Behavior of Neutral Particles under Charge Conjugation

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Some properties are discussed of the θ^0, a heavy boson that is known to decay by the process $\theta^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the θ^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all θ^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

I

IT is generally accepted that the microscopic laws of physics are invariant to the operation of charge conjugation (CC); we shall take the rigorous validity of this postulate for granted. Under CC, every particle is carried into what we shall call its "antiparticle". The principle of invariance under CC implies, among other things, that a particle and its antiparticle must have exactly the same mass and intrinsic spin and must have equal and opposite electric and magnetic moments.

A charged particle is thus carried into one of opposite charge. For example, the electron and positron are each other's antiparticles; the π^+ and π^- and the μ^+ and μ^- mesons are supposed to be pairs of antiparticles; and the proton must possess an antiparticle, the "antiproton".

Neutral particles fall into two classes, according to their behavior under CC:

(a) Particles that transform into themselves, and which are thus their own antiparticles. For instance the photon and the π^0 meson are bosons that behave in this fashion. It is conceivable that fermions, too, may belong to this class. An example is provided by the Majorana theory of the neutrino.

In a field theory, particles of class (a) are represented by "real" fields, i.e., Hermitian field operators. There is an important distinction to be made within this class, according to whether the field takes on a plus or a minus sign under CC. The operation of CC is performed by a unitary operator $\mathfrak{C}$. The photon field operator $A_\mu(x)$ satisfies the relation

$$\mathfrak{C}A_\mu(x)\mathfrak{C}^{-1} = -A_\mu(x), \tag{1}$$

while for the π^0 field operator $\phi(x)$ we have

$$\mathfrak{C}\phi(x)\mathfrak{C}^{-1} = \phi(x). \tag{2}$$

Equation (1) expresses the obvious fact that the electromagnetic field changes sign when positive and negative charges are interchanged; that the π^0 field must not change sign can be inferred from the observed two-photon decay of the π^0.

We are effectively dealing here with the "charge conjugation quantum number" C, which is the eigenvalue of the operator $\mathfrak{C}$, and which is rigorously conserved in the absence of external fields. If only an odd (even) number of photons is present, we have $C=-1(+1)$; if only π^0's are present, $C=+1$; etc. As a trivial example of the conservation of C, we may mention that the decay of the π^0 into an odd number of photons is forbidden.[1]

We may recall that a state of a neutral system composed of charged particles may be one with a definite value of C. For example, the $^1S_0$ state of positronium has $C=+1$; a state of a π^+ and a π^- meson with relative orbital angular momentum l has $C=(-1)^l$; etc.

For fermions, as for bosons, a distinction may be made between "odd" and "even" behavior of neutral fields of class (a) under CC. However, the distinction is then necessarily a relative rather than an absolute one.[2] In other words, it makes no sense to say that a single such fermion field is "odd" or "even", but it does make sense to say that two such fermion fields have the same behavior under CC or that they have opposite behavior.

(b) Neutral particles that behave like charged ones in that: (1) they have antiparticles distinct from themselves; (2) there exists a rigorous conservation law that prohibits virtual transitions between particle and antiparticle states.

A well-known member of this class is the neutron N, which can obviously be distinguished from the antineutron $\bar{N}$ by the sign of its magnetic moment. The law that forbids the virtual processes $N \rightleftarrows \bar{N}$ is the law

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[1] For other consequences of invariance under charge conjugation see A. Pais and R. Jost, Phys. Rev. 87, 871 (1952); L. Wolfenstein and D. G. Ravenhall, Phys. Rev. 88, 279 (1952); L. Michel, Nuovo cimento 10, 319 (1953).

[2] This is due to the fact that fermion fields can interact only bilinearly. For example, one easily sees that the interactions responsible for $P \rightarrow N + e^+ + \nu$ would not lead to physically distinguishable results if ν were either an even or an odd Majorana neutrino.

of conservation of baryons,[3] which is, so far as we know, exact, and which states that n, the number of baryons minus the number of antibaryons, must remain unchanged. Clearly all neutral hyperons likewise belong to this class. Although we know of no "elementary" bosons in the same category, we have no *a priori* reason for excluding their existence. [Note that the H atom is an example of a "non-elementary" boson of class (b).]

Particles in this class are represented by "complex" fields, and the operation of charge conjugation transforms the field operators into their Hermitian conjugates.

It is the purpose of this note to discuss the possible existence of neutral particles that seem, at first sight, to belong neither to class (a) nor to class (b).

II

Recently, attempts have been made to interpret hyperon and K-particle phenomena by distinguishing sharply between strong interactions, to which essentially all production of these particles is attributed, and weak interactions, which are supposed to induce their decay. It is necessary to assume that the strong interactions give rise to "associated production "exclusively.[4]

Certain detailed schemes[5] which meet this requirement lead to further specific properties of particles and interactions. In particular, a suggestion has been made about the θ^0 particle, a heavy boson that is known to decay according to the scheme:

$$\theta^0 \rightarrow \pi^+ + \pi^- + (\sim 215 \text{ Mev}). \qquad (3)$$

It has been proposed that the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself, and that in the absence of the weak decay interactions, there is a conservation law that prohibits the virtual transitions $\theta^0 \rightleftarrows \bar{\theta}^0$. [In our present language, we would say that the θ^0 belongs to class (b) if the weak interactions are turned off.] This conservation law also leads to stability of the θ^0 and $\bar{\theta}^0$; moreover, while it permits the reaction $\pi^- + P \rightarrow \Lambda^0 + \theta^0$ it forbids the analogous process $\pi^- + P \rightarrow \Lambda^0 + \bar{\theta}^0$. In the schemes under consideration this is the same law that forbids the reaction: 2 neutrons$\rightarrow 2\Lambda^0$.

1a) The weak interactions that must be invoked to account for the observed decay (3) evidently cause the conservation law to break down (a fact that is, of course, of little importance for production). This breakdown makes the forbiddenness of the processes $\theta^0 \rightleftarrows \bar{\theta}^0$ no longer absolute, as can be seen from the following argument: In the decay (3) the pions are left in a state with a definite relative angular momentum and therefore with a definite value of the charge-conjugation quantum number C. The charge-conjugate process,

$$\bar{\theta}^0 \rightarrow \pi^+ + \pi^-, \qquad (4)$$

[3] Nucleons and hyperons are collectively referred to as baryons.
[4] A. Pais, Phys. Rev. 86, 663 (1952).
[5] M. Gell-Mann, Phys. Rev. 92, 833 (1953); A. Pais, Proc. Nat, Acad. Sci. U. S. 40, 484, 835 (1954); M. Gell-Mann and A. Pais. *Proceedings of the International Conference Glasgow* (Pergamon Press, London, to be published).

must also occur and must leave the pions in the same state; moreover the reverse of (4) must also be possible, at least as a virtual process. Therefore the virtual transition $\theta^0 \rightleftarrows \pi^+ + \pi^- \rightleftarrows \bar{\theta}^0$ is induced by the weak interactions, and we are no longer dealing exactly with case (b). 1b)

In order to treat this novel situation, we shall find it convenient to introduce a change of representation. Since the θ^0 and $\bar{\theta}^0$ are distinct, they are associated, in a field theory, with a "complex" field ψ (a non-Hermitian field operator), just as in case (b). Under charge conjugation ψ must transform according to the law:

$$\mathcal{C}\psi\mathcal{C}^{-1} = \psi^+, \quad \mathcal{C}\psi^+\mathcal{C}^{-1} = \psi, \qquad (5)$$

where ψ^+ is the Hermitian conjugate of ψ. Let us now define

$$\psi_1 \equiv (\psi + \psi^+)/\sqrt{2} \qquad (6)$$

and

$$\psi_2 \equiv (\psi - \psi^+)/\sqrt{2}i, \qquad (7)$$

so that ψ_1 and ψ_2 are Hermitian field operators satisfying

$$\mathcal{C}\psi_1\mathcal{C}^{-1} = \psi_1, \qquad (8)$$

and

$$\mathcal{C}\psi_2\mathcal{C}^{-1} = -\psi_2. \qquad (9)$$

The fields ψ_1 and ψ_2 evidently correspond to class (a); in fact ψ_1 is "even" like the π^0 field and ψ_2 is "odd" like the photon field. Corresponding to these real fields there are quanta, which we shall call θ_1^0 and θ_2^0 quanta. 1c)
The relationship that these have to the quanta of the complex ψ field, which we have called θ^0 and $\bar{\theta}^0$, may be seen from an example: Let Ψ_1 be the wave-functional representing a single θ_1 quantum in a given state, while Ψ_0 and Ψ_0' describe a θ^0 and a $\bar{\theta}^0$, respectively, in the same state. Then we have

$$\Psi_1 = (\Psi_0 + \Psi_0')/\sqrt{2}.$$

Thus the creation of a θ_1 (or, for that matter, of a θ_2) corresponds physically to the creation, with equal probability and with prescribed relative phase, of either a θ^0 or a $\bar{\theta}^0$. Conversely, the creation of a θ^0 (or of a $\bar{\theta}^0$) corresponds to the creation, with equal probability and prescribed relative phase, of either a θ_1^0 or a θ_2^0.

The transformation (6), (7) to two real fields could equally well have been applied to a complex field of class (b), such as that associated with the neutron. However, this would not be particularly enlightening. It would lead us, for instance, to describe phenomena involving neutrons and antineutrons in terms of "N_1 and N_2 quanta". Now a state with an N_1 (or N_2) quantum is a mixture of states with different values of the quantum number n, the number of baryons minus the number of antibaryons. But the law of conservation of baryons requires this quantity to be a constant of the motion, and so a mixed state can never arise from a pure one. Since in our experience we deal exclusively 1d) with states that are pure with respect to n, the introduction of N_1 and N_2 quanta can only be a mathematical device that distracts our attention from the truly physical particles N and $\bar{N}$.

On the other hand, it can obviously not be argued in a similar way that the θ_1^0 and θ_2^0 quanta are completely unphysical, for the corresponding conservation law in that case is not a rigorous one. Always assuming the correctness of our model of the θ^0, we still have the θ^0 and $\bar{\theta}^0$ as the primary objects in production phenomena. But we shall now show that the decay process is best described in terms of θ_1^0 and θ_2^0.

The weak interactions, in fact, must lead to very different patterns of decay for the θ_1^0 and θ_2^0 into pions and (perhaps) γ rays; any state of pions and/or γ rays that is a possible decay mode for the θ_1^0 is not a possible one for the θ_2^0, and *vice versa*. This is because, according to the postulate of rigorous CC invariance, the quantum number C is conserved in the decay; the θ_1^0 must go into a state that is even under charge conjugation, while the θ_2^0 must go into one that is odd. Since the decay modes are different and even mutually exclusive for the θ_1^0 and θ_2^0, their rates of decay must be quite unrelated. There are thus two independent lifetimes, one for the θ_1^0, and one for the θ_2^0.

An important illustration of the difference in decay modes of the θ_1^0 and θ_2^0 is provided by the two-pion disintegration. We know that reaction (3) occurs; therefore at least one of the two quanta θ_1^0 and θ_2^0, say θ_1^0, must be capable of decay into two charged pions. The final state of the two pions in the θ_1^0 decay is then even under charge conjugation like the θ_1^0 state itself. These two pions are thus in a state of even relative angular momentum and therefore of even parity. So the θ_1^0 must have even spin and even parity. Now we assume that the θ^0 has a definite intrinsic parity, and therefore the parity and spin of the θ_2^0 must be the same as those of the θ_1^0, both even. If the θ_2^0 were to decay into two pions, these would again be in a state of even relative angular momentum and thus even with respect to charge conjugation. However, the θ_2^0 is itself odd under charge conjugation; its decay into two pions is therefore forbidden.

Alternatively, if the θ_2^0 is the one that actually goes into two pions, then the spin and parity of θ_1^0 and the θ_2^0 are both odd, and so the θ_1^0 cannot decay into two pions.

Of the θ_1^0 and the θ_2^0, that one for which the two-pion decay is forbidden may go instead into $\pi^+ + \pi^- + \gamma$ or possibly into three pions (unless the spin and parity of the θ^0 are $0+$), etc.

While we have seen that the θ_1^0 and θ_2^0 may each be assigned a lifetime, this is evidently not true of the θ^0 or $\bar{\theta}^0$. Since we should properly reserve the word "particle" for an object with a unique lifetime, it is the θ_1^0 and θ_2^0 quanta that are the true "particles". The θ^0 and the $\bar{\theta}^0$ must, strictly speaking, be considered as "particle mixtures."

1e)

It should be remarked that the θ_1^0 and the θ_2^0 differ not only in lifetime but also in mass, though the mass difference is surely tiny. The weak interactions responsible for decay cause the θ_1^0 and the θ_2^0 to have their respective small level widths and correspondingly must produce small level shifts which are different for the two particles.

To sum up, our picture of the θ^0 implies that it is a particle mixture exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that *not more than half of all* θ^0*'s* can undergo the familiar decay into two pions.[6]

We know experimentally that the lifetime τ for the decay mode (3) (and hence for all decay modes that may compete with this one) is about 1.5×10^{-10} sec. The present qualitative considerations, even if at all correct in their underlying assumptions, do not enable us to predict the value of the "second lifetime" τ' of the θ^0. Nevertheless, the examples given above of decays responsible for the second lifetime lead one to suspect that[7] $\tau' \gg \tau$. As an illustration of the experimental implications of this situation consider the study of the reaction $\pi^- + P \rightarrow \Lambda^0 + \theta^0$ in a cloud chamber. If the reaction occurs and subsequently $\Lambda^0 \rightarrow P + \pi^-$, $\theta^0 \rightarrow \pi^+ + \pi^-$, there should be a reasonable chance to observe this whole course of events in the chamber, as the lifetime for the Λ^0 decay ($\sim 3.5\times10^{-10}$ sec) is comparable to τ. However, if it is true that $\tau' \gg \tau$, it would be very difficult to detect the decay with the second lifetime in the cloud chamber with its characteristic bias for a limited region of lifetime values.[8] Clearly this also means an additional complication in the determination from cloud chamber data as to whether or not production always occurs in an associated fashion. In some such cases the analysis of the reaction $\pi^- + P \rightarrow \Lambda^0 + ?$ may still be pushed further, however, if one assumes that besides the Λ^0 only one other neutral object is formed.[9]

1f)

At any rate, the point to be emphasized is this: a neutral boson may exist which has the characteristic θ^0 mass but a lifetime $\neq \tau$ and which may find its natural place in the present picture as the second component of the θ^0 mixture.

1g)

One of us, (M. G.-M.), wishes to thank Professor E. Fermi for a stimulating discussion.

[6] Note that if the spin and parity of the θ^0 are even, then the θ_1^0 may decay into $2\pi^0$'s as well as into $\pi^+ + \pi^-$.

[7] The process $\theta^0 \rightarrow \pi^+ + \pi^- + \gamma$ may occur as a radiative correction to the allowed decay into $\pi^+ + \pi^-$ connected with the lifetime τ; see S. B. Treiman, Phys. Rev. **95**, 1360 (1954). The process may also occur as one of the principal decay modes associated with the second lifetime τ'. The latter case may be distinguished from the former not only by the distinct lifetime but also by a different energy spectrum which probably favors higher γ-ray energies; such a spectrum is to be expected in a case where the emission of the γ ray is not just part of the "infrared catastrophe", but is an integral part of the decay process.

[8] See, e.g., Leighton, Wanlass, and Anderson, Phys. Rev. **89**, 148 (1953), Sec. III.

[9] See Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **91**, 1287 (1953).

The location of the curve on the θ axis can be shifted to larger angles by increasing V_2 and R (thus maintaining the well-known VR ambiguity in the optical model) and to smaller angles by increasing V_1 and $|\eta|$, the energy difference between entrance and exit channels, which is determined experimentally and not treated as a parameter. The effect of varying V_2 is much larger than that of varying V_1, since V_2 determines two optical-model wave functions, V_1 only determines one. It was found that a large difference between V_2 and V_1 was necessary to locate the curves properly. The values quoted are not unique.

The over-all width is determined almost exclusively by R_b. Increasing R_b decreases the over-all width and increases the magnitude of the cross section at the center of the curve. It is found that when the best value of R_b is used in each state, the relative magnitudes are automatically fitted well.

The effects of increasing W_1, W_2, and a are small. Increasing W_1 and W_2 decreases the magnitude of both curves slightly. In fitting the p-state curve, V_2 and V_1 have opposite effects on the ratio of peak heights. Increasing V_2 increases the ratio. Increasing both V_1 and V_2 reduces the depth of the minimum by a very small amount.

The physical conclusions which we tentatively draw from this calculation are rather significant. For finite potentials there cannot be significant differences between single-particle wave functions whose principal quantum number, angular momentum, binding energy, and rms radius are given. Hence it seems that a distorted-wave analysis of $(p, 2p)$ experiments determines the single-particle wave functions very well.

The rms radius of the charge distribution in C^{12} given by our empirical values of R_b is 2.5 F. The experimental value obtained from electron scattering is 2.4 F. The rms radius for s-state protons is 1.7 F, which is the experimental value for the α particle. Whether this is true for s states in other light nuclei is, at present, being investigated by a systematic study of the available data. Finer points concerning curve fitting are also being investigated.

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UNITARY SYMMETRY AND LEPTONIC DECAYS

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We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the V-A theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \to n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-

current conservation.[2]

Together with the octet of vector currents, j_μ, we assume an octet of axial currents, g_μ. In each of these octets we have a current with $\Delta S = 0$, $\Delta Q = 1$, $j_\mu^{(0)}$, and $g_\mu^{(0)}$, and a current with $\Delta S = \Delta Q = 1$, $j_\mu^{(1)}$, and $g_\mu^{(1)}$. Their isospin selection rules are, respectively, $\Delta I = 1$ and $\Delta I = 1/2$.

From our first assumption we then get

$$J_\mu = a(j_\mu^{(0)} + g_\mu^{(0)}) + b(j_\mu^{(1)} + g_\mu^{(1)}). \tag{1}$$

A restriction $a = b = 1$ would not ensure universality in the usual sense (equal coupling for all currents), because if J_μ [as given in Eq. (1)] is coupled, we can build a current, $b(j_\mu^{(0)} + g_\mu^{(0)}) - a(j_\mu^{(1)} + g_\mu^{(1)})$, which is not coupled. We want, however, to keep a weaker form of universality, by requiring the following:

(3) J_μ has "unit length," i.e., $a^2 + b^2 = 1$.

We then rewrite J_μ as[4]

$$J_\mu = \cos\theta(j_\mu^{(0)} + g_\mu^{(0)}) + \sin\theta(j_\mu^{(1)} + g_\mu^{(1)}), \tag{2}$$

where $\tan\theta = b/a$. Since J_μ, as well as the baryons and the pseudoscalar mesons, belongs to the octet representation of SU_3, we have relations (in which θ enters as a parameter) between processes with $\Delta S = 0$ and processes with $\Delta S = 1$.

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

2b)

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu) = \tan^2\theta M_K(1 - M_\mu{}^2/M_K{}^2)^2/M_\pi(1 - M_\mu{}^2/M_\pi{}^2)^2. \tag{3}$$

From the experimental data, we then get[5,6]

$$\theta = 0.257. \tag{4}$$

For an independent determination of θ, let us consider $K^+ \to \pi^0 + e^+ + \nu$. The matrix element for this process can be connected to that for $\pi^+ \to \pi^0 + e^+ + \nu$, known from the conserved vector-current hypothesis (2nd assumption). From the rate[6] for $K^+ \to \pi^0 + e^+ + \nu$, we get

$$\theta = 0.26. \tag{5}$$

The two determinations coincide within experimental errors; in the following we use $\theta = 0.26$.

We go now to the leptonic decays of the baryons, of the type $A \to B + e + \nu$. The matrix element of any member of an octet of currents among two baryon states (also members of octets) can be expressed in terms of two reduced matrix elements[7]

$$\langle A | j_\mu^{(i)} + g_\mu^{(i)} | B \rangle = i f_{ABi} O_\mu + d_{ABi} E_\mu; \tag{6}$$

the f's and d's are coefficients defined in Gell-Mann's paper.[1,7] It is sufficient to consider only allowed contributions and write

$$O_\mu, E_\mu = F^{O,E}\gamma_\mu + H^{O,E}\gamma_\mu\gamma_5. \tag{7}$$

From the connection with the electromagnetic current we get the vector coefficients: $F^O = 1$, $F^E = 0$; from neutron decay we get

$$H^O + H^E = 1.25. \tag{8}$$

We remain with one parameter which can be determined from the rate for $\Sigma^- \to \Lambda + e^- + \bar\nu$. The relevant matrix element for this is

$$\cos\theta\langle\Sigma^- | j_\mu^{(0)} + g_\mu^{(0)} | \Lambda\rangle = \cos\theta(\tfrac{2}{3})^{1/2} E_\mu = (\tfrac{2}{3})^{1/2}\cos\theta H^E \gamma_\mu\gamma_5. \tag{9}$$

Taking the branching ratio for this mode to be 0.9×10^{-4},[8] we get

$$H^E = \pm 0.95. \tag{10}$$

The negative solution can be discarded because it produces a large branching ratio for $\Sigma^- \to n + e^- + \bar\nu$, of the order of 1%. The positive solution ($H^E = 0.95$, $H^O = 0.30$) is good, because it produces a cancellation of the axial contribution to this process. This explains the experimental result that this mode is more depressed than the $\Lambda \to p + e^- + \bar\nu$ in respect to the predictions of Feynman and Gell-Mann.[2] In Table I we give a summary of our predictions for the electron modes with $\Delta S = 1$. The branching ratios for $\Lambda \to p + e^- + \bar\nu$ and $\Sigma^- \to n + e^- + \bar\nu$ are in good agreement with experimental data.[9]

As a final remark, the vector-coupling constant for β decay is not G, but $G\cos\theta$. This gives a correction of 6.6% to the ft value of Fermi transitions, in the right direction to eliminate the discrepancy between O^{14} and muon lifetimes.

Table I. Predictions for the leptonic decays of hyperons.

Decay	Branching ratio From reference 2	Branching ratio Present work	Type of interaction
$\Lambda \to p + e^- + \bar\nu$	1.4 %	0.75×10^{-3}	$V - 0.72A$
$\Sigma^- \to n + e^- + \bar\nu$	5.1 %	1.9×10^{-3}	$V + 0.65A$
$\Xi^- \to \Lambda + e^- + \bar\nu$	1.4 %	0.35×10^{-3}	$V + 0.02A$
$\Xi^- \to \Sigma^0 + e^- + \bar\nu$	0.14 %	0.07×10^{-3}	$V - 1.25A$
$\Xi^0 \to \Sigma^+ + e^- + \bar\nu$	0.28 %	0.26×10^{-3}	$V - 1.25A$

2c)

The correction is, however, too large, leaving about 2% to be explained.[10]

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[1]M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[3]R. E. Marshak and E. C. G. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957 (Società Italiana di Fisica, Padua-Venice, 1958); Phys. Rev. 109, 1860 (1958).

[4]Similar considerations are forwarded in M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1958).

[5]The lifetimes from W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 878. The branching ratio for $K^+ \rightarrow \mu^+ + \nu$ is taken as 57.4%. W. Becker, M. Goldberg, E. Hart, J. Leitner, and S. Lichtman (to be published).

[6]B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961). These authors give the branching ratio for $K^+ \rightarrow \mu^+ + \nu$ as 64%, from which $\theta = 0.269$. Also this value agrees with that from $K^+ \rightarrow \pi^0 + e^+ + \nu$ within experimental errors.

[7]N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961). Our notation for the currents is different from the one used in this reference and by Gell-Mann; the connection is $j_\mu^{(0)} = j_\mu^1 + ij_\mu^2$, $j_\mu^{(1)} = j_\mu^4 + ij_\mu^5$.

[8]W. Willis et al. reported at the Washington meeting of the American Physical Society, 1963 [W. Willis et al., Bull. Am. Phys. Soc. 8, 349 (1963)] this branching ratio as $(0.9^{+0.5}_{-0.4}) \times 10^{-4}$. If it is allowed to vary between these limits, our predictions for the $\Sigma^- \rightarrow ne^-\bar{\nu}$ varies between 0.8×10^{-3} and 4×10^{-3}, and that for $\Lambda^0 \rightarrow pe^-\bar{\nu}$ between 1.05×10^{-3} and 0.56×10^{-3}. I am grateful to the members of this group for prepublication communication of their results.

[9]R. P. Ely, G. Gidal, L. Oswald, W. Singleton, W. M. Powell, F. W. Bullock, G. E. Kalmus, C. Henderson, and R. F. Stannard [Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 445] give the branching ratio for $\Lambda \rightarrow p + e^- + \bar{\nu}$ as $(0.85 \pm 0.3) \times 10^{-3}$, while that for $\Sigma^- \rightarrow n + e^- + \bar{\nu}$ is given (see preceding reference) as $(1.9 \pm 0.9) \times 10^{-3}$.

[10]R. P. Feynman, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 501. Recent measurements of the muon lifetime have slightly increased the discrepancy. We think that more information will be needed to decide whether our 3rd assumption can be maintained.

EXPERIMENTAL EVIDENCE ON π - π SCATTERING NEAR THE ρ AND f^0 RESONANCES, FROM $\pi^- + p \rightarrow \pi + \pi +$ NUCLEON, AT 3 BeV/c†

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This note reports some preliminary results on π - π scattering, near the 770-MeV ρ and 1250-MeV f^0 resonances. The experiment is the one reported earlier[1]; with more data measured (now about 75% of the two-prong events), we have examined the data to see to what extent they seem analyzable in terms of π - π scattering. We give a brief summary of the results, and then a few details. A more detailed report will be available later.

(1) There is evidence of a major contribution from the one-pion-exchange mechanism ("peripheral collision"), for low nucleon recoil momentum. We take the region of $\Delta^2 < \Delta_{\rm min}^2 + 10$ to be interpretable in terms of π - π scattering. (Δ^2 is the square of the four-momentum transfer to the nucleon, in units of the pion mass squared; $\Delta_{\rm min}^2$ is the lower kinematic limit, which is a function of the π - π "mass" and the incident energy)

(2) We then consider these "peripheral" (i.e., peripheral-collision) events to be representative of the angular distribution of π - π scattering. Two obvious points of caution must be mentioned here: (a) Interference effects arise from nucleon isobar production, and (b) the effective π - π scattering is off the energy shell. From detailed examination of the data, we believe neither of these effects is so severe as to grossly affect the further conclusions below. A third possible complicating effect is interference from two-pion decay of the ω, into $\pi^+\pi^-$; the possible magnitude of this effect is at present difficult for us to estimate.

(3) The spin of the f^0 is greater than zero, as reported earlier by Veillet et al.[2] We believe it is difficult to draw any conclusion from these data as to whether the spin is 2 or greater than 2. (Isospin arguments, and the data directly, exclude spin 1.)

(4) The π^- - π^0 scattering in the ρ region is con-

exchange reactions[18] ($\pi^+p \to K^+\Sigma^+$, $\pi^-p \to K^0\Lambda^0$, etc.) gives the intercepts $\alpha_{0q}=0.35$ and $\alpha_{0Q}=0.24$ (with uncertain errors). The intercepts resulting from an analysis of total cross-section data are also consistent with the values of the present analysis provided we postulate[19] that the Pomeranchuk trajectory has a small $I=0$ octet component in addition to the usual $SU(3)$ singlet component. Table I summarizes the situation on the intercepts of the q and Q trajectories.

In conclusion, the following comments may be made: Although the quality of the fits in the present case is not comparable with those which can be made with the Δ-production data, it nevertheless demonstrates that $SU(3)$ symmetry for Regge vertices and Regge behavior are consistent with the data. Further, the same mechanism seems to be operative in the production of these members of the $\frac{3}{2}^+$ decuplet. The q and Q trajectories do not seem to be degenerate,[20] and the values determined from the analysis of the $Y_1^*(1385)$-production reactions are consistent with earlier determinations from other reactions.

[18] D. D. Reeder and K. V. L. Sarma, Phys. Rev. 172, 1566 (1968).

[19] K. V. L. Sarma and G. H. Renninger, Phys. Rev. Letters 20, 399 (1969).

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[20] K. W. Lai and J. Louie [Nucl. Phys. **B19**, 205 (1970)] have examined reactions (1) and (2) with a view to testing the exchange degeneracy of the K^* and K^{**} exchanges. They find that exchange degeneracy is not indicated in these reactions. D. J. Crennell *et al.* [Phys. Rev. Letters **23**, 1347 (1969)] and P. R. Auvil *et al.* [Phys. Letters **31B**, 303 (1970)] have found that the data on meson-baryon hypercharge exchange reactions similarly do not indicate exchange degeneracy for these exchanges.

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Weak Interactions with Lepton-Hadron Symmetry*

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We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

INTRODUCTION

Weak-interaction phenomena are well described by a simple phenomenological model involving a single charged vector boson coupled to an appropriate current. Serious difficulties occur only when this model is considered as a quantum field theory, and is examined in other than lowest-order perturbation theory.[1] These troubles are of two kinds. First, the theory is too singular to be conventionally renormalized. Although our attention is not directed at this problem, the model of weak interactions we propose may readily be extended to a massive Yang-Mills model, which may be amenable to renormalization with modern techniques. The second problem concerns the selection rules and the relationships among coupling constants which are carefully and deliberately incorporated into the original phenomenological Lagrangian. Our principal concern is the fact that these properties are not necessarily maintained by higher-order weak interactions.

Weak-interaction processes, and their higher-order weak corrections, may be classified[2] according to their dependence upon a suitably introduced cutoff momentum Λ. Contributions to the S matrix of the form

$$\sum_{n=1}^{\infty} A_n(G\Lambda^2)^n$$

(where G is the usual Fermi coupling constant and A_n are dimensionless parameters) are called zeroth-order

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[1] B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. 6, 828 (1967) [Soviet J. Nucl. Phys. 6, 603 (1968)]; Z. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 6, 978 (1967) [Soviet Phys. JETP Letters 6, 390 (1967)]; R. N. Mohapatra, J. Subba Rao, and R. E. Marshak, Phys. Rev. Letters 20, 1081 (1968); Phys. Rev. 171, 1502 (1968); F. E. Low, Comments Nucl. Particle Phys. 2, 33 (1968); R. N. Mohapatra and P. Olesen, Phys. Rev. 179, 1917 (1969).

[2] T. D. Lee, Nuovo Cimento 59A, 579 (1969).

weak effects, terms of the form

$$G\sum_{n=0}^{\infty} B_n(G\Lambda^2)^n$$

are called first-order weak effects, and generally, terms of the form

$$G^l\sum_{n=0}^{\infty} C_{ln}(G\Lambda^2)^n$$

are called lth order. (We are disregarding possible logarithmic dependences on the cutoff.) The zeroth-order terms present us with the dangerous possibility of serious violations of parity and hypercharge in strong interactions. First-order terms include the usual lowest-order contributions (order G) to leptonic and semileptonic processes. However, other first-order terms may yield violations of observed selection rules: There can be $\Delta S=2$ amplitudes, yielding a K_1-K_2 mass splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2\to\mu^++\mu^-$, $K^+\to\pi^++l+\bar{l}$, etc., involving neutral lepton pairs, or departures from the leptonic $\Delta S=\Delta Q$ law. We shall say of a model that its divergences are properly ordered if it is true that the zeroth-order terms *do not* yield violations of parity or hypercharge, and if the first-order terms *do* satisfy the observed selection rules of weak-interaction phenomena.

3 a)

In most conventional formulations of a weak-interaction field theory (say, a vector boson coupled to a quark triplet), the divergences are not properly ordered. Defenders of such theories must argue that there is an effective weak-interaction cutoff which guarantees that the induced higher-order effects are as small as experiment indicates. A remarkably small cutoff,[1] not greater than 3 or 4 GeV, seems necessary. Should such a cutoff be justified, the problem of higher-order departures from known selection rules is solved; all such departures are small.

Others feel that such a small cutoff is implausible and unrealistic, and that one must confront the possibility that $G\Lambda^2$ is large—perhaps obtaining sensible results in the limit $G\Lambda^2\to\infty$. In this case, one may regard all the first-order terms as having the same general magnitude, that of observed weak phenomena, and nth-order terms as having the magnitude naively expected of nth-order weak interactions.

An elegant solution to the problem of the zeroth-order terms was recently discovered, removing the specter of strong violations of parity and hypercharge.[3,4] One assumes a particular form for the breakdown of chiral $SU(3)$: The symmetry-breaking term must transform like the $(3,\bar{3})+(\bar{3},3)$ representation[5]; in a quark model, like the quark mass term. In this case, the zeroth-order weak interactions may be identified as an object belonging to the same representation as the symmetry-breaking term. After an appropriate $SU(3)\times SU(3)$ transformation, their only effect is to cause a renormalization of the symmetry-breaking terms, giving renormalized quark masses.[4] There is no violation of hypercharge or parity. Indeed, from a speculative stability requirement of the symmetry-breaking term under weak and electromagnetic corrections, the correct value of the Cabibbo angle may be deduced.[4]

Although the zeroth-order terms are controlled with an appropriate model of strong interactions, the first-order terms remain troublesome. Indeed, with a quark model, we immediately encounter strangeness-violating couplings of neutral lepton currents and contributions to the neutral kaon mass splitting to order $G(G\Lambda^2)$.[6] (In such a model, departures from $\Delta S=\Delta Q$ first appear at second order.) For this reason, it appears necessary to depart from the original phenomenological model of weak interactions. One suggestion[7] involves the introduction of a large number of intermediaries of spins one and zero, so coupled that the leading divergences are associated with only the diagonal symmetry-preserving interactions; in this fashion a proper ordering of divergences is readily obtained. But this model is an awkward one involving many intermediaries with different spins but degenerate coupling strengths. Few would concede so much sacrifice of elegance to expediency.[8]

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are mediated by just one charged vector boson. The weak hadronic current is constructed in precise analogy with the weak lepton current, thereby revealing suggestive lepton-quark symmetry. The extra quark is the simplest modification of the usual model leading to the proper ordering of divergences. Just as importantly, we argue that universality is preserved, in the sense that the

[3] C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento 56A, 1150 (1968); J. Iliopoulos, *ibid.* 62A, 209 (1969); R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968); Nuovo Cimento Letters 1, 1 (1969).

[4] N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968); Phys. Rev. D 1, 707 (1970).

[5] S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[6] Of course, one cannot exclude *a priori* the possibility of a cancellation in the sum of the relevant perturbation expansion in the limit $\Lambda\to\infty$.

[7] M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969).

[8] For other departures from the conventional theory, see, for example, C. Fronsdal, Phys. Rev. 136B, 1190 (1964); W. Kummer and G. Segrè, Nucl. Phys. 64, 585 (1965); G. Segrè, Phys. Rev. 181, 1996 (1969); L. F. Li and G. Segrè, *ibid.* 186, 1477 (1969); N. Christ, *ibid.* 176, 2086 (1968). It should be understood that the ingenious conjecture of T. D. Lee and G. C. Wick [Nucl. Phys. B9, 209 (1969)] for removing divergences is logically independent of our analysis. If their hypothesis is correct, the role of the cutoff momentum is played by M_W. Only if M_W is small (~3–4 GeV) would the problems associated with ordering of divergences be solved; otherwise, a modification of the coupling scheme, such as ours, is still necessary.

leading divergent corrections (i.e., the first-order terms) yield a *common* renormalization to each of the various observed coupling constants.

The new model is discussed in Sec. I. Since Cabibbo's algebraic notion of universality[9] is maintained, that is to say, the entire weak charges generate the algebra of $SU(2)$, we observe in Sec. II that an extension to a three-component Yang-Mills model may be feasible. In contradistinction to the conventional (three-quark) model, the couplings of the neutral intermediary—now hypercharge conserving—cause no embarrassment. The possibility of a synthesis of weak and electromagnetic interactions is also discussed.

In Sec. III we briefly note some of the implications of the existence of a fourth quark, and finally, in Sec. IV we discuss some of the experimental tests of our model of weak interactions.

I. NEW MODEL

3b)
3c)

We begin by introducing four quark fields.[10] The three quarks $\mathcal{P}$, $\mathcal{N}$, and λ form an $SU(3)$ triplet, and the fourth, $\mathcal{P}'$, has the same electric charge as $\mathcal{P}$ but differs from the triplet by one unit of a new quantum number $\mathcal{C}$ for charm. The strong-interaction Lagrangian is supposed to be invariant under chiral $SU(4)$, except for a symmetry-breaking term transforming, like the quark masses, according to the $(4,\bar{4})+(\bar{4},4)$ representation. This term may always be put in real diagonal form by a transformation of $SU(4)\times SU(4)$, so that B, Q, Y, $\mathcal{C}$, and parity are necessarily conserved by these strong interactions.

3d)

The extra quark completes the symmetry between quarks and the four leptons ν, ν', e^-, and μ^-. Both quadruplets possess unexplained unsymmetric mass spectra, and consist of two pairs separated by one in electric charge.

The weak lepton current may be expressed as

$$J_\mu{}^L=\bar{l}C_L\gamma_\mu(1+\gamma_5)l, \tag{1}$$

where l is a column vector consisting of the four lepton fields (ν, ν', e^-, μ^-) and the matrix C_L is given by

$$C_L=\begin{pmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}. \tag{2}$$

This is a convenient way to rewrite the conventional current. In analogy with this expression, we define the weak hadron current to be

$$J_\mu{}^H=\bar{q}C_H\gamma_\mu(1+\gamma_5)q, \tag{3}$$

where q is the quark column vector $(\mathcal{P}',\mathcal{P},\mathcal{N},\lambda)$ and the matrix C_H must be of the form

$$C_H=\left(\begin{array}{cc|c}0&0&\\0&0&\multicolumn{1}{c}{U}\\\hline 0&0&0\quad 0\\0&0&0\quad 0\end{array}\right) \tag{4}$$

in order for $J_\mu{}^H$ to carry unit charge. Pursuing the analogy further, we demand that the 2×2 submatrix U be unitary, so that the matrix C_H is equivalent to C_L under an $SU(4)$ rotation. Of course, it is not convenient to carry out the transformation making C_H and C_L coincide, for this would destroy the diagonalization of the $SU(4)$-breaking term, the quark masses. Nevertheless, suitable redefinitions of the relative phases of the quarks may be performed in order to make U real and orthogonal, so without loss of generality we write

$$U=\begin{bmatrix}-\sin\theta&\cos\theta\\\cos\theta&\sin\theta\end{bmatrix}. \tag{5}$$ 3e)

This is just the form of the weak current suggested in an earlier discussion of $SU(4)$ and quark-lepton symmetry.[10] What is new is the observation that this model is consistent with the phenomenological selection rules and with universality even when all divergent first-order terms [i.e., $G(G\Lambda^2)^n$] are considered.

To see this, we proceed diagrammatically in the quark model ignoring the strong $SU(4)$-invariant interactions.[11] Zeroth-order terms occur only in diagrams with only one external quark line, and give contributions to the quark mass operator of the form

$$\delta M(\gamma k)=\sum A_n(G\Lambda^2)^n\bar{q}M_n\gamma\cdot k(1+\gamma_5)q. \tag{6}$$

The A_n are dimensionless parameters, and the matrix M_n is a symmetric homogeneous polynomial of order n in C_H and of order n in $C_H{}^\dagger$. From the definition of C_H, it is seen that M_n must be a multiple of the unit matrix—again in contradistinction to the $SU(3)$ situation. Now, the zeroth-order terms are $SU(4)$ invariant.

There remains an apparent zeroth-order violation of parity, which may be transformed away because of the simple fashion of chiral $SU(4)$ breaking we have assumed. We simply define new quark fields

$$q_i'=(\alpha+\beta\gamma_5)q_i \tag{7}$$

with the real cutoff-dependent parameters α and β chosen so that the entire (bare plus zeroth-order) mass operator, in terms of q_i', is diagonal and parity conserving. The $SU(4)\times SU(4)$-invariant strong interactions are left unchanged. The procedure is analogous

[9] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[10] B. J. Bjorken and S. L. Glashow, Phys. Letters 11, 255 (1964).

[11] All our results about the zero- and first-order selection rules are trivially extended to the case of an $SU(4)$-invariant strong interaction which consists of a neutral vector boson coupled to quark number, the so-called "gluon" model. The only results of this paper which might be affected by such an interaction are the universality conditions in Eq. (9).

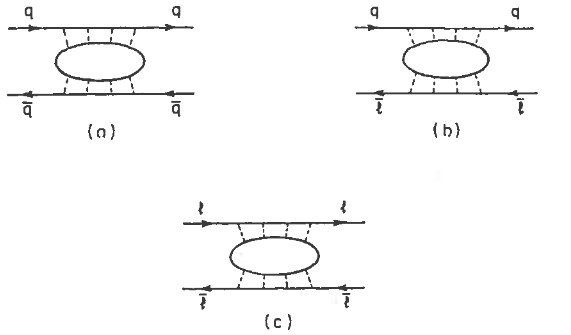

FIG. 1. (a) Connected part of the $q\bar{q}\to q\bar{q}$ amplitude. The crossed (annihilation) channel is also understood. (b) Connected part of the $ql\to ql$ amplitude. (c) Connected part of the $l\bar{l}\to l\bar{l}$ amplitude.

to that of Ref. 4, with the difference that the transformation (7) is $SU(4)$ invariant and does not change the definition of strangeness (or charm), or of the Cabibbo angle. An important consequence of the fact that M_n does not depend on the Cabibbo angle is that, unlike the situation in Ref. 4, it is impossible in our case to evaluate the Cabibbo angle by imposing a condition on the leading divergences. We conclude that zeroth-order weak effects are not significant.

We now consider the first-order $G(G\Lambda^2)^n$ terms which are of four types: (i) next-to-the-leading contributions to the quark and lepton mass operators, (ii) leading contributions to quark-quark or quark-antiquark scattering, (iii) leading contributions to quark-lepton scattering, and (iv) leading contributions to lepton-lepton scattering. Graphs with more than two external fermion lines yield no larger than second-order effects. Terms of type (i) are harmless: They contribute to observable nonleptonic $\Delta I=\frac{1}{2}$ processes, but since they cannot give $\Delta Y=2$, they do not produce a K_1K_2 mass splitting. On the other hand, type-(ii) diagrams could lead to $\mathcal{N}\bar{\lambda}\to\bar{\mathcal{N}}\lambda$, possibly giving rise to first-order contributions to the K_1K_2 mass difference, contrary to experiment. Let us show that they do not.

Graphs contributing to type (ii) effects are of the general form shown in Fig. 1(a), where the bubble includes any possible connections among the boson lines, and any number of closed fermion loops. The leading divergent contributions to q-$\bar{q}$ scattering from these graphs have the form

$$T_{HH}=G\sum_{n=2}^{\infty}B_n(G\Lambda^2)^{n-1}[\bar{q}\gamma_\mu(1+\gamma_5)\times B_H^{(n)}q\bar{q}\gamma^\mu(1+\gamma_5)B_H^{(n)\dagger}q],\quad(8)$$

where the B_n are finite dimensionless parameters independent of masses or momenta. It is clear that these first-order terms are independent of all external momenta. The matrix $B_H^{(n)}$ is a polynomial in C_H and $C_H^\dagger$ of order k and l, respectively, with $k+l\leq n$. Furthermore, the charge structure of the quark multiplets allows a change of charge no greater than unity, so that $|k-l|$ must be zero or one, and the matrices $B_H^{(n)}$ are easily computed (see the Appendix) to be

$$B_H^{(n)}=C_H \text{ or } C_H^\dagger\quad(k=l\pm1)\qquad(8')$$
$$=[C_H,C_H^\dagger]\quad(k=l)\,.\qquad(8'')$$

Thus, T_{HH} gives rise to contributions with $|\Delta Y|\leq1$ and, in particular, it does not yield a first-order K_1K_2 mass splitting. Of course, the next-to-the-leading divergences of these graphs will give $\Delta Y=2$, and do contribute to a second-order K_1K_2 mass difference, agreeing with experiment.

The leading divergences of types (iii) and (iv) give first-order contributions T_{HL} and T_{LL}, to semileptonic and leptonic processes. There will be a 1-to-1 correspondence among the graphs contributing to T_{LL}, T_{HL} [Figs. 1(b) and 1(c)], and T_{HH}. Because the algebraic properties of C_H and C_L are identical, we construct T_{HL} and T_{LL} from T_{HH} by the appropriate substitutions of $q\to L$ and $C_H\to C_L$.

In processes where the lepton charge changes, no violations of observed selection rules occur, but the first-order terms cause a renormalization of observed coupling constants. It is important to note that these renormalizations are common to leptonic and semileptonic processes, so that the relations

$$\begin{aligned}G_V(\Delta S=0)&=G_\mu\cos\theta\,,\\G_V(\Delta S=1)&=G_\mu\sin\theta\end{aligned}\qquad(9)$$

remain true when all first-order terms are included. This renormalization is given by the factor $1+\sum B_n(G\Lambda^2)^{n-1}$. A sufficient condition for these renormalizations to be common is the algebraic version of universality—a condition which is satisfied by our model, as well as by the usual three-quark model.

Next, we turn to the induced first-order couplings of hadrons to neutral lepton currents and self-couplings of neutral lepton currents. The neutral lepton currents are generated by the matrix C_L^0 and the neutral hadron currents by the matrix C_H^0, where

$$C_L^0=[C_L,C_L^\dagger]=\begin{bmatrix}1&0\\0&-1\end{bmatrix}=[C_H,C_H^\dagger]=C_H^0.\quad(10)$$

Evidently, there are no induced couplings of neutral lepton currents to strangeness-changing currents. The induced couplings involve the strangeness-conserving current

$$\begin{aligned}J_\mu^0=&\bar{q}\gamma_\mu C_H^0(1+\gamma_5)q+\bar{l}\gamma_\mu C_L^0(1+\gamma_5)l\\=&\bar{\mathcal{P}}'\gamma_\mu(1+\gamma_5)\mathcal{P}'+\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{P}-\bar{\mathcal{N}}\gamma_\mu(1+\gamma_5)\mathcal{N}\\&-\bar{\lambda}\gamma_\mu(1+\gamma_5)\lambda+\bar{\nu}'\gamma_\mu(1+\gamma_5)\nu'+\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\\&-\bar{e}\gamma_\mu(1+\gamma_5)e-\bar{\mu}\gamma_\mu(1+\gamma_5)\mu\,.\end{aligned}\qquad(11)$$

The coupling constant for this new neutral current-current interaction is a first-order expression of the form

$$G\sum_{n=2}^{\infty} C_n(G\Lambda^2)^{n-1}.$$

We anticipate that its strength should be comparable to the strength of the charged leptonic interactions. The new coupling plays no role in observed decay modes, but is should be detectable in accelerator experiments.

In Sec. II we discuss the possible extension of our model to a Yang-Mills model, where the coupling strength of the neutral W to its current is uniquely determined. These neutral lepton couplings constitute the most characteristic and interesting feature of our model. Relevant experimental evidence is discussed in Sec. IV.

II. YANG-MILLS MODEL OF WEAK INTERACTIONS

Divergences appear in our model of weak interactions, but they are properly ordered; observed selection rules are broken only in order $G^2(G\Lambda^2)^n$. But, the model is certainly not renormalizable. There is at least a possibility that a Yang-Mills model of weak interactions may be less singular.[12] In this section, we show how our model can be extended to include a symmetrically coupled triplet of W's. It is possible that W self-couplings can be introduced to give a complete Yang-Mills theory.

The Lagrangian with which we work may be written, in the four-quark model, without electromagnetism,

$$\mathfrak{L}=\mathfrak{L}_{\rm kin}+\mathfrak{L}_s+\mathfrak{L}_M+\mathfrak{L}_w\,, \tag{12}$$

where $\mathfrak{L}_{\rm kin}$ is the purely kinematic term

$$\mathfrak{L}_{\rm kin}=\bar{q}\gamma\cdot pq+\bar{l}\gamma\cdot pl+G_{\mu\nu}G^{\mu\nu}+W_{\mu\nu}{}^{\dagger}W^{\mu\nu} \tag{13}$$

describing four free massless quarks, four leptons, and their strong and weak intermediaries ($X_{\mu\nu}$ denotes the antisymmetric curl of X_μ). $\mathfrak{L}_s$ denotes the $SU(4)$-invariant strong interaction, most simply

$$\mathfrak{L}_s=fG_\mu\bar{q}\gamma^\mu q\,, \tag{14}$$

and $\mathfrak{L}_w$ is the weak interaction

$$\mathfrak{L}_w=gW_\mu{}^{\dagger}[\bar{q}C_H\gamma^\mu(1+\gamma_5)q+\bar{l}C_L\gamma^\mu(1+\gamma_5)l+\text{H.a.}]\,. \tag{15}$$

The bare-mass term $\mathfrak{L}_M$ produces the observed masses of the leptons, the masses of W and G, and gives rise to the observed hierarchy of hadron symmetry,

$$\mathfrak{L}_M=\bar{q}M_Hq+\bar{l}M_Ll+m^2G_\mu G^\mu+M^2W_\mu W^\mu\,, \tag{16}$$

where M_H and M_L are 4×4 matrices. This model gives a complete description of weak-interaction phenomena. The most important new feature is the appearance of neutral currents generated by the most divergent parts of diagrams containing an exchange of W^+, W^- pairs between two fermion lines. The effective coupling strength of these currents is expected to be of order G but, at this stage, we cannot predict its precise numerical value since we are unable to sum the perturbation series. In order to extend this model to a more symmetric one, we introduce an additional weak intermediary W_0 with appropriate couplings. 3 f)

3 g)

The couplings of W_0 to hadrons and leptons must be taken to be

$$2^{-1/2}gW_0{}^\mu\{\bar{q}[C_H{}^\dagger,C_H]\gamma_\mu(1+\gamma_5)q+\bar{l}[C_L{}^\dagger,C_L]\gamma_\mu(1+\gamma_5)l\}\,. \tag{17}$$

We emphasize that the introduction of W_0 is by no means necessary in our model; however, we think that it gives a much more symmetric and aesthetically appealing theory.

In the conventional model of weak interactions, the extension to a three-component vector-meson theory cannot be made without contradicting experiment: The neutral boson leads to strangeness-changing decays involving neutral-lepton currents and to $\Delta S=2$ at order G. This is because the commutator of the conventional weak charge with its adjoint yields a strangeness-violating neutral charge. In our case, the corresponding operator is diagonal, and these difficulties are absent.

It is straightforward to show that the introduction of the neutral current does not spoil the proper ordering of divergences: The observed selection rules are preserved by all terms of order $G(G\Lambda^2)^n$. This is shown in the Appendix.

We note that W_0 is coupled to precisely the same neutral current appearing in the last section as an induced coupling. In the symmetric three-W model, its strength is uniquely predicted. Universality now applies to both charged and neutral couplings. That is to say, the leading divergent corrections to each are the same. The bare relationship

$$G_0=\tfrac{1}{2}G \tag{18}$$

is preserved by the renormalizations, to first order [i.e., including all terms of order $G(G\Lambda^2)^n$]. This assertion is proved in the Appendix.

The introduction of a neutral W opens the possibility of formulating the weak interactions into a Yang-Mills theory. Self-couplings must be introduced among the W triplet in order to ensure the gauge symmetry. This is accomplished if we choose the Lagrangian in a manifestly gauge-invariant fashion:

$$\mathfrak{L}=\bar{q}\gamma\Pi_Hq+\bar{l}\gamma\Pi_Ll+\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}+G_{\mu\nu}G^{\mu\nu}+\mathfrak{L}_M+\mathfrak{L}_s\,, \tag{19}$$

where

$$\Pi_H{}^\mu=\partial^\mu+ig(\mathbf{C}_H\cdot\mathbf{W}^\mu)(1+\gamma_5)\,, \tag{19'}$$

$$\Pi_L{}^\mu=\partial^\mu+ig(\mathbf{C}_L\cdot\mathbf{W}^\mu)(1+\gamma_5)\,, \tag{19''}$$

[12] See, for example, S. Mandelstam, Phys. Rev. **175**, 1580 (1969); M. Veltman, Nucl. Phys. **B7**, 637 (1968); H. Reiff and M. Veltman, *ibid.* **B13**, 545 (1969); D. Boulware, Ann. Phys. (N. Y.) **56**, 140 (1970); A. A. Slavnor, University of Kiev Report No. ITP 69/20 (unpublished); E. S. Fradkin and I. V. Tyutin, Phys. Letters **30B**, 562 (1969). Notice, however, that none of these references consider the far more difficult case of vector mesons coupled to nonconserved currents.

TABLE I. Quark quantum numbers.

	Fractional assignment			Integral assignment		
	Q	Y	$\mathcal{C}$	Q	Y	$\mathcal{C}$
$\mathcal{P}'$	$\frac{2}{3}$	$-\frac{2}{3}$	1	0	0	0
$\mathcal{P}$	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	-1
$\mathfrak{N}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	-1	0	-1
λ	$-\frac{1}{3}$	$-\frac{2}{3}$	0	-1	-1	-1

and

$$W^{\mu\nu}=\Pi_W{}^\mu W^\nu-\Pi_W{}^\nu W^\mu, \tag{19'''}$$

where

$$(\Pi_W{}^\mu)_{ij}=\delta_{ij}\partial^\mu+ig2^{-1/2}(\mathbf{t}\cdot\mathbf{W}^\mu)_{ij}. \tag{19''''}$$

The matrix-valued vectors $\mathbf{C}_H$ and $\mathbf{C}_L$ have components $(C,C^\dagger,2^{-1/2}[C^\dagger,C])$ in a basis where charge is diagonal, and $\mathbf{t}$ are the usual 3×3 generators of $O(3)$, with t_3 diagonal. The gauge group thus introduced is an exact symmetry of the entire Lagrangian excepting both $\mathcal{L}_M$ and electromagnetism.

The Yang-Mills model is undoubtedly the most attractive way to couple a triplet of vector mesons and the only one for which people have expressed some hope of constructing a renormalizable theory. The massless case has been proved to be renormalizable[12]; however, very little is known about the physically more interesting massive theory. In fact, the naive power counting shows that the highest divergence in a Yang-Mills theory is $g^{2n}\Lambda^N$ with $N=6n$. Notice that in the absence of the self-couplings the corresponding divergences are given, as we have already seen, by $N=2n$. So, at first sight, the Yang-Mills theory seems to be much more divergent than the ordinary coupling of the vector mesons with the currents. However, one can show that the naive limit $N=6n$ can be considerably lowered. We have already been able to show that $N\leq3n$ and we believe that one can still lower this limit to at least $N=2n$. In other words, we believe that the introduction of the self-couplings does not make the theory more divergent.

3g) Let us briefly consider a more daring speculation. It has long been suspected[13] that there may be a fundamental unity of weak and electromagnetic interactions, reflected phenomenologically by the common vectorial character of their couplings. For this reason, it may have been wrong for us to introduce a gauge symmetry for the weak interactions not shared by electromagnetism. As a more speculative alternative, consider the possibility of a four-parameter gauge group involving $\mathbf{W}$, and an additional Abelian singlet W_S, broken only by the mass term $\mathcal{L}_M$. Suppose, however, that a one-parameter gauge symmetry, corresponding to a linear combination A of W_0 and W_S remains unbroken. Then A must be massless, and may be identified as the photon. The orthogonal neutral combination B is massive, and acts as an intermediary of weak interactions along with $W^\pm$. This model could be correct only if the weak bosons are very massive (100 GeV) so that the weak and electromagnetic coupling constants could be comparable. With this model, the relation (18) would not persist, and the weak neutral current would involve $(1-\gamma_5)$ as well as $(1+\gamma_5)$ currents. The precise form of the model would depend on what linear combination of W_0 and W_S is the photon.

[13] J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1957); S. L. Glashow, Nucl. Phys. **10**, 107 (1959); **22**, 579 (1961)

III. ANOTHER QUARK MAKES $SU(4)$

Having introduced four quarks, we must consider strong interactions which admit the algebra of chiral $SU(4)$. Does this mean we should expect $SU(4)$ to be an approximate symmetry of nature? Nothing in our argument depends on how much $SU(4)$ is broken; the divergences are necessarily properly ordered. However, for the higher-order nonleading divergences to be as small as they must be, the breaking of $SU(4)$ cannot be too great: The limit on the cutoff Λ is replaced by a limit on Δ, a parameter measuring $SU(4)$ breaking; and from the observed K_1K_2 mass difference we now conclude that Δ must be not larger than 3–4 GeV. Thus, some residue of $SU(4)$ symmetry should persist.

We expect the appearance of charmed hadron states.[10] Meson multiplets, made up of a quark-antiquark pair, must belong to the 15-dimensional adjoint representation of $SU(4)$, consisting of an uncharmed $SU(3)$ singlet and octet, as well as two $SU(3)$ triplets of charm ±1. The structure of baryons depends on the quantum numbers assigned to the quarks. The two simplest possibilities are shown in Table I. For the more conventional fractional charge assignment, the baryons are made up of three quarks, and must belong to one of the representations contained in $4\times4\times4$. The only possibility is a 20-dimensional representation, which contains, besides the baryon octet, a triplet and sextet of charmed states and a doubly charmed triplet. The $j=\frac{3}{2}^+$ baryon decuplet belongs to another 20-dimensional representation with a charmed sectet, a doubly charmed triplet, and a triply charmed singlet.

With the integral-charge assignment, the baryon octet must be made of two quarks and an antiquark, the decuplet of three quarks and two antiquarks. The lepton and quark charged spectra now coincide, and the synthesis of weak and electromagnetic interactions appears more plausible. Moreover, there is no difficulty in obtaining the correct value for the π^0 lifetime.

Why have none of these charmed particles been seen? Suppose they are all relatively heavy, say ~2 GeV. Although some of the states must be stable under strong (charm-conserving) interactions, these will decay rapidly ($\sim10^{+13}$ sec^{-1}) by weak interactions into a very wide variety of uncharmed final states (there are about a hundred distinct decay channels). Since the charmed particles are copiously produced only in associated production, such events will necessarily be of very complex topology, involving the plentiful decay prod- 3h) ucts of both charmed states. Charmed particles could easily have escaped notice.

Finally, we briefly comment on the leptonic decay rates of ρ, ω, and ϕ (Γ_ρ, Γ_ω, and Γ_ϕ). Our electric current contains $SU(3)$ singlet as well as octet terms, so that the inequality

$$m_\omega\Gamma_\omega+m_\phi\Gamma_\phi\geq\tfrac{1}{3}m_\rho\Gamma_\rho \tag{20}$$

may be deduced from the Weinberg spectral function sum rules and ω, ϕ, ρ dominance.[14] A stronger result is obtained if we extend Weinberg's Schwinger-term hypothesis to the vector currents of $SU(4)$:

$$m_\omega\Gamma_\omega+m_\phi\Gamma_\phi\geq m_\rho\Gamma_\rho. \tag{21}$$

This result is in poor agreement with experiment, which favors the equality in (20). A resolution of this difficulty that does not abandon the Schwinger-term symmetry requires the introduction of a third $Y=T=0$ vector meson, another partner of ω and ϕ, corresponding to the $SU(4)$ singlet vector current.

IV. EXPERIMENTAL SUGGESTIONS

In this section, we discuss some of the observable effects characteristic of our picture of strong and weak interactions. Firstly, consider the experimental implications of the existence of a new quantum number—
3?) charm—broken only by weak interactions. The charmed particles, because they are heavy, are too short lived to give visible tracks. However, they should be copiously produced in hardonic collisions at accelerator energies:

$$(\text{hadron or }\gamma)+(\text{hadron})\to X^{(+)}+X^{(-)}+\cdots,$$

where $X^{(\pm)}$ are oppositely charmed particles, each rapidly decaying into uncharmed hadrons with or without a charged lepton pair. The purely hadronic decay modes could provide illusory violations of hypercharge conservation in strong interactions. The leptonic decay modes provide a mechanism for the seemingly direct production of one or two charged leptons in hadron-hadron collisions.[15] Conceivably, muons thus produced may be responsible for the anomalous observed angular distribution of cosmic-ray muons in the 10^{12}-eV range,[16] where these directly produced muons may dominate the sea-level muon flux.

Should this last speculation about cosmic rays be correct, we need to revise radically estimates of the flux of ν and $\bar{\nu}$ in this energy range. We expect the charmed particle decays to yield equal numbers of each lepton variety; this gives a flux of electron neutrinos and antineutrinos equal to the muon flux, and 10–100 times greater than other estimates. This fact is of crucial importance to the possible detection of the resonance scattering[17]

$$\bar{\nu}+e^-\to\bar{\nu}'+\mu^-.$$

Charmed particles may be produced singly by neutrinos in such reactions as

$$\nu'+N\to\mu^-+X,\quad \bar{\nu}'+N\to\mu^++X,$$

where the charmed particle X would have a variety of decay modes, including leptonic ones. With the fractional charge assignment, the neutrino processes are suppressed by $\sin^2\theta$ and the antineutrino processes are forbidden. On the other hand, with the integral-charge assignment, the neutrino processes are again proportional to $\sin^2\theta$ while the antineutrino processes are proportional to $\cos^2\theta$.

The second new feature of our model is the appearance of neutral leptonic and semileptonic couplings involving a specified ($Y=0$) current and with a coupling constant comparable with the Fermi constant. Without the introduction of a W_0, we may say only $G_0\sim G$. To be more definite, we shall phrase our arguments in terms of the value $G_0=\frac{1}{2}G$ of Eq. (18).

Let us summarize the presently available data about these interactions.[18] Consider the following three reactions induced by muon neutrinos:

$$\text{(i)}\quad \nu'+e^-\to\nu'+e^-,$$
$$\text{(ii)}\quad \nu'+p\to\nu'+p,$$
$$\text{(iii)}\quad \nu'+p\to\nu'+\pi^++n.$$

None of these neutral couplings have been observed; experimentally, we can only quote limits. From the absence of observed forward energetic electrons in the CERN bubble-chamber experiments, we may conclude

$$G_0\leq G,$$

a limit which is close to but consistent with our prediction.

For reaction (ii), it is found that

$$R=\sigma(\nu'p\to\nu'p)/\sigma(\bar{\nu}'p\to\mu^+n)\leq0.5.$$

Because our neutral current contains both $I=0$ and $I=1$ parts, we cannot unambiguously predict this ratio. In a naive quark model, where the proton consists of only $\mathfrak{N}$ and $\mathcal{P}$ quarks, we find $R=\frac{1}{4}$, again close but consistent.

Finally, we quote the experimental limit on reaction (iii):

$$R'=\sigma(\nu'+p\to\pi^++n+\nu')/\sigma(\nu'+p\to\pi^++p+\mu^-)\leq0.08.$$

[14] S. Weinberg, Phys. Rev. Letters 18, 507 (1967); T. Das, V. Mathur, and S. Okubo, *ibid.* 19, 470 (1967).

[15] In a recent experiment, P. J. Wanderer *et al.* [Phys. Rev. Letters 23, 729 (1969)] have performed a search for W's by measuring the intensity and polarization of prompt energetic muons from the interaction of 28-GeV protons with nuclei. Their results are compatible with the assumption that all 25-GeV prompt muons have electromagnetic origin. There is no indication of the existence of W's. However, the published evidence does not seem to be relevant to the existence of charmed particles, which are produced in pairs, decay into many final states, and are not expected to produce many very energetic muons.

[16] H. E. Bergeson *et al.*, Phys. Rev. Letters 21, 1089 (1968).

[17] M. G. K. Menon *et al.*, Proc. Roy. Soc. (London) **A301**, 137 (1967).

[18] See D. H. Perkins, in Proceedings of the Topical Conference in Weak Interactions, CERN, 1969 [CERN Report No. 69-7], pp. 1–42 (unpublished).

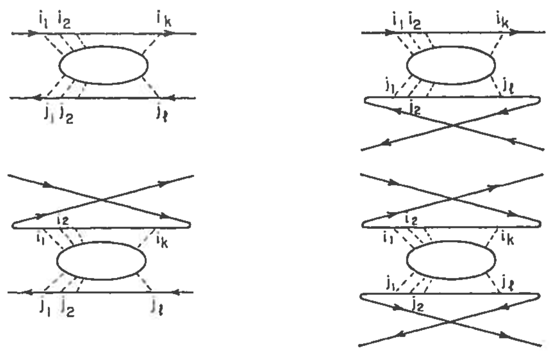

FIG. 2. Decomposition of the $q\bar{q}\to q\bar{q}$ connected amplitude by crossing the external fermion lines.

Because this transition is $\Delta I=1$, we unambiguously predict $R'=\frac{1}{9}$ under the hypothesis of $\Delta(1238)$ dominance. In each of these three reactions, experiment is very close to a decisive test of our model.

In our model, the parity-violating nonleptonic interaction is also changed. In particular, the parity-violating one-pion-exchange nuclear force is no longer suppressed by $\sin^2\theta$.

Next we consider some experiments which could discover the existence of W_0. A simple and attractive possibility is the search for muon tridents in the semiweak reaction[19]

$$\mu^-+Z\to\mu^-+W_0+Z\,,$$

with the subsequent muonic decay of W_0. Another possibility is the reaction[20]

$$e^+e^-\to\mu^+\mu^-\,.$$

The interference between the W^0 and photon contributions causes an asymmetry of the μ^+ angular distribution relative to the momentum of the incident e^+ given by

3)

$$\delta=\frac{N_F-N_B}{N_F+N_B}=\frac{3M_W{}^2}{16\sqrt{2}}\frac{G}{\alpha\pi}\frac{s}{s-M_W{}^2}\,,$$

where

$$G=10^{-5}M_p{}^{-2}\,,\quad \alpha=1/137\,,\quad\text{and}\quad s=4E_e{}^2\,.$$

Away from the W^0 pole, the effect is rather small (less than 1% for $E_e=3.5$ GeV) and it is masked by a similar effect due to the two-photon contribution. However, the factor $s/(s-M_W{}^2)$ makes the asymmetry increase sharply and change sign near M_W. Therefore, this reaction is an excellent tool to sweep a substantial mass range looking for W's. Another effect, much harder to detect, would be the direct observation of parity violation in $e^+e^-\to\mu^+\mu^-$. This requires the measurement of μ polarization.

Finally, we recall from Sec. III that the $SU(4)$ description of leptonic decays of vector mesons suggested the existence of another strongly coupled neutral $I=0$ vector meson with considerable coupling to lepton pairs. Evidence for its existence could come from colliding beam experiments.

[19] M. Tannenbaum (private communication).

[20] N. Cabibbo and R. Gatto, Phys. Rev. 129, 1577 (1961).

APPENDIX

In this appendix we determine the form of the leading weak corrections to the q-$\bar{q}$, q-$\bar{l}$, and l-$\bar{l}$ amplitudes.

We have already shown that the wave-function renormalization of spinors is the same for both quarks and leptons and contributes a common factor to T_{HH}, T_{HL}, and T_{LL}. Therefore we need consider only the q-$\bar{q}$ amplitude T_{HH}. The other amplitudes T_{HL} and T_{LL} can be obtained from T_{HH} by appropriate substitutions. In the following, we shall omit the common wave-function renormalization factors.

For the sake of clarity, let us first consider our model of weak interactions, where we have three vector bosons symmetrically coupled.

The graphs of Fig. 1(a) can be decomposed into four classes of terms, obtained by keeping the boson lines fixed and reversing the fermion lines, as shown in Fig. 2. We then obtain for the contribution to T_{HH} corresponding to these four classes of diagrams

$$\begin{aligned}T_{HH}{}^{(n,k,l)}&=\bar{q}\gamma_\mu(1+\gamma_5)[C_{i_1}C_{i_2}\cdots C_{i_k}-(-1)^kC_{i_k}C_{i_{k-1}}\cdots C_{i_1}]q\\&\quad\times P_{j_l\cdots j_1;i_1\cdots i_k}\bar{q}^\mu(1+\gamma_5)\\&\quad\times[C_{j_1}C_{j_2}\cdots C_{j_l}-(-1)^lC_{j_l}C_{j_{l-1}}\cdots C_{j_1}]q\,,\\&\qquad k+l\leq n\,,\quad k,l\geq1\,.\end{aligned}\tag{A1}$$

All the i's and j's go from 1 to 3 and the sum over all indices is understood. $P_{j_l\cdots j_1;i_1\cdots i_k}$ is a tensor made out of the invariant tensors δ_{ij} and ϵ_{ijk}.

It is easy to show that for any k

$$\mathrm{Tr}[C_{i_1}C_{i_2}\cdots C_{i_k}-(-)^kC_{i_k}C_{i_{k-1}}\cdots C_{i_1}]=0\,.\tag{A2}$$

Therefore, since the interaction is $O(3)$ invariant, the connected part of T_{HH} has the form

$$T_{HH}=G\sum_{n=0}^{\infty}b_n(G\Lambda^2)^n\times(\bar{q}\gamma_\mu(1+\gamma_5)\mathbf{C}_Hq)\cdot(\bar{q}\gamma^\mu(1+\gamma_5)\mathbf{C}_Hq)\,.\tag{A3}$$

In the case where we have only charged bosons, the argument is even simpler. Each of the indices $i_1\cdots i_k$, $j_1\cdots j_l$ appearing in Eq. (A1) takes only two possible values. With the relations

$$\begin{aligned}(C_H)^2&=(C_H{}^\dagger)^2=0\,,\\(C_HC_H{}^\dagger)^2&=C_HC_H{}^\dagger\,,\\(C_H{}^\dagger C_H)^2&=C_H{}^\dagger C_H\,,\end{aligned}\tag{A4}$$

Eq. (A1) explicitly reads

$$\begin{aligned}T_{HH}{}^{(n;k,l)}&=(\bar{q}\gamma_\mu(1+\gamma_5)[C_H,C_H{}^\dagger]q)\\&\quad\times(\bar{q}\gamma^\mu(1+\gamma_5)[C_H,C_H{}^\dagger]q)\,,\quad k=l\\T_{HH}{}^{(n,k,l)}&=(\bar{q}\gamma_\mu(1+\gamma_5)C_Hq)(\bar{q}\gamma^\mu(1+\gamma_5)C_H{}^\dagger q)\,,\\&\qquad k=l+1\,.\quad\text{Q.E.D.}\end{aligned}$$

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CP-Violation in the Renormalizable Theory of Weak Interaction

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In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

When we apply the renormalizable theory of weak interaction[1] to the hadron system, we have some limitations on the hadron model. It is well known that 4a) there exists, in the case of the triplet model, a difficulty of the strangeness changing neutral current and that the quartet model is free from this difficulty. 4b, c, d) Furthermore, Maki and one of the present authors (T.M.) have shown[2] that, in the latter case, the strong interaction must be chiral $SU(4)\times SU(4)$ invariant as precisely as the conservation of the third component of the iso-spin I_3. In addition to these arguments, for the theory to be realistic, *CP*-violating interactions should be incorporated in a gauge invariant way. This requirement will impose further limitations on the hadron model and the *CP*-violating interaction itself. The purpose of the present paper is to investigate this problem. In the following, it will be shown that in the case of the above-mentioned quartet model, we cannot make a *CP*-violating interaction without introducing any other new fields when we require the following conditions: a) The mass of the fourth member of the quartet, which we will call ζ, is sufficiently large, b) the model should be consistent with our well-established knowledge of the semi-leptonic processes. After that some possible ways of bringing *CP*-violation into the theory will be discussed.

We consider the quartet model with a charge assignment of Q, $Q-1$, $Q-1$ and Q for p, n, λ and ζ, respectively, and we take the same underlying gauge group $SU_{\text{weak}}(2)\times SU(1)$ and the scalar doublet field φ as those of Weinberg's original model.[1] Then, hadronic parts of the Lagrangian can be devided in the following way:

$$\mathcal{L}_{\text{had}}=\mathcal{L}_{\text{kin}}+\mathcal{L}_{\text{mass}}+\mathcal{L}_{\text{strong}}+\mathcal{L}',$$ 4 e)

where $\mathcal{L}_{\text{kin}}$ is the gauge-invariant kinetic part of the quartet field, q, so that it contains interactions with the gauge fields. $\mathcal{L}_{\text{mass}}$ is a generalized mass term of q, which includes Yukawa couplings to φ since they contribute to the mass of q through the spontaneous breaking of gauge symmetry. $\mathcal{L}_{\text{strong}}$ is a strong-interaction part which conserves I_3 and therefore chiral $SU(4)\times SU(4)$ invariant.[2] We assume C- and P-invariance of $\mathcal{L}_{\text{strong}}$. The last term denotes residual interaction parts if they exist. Since $\mathcal{L}_{\text{mass}}$ includes couplings with φ, it has possibilities of violating CP-conservation. As is known as Higgs phenomena,[3] three massless components of φ can be absorbed into the massive gauge fields and eliminated from the Lagrangian. Even after this has been done, both scalar and pseudoscalar parts remain in $\mathcal{L}_{\text{mass}}$. For the mass term, however, we can eliminate such pseudoscalar parts by applying an appropriate constant gauge transformation on q, which does not affect on $\mathcal{L}_{\text{strong}}$ due to gauge invariance.

Now we consider possible ways of assigning the quartet field to representations of the $SU_{\text{weak}}(2)$. Since this group is commutative with the Lorentz transformation, the left and right components of the quartet field, which are respectively defined as $q_L=\frac{1}{2}(1+\gamma_5)q$ and $q_R=\frac{1}{2}(1-\gamma_5)q$, do not mix each other under the gauge transformation. Then, *each* component has three possibilities:

A) $\mathbf{4}=\mathbf{2}+\mathbf{2}$,

B) $\mathbf{4}=\mathbf{2}+\mathbf{1}+\mathbf{1}$, 4 f)

C) $\mathbf{4}=\mathbf{1}+\mathbf{1}+\mathbf{1}+\mathbf{1}$,

where on the r.h.s., $\mathbf{n}$ denotes an n-dimensional representation of $SU(2)$. The present scheme of charge assignment of the quartet does not permit representations of $n\geq 3$. As a result, we have nine possibilities which we will denote by (A, A), (A, B), $\cdots$, where the former (latter) in the parentheses indicates the transformation properties of the left (right) component. Since all members of the quartet should take part in the weak interaction, and size of the strangeness changing neutral current is bounded experimentally to a very small value, the cases of (B, C), (C, B) and (C, C) should be abandoned. The models of (B, A) and (C, A) are equivalent to those of (A, B) and (A, C), respectively, except relative signs between vector and axial vector parts of the weak current. Since g_A/g_V ratios are measured only for composite states, this difference of the relative signs would be reduced to a dynamical problem of the composite system. So, we investigate in detail the cases of (A, A), (A, B), (A, C) and (B, B).

i) *Case* (A, C) 4 g)

This is the most natural choice in the quartet model. Let us denote two $(SU_{\text{weak}}(2))$ doublets and four singlets by L_{d1}, L_{d2}, $R_{s1}^{(p)}$, $R_{s2}^{(p)}$, $R_{s1}^{(n)}$ and $R_{s2}^{(n)}$, where superscript $p(n)$ indicates p-like (n-like) charge states. In this case, $\mathcal{L}_{\text{mass}}$ takes, in general, the following form:

$$\mathcal{L}_{\text{mass}}=\sum_{i,j=1,2}[M_{ij}^{(n)}\bar{L}_{di}\varphi R_{sj}^{(n)}+M_{ij}^{(p)}\bar{L}_{di}\varepsilon\varphi^* R_{sj}^{(p)}]+\text{h.c.},$$

$$\varphi^*\equiv\begin{pmatrix}\varphi^-\\ \bar{\varphi}^0\end{pmatrix},\qquad \varepsilon\equiv\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\tag{1}$$

where $M_{ij}^{(n)}$ and $M_{ij}^{(p)}$ are arbitrary complex numbers. We can eliminate three Goldstone modes ϕ_i by putting

$$\varphi = e^{i\phi_i\tau_i}\begin{pmatrix} 0 \\ \lambda+\sigma \end{pmatrix}, \tag{2}$$

where λ is a vacuum expectation value of φ^0 and σ is a massive scalar field. Thereafter, performing a diagonalization of the remaining mass term, we obtain

$$\mathcal{L}_{\text{mass}} = \bar{q}mq\left(1+\frac{\sigma}{\lambda}\right),$$

$$m = \begin{pmatrix} m_p & 0 & 0 & 0 \\ 0 & m_n & 0 & 0 \\ 0 & 0 & m_\zeta & 0 \\ 0 & 0 & 0 & m_\lambda \end{pmatrix}, \qquad q = \begin{pmatrix} p \\ n \\ \zeta \\ \lambda \end{pmatrix}. \tag{3}$$

4 h)

Then, the interaction with the gauge field in $\mathcal{L}_{\text{kin}}$ is expressed as

$$\sum_{j=1}^{3} A_\mu{}^j i\bar{q}\Lambda_j\gamma_\mu \frac{1+\gamma_5}{2} q\,. \tag{4}$$

Here, Λ_j is the representation matrix of $SU_{\text{weak}}(2)$ for this case and explicitly given by

$$\Lambda_+ = \frac{\Lambda_1+i\Lambda_2}{2} = K\begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}K^{-1}, \quad \Lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

where U is a 2×2 unitary matrix. Here and hereafter we neglect the gauge field corresponding to $U(1)$ which is irrelevant to our discussion. With an appropriate phase convention of the quartet field we can take U as

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{6}$$

Therefore, if $\mathcal{L}'=0$, no CP-violations occur in this case. It should be noted, however, that this argument does not hold when we introduce one more fermion doublet with the same charge assignment. This is because all phases of elements of a 3×3 unitary matrix cannot be absorbed into the phase convention of six fields. This possibility of CP-violation will be discussed later on.

ii) *Case* (A, B)

This is a rather delicate case. We denote two left doublets, one right doublet and two singlets by L_{d1}, L_{d2}, R_d, $R_s{}^{(p)}$ and $R_s{}^{(n)}$, respectively. The general form of $\mathcal{L}_{\text{mass}}$ is given by

$$\mathcal{L}_{\text{mass}} = \sum_{i=1,2} [m_i \bar{L}_{di} R_d + M_i^{(n)} \bar{L}_{di} \varphi R_s^{(n)} + M_i^{(p)} \bar{L}_{di} \varepsilon \varphi^* R_s^{(p)}] + \text{h.c.},$$

where m_i, $M_i^{(n)}$ and $M_i^{(p)}$ are arbitrary complex numbers. After diagonalization of mass terms (in this case, the CP-odd part of coupling with σ does not disappear in general) each multiplet can be expressed as follows:

$$L_{d1} = \frac{1+\gamma_5}{2} \begin{pmatrix} p \\ \cos\theta e^{i\alpha} n + \sin\theta e^{i\beta} \lambda \end{pmatrix}, \qquad L_{d2} = \frac{1+\gamma_5}{2} \begin{pmatrix} e^{i\gamma} \zeta \\ -\sin\theta e^{i\alpha} n + \cos\theta e^{i\beta} \lambda \end{pmatrix},$$

$$R_d = \frac{1-\gamma_5}{2} \begin{pmatrix} \sin\xi \cdot p + \cos\xi \cdot \zeta \\ \sin\eta \cdot n + \cos\eta \cdot \lambda \end{pmatrix}, \qquad R_s^{(p)} = \frac{1-\gamma_5}{2} (\cos\xi \cdot p - \sin\xi \cdot \zeta),$$

$$R_s^{(n)} = \frac{1-\gamma_5}{2} (\cos\eta \cdot n - \sin\eta \cdot \lambda), \qquad (7)$$

where phase factors α, β and γ satisfy two relations with the masses of the quartet:

$$e^{i\gamma} m_\zeta \sin\theta \cos\xi = m_p \cos\theta \sin\xi - c^{i\alpha} m_n \sin\eta,$$

$$e^{i\gamma} m_\zeta \cos\theta \cos\xi = -m_p \sin\theta \cos\xi + e^{i\beta} m_\lambda \cos\eta. \qquad (8)$$

Owing to the presence of phase factors, there exists a possibility of CP-violation also through the weak current. However, the strangeness changing neutral current is proportional to $\sin\eta \cos\eta$ and its experimental upper bound is roughly

$$\sin\eta \cos\eta < 10^{-2\sim-3}. \qquad (9)$$

Thus, making an approximation of $\sin\eta \sim 0$ (the other choice $\cos\eta \sim 0$ is less critical) we obtain from Eq. (8)

$$m_\zeta / m_p \sim \cot\theta \cdot \tan\xi,$$

$$m_\lambda / m_n \sim \sin\xi / \sin\theta. \qquad (10)$$

We have no low-lying particle with a quantum number corresponding to ζ, so that m_ζ, which is a measure of chiral $SU(4) \times SU(4)$ breaking, should be sufficiently large compared to the masses of the other members. However, the present experimental results on the g_A/g_V ratios of the octet baryon β-decay would not permit $\sin\xi > \sin\theta$. Thus, it seems difficult to reconcile the hierarchy of chiral symmetry breaking with the experimental knowledge of the semileptonic processes.

iii) *Case* (B, B)

As a previous one, in this case also, occurrence of CP-violation is possible, but in order to suppress $|\Delta S| = 1$ neutral currents, coefficients of the axial-vector part of $\Delta S = 0$ and $|\Delta S| = 1$ weak currents must take signs oppossite to each other. This contradicts again the experiments on the baryon β-decay.

iv) *Case* (A, A)

In a similar way, we can show that no CP-violation occurs in this case as far as $\mathcal{L}'=0$. Furthermore this model would reduce to an exactly $U(4)$ symmetric one.

Summarizing the above results, we have no realistic models in the quartet scheme as far as $\mathcal{L}'=0$. Now we consider some examples of CP-violation through $\mathcal{L}'$. Hereafter we will consider only the case of (A, C). The first one is to introduce another scalar doublet field ψ. Then, we may consider an interaction with this new field

4 e)

$$\mathcal{L}'=\bar{q}\psi C\frac{1-\gamma_5}{2}q+\text{h.c.}\,, \tag{11}$$

$$\psi\equiv\begin{pmatrix} \bar{\psi}^0 & \psi^+ & 0 & 0 \\ -\psi^- & \psi^0 & 0 & 0 \\ 0 & 0 & \bar{\psi}^0 & \psi^+ \\ 0 & 0 & -\psi^- & \psi^0 \end{pmatrix}, \qquad C\equiv\begin{pmatrix} c_{11} & 0 & c_{12} & 0 \\ 0 & d_{11} & 0 & d_{12} \\ c_{21} & 0 & c_{22} & 0 \\ 0 & d_{21} & 0 & d_{22} \end{pmatrix},$$

where c_{ij} and d_{ij} are arbitrary complex numbers. Since we have already made use of the gauge transformation to get rid of the CP-odd part from the quartet mass term, there remains no such arbitrariness. Furthermore, we note that an arbitrariness of the phase of ψ cannot absorb all the phases of c_{ij} and d_{ij}. So, this interaction can cause a CP-violation.

Another one is a possibility associated with the strong interaction. Let us consider a scalar (pseudoscalar) field S which mediates the strong interaction. For the interaction to be renormalizable and $SU_{\text{weak}}(2)$ invariant, it must belong to a $(4, 4^*)+(4^*, 4)$ representation of chiral $SU(4)\times SU(4)$ and interact with q through scalar and pseudoscalar couplings. It also interacts with φ and possible renormalizable forms are given as follows:

$$\begin{aligned} &\operatorname{tr}\{G_0S^+\boldsymbol{\varphi}\}+\text{h.c.}\,, \\ &\operatorname{tr}\{G_1S^+\boldsymbol{\varphi}G_2\boldsymbol{\varphi}^+S\}+\text{h.c.}\,, \\ &\operatorname{tr}\{G_1'S^+\boldsymbol{\varphi}G_2'S^+\boldsymbol{\varphi}\}+\text{h.c.}\,, \end{aligned} \tag{12}$$

with

$$\boldsymbol{\varphi}\equiv\begin{pmatrix} \bar{\varphi}^0 & \varphi^+ & 0 & 0 \\ -\varphi^- & \varphi^0 & 0 & 0 \\ 0 & 0 & \bar{\varphi}^0 & \varphi^+ \\ 0 & 0 & -\varphi^- & \varphi^0 \end{pmatrix},$$

where G_i is a 4×4 complex matrix and we have used a 4×4 matrix representation for S. It is easy to see that these interaction terms can violate CP-conservation.

Next we consider a 6-plet model, another interesting model of CP-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have CP-violating effects through the interference among these different current components. An interesting feature of this model is that the CP-violating effects of lowest order appear only in $\Delta S\neq 0$ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, $\Delta S=0$ non-leptonic and pure-leptonic processes. 4 i)

4 j)

So far we have considered only the straightforward extensions of the original Weinberg's model. However, other schemes of underlying gauge groups and/or scalar fields are possible. Georgi and Glashow's model[4] is one of them. We can easily see that CP-violation is incorporated into their model without introducing any other fields than (many) new fields which they have introduced already.

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