Extrapolating sensitivity at infinite statistics

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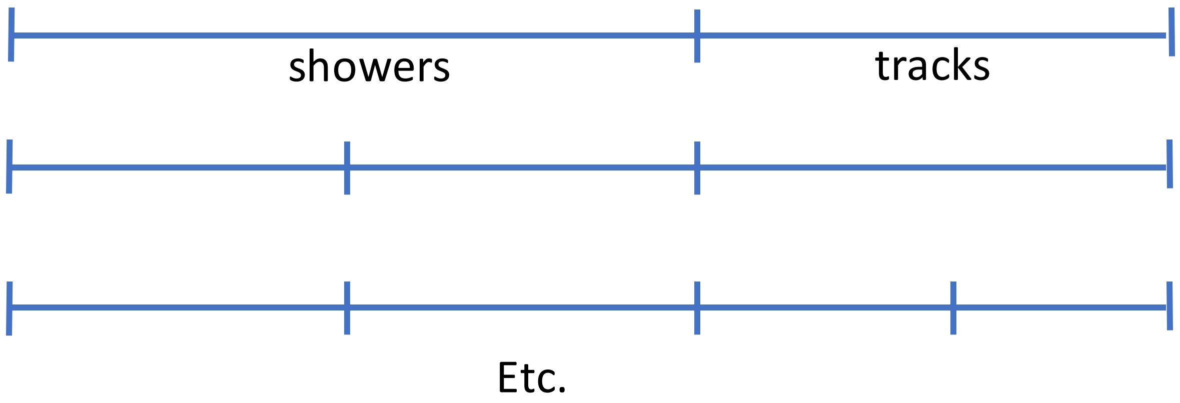
Group meeting

2019-05-09

Goal

- Goal:
  Show whether the Neutrino Mass Ordering sensitivity increases when using multiple categories in the PID variable q

- Method: introduce some binning scheme in PID variable q and calculate the sensitivity



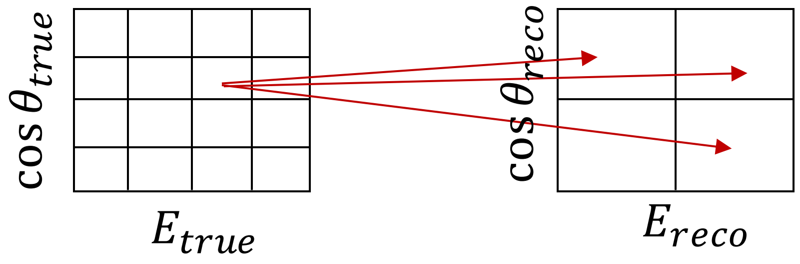
Issue

- We see: Sensitivity increases with more `PID categories`

- Due to signal in PID variable q
- Due to statistics of response matrix
  - More categories → less events per Response Matrix (track, shower, etc.)
  - Response matrix mapping gets worse with less events
  - Event distributions deviate more from a true detector response
  - Easier to distinguish two hypotheses H0, H1
  - Sensitivity to hypothesis H1 goes up

- Decouple these 2 effects

Detector Response Matrix



Map true events to reco events

Response matrix property

- When calculating the average chi2 for a RM [1]:

$$\langle \Delta\chi^2 \rangle(N) = \langle \Delta\chi^2_\infty \rangle + \sum_i^{true} \sum_j^{reco} \frac{K_{ij}}{N_i} + O(N^{-\frac{3}{2}})$$

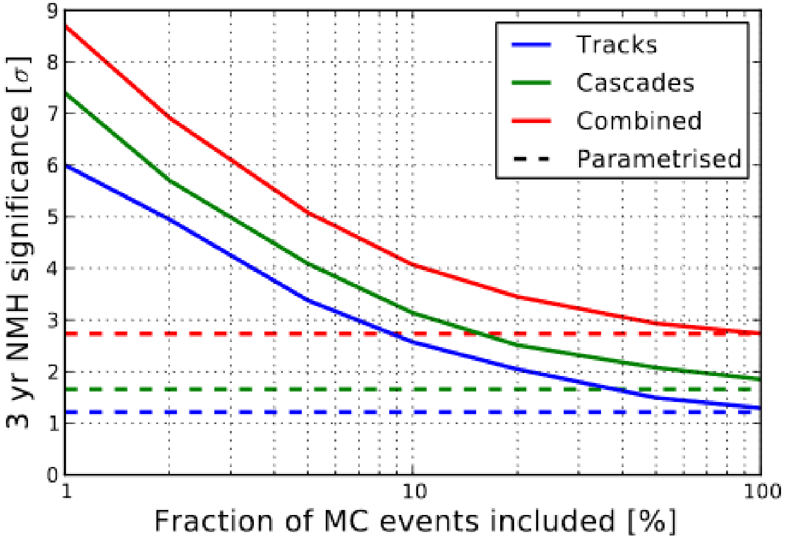
$$K_{ij} \approx N_j^A \left(\frac{n_i^A}{N_j^A} - \frac{n_i^B}{N_j^B} \right) \Phi_i(X_j) V_j e_i$$

$$\langle \Delta\chi^2 \rangle(N) \approx \langle \Delta\chi^2_\infty \rangle + \frac{K}{N_{MC}}$$

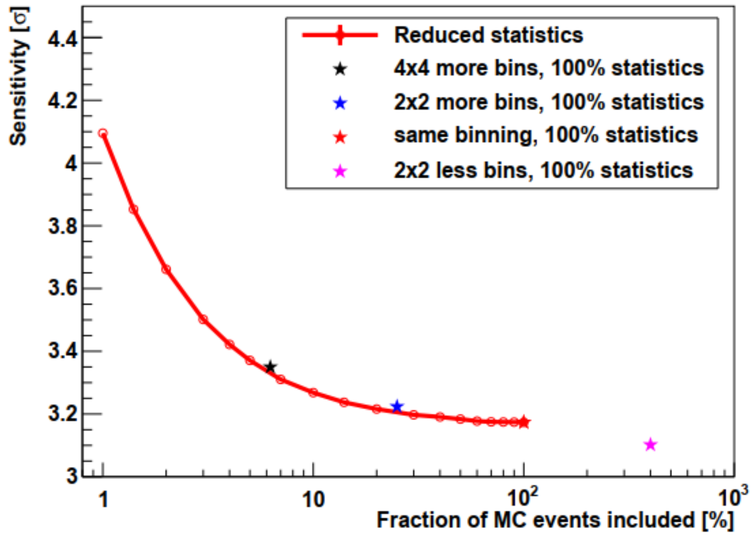
- See backup for details on K_{ij}

[1] Neutrino oscillations and Earth tomography with KM3NeT-ORCA, S. Bourret, 2018

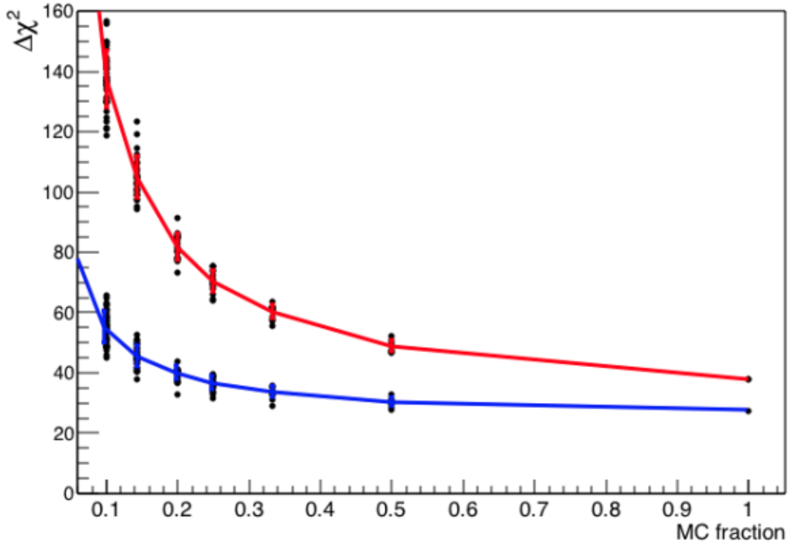
Literature



Determining the Neutrino Mass Hierarchy with the Precision IceCube Next Generation Upgrade, *L. Schulte*, 2015



Measuring the neutrino mass hierarchy with the future KM3NeT/ORCA detector, *J. Hofestädt*, 2017



Neutrino oscillations and Earth tomography with KM3NeT-ORCA, *S. Bourret*, 2018

Fit to chi2 of sampled response matrix

1. Create response matrix with `fraction` of total MC events (4M)
2. Calculate sensitivity with `sampled` RM
3. Do 1. and 2. many times
4. Sensitivity should follow

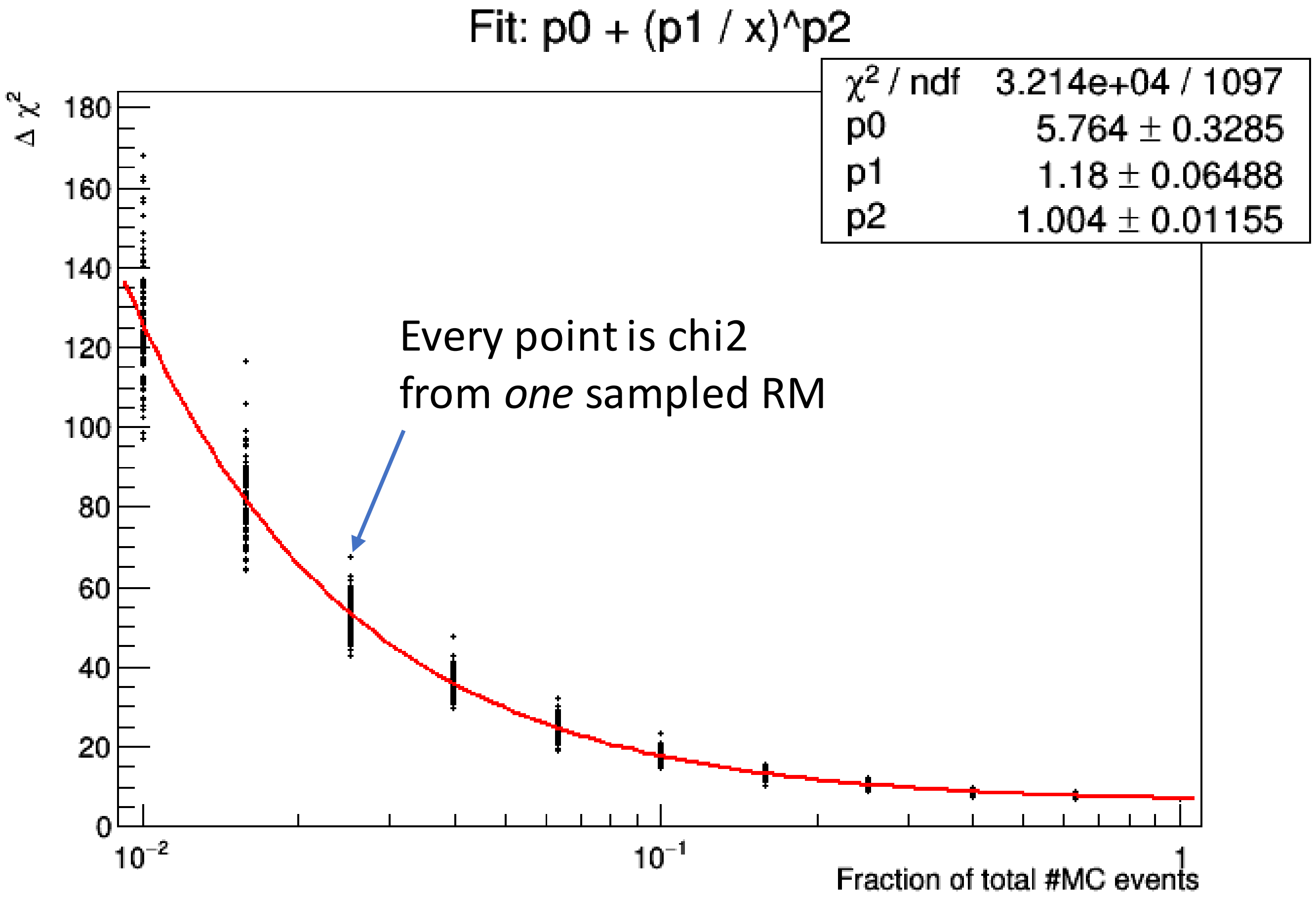
$$\langle \Delta\chi^2 \rangle \approx \langle \Delta\chi^2_\infty \rangle + \frac{K}{N_{MC}}$$

5. Fit to function `p0 + (p1 / x)^p2` as a check on the behavior: p2 = 1

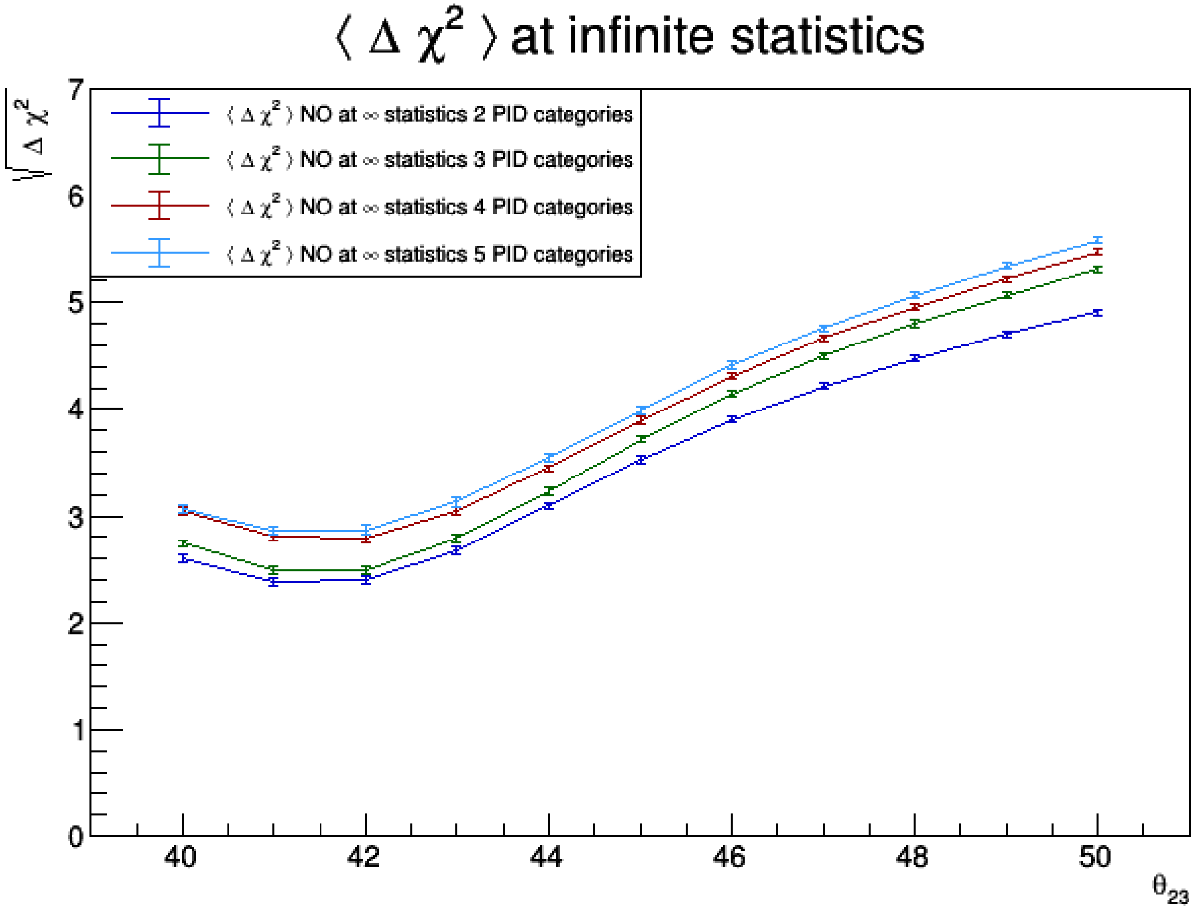
6. Fit to function `p0 + (p1 / x)`

$$\Delta\chi^2(1) = 7.65$$

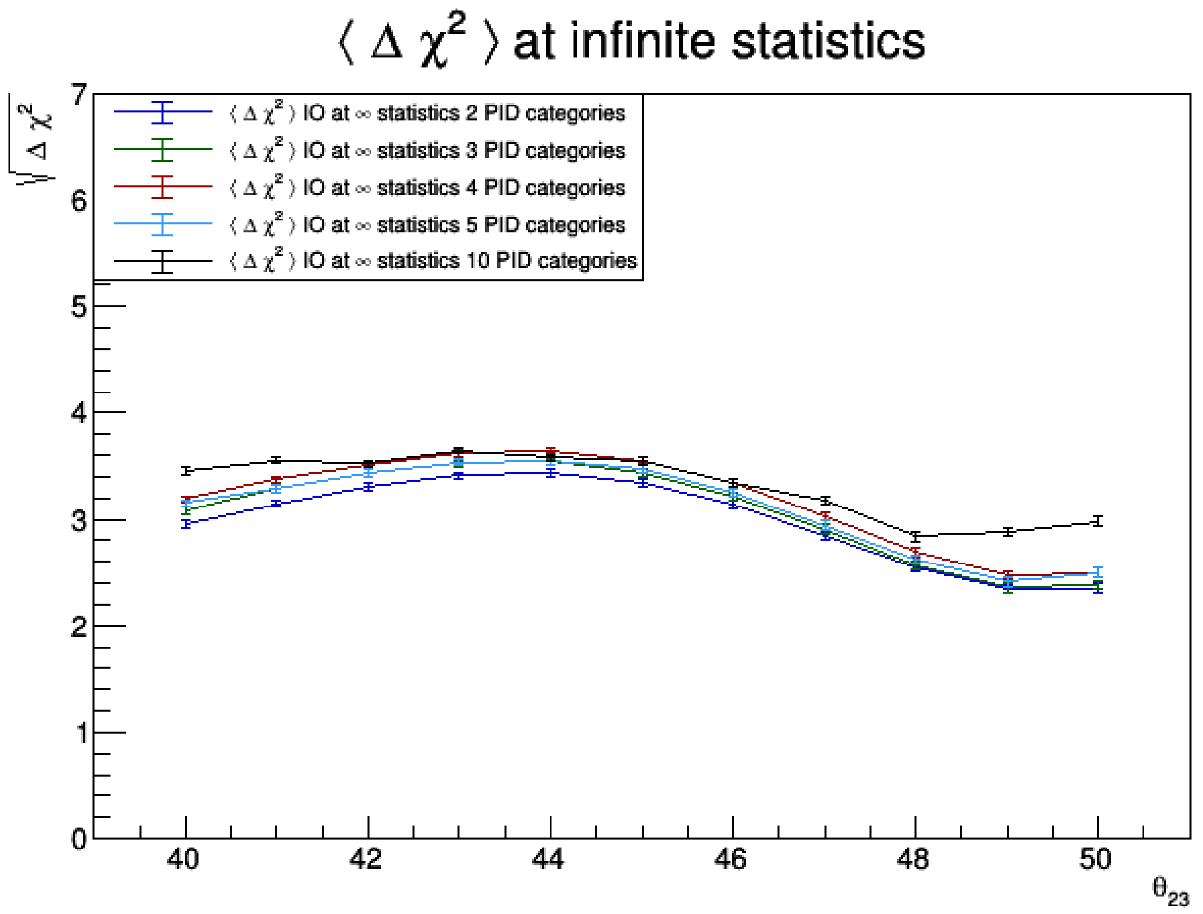
$$\langle \Delta\chi^2_\infty \rangle = 5.66$$



Apply method to sensitivity curve



Apply method to sensitivity curve



Backup

$$\langle \Delta\chi^2 \rangle(N) = \langle \Delta\chi^2_\infty \rangle + \sum_i^{true} \sum_j^{reco} \frac{K_{ij}}{N_i} + O(N^{-\frac{3}{2}})$$

$$K_{ij} \approx N_j^A \Big(\frac{n_i^A}{N_j^A} - \frac{n_i^B}{N_j^B} \Big) \Phi_i(X_j) V_j e_i$$

N_j^A: average number of events in reco bin j under hypothesis A

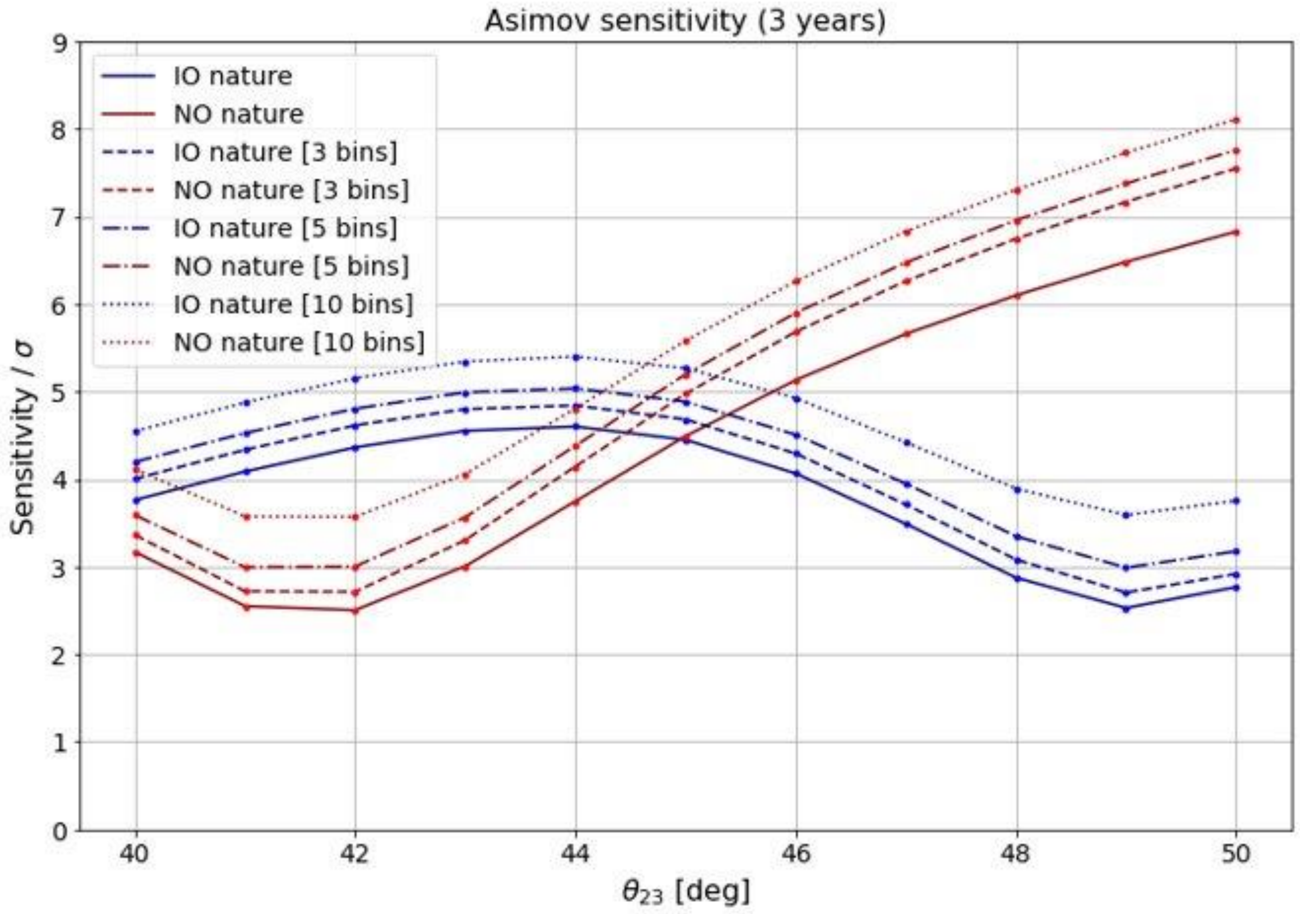
n_i^B: average number of events in true bin i under hypothesis B

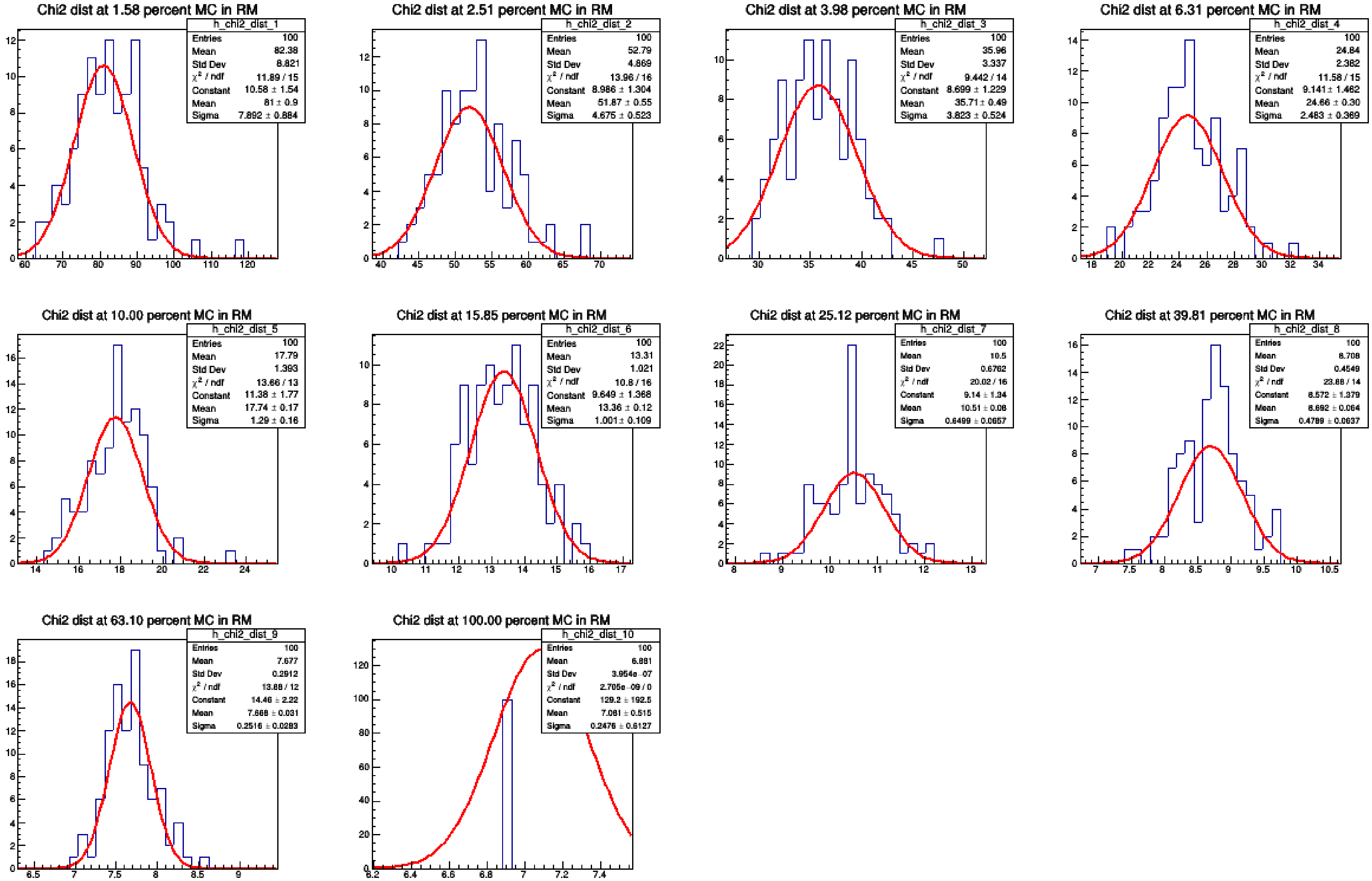
Φ_i: the pdf that maps true events from bin i to bin j

X_j: bin center of reco bin j

V_j: bin volume of reco bin j

e_i: detector efficiency at true bin i


Asimov sensitivity (3 years)
IO nature
NO nature
IO nature [3 bins]
NO nature [3 bins]
IO nature [5 bins]
NO nature [5 bins]
IO nature [10 bins]
NO nature [10 bins]
Sensitivity / σ
θ23 [deg]
0
1
2
3
4
5
6
7
8
9
40
42
44
46
48
50


Chi2 dist at 1.58 percent MC in RM
h_chi2_dist_1
Entries 100
Mean 82.38
Std Dev 8.821
χ² / ndf 11.89 / 15
Constant 10.58 ± 1.54
Mean 81 ± 0.9
Sigma 7.892 ± 0.884
Chi2 dist at 2.51 percent MC in RM
h_chi2_dist_2
Entries 100
Mean 52.79
Std Dev 4.869
χ² / ndf 13.96 / 16
Constant 8.986 ± 1.304
Mean 51.87 ± 0.55
Sigma 4.675 ± 0.523
Chi2 dist at 3.98 percent MC in RM
h_chi2_dist_3
Entries 100
Mean 35.96
Std Dev 3.337
χ² / ndf 9.442 / 14
Constant 8.699 ± 1.229
Mean 35.71 ± 0.49
Sigma 3.823 ± 0.524
Chi2 dist at 6.31 percent MC in RM
h_chi2_dist_4
Entries 100
Mean 24.84
Std Dev 2.382
χ² / ndf 11.58 / 15
Constant 9.141 ± 1.462
Mean 24.66 ± 0.30
Sigma 2.483 ± 0.369
Chi2 dist at 10.00 percent MC in RM
h_chi2_dist_5
Entries 100
Mean 17.79
Std Dev 1.393
χ² / ndf 13.66 / 13
Constant 11.38 ± 1.77
Mean 17.74 ± 0.17
Sigma 1.29 ± 0.16
Chi2 dist at 15.85 percent MC in RM
h_chi2_dist_6
Entries 100
Mean 13.31
Std Dev 1.021
χ² / ndf 10.8 / 16
Constant 9.649 ± 1.368
Mean 13.36 ± 0.12
Sigma 1.001 ± 0.109
Chi2 dist at 25.12 percent MC in RM
h_chi2_dist_7
Entries 100
Mean 10.5
Std Dev 0.6762
χ² / ndf 20.02 / 16
Constant 9.14 ± 1.34
Mean 10.51 ± 0.08
Sigma 0.6499 ± 0.0657
Chi2 dist at 39.81 percent MC in RM
h_chi2_dist_8
Entries 100
Mean 8.708
Std Dev 0.4549
χ² / ndf 23.88 / 14
Constant 8.572 ± 1.379
Mean 8.692 ± 0.064
Sigma 0.4789 ± 0.0637
Chi2 dist at 63.10 percent MC in RM
h_chi2_dist_9
Entries 100
Mean 7.677
Std Dev 0.2912
χ² / ndf 13.88 / 12
Constant 14.46 ± 2.22
Mean 7.668 ± 0.031
Sigma 0.2516 ± 0.0283
Chi2 dist at 100.00 percent MC in RM
h_chi2_dist_10
Entries 100
Mean 6.881
Std Dev 3.954e-07
χ² / ndf 2.705e-09 / 0
Constant 129.2 ± 192.5
Mean 7.061 ± 0.515
Sigma 0.2476 ± 0.6127