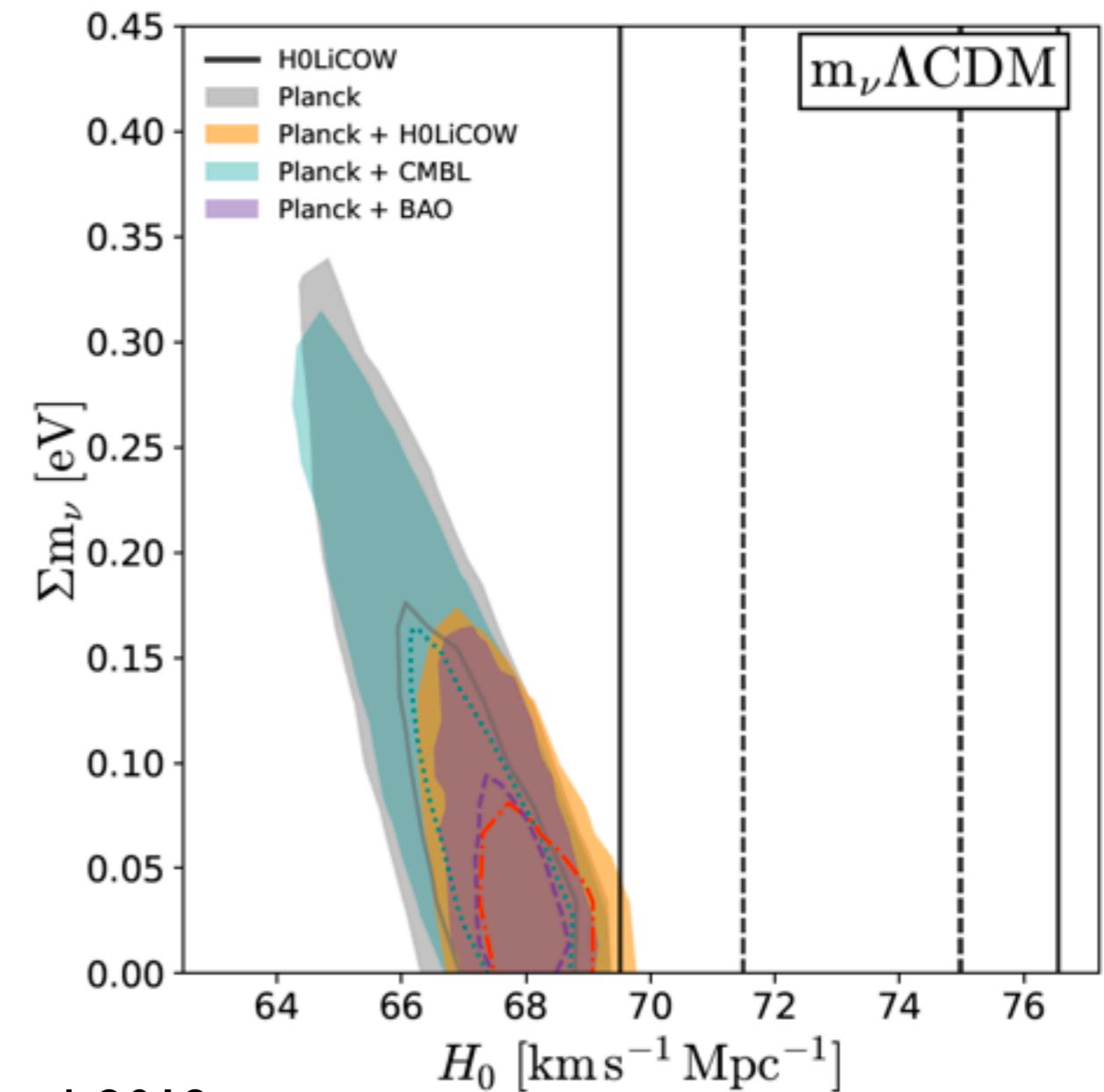
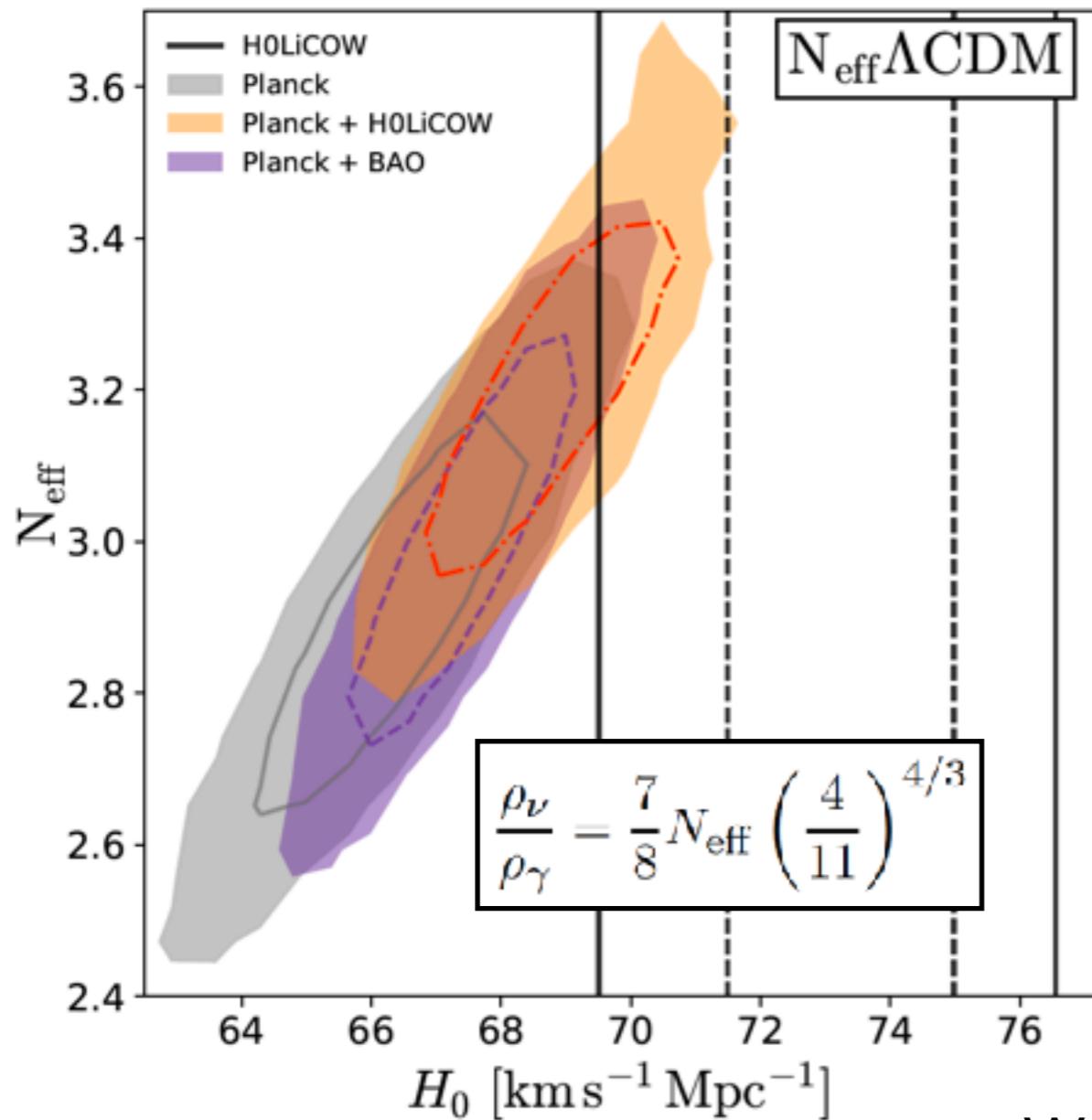
Wong et al. 2019

Evolving DE equation of state

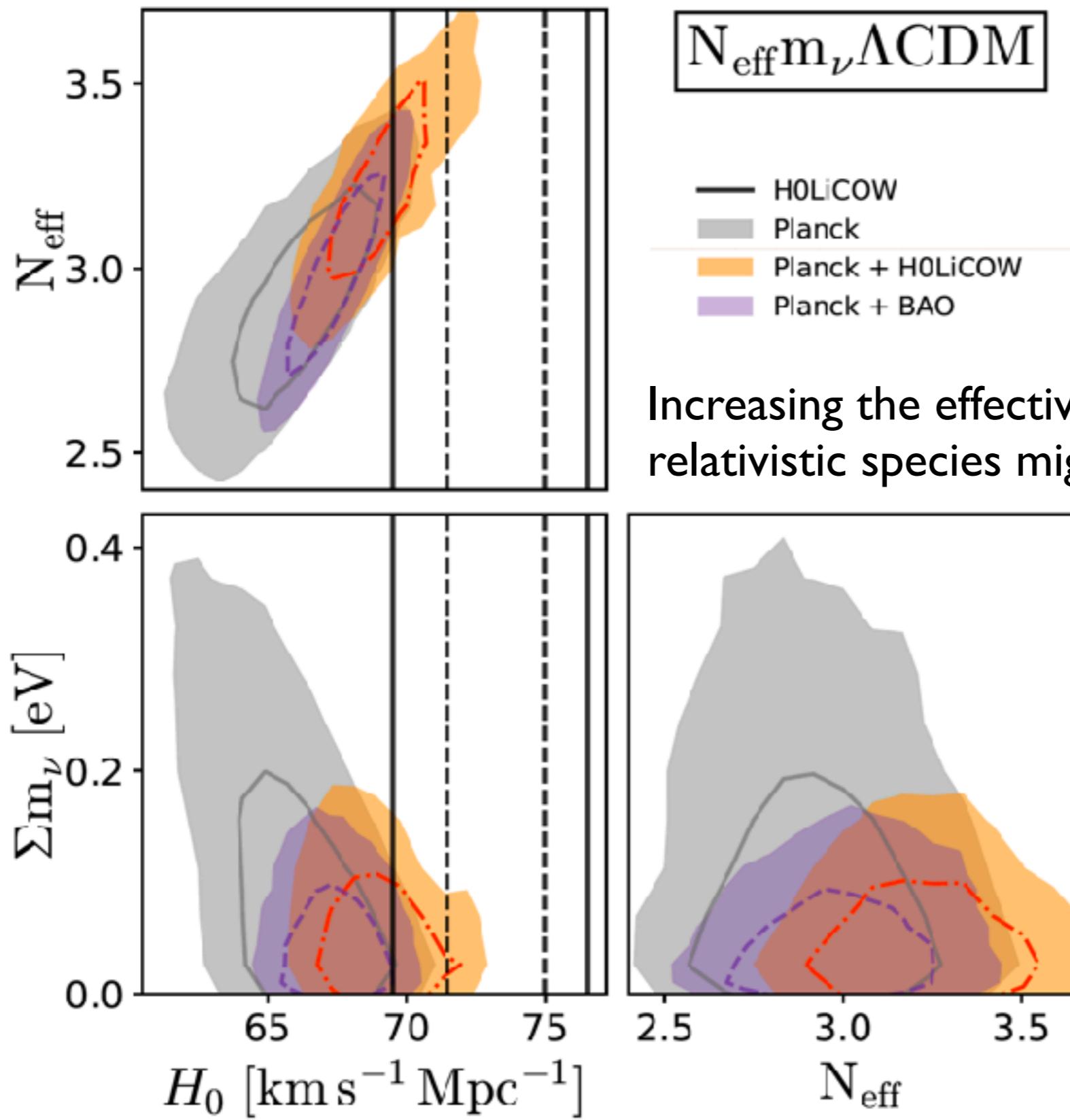
# It's a little easier to increase $H_0$ from CMB+BAO

*Playing with the effective number of relativistic species (i.e. beyond the neutrino species) or neutrino masses.*

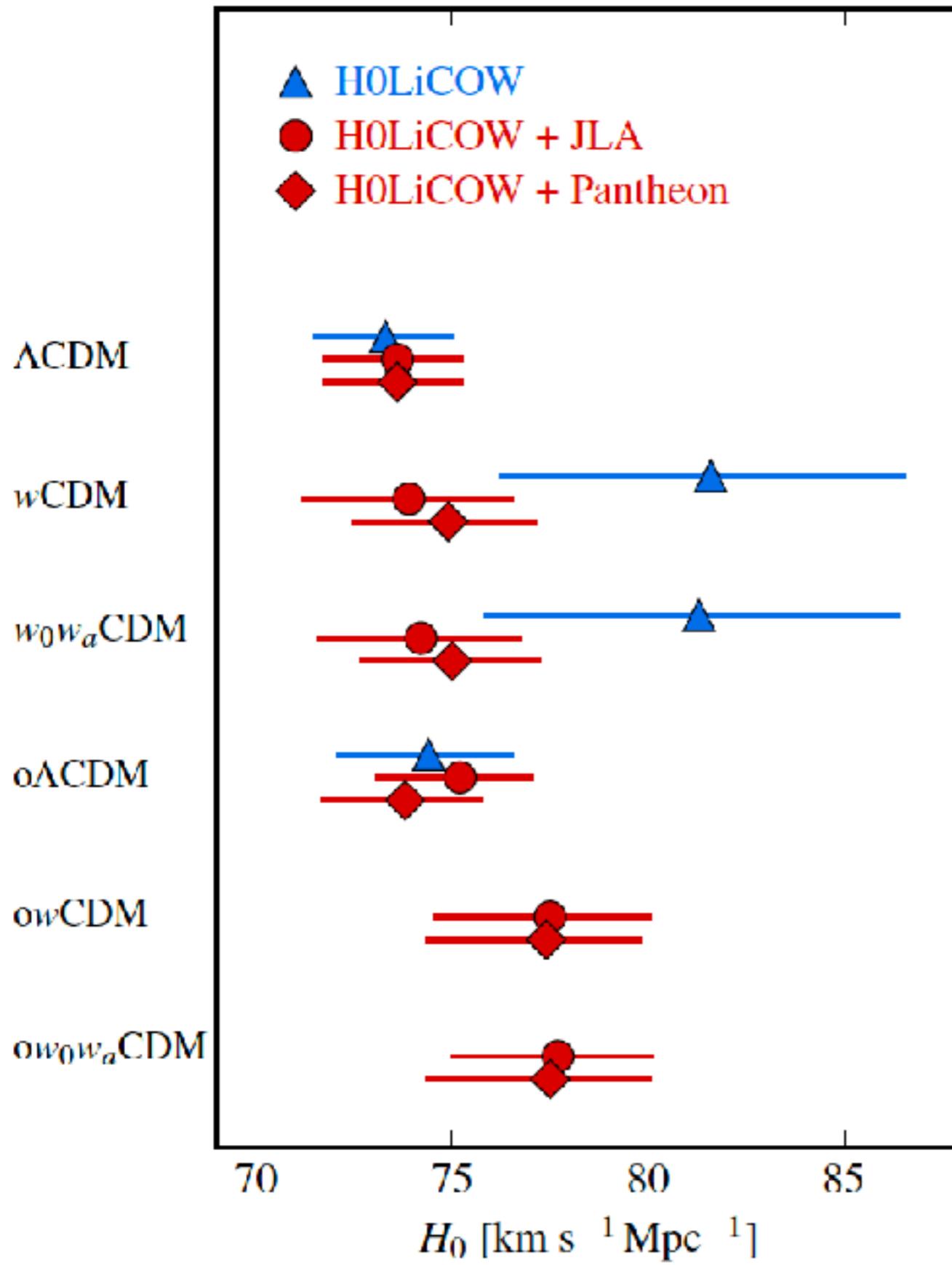
*This affects the early universe and not lensing. Increasing  $N_{\text{eff}}$  might help a little*



# It's a little easier to increase $H_0$ from CMB+BAO



$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\text{eff}} \left( \frac{4}{11} \right)^{4/3}$$



## In summary

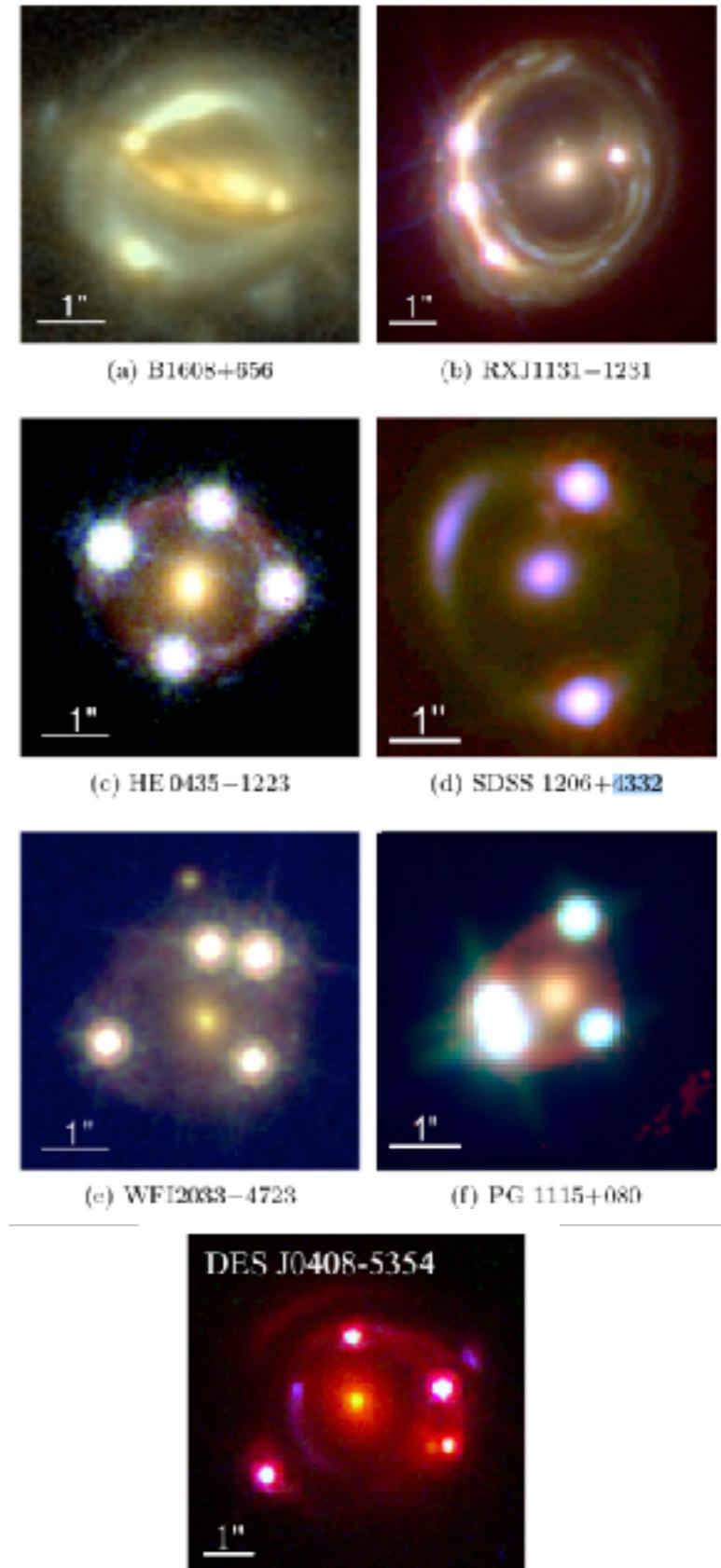
It appears that any deviation from the standard model in the late Universe increases the tension.

However, changes in the early Universe might alleviate it, but **requires some radical changes in the standard model**, in particle physics, or even giving up the flatness of the Universe.

Or a combination of several!

# Summary — I

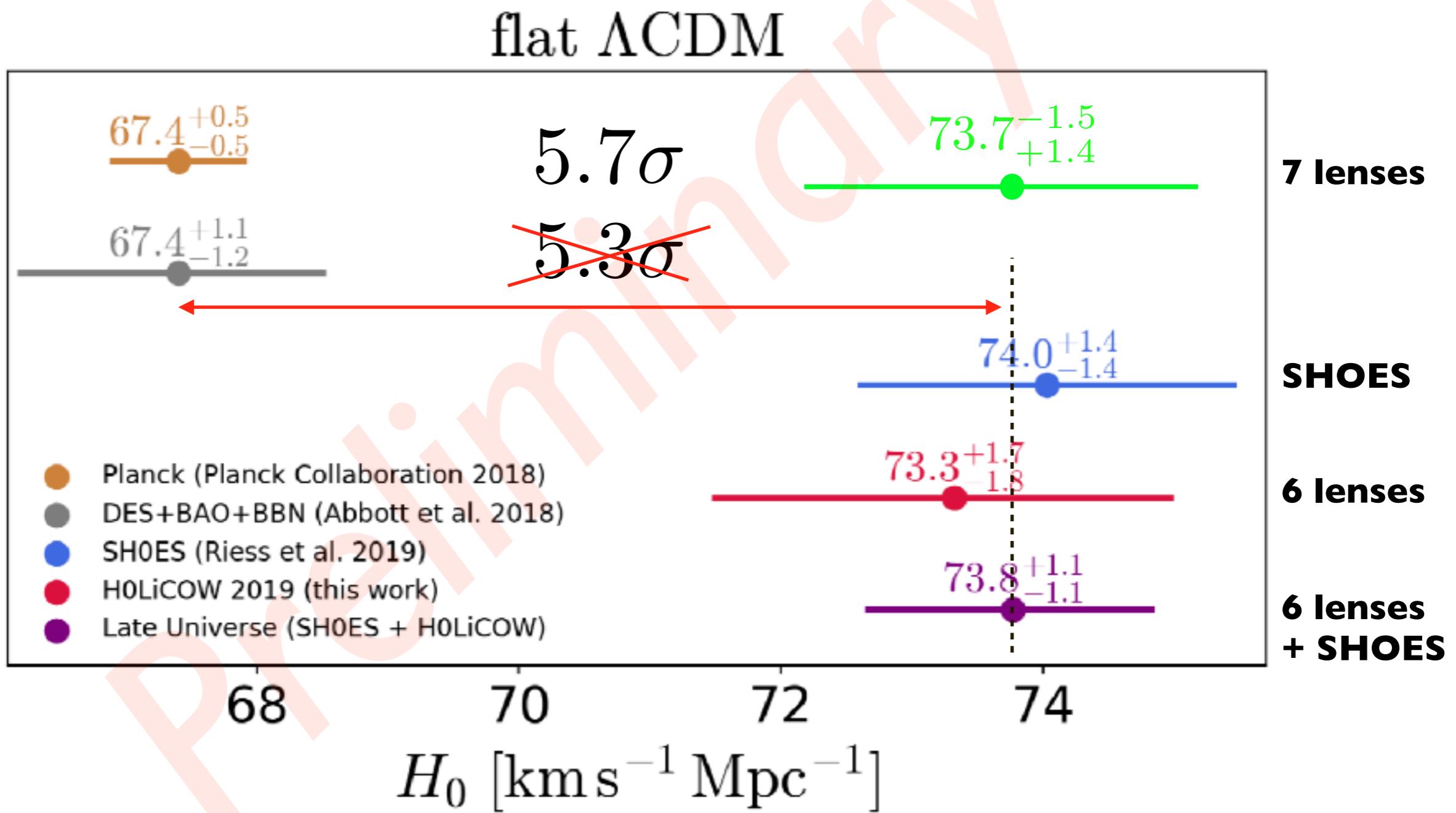
- Strong lensing time delays provide an absolute distance scale and a direct measure of  $H_0$  independent from distance-ladder or CMB/BAO physics/methods.
- Time delays are currently measured to a few percents within one single season and mass models of the lens, field and line of sight to a similar precision.
- DES, KIDS, HSC, PanSTARSS EUCLID, LSST, Gaia, are and are discovering hundreds of new suitable targets in the coming years. Time-delay cosmography is very promising!
- Seven lenses now give  $H_0$  with accuracy and precision comparable to e.g. supernovae and are independent.



In flat  $\Lambda$ CDM:  $H_0 = 73.7 \pm 2.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ; a dozen of additional objects already « in the pipeline »; All analyses are done blindly !!

# Summary — 2

Currently a clear and persistent tension between CMB & BOA and strong-lensing and other local-universe inferences of  $H_0$  remains, \*if\* a flat  $\Lambda$ CDM universe is assumed. Alleviating this might need new physics!



Thank you!





Thank you!