

Theory and phenomenology of neutrino masses and mixings

Lecture I



Topical Lectures Research School Subatomic Physics (Nikhef, Amsterdam, Dec. 2019)

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The lectures are intended for a broad audience of students with competences in different fields in particle physics

The goal is to “get you (more) interested” in ν physics, by moving from basic neutrino properties and phenomena to more advanced topics at the current frontier of the field.

Some exercises are also proposed on ν oscill. probabilities. (see Tutorial pdf file). Others are contained in the slides.

People interested in further reading can usefully browse the “Neutrino Unbound” website: www.nu.to.infn.it, or just email me for advice about specific topics: elgio.lisi@ba.infn.it

Outline of lectures:

Lecture I

Pedagogical intro + warm-up case study for oscillations

Lecture II

Standard 3 ν oscillations: evolution and current status

Lecture III

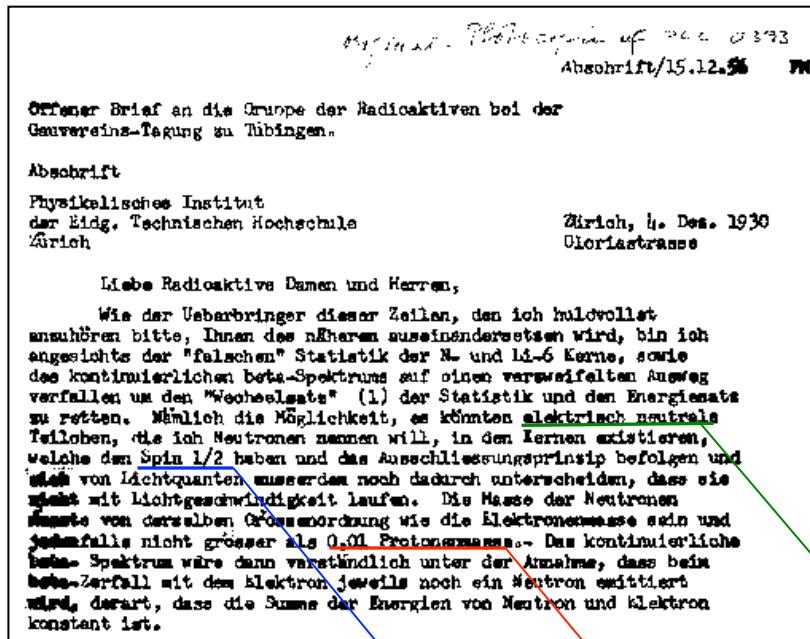
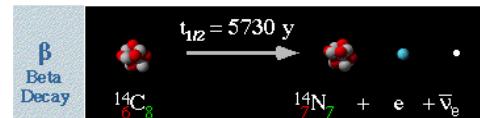
Neutrino absolute masses + open problems in ν physics

Feel free to stop me and ask questions at any time!

Pedagogical Introduction

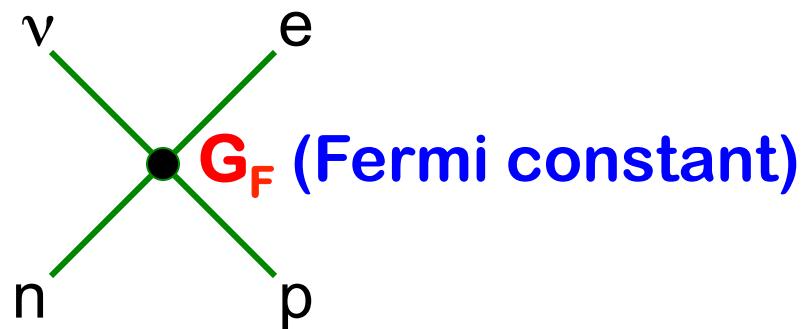
The neutrino celebrated its 89th birthday on December 4th!

This particle was invented in 1930 by Wolfgang Pauli as a “desperate remedy” to explain the continuous β -ray spectrum via a 3-body decay, e.g.,



Kinematics: spin 1/2, tiny mass, zero electric charge

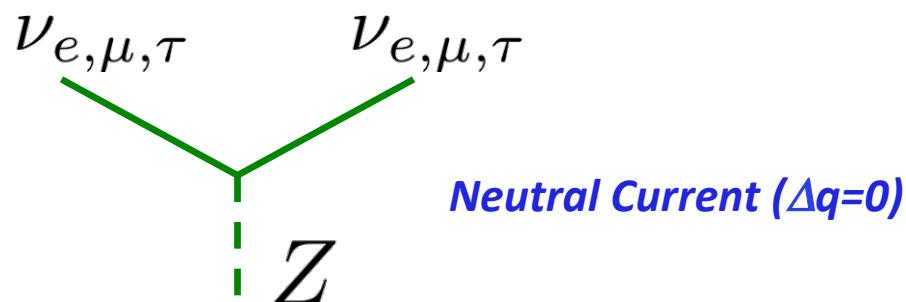
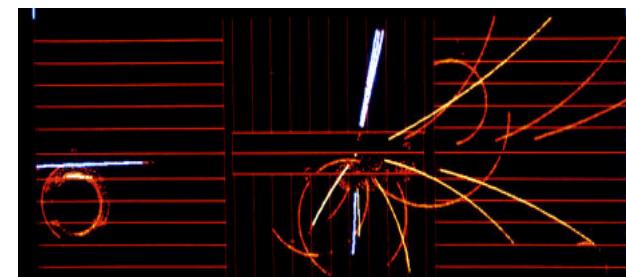
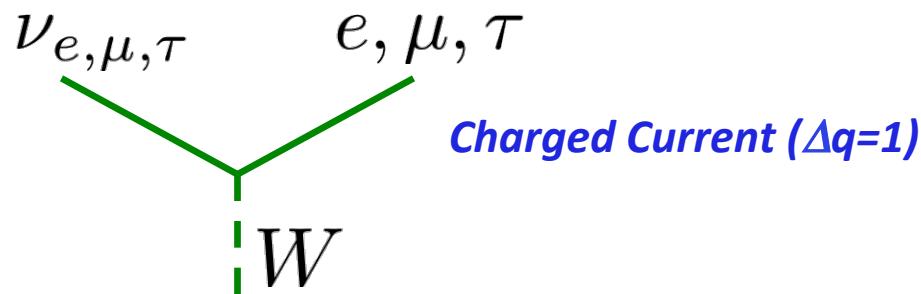
The name “neutrino” (=“little neutral one”, in Italian) was actually invented by Enrico Fermi, who first proposed in 1933-34 a theory for its **dynamics** (weak interactions)



Many decades of research have revealed other properties of the neutrino. For instance, there are **3 different ν “flavors” e μ τ**

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \leftarrow \quad q = 0 \quad (\Delta q = 1) \\ \leftarrow \quad q = -1$$

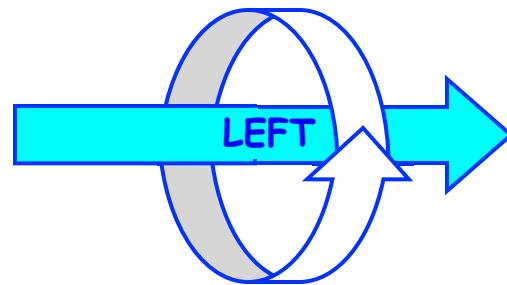
and their Fermi interactions are mediated by a charged vector boson **W**, with a neutral counterpart, the **Z boson**



Such interactions are chiral (= not mirror-symmetric):

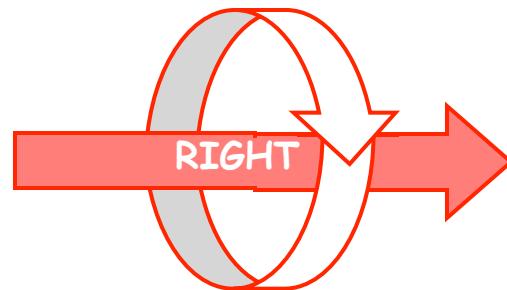
Neutrinos are created in
a left-handed (LH) state

ν



Anti-nus are created in
a right-handed (RH) state

$\bar{\nu}$



The handedness is a constant of motion only for massless neutrinos.
It is “almost” constant –at $O(m/E)$ - for ultrarelativistic ν with mass m.

We shall “forget” about neutrino handedness and spin for a while,
until mass effects at $O(m/E)$ will be reconsidered in Lecture III.

Most of the recent progress in ν physics has actually been driven by another kind of effects, at $O(m^2/E)$: **neutrino flavor oscillations**

The starting point is a century-old equation ...

Die Ruhe - Energie ändert sich also (additiv) mit der Masse. Da erstere ihren Beigefüfe nach nur bis auf eine additive Konstante bestimmt, so kann man festsetzen, dass c_0 ~~zusammen~~ mit m verschwindet.
Dann ist einfacher

$$c_0 = m,$$

was der Äquivalenz - Satz von troyer Masse und Ruhe-Energie ausspricht.

~~Hätten wir oben nicht die Massenkonstante des Impulses gleich gesetzt?~~

... namely, for $p \neq 0$: $E = \sqrt{m^2 + p^2}$

(in natural units)

Our ordinary experience takes place in the limit: $p \ll m$

$$E \simeq m + \frac{p^2}{2m}$$

... while for neutrinos the proper limit is: $p \gg m$

$$E \simeq p + \frac{m^2}{2p}$$

Energy difference between two neutrinos ν_i e ν_j with mass m_i e m_j in the same beam ($p_i = p_j \simeq E$) :

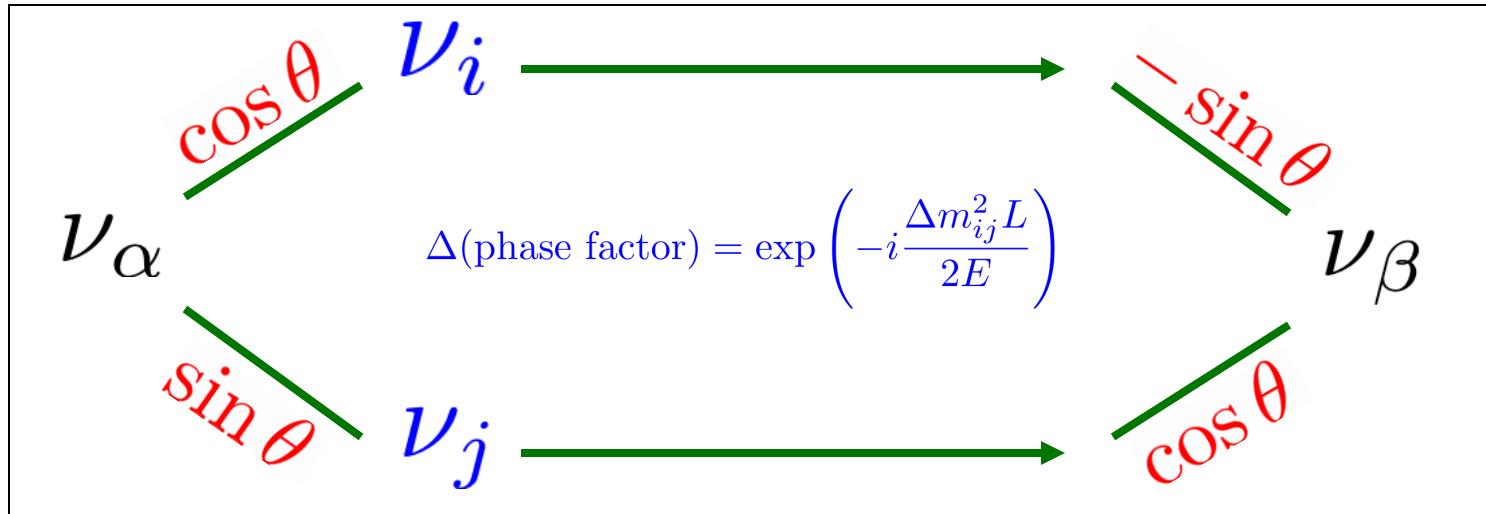
$$\Delta E \simeq \frac{\Delta m_{ij}^2}{2E}$$

PMNS*: neutrinos with definite mass (ν_i and ν_j) might have NO definite flavor (ν_α e ν_β), e.g.,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_j \end{pmatrix}$$

*Pontecorvo; Maki, Nakagawa & Sakata

Analogy with a two-slit interference experiment in vacuum:



This is the simplest case (only 2ν involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation.

Indeed, it changes (“oscillates”) significantly over a distance L ($=x \approx \Delta t$) dictated by the uncertainty relation:

$$1 \sim \Delta E \Delta t \simeq \frac{m_i^2 - m_j^2}{2E} L$$

One can easily derive that a neutrino created with **flavor** α can develop in vacuum a different **flavor** β with periodical oscillation probability in L/E :

$$P(\nu_\alpha \rightarrow \nu_\beta) = 4 \sin^2 \theta \cos^2 \theta \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \quad (\text{B. Pontecorvo})$$

Amplitude

(vanishes for $\theta=0$ or $\pi/2$,
is maximal for $\theta=\pi/4$)

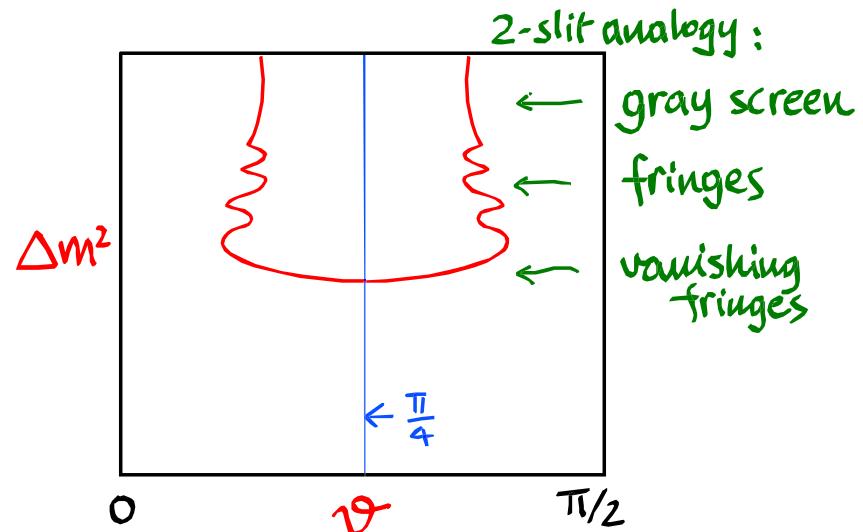
Oscillation phase

(vanishes for degenerate
masses or small L/E)

Note 1 : This is the flavor “appearance” probability. The “disappearance” probability is the complement to 1.

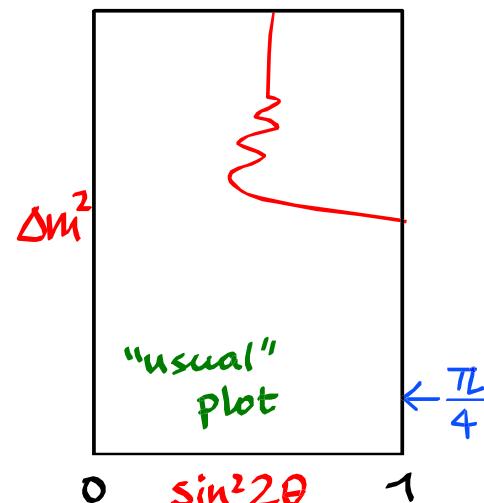
Note 2: The oscillation effect depends on the *difference* of (squared) masses, not on the *absolute masses themselves*.

Typical iso- $\langle P_{\alpha\beta} \rangle$ contours



Octant symmetry: $\theta \rightarrow \frac{\pi}{2} - \theta$ in $P_{\mu\mu}$

If 2nd octant folded onto the 1st one :



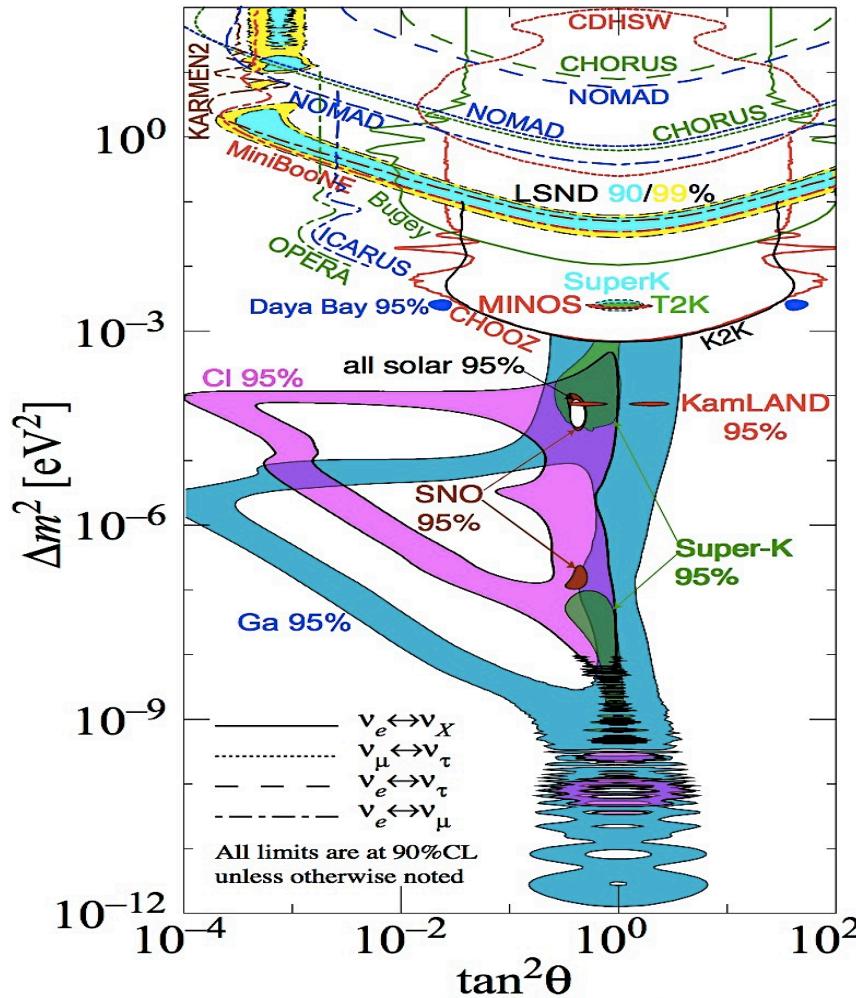
Basically obsolete

In general, better to use
(preserve octant-symmetry)

$\log \tan^2 \theta$
or $\sin^2 \theta$

(Note: 2v Octant symmetry broken by 3v and/or matter effects)

Octant (a)symmetric 2ν contours from PDG Review:



But... patching 2ν approximations in different oscillation channels, in order to get a full 3ν picture, is no longer a useful approach:
better to go the other way around, from the full 3ν case to 2ν limits

The “standard” 3ν oscillation framework

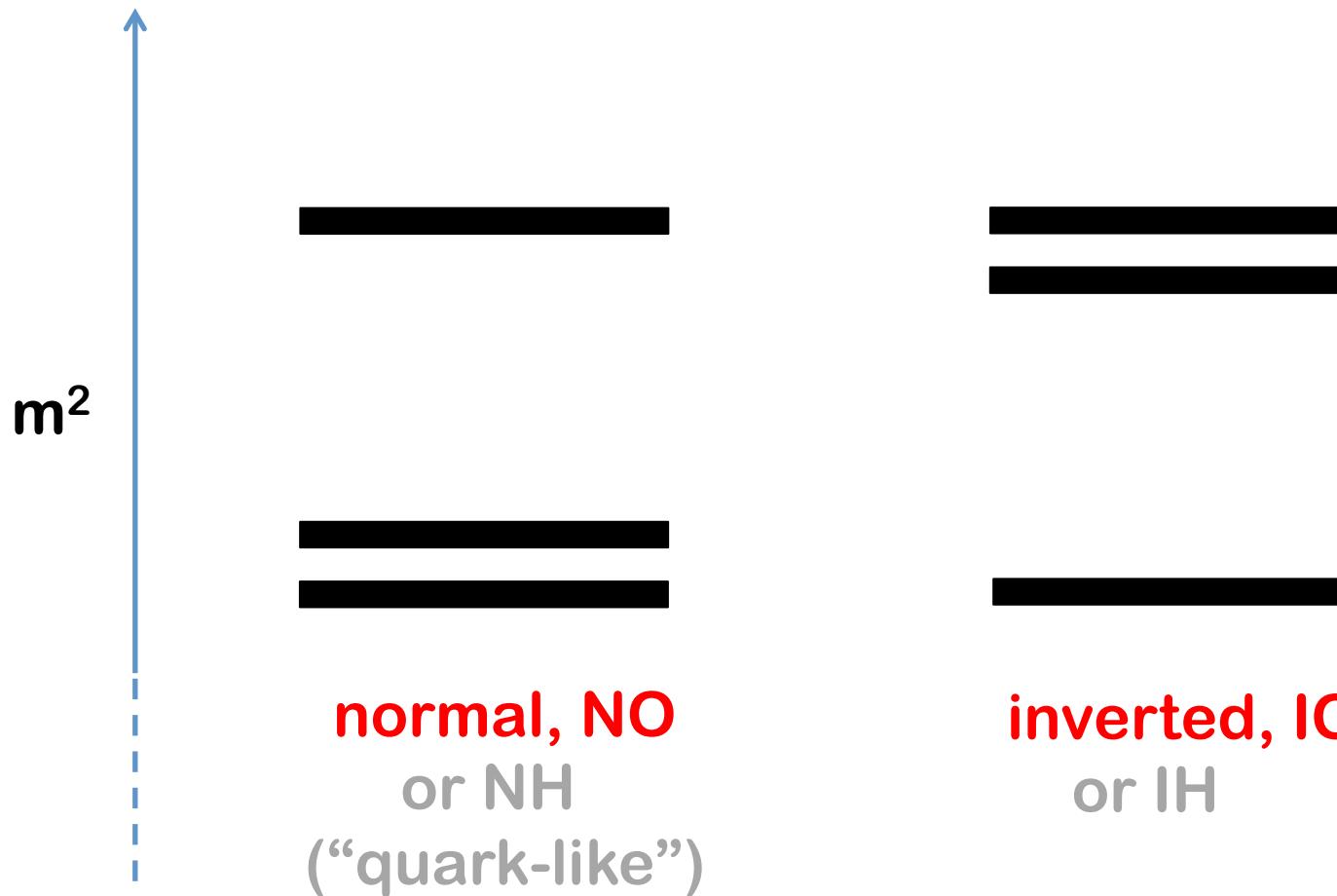
Physics facts and mass notation:

- There are three mass states ν_1, ν_2, ν_3 with masses m_1, m_2, m_3
- For ultrarelativistic ν in vacuum, $E = \sqrt{m_i^2 + p^2} \simeq p + \frac{m_i^2}{2p}$
- Neutrino oscillations probe the differences $\Delta E \propto \Delta m_{ij}^2$
- 3 neutrinos → two independent Δm_{ij}^2 , say, δm^2 and Δm^2
- Experimentally, very different scales: $\delta m^2 / \Delta m^2 \sim 1/30$
Difficult to observe both! Current expts sensitive to a dominant one.

$$\delta m^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2 \leftarrow \text{“small” or “solar” splitting}$$

$$\Delta m^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2 \leftarrow \text{“large” or “atmospheric” splitting}$$

Two possible mass *orderings* (or *hierarchies*)



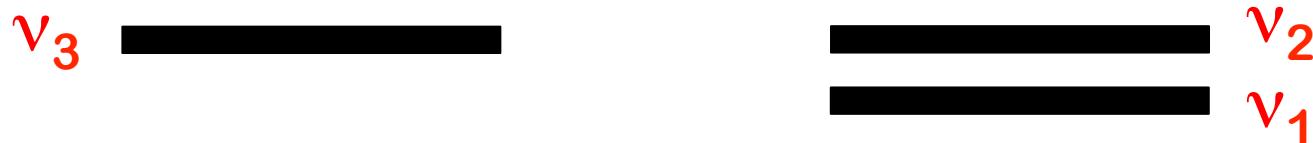
0 ?

Absolute mass scale still unknown,
but upper limits exist: $< O(0.1\text{-}1) \text{ eV}^2$

PDG convention for 3ν masses:

(ν_1, ν_2) = “close” states, with $m_2 > m_1$ in NO and IO

ν_3 = “lone” state, with $m_3 > m_{1,2}$ ($< m_{1,2}$) in NO (IO)



Our notation for splittings: Define as independent ones

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

$$\Delta m^2 = \frac{1}{2}(\Delta m_{31}^2 + \Delta m_{32}^2) > 0 \quad (\text{NH})$$

$$< 0 \quad (\text{IH})$$

PDG convention for 3ν mixing:

Three Euler rotations, one being complex

$$\nu_\alpha = U_{\alpha i} \nu_i \quad \begin{array}{l} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{array}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

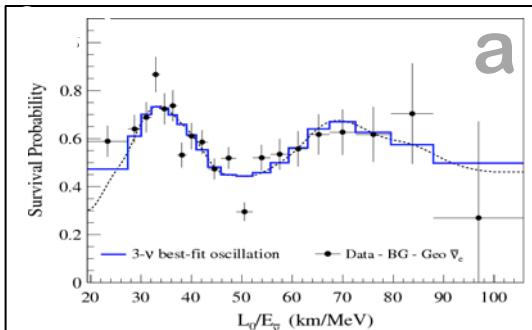
This ordering happens to be particularly useful for phenomenologically interesting limits

$$\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{bmatrix}$$

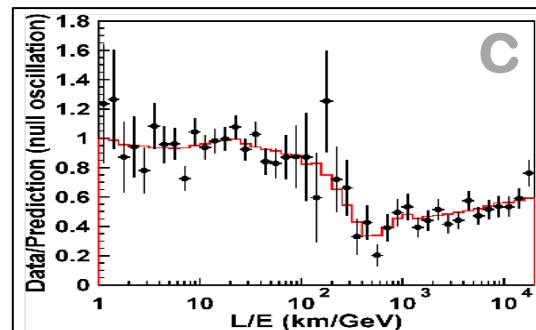
$$UU^\dagger = 1 \quad U \rightarrow U^* \text{ for } \bar{\nu} \quad c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij}$$

3 ν can explain $\alpha \rightarrow \beta$ oscillations seen in vacuum and matter...

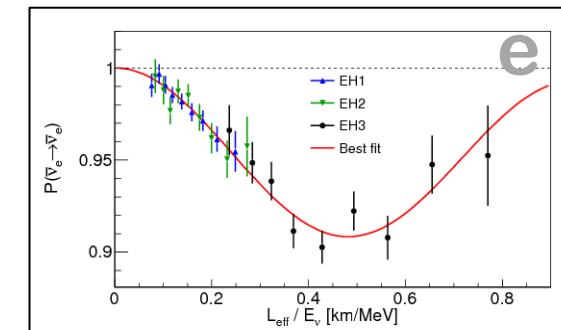
$e \rightarrow e$ (KamLAND, KL)



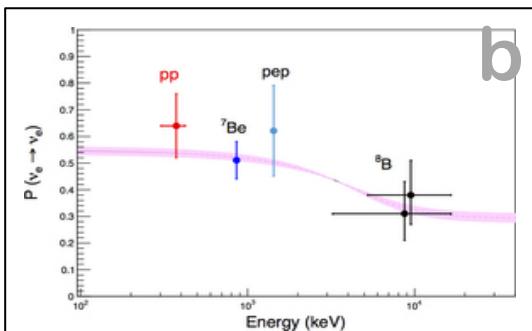
$\mu \rightarrow \mu$ (Atmospheric)



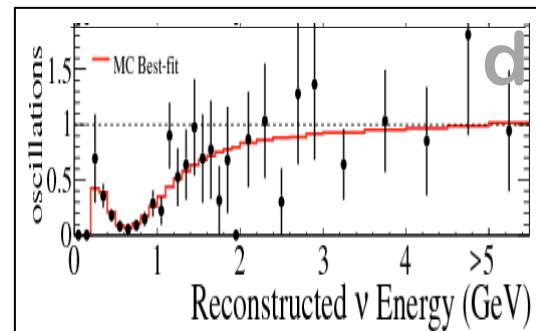
$e \rightarrow e$ (SBL Reac.)



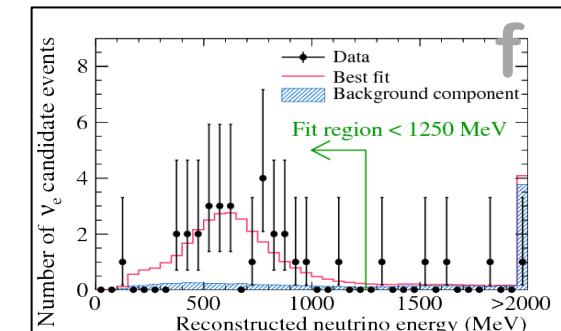
$e \rightarrow e$ (Solar)



$\mu \rightarrow \mu$ (LBL Accel)



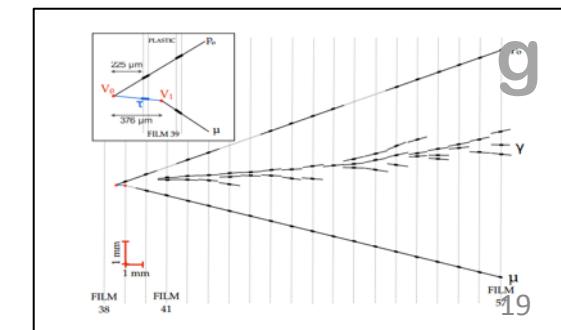
$\mu \rightarrow e$ (LBL Accel)



Data from various types of neutrino experiments: (a) solar, (b) long-baseline reactor, (c) atmospheric, (d) long-baseline LBL accelerator, (e) short-baseline reactor, (f,g) long baseline accelerator (and, in part, atmospheric).

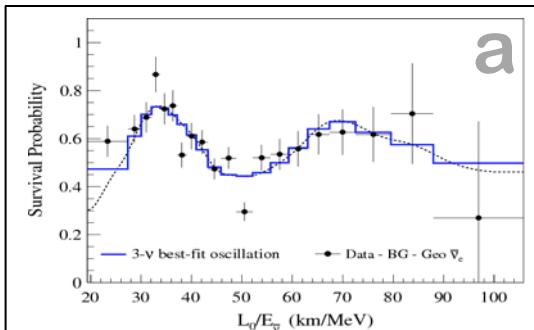
(a) KamLAND [[plot](#)]; (b) Borexino [[plot](#)], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [[plot](#)], DeepCore, MACRO, MINOS etc.; (d) T2K [[plot](#)], NOvA, MINOS, K2K; (e) Daya Bay [[plot](#)], RENO, Double Chooz; (f) T2K [[plot](#)], MINOS, NOvA; (g) OPERA [[plot](#)], Super-K and IC-CD atmospheric.

$\mu \rightarrow \tau$ (OPERA, SK, DC)

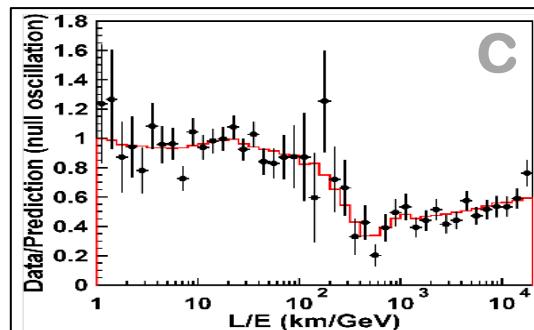


...with dominant 3ν parameters:

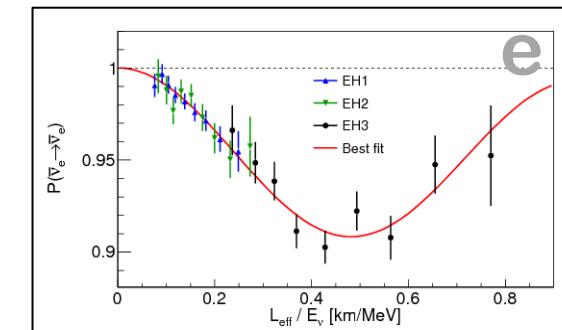
$e \rightarrow e$ ($\delta m^2, \theta_{12}$)



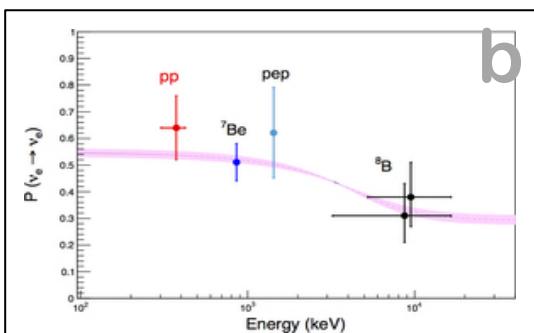
$\mu \rightarrow \mu$ ($\Delta m^2, \theta_{23}$)



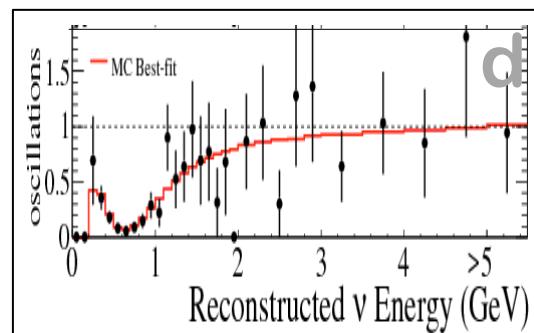
$e \rightarrow e$ ($\Delta m^2, \theta_{13}$)



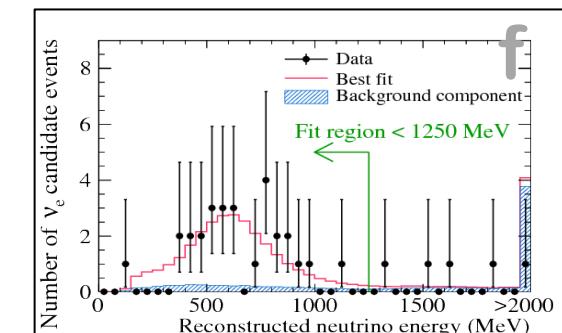
$e \rightarrow e$ ($\delta m^2, \theta_{12}$)



$\mu \rightarrow \mu$ ($\Delta m^2, \theta_{23}$)



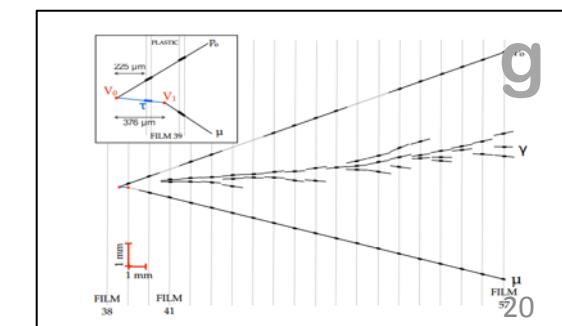
$\mu \rightarrow e$ ($\Delta m^2, \theta_{13}, \theta_{23}$)



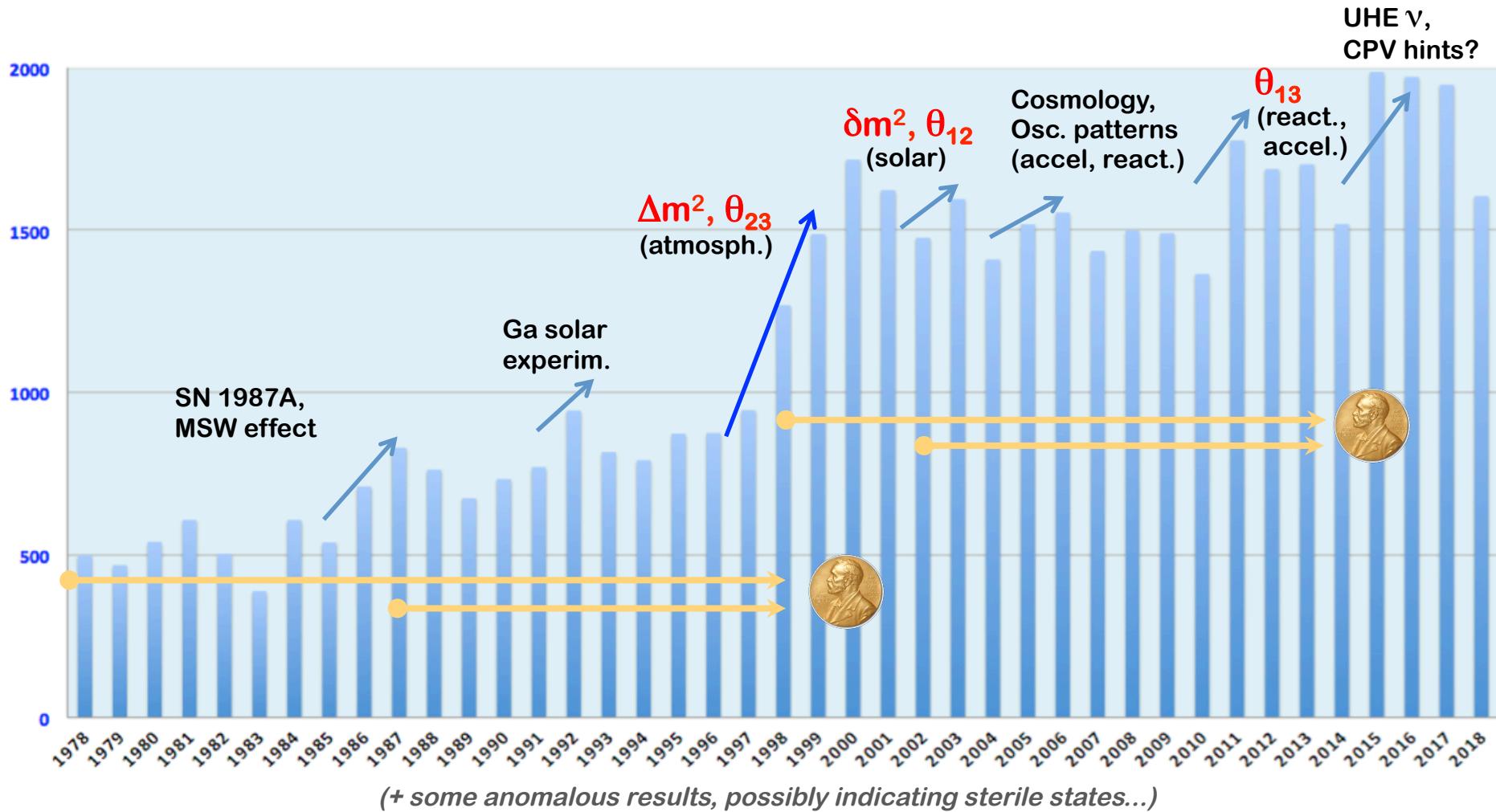
Established so far:

δm^2 $|\Delta m^2|$ θ_{12} θ_{23} θ_{13}

$\mu \rightarrow \tau$ ($\Delta m^2, \theta_{23}$)



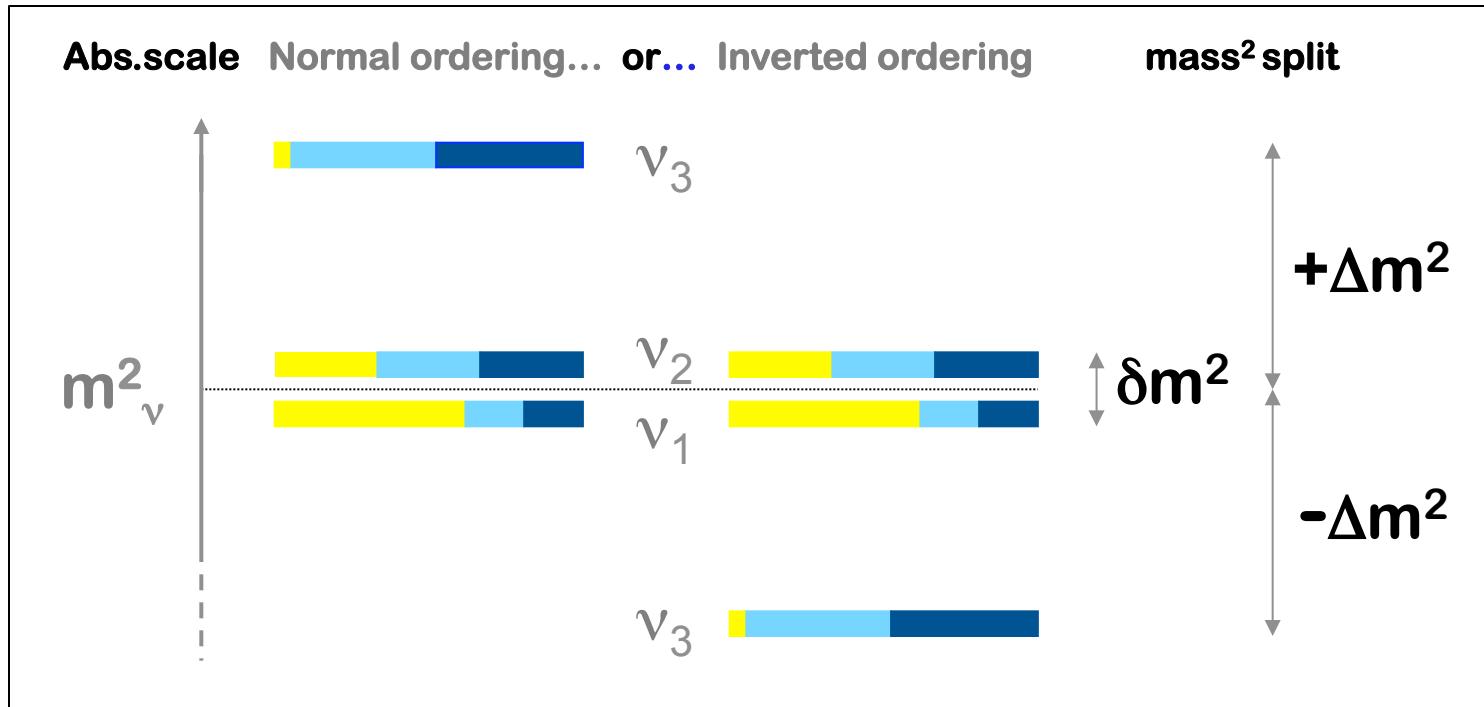
Preprints with #neutrino# in title (from InSpire)



Oscillation discoveries shaped the field so far. Future?

Present 3ν knowledge in one slide (with 1-digit accuracy)

e μ τ



We have seen:

$$\begin{aligned}\delta m^2 &\sim 7 \times 10^{-5} \text{ eV}^2 \\ \Delta m^2 &\sim 2 \times 10^{-3} \text{ eV}^2 \\ \sin^2 \theta_{12} &\sim 0.3 \\ \sin^2 \theta_{23} &\sim 0.5 \\ \sin^2 \theta_{13} &\sim 0.02\end{aligned}$$

We would like to see:

δ (CP)
 $\text{sign}(\Delta m^2) = \text{ordering}$
 $\text{octant}(\theta_{23})$
 absolute mass scale
 Dirac/Majorana nature

+ Physics beyond 3ν?
 (anomalies, new states or interactions)

We shall now look into neutrino flavor evolution in more detail, both in vacuum and in matter.

The presence of two small parameters,

$$\delta m^2 / \Delta m^2 \sim 3 \times 10^{-2}$$

$$\sin^2 \theta_{13} \sim 2 \times 10^{-2}$$

will often simplify formalism & understanding.

Neutrino flavor evolution

- It is $m_i \ll E$ in almost all cases of phenomenological interest
- We can then set $\beta = v/c \simeq 1$, $x \simeq t$, $\partial_x \simeq \partial_t$
- Chirality flips LH \rightarrow RH of amplitude $O(m_i/E)$ can be ignored
- For propagation purposes, neutrinos akin to “scalar” states $|\nu\rangle$
- State evolution governed by Hamiltonian: $i\frac{d}{dx}|\nu\rangle = \hat{H}|\nu\rangle$
- Formal solution (evolution operator): $|\nu(x)\rangle = \hat{S}(x, 0)|\nu(0)\rangle$
- $S_{\beta\alpha}$ components in flavor basis = amplitudes for $\nu_\alpha \rightarrow \nu_\beta$
- Flavor evolution probabilities: $P_{\alpha\beta} = P(\nu_\alpha \rightarrow \nu_\beta) = |S_{\beta\alpha}|^2$
- $\alpha=\beta$: flavor disappearance channel, $P_{\alpha\alpha} \leq 1$
- $\alpha \neq \beta$: flavor appearance channel, $P_{\alpha\beta} \geq 0$

Three-neutrino flavor evolution in vacuum

The hamiltonian is exceedingly simple in the mass basis $(\nu_1, \nu_2, \nu_3)^T$:

$$H_{\text{mass}} = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{pmatrix} \simeq p \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

(and even simpler than that, since terms proportional to unity decouple)

In flavor basis $(\nu_e, \nu_\mu, \nu_\tau)^T$ it becomes nondiagonal (\rightarrow flavor not conserved):

$$H_{\text{flavor}} = U H_{\text{mass}} U^\dagger$$

Let us work out (tutorials) and discuss (slides)
some implications of this simple hamiltonian \rightarrow

3ν oscillations in vacuum: general case

Prove that

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \operatorname{Re} J_{\alpha\beta}^{ij} \sin^2 \left(\frac{\Delta m_{ij}^2 x}{4E} \right) - 2 \sum_{i < j} \operatorname{Im} J_{\alpha\beta}^{ij} \sin \left(\frac{\Delta m_{ij}^2 x}{2E} \right)$$

where

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

$$J_{\alpha\beta}^{ij} = U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}$$

$$\frac{\Delta m_{ij}^2}{4E} = 1.267 \left(\frac{\Delta m_{ij}^2}{\text{eV}^2} \right) \left(\frac{x}{m} \right) \left(\frac{\text{MeV}}{E} \right)$$

← Jarlskog invariant

(See tutorials)

In general, $P_{\alpha\beta}$ is not an observable...

$$R_\beta \sim \int \Phi_\alpha \otimes P_{\alpha\beta} \otimes \sigma_\beta \otimes \epsilon_\beta$$

Observable
event rate

Source flux
(production)

Propagation
(flavor change)

Interaction
and detection

→ need to take into account detailed phenomenology

Many open research problems for each of these ingredients, in all subfields of neutrino physics.

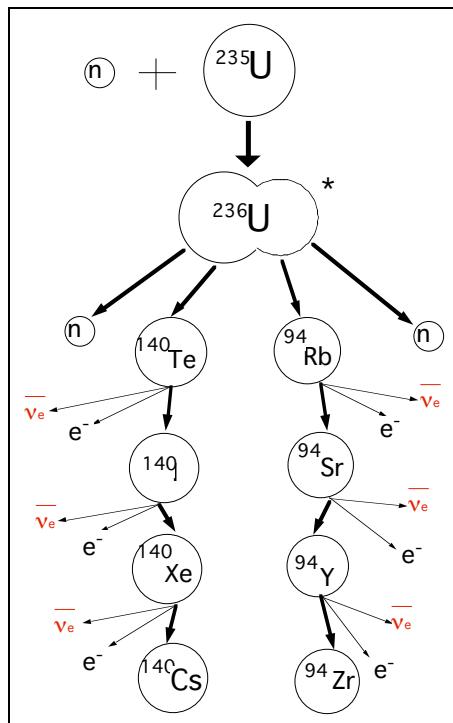
**Warm-up case study
for oscillation phenomenology:**

Short-baseline (SBL) reactors

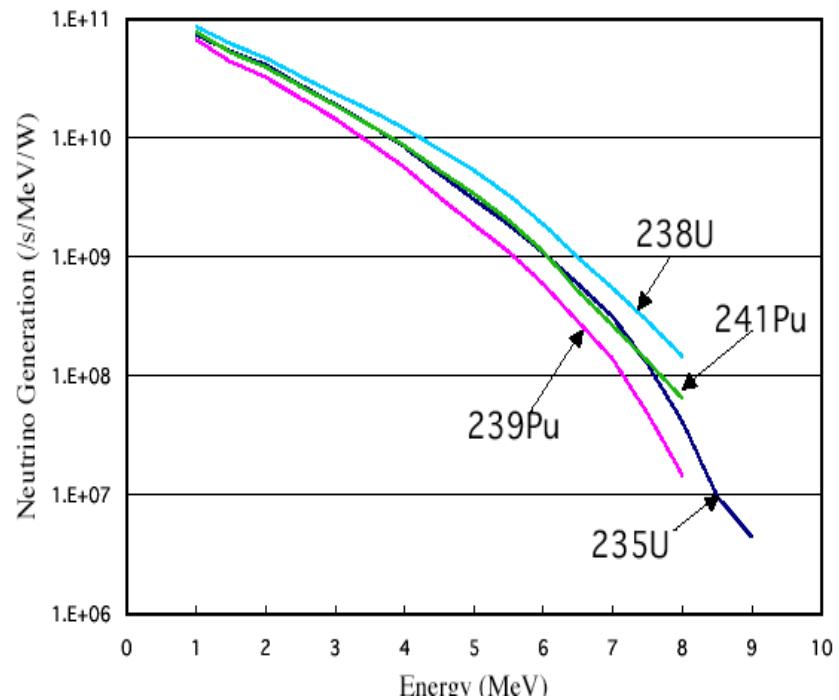
Production

Reactors: Intense sources of anti- ν_e ($\sim 6 \times 10^{20}/s/\text{reactor}$)

Typically, 6 neutron decays to reach stable matter from fission:

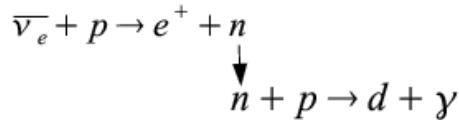


$\sim 200 \text{ MeV per fission / 6 decays:}$
Typical available neutrino energy
 $E \sim \text{few MeV}$



Detection

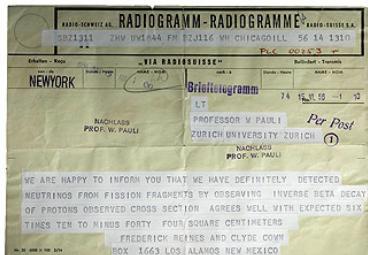
Reaction Process: inverse β -decay



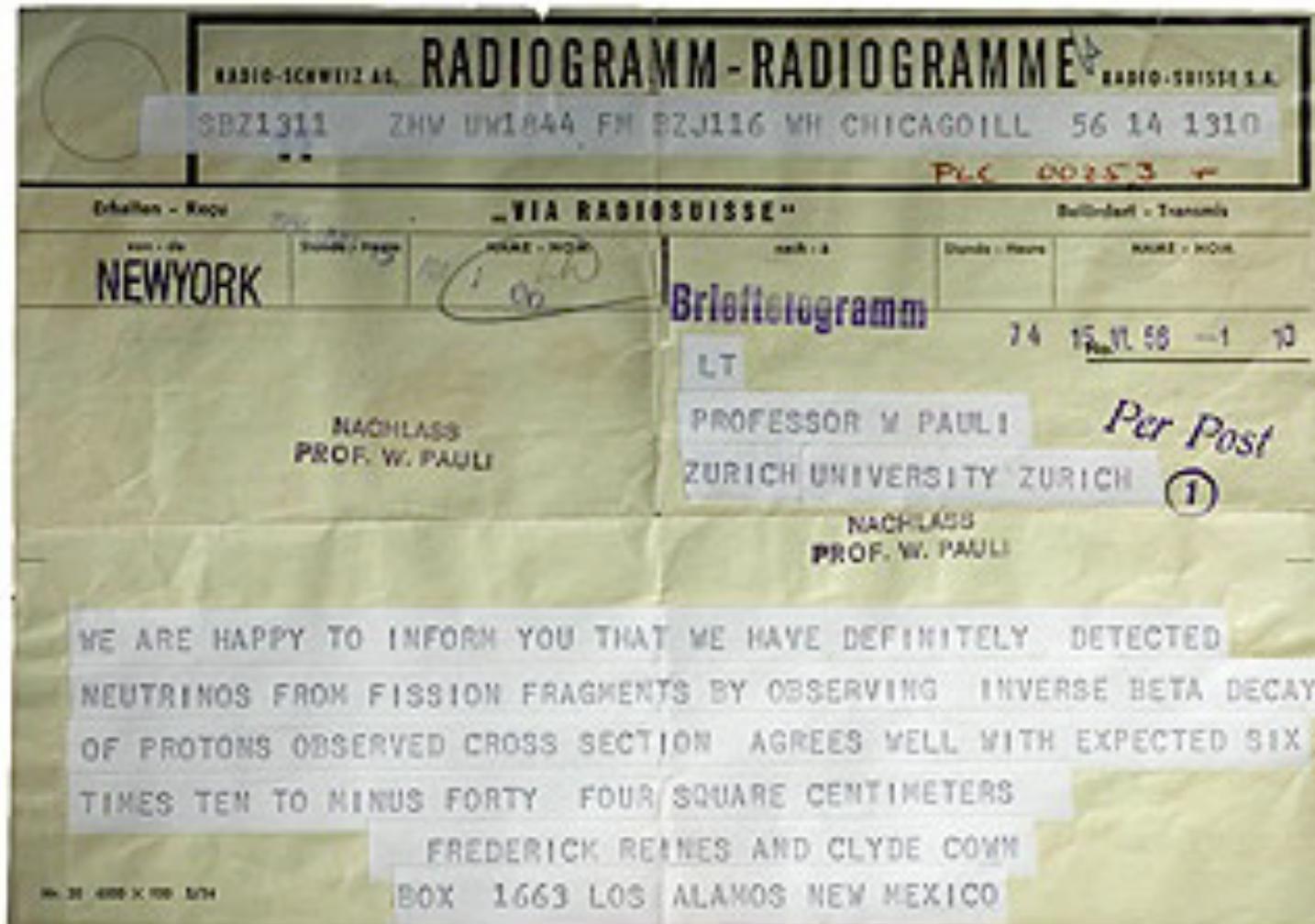
Scintillator is target and detector

- Distinct two-step signature:
 - prompt event: positron
 $E_\nu \approx E_{e^+} + 0.8 \text{ MeV}$
 - delayed event: neutron capture after $\sim 210\mu\text{s}$
 - 2.2 MeV gamma

Delayed coincidence: good background rejection



← This reaction allowed experimental
✓ discovery in 1956 (Reines & Cowan)



Reply by telegram:

**"Thanks for message. Everything comes
to him who knows how to wait. Pauli."**

Propagation

Exercise: $3\nu \rightarrow 2\nu$ reduction for SBL reactor expts.

Short-baseline reactor experiments look for $\bar{\nu}_e$ oscillations at $x = L \sim O(1\text{ km})$ and $E \sim \text{few MeV}$. At these energies, CC reactions in the final state can produce e^+ but not μ^+ or τ^+ ; therefore, only $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ is observable (disappearance) but not $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ or $P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau)$ (appearance). Moreover, it is $\Delta m^2 L / 4E \ll 1$, while $\Delta m^2 L / 4E \sim \mathcal{O}(1)$.

Prove that, in the limit $\Delta m^2 \approx 0$, effective 2ν oscillations occur:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$\underbrace{\hspace{2cm}}$ $\underbrace{\hspace{2cm}}$
oscillation amplitude oscillating factor

Try to get an intuitive understanding of the dependence on θ_{13} only.

Solution - From the previous exercise (with $U \rightarrow U^*$), the only nonzero oscillating terms for $\alpha\beta = ee$ and $\Delta m^2 = m_2^2 - m_1^2 \approx 0$ are multiplied by:

$$J_{ee}^{13} = |U_{e1}|^2 |U_{e3}|^2 |U_{e3}^*| |U_{e1}| = |U_{e1}|^2 |U_{e3}|^2 \text{ and } J_{ee}^{23} = |U_{e2}|^2 |U_{e3}|^2 |U_{e3}^*| |U_{e2}|. \text{ Then:}$$

$$\text{Im}(J_{ee}^{13}) = 0 = \text{Im}(J_{ee}^{23}) \text{ and}$$

$$\begin{aligned} P(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - 4(|U_{e1}|^2 |U_{e3}|^2 + |U_{e2}|^2 |U_{e3}|^2) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\ &= 1 - 4 |U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad \leftarrow |U_{e3}|^2 = \sin^2 \theta_{13} \\ &= 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \end{aligned}$$

Intuitively : two of the three mixing rotations have \sim no effect

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} (23) & (13) & (12) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \rightarrow \text{only } (13) \text{ physical}$$

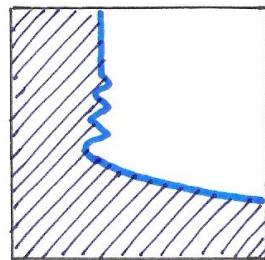
↑ ↑
 mixes mixes
 unobservable ~degenerate
 flavors (ν_μ, ν_τ) states (ν_1, ν_2)

Note that, in this case, δ is unobservable, as well as $\text{sign}(\pm \Delta m^2)$, and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P(\nu_e \rightarrow \nu_e)$.

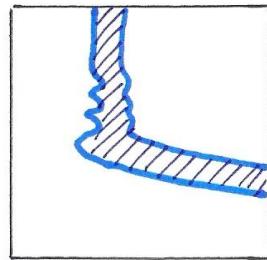
Generic experimental constraints in 2ν approxim.

- Experiments measure some "averaged" $P_{\alpha\beta} \approx \sin^2 2\theta \langle \sin^2 \left(\frac{\Delta m_{ij}^2 x}{4E} \right) \rangle$
 - Curve of $\text{iso} - P_{\alpha\beta}$:
-
- Δm_{ij}^2
- $\sin^2 2\theta$
- $\leftarrow \frac{\Delta m_{ij}^2 x}{4E} \gg 1, \langle \dots \rangle \sim \frac{1}{2}, \text{ fast oscillations}$
- $\leftarrow \frac{\Delta m_{ij}^2 x}{4E} \sim O(1)$
- $\leftarrow \frac{\Delta m_{ij}^2 x}{4E} \ll 1, \text{ vanishing oscillations}$

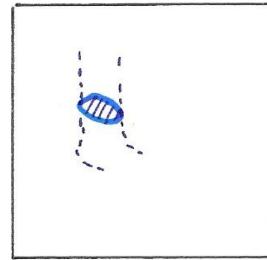
- Possible expt. constraints:



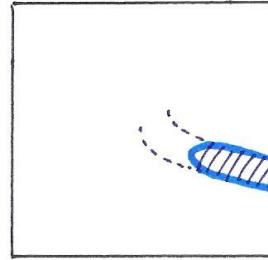
No signal



Signal

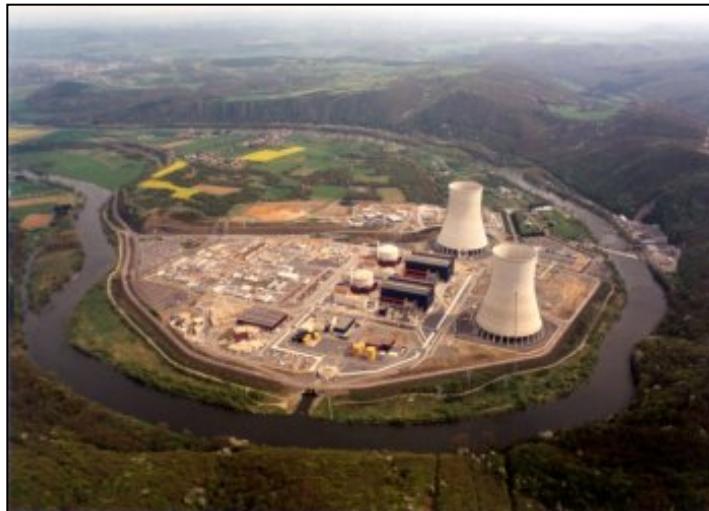


Precise signal
at small mixing
(need ≥ 2 expts or spectral data in 1 expt)

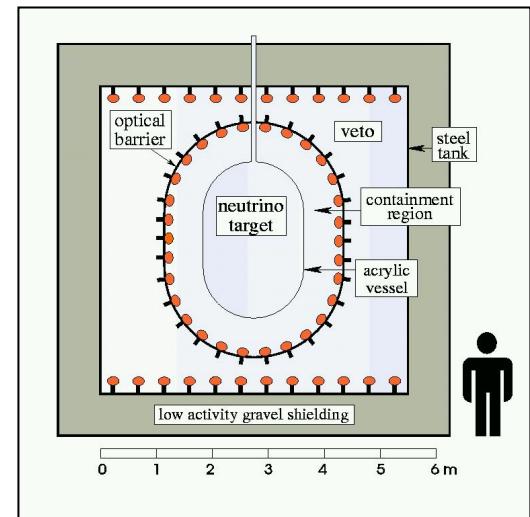


Precise signal
at large mixing

The short-baseline reactor experiment CHOOZ (1998+)



$L \sim 1 \text{ km} \rightarrow$



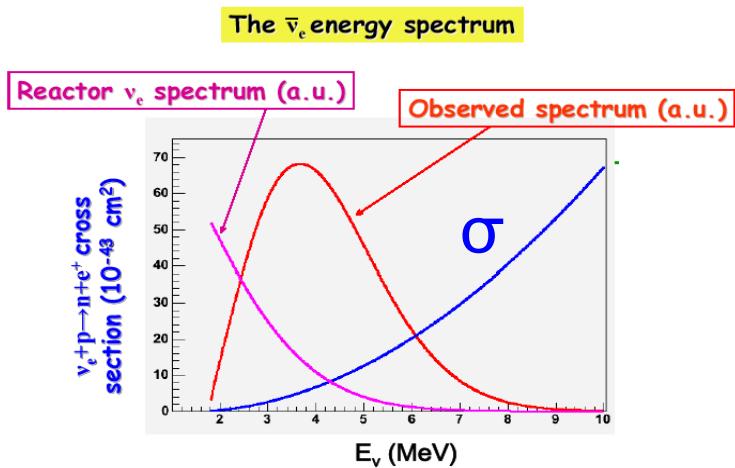
Probably (one of) the most cited **negative** results ever!

First data: Phys. Lett. B 466, 415 (1999) >1900 cites

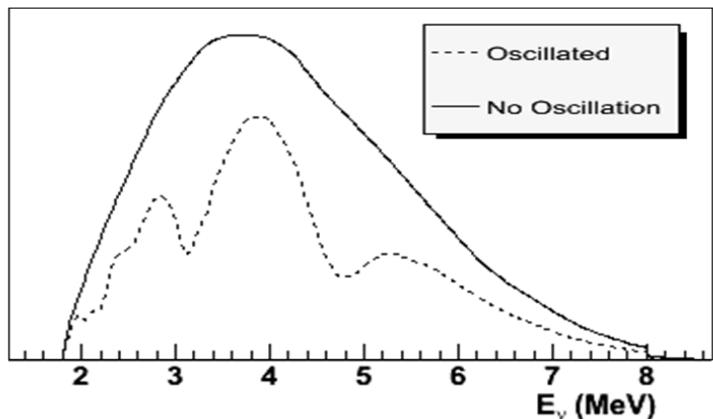
Final data: Eur. Phys. J. C 27, 331 (2003) >1300 cites

CHOOZ reactor results (1998-2003)

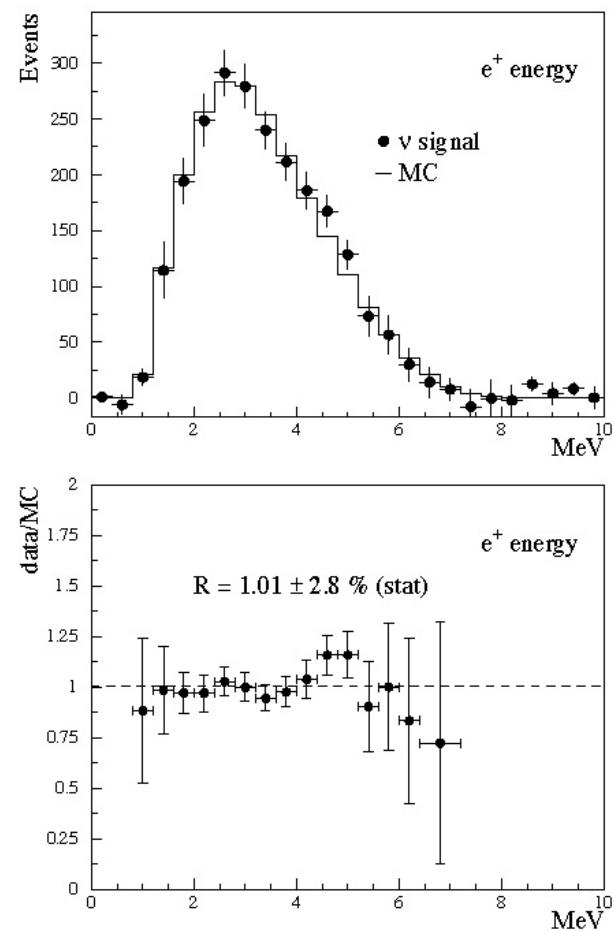
Expected spectrum (no oscill.):



With oscillations (qualitative):



CHOOZ: no oscillations
within few % error



Interpretation

One mass scale dominance:

$$P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L / 4E_\nu)$$

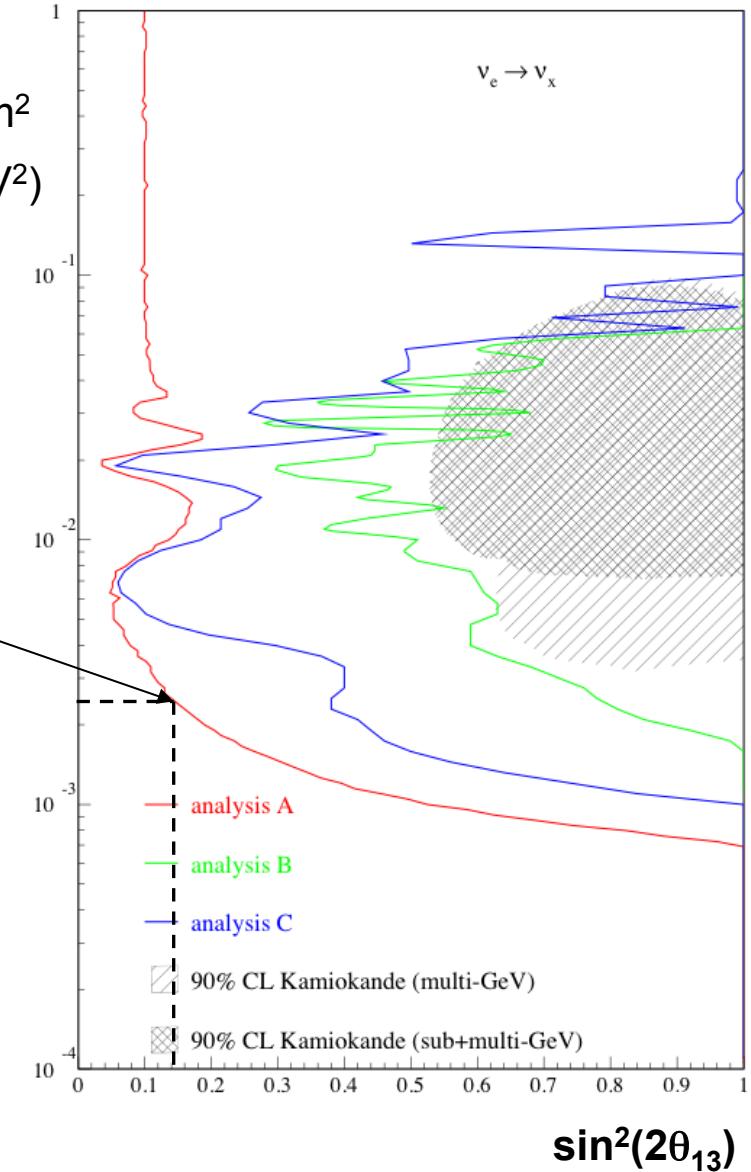
For any value of Δm^2 in the range allowed by atmospheric data (next Lect.), get stringent upper bound on θ_{13}

**$\sin^2 \theta_{13} < \text{few \%}$
(depending on Δm^2)**

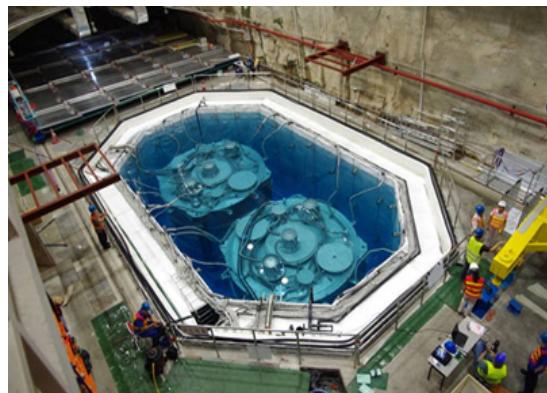
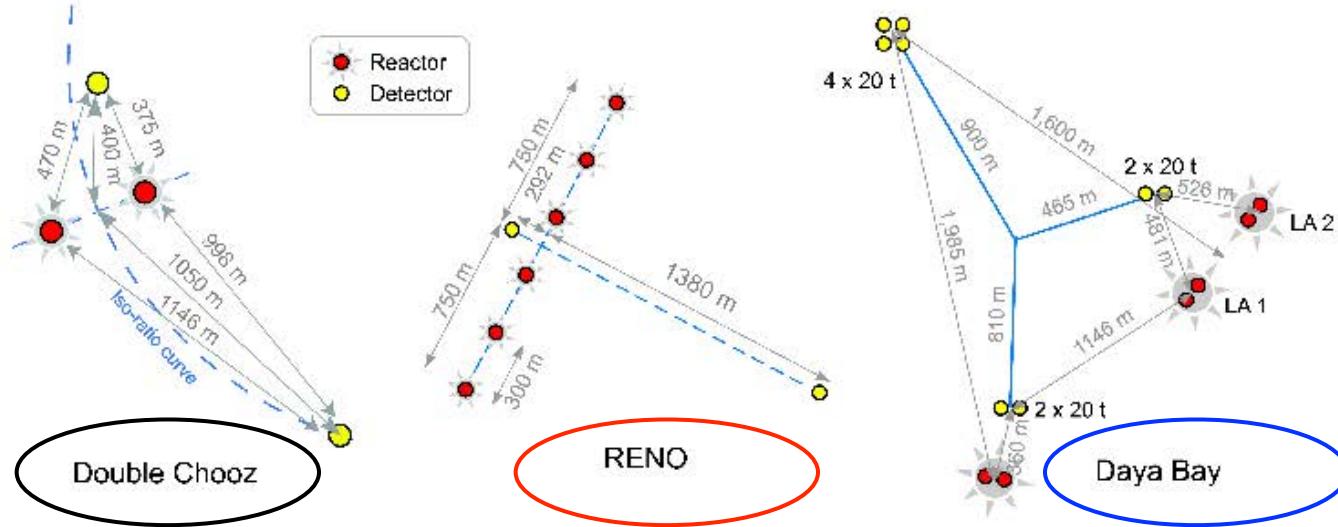
... Nobody could know at that time, but θ_{13} was just behind the corner (less than a factor of two in sensitivity!)

In any case, it was clear that, to reach higher θ_{13} sensitivity, need to use a second (close) detector to reduce systematics by far/near comparison →

CHOOZ exclusion plot



Current SBL reactor expts with near & far detectors (ND & FD)



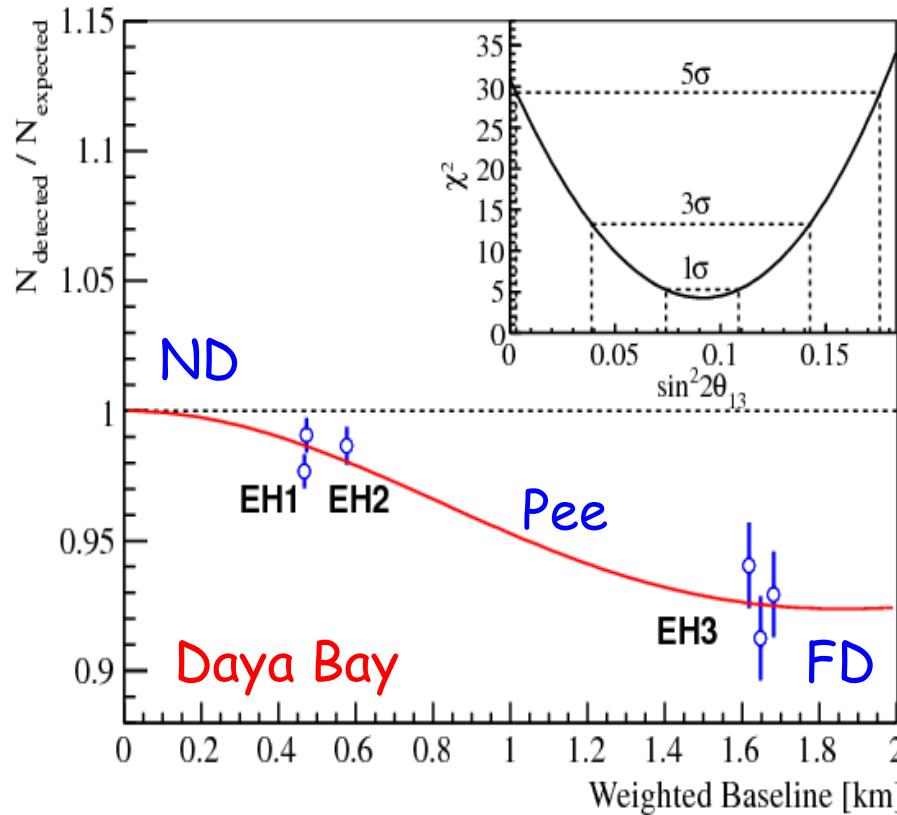
E.g, for
Daya Bay:

← ND

FD →



2012: discovery of $\theta_{13} > 0$! ($\sin^2 \theta_{13} \sim 0.022$ at \sim fixed Δm^2)

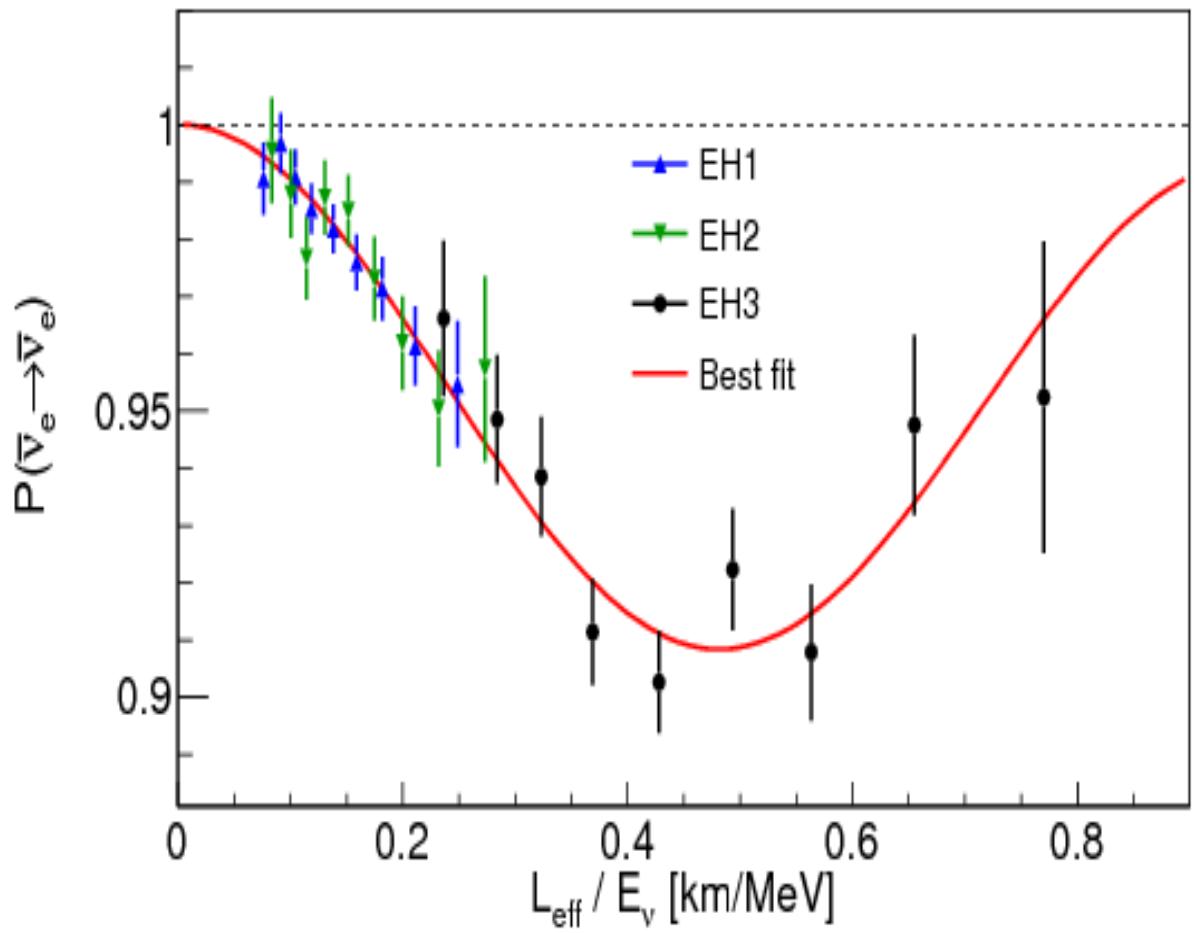
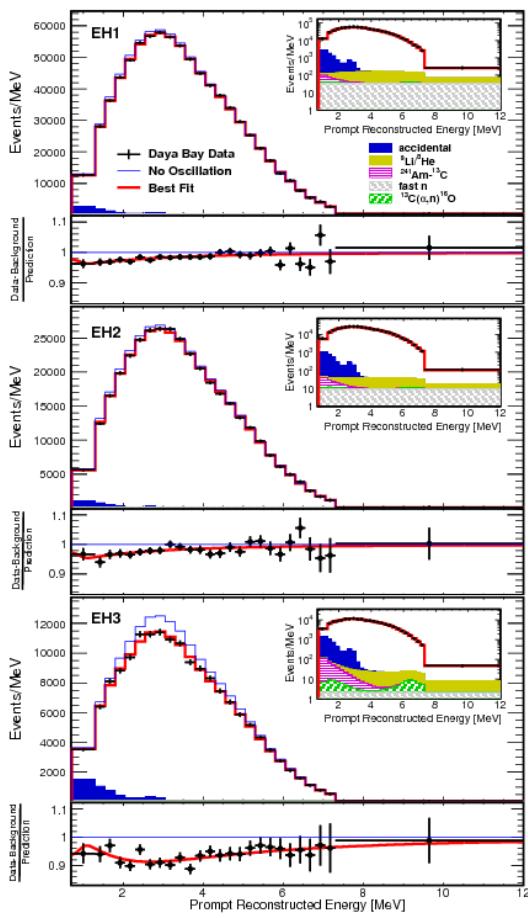


Daya Bay (& RENO): disappearance at FD w.r.t. \sim unoscillated at ND

Double Chooz results (FD only) were also consistent with Daya Bay & RENO.

Interestingly, approximate value of θ_{13} was previously hinted from other data: weaker signals were also coming from other experiments before 2012 (see Lec. II).

2013: more data → spectral analys. → $\frac{1}{2}$ osc. cycle in L/E!

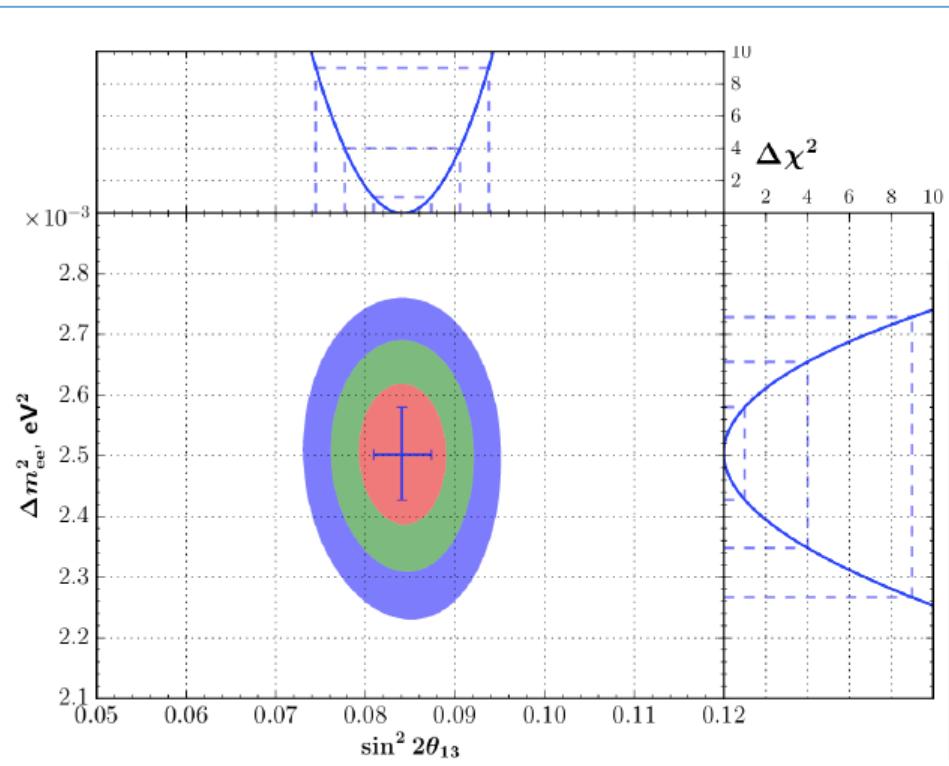


(Above: Daya Bay data. 1/2 cycle also observed in RENO, with less statistics)

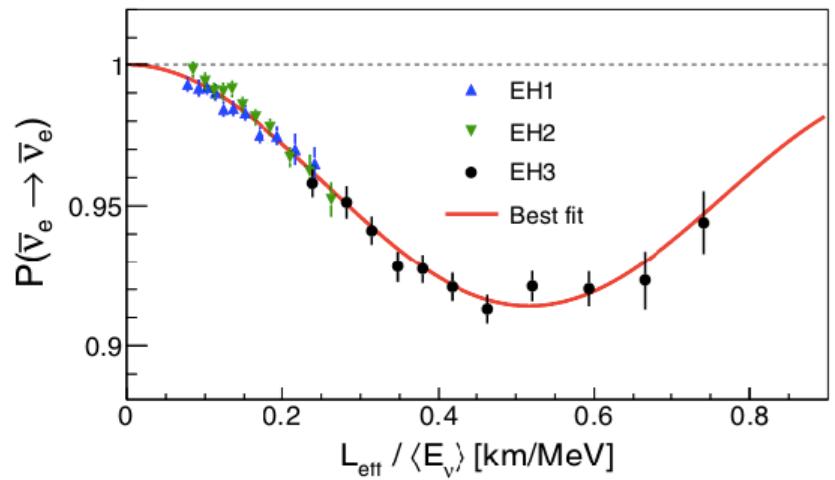
Daya Bay results ~2016 (note: still running)

(see also RENO and Double Chooz)

Current accuracy requires to go beyond the 2ν approximation
 (see exercises in tutorial): 



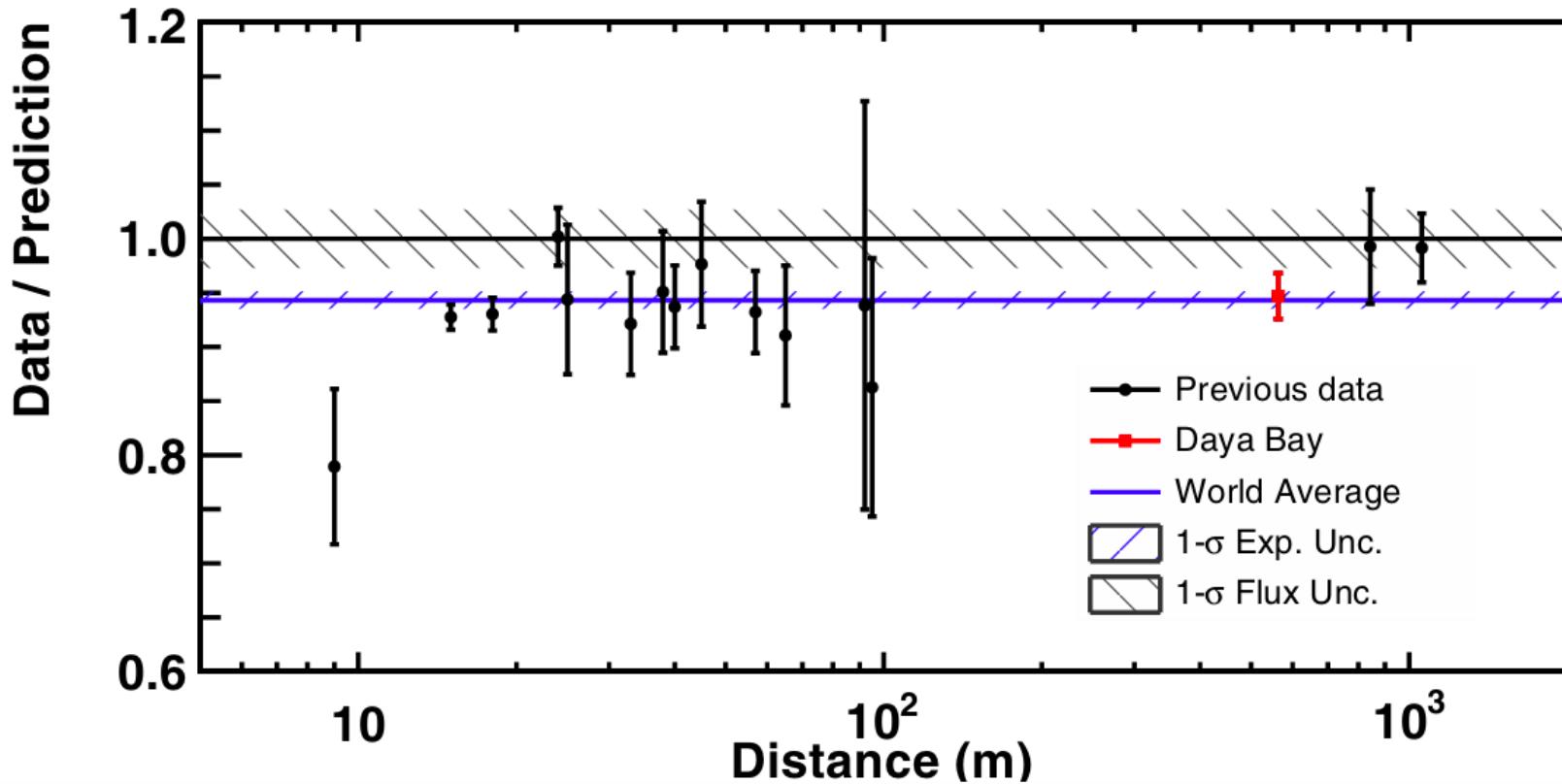
$$P = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \frac{1.267 \Delta m_{21}^2 L}{E} - \sin^2 2\theta_{13} \sin^2 \frac{1.267 \Delta m_{ee}^2 L}{E}.$$



 Precise measurement of both Δm^2 ("mass") and θ_{13} ("mixing") oscill. parameters in $\bar{\nu}_e \rightarrow \nu_e$ channel

 Position of oscillation dip in L/E determines Δm^2 , while depth fixes θ_{13}

Some open problems in this field... (1)



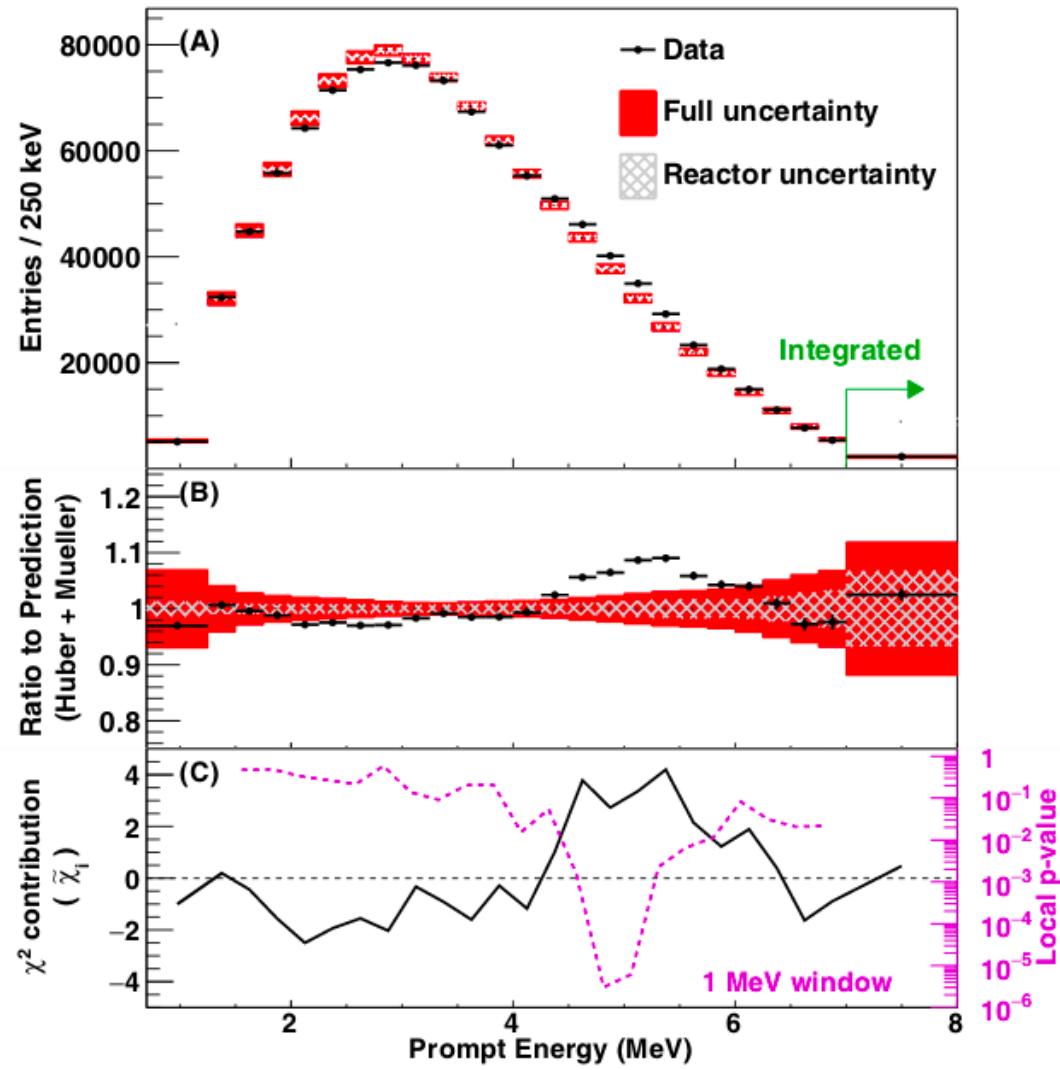
Absolute flux normalization somewhat below expectations:
overestimated flux, or fast disappearance into a new ν state?
("reactor ν anomaly" and oscillations into "sterile neutrinos")

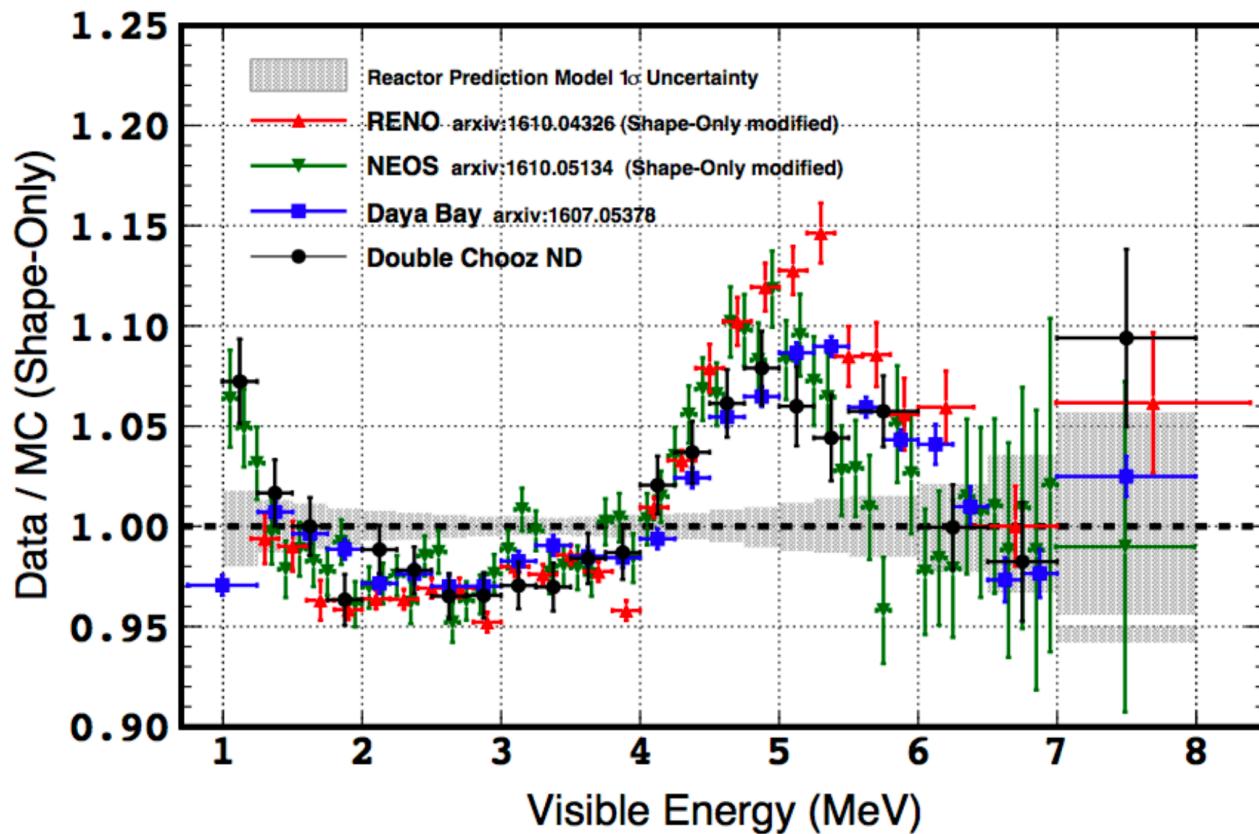
Some open problems in this field... (2)

Flux spectrum shape somewhat different from expectations:
~5 MeV “bump”

Incomplete nuclear physics description of decay chains or spectra?
Miscalibrations?

(probably unrelated to oscillations, but affects systematics)





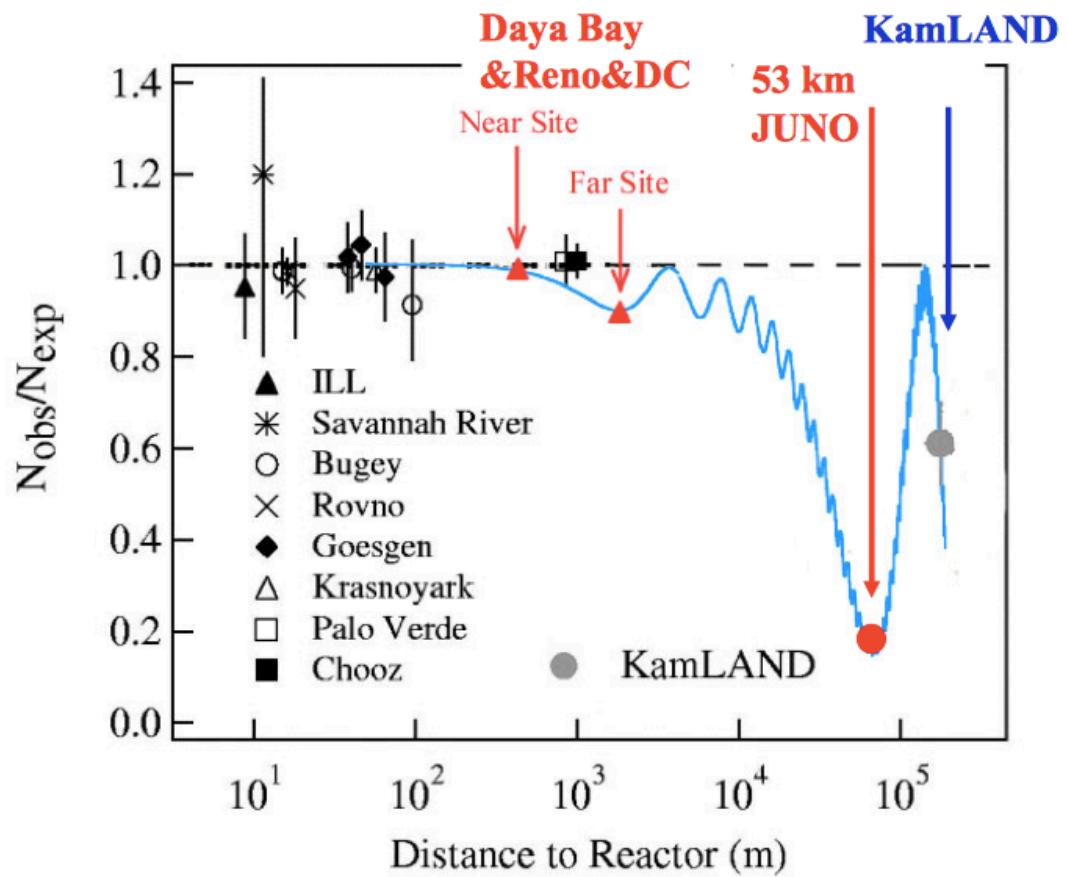
**“Bump” is seen by all experiments.
Must take into account “spectral shape” errors.**

Some open problems in this field... (3)

Can one reach the accuracy needed to observe also the oscillations driven by δm^2 , as well as the interference between δm^2 and $\pm \Delta m^2 \rightarrow$ mass ordering effects?

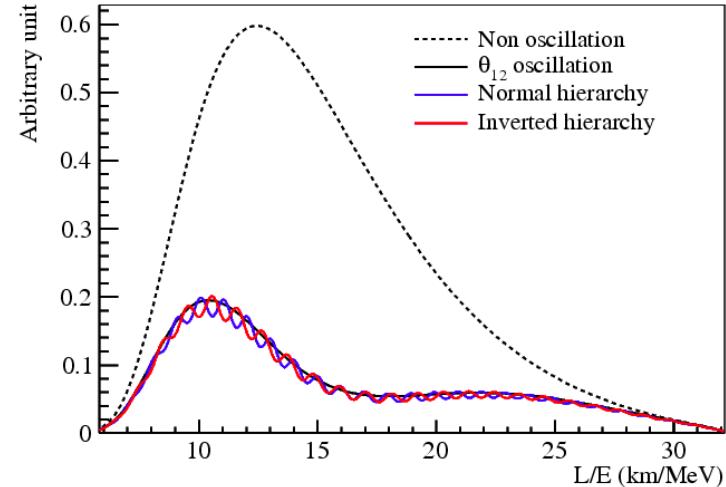
Not only a problem of accuracy but also of L/E \rightarrow L~50 km ("medium baselines") needed to observe both oscillations at the same time within the reactor spectrum band width in energy

\rightarrow JUNO



From JUNO proposal,
arXiv:1507.05613v2, p. 36:

Hierarchy effects → advancement or retardation of “phase” $\pm\phi$ for the fast-oscillating component



JUNO is designed to resolve the neutrino MH using precision spectral measurements of reactor antineutrino oscillations. Before giving the quantitative calculation of the MH sensitivity, we shall briefly review the principle of this method. The electron antineutrino survival probability in vacuum can be written as [69, 79, 94]:

$$\begin{aligned} P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} &= 1 - \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \quad (2.1) \\ &= 1 - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \phi) \right] - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}, \end{aligned}$$

where $\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4E$, in which L is the baseline, E is the antineutrino energy,

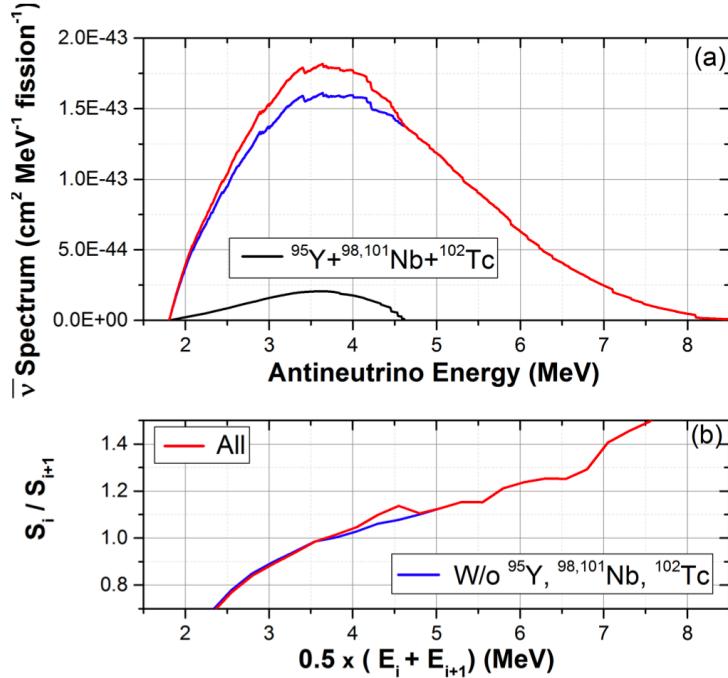
$$\sin \phi = \frac{c_{12}^2 \sin(2s_{12}^2 \Delta_{21}) - s_{12}^2 \sin(2c_{12}^2 \Delta_{21})}{\sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}}}, \quad \cos \phi = \frac{c_{12}^2 \cos(2s_{12}^2 \Delta_{21}) + s_{12}^2 \cos(2c_{12}^2 \Delta_{21})}{\sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}}},$$

and [95, 96]

$$\Delta m_{ee}^2 = \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2. \quad (2.2)$$

(see also tutorial). **Very challenging goal, at the frontier of the field!**

At this level of accuracy, need to account for the spectral “sawtooth” structure, due to hundreds of beta decay branches
 → need near hi-resolution detector (JUNO-TAO); improve theory?



figures from 1710.00092

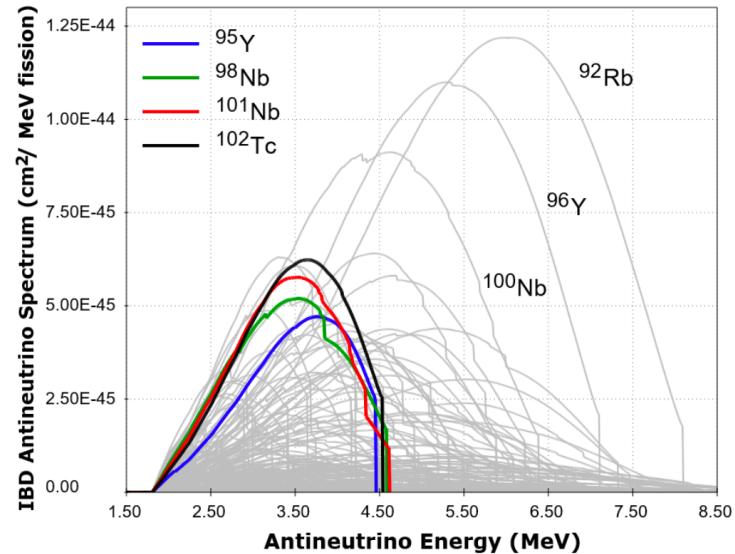
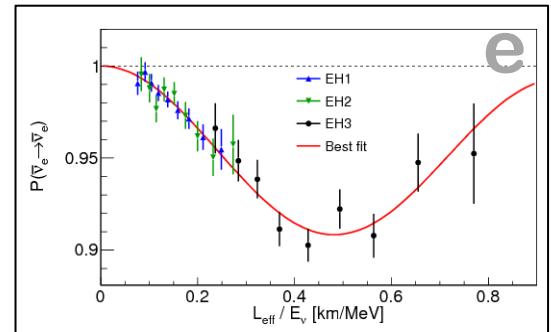


FIG. 6. Calculated IBD antineutrino spectra from all the fission products, highlighting the ^{95}Y , $^{98,101}\text{Nb}$, and ^{102}Tc ones.

This is just one example of the general need for improved understanding of nuclear / hadronic / strong interaction effects in neutrino physics
 → emerging interdisciplinary field of “Electroweak Nuclear Physics”

1st lecture summary: Intro + ...

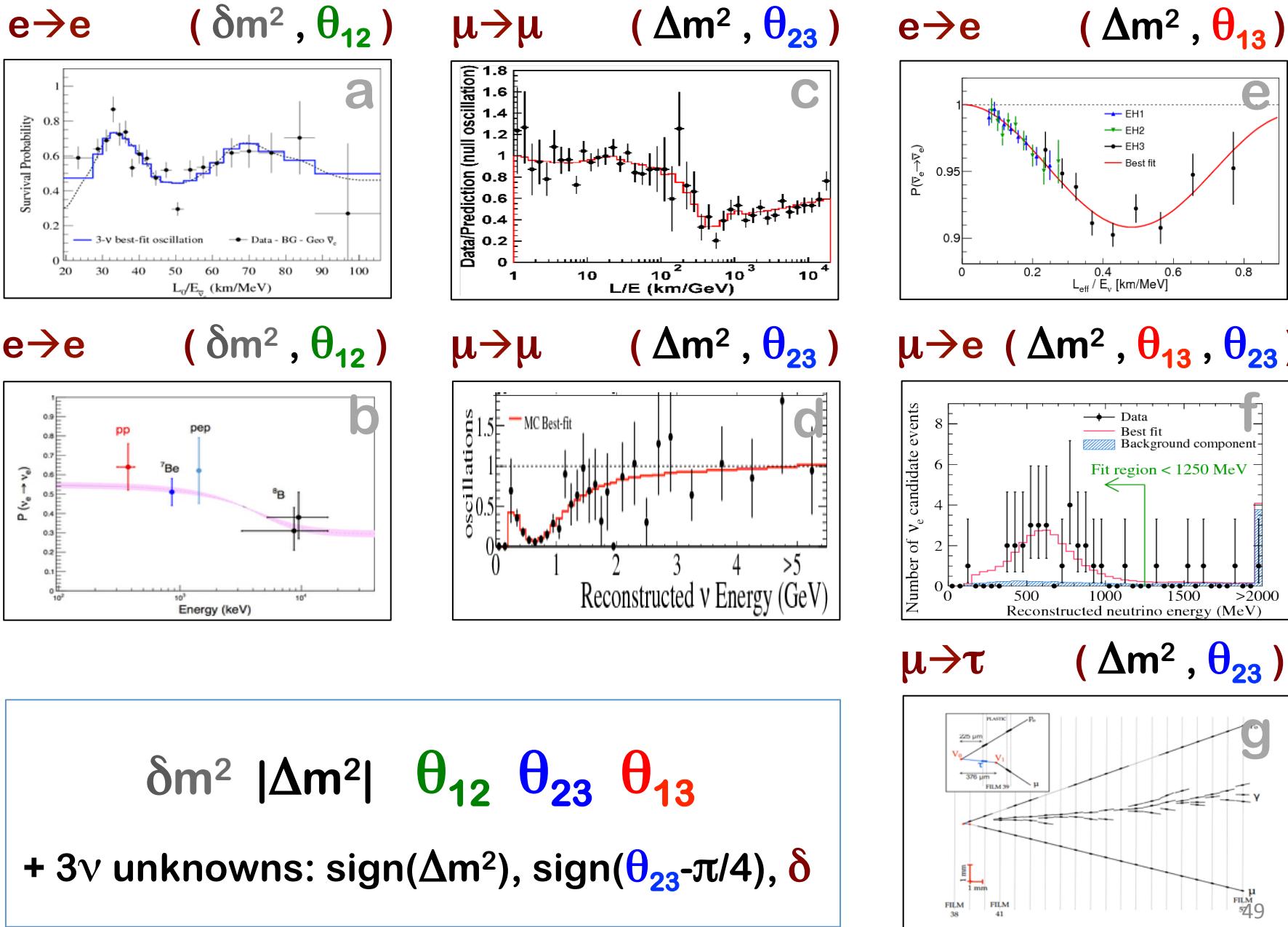
$e \rightarrow e$ (Δm^2 , θ_{13})



$|\Delta m^2|$

θ_{13}

2nd lecture plan (rather intensive...)



Worked-out exercises

PhD School Tutorial on ν oscillation probabilities

Elvio Lisi
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- Most of our current knowledge of neutrino properties is based on the observed phenomenon of ν flavor oscillations: $\nu_\alpha \rightarrow \nu_\beta$.
- This tutorial contains a set of worked-out exercises of ν oscillation probabilities in vacuum and matter, with increasing difficulty.
- Basically all the results of the exercises are of interest for the current phenomenology, e.g., as described in the review on Neutrino masses & mixings of the Particle Data Group (PDG).
- Some exercises will bring the student at the frontier of the current discussion of precision oscillation experiments; e.g., we shall prove eq. (2.1) of arXiv: 1507.05613 (interesting for future reactor experiments such as JUNO or RENO-50) and eq. (3.5) of arXiv: 1512.06148 (interesting for current and future accelerator experiments such as DUNE).
- Apologies for hand-writing and possible typos!

Exercises related to the 1st Lecture:

INDEX:

- Conventions about mixing matrix \mathbb{U}
- Conventions about mixing: \mathbb{U} versus \mathbb{U}^*
- Conventions about mass states
- Conventions about flavor indices
- Exercise: 3ν oscillations in vacuum - general case for $P_{\alpha\beta}$
- Exercise: changing units in vacuum
- Exercise: CP(T) properties of $P_{\alpha\beta}$ in vacuum
- Exercise: conditions to observe \mathcal{CP} in vacuum
- Exercise: $P_{\alpha\beta}$ in vacuum for $(\Delta m^2 x / 4E) \sim \Theta(1)$ and $(\delta m^2 x / 4E) \ll 1$
- Exercise: $P_{\alpha\beta}$ in vacuum for $(\Delta m^2 x / 4E) \gg 1$ and $(\delta m^2 x / 4E) \sim \Theta(1)$
- Exercise: P_{ee} in vacuum, general case (3ν)
- Exercise: P_{ee} in vacuum, general case in alternative formulation \leftarrow Useful for JUNO, RENO-50
- Sketchy proof of $V = \sqrt{2} G_F N_e$ in matter
- Exercise: changing units in matter
- Exercise: 2ν oscillations in matter with constant density
- Exercise: 2ν oscillations in matter with slowly varying density
- 3ν oscillations in Matter: general reduction tools
- Calculation of $P_{e\mu}^{3\nu}$ in matter at 2nd order in θ_{13} , δm^2 \leftarrow Useful for precision LBL accelerators