AdS/CFT and Heavy Ion Physics

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Utrecht

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References

- Based on the papers

- Reviews on Heavy Ion Collisions and AdS/CFT
  arXiv:1101.0618 — Exhaustive, emphasis on AdS/CFT
  arXiv:0902.3663—Hydrodynamics for HIC
  arXiv:1102.3010—RHIC/LHC results and elliptic flow
  arXiv:1006.546—Non-conformal holographic QCD approach
Outline

- Lecture I:
  - AdS/CFT for QCD
  - Deformations of AdS/CFT
  - Bottom-up approach to AdS/CFT
  - Improved Holographic QCD
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- **Lecture I:**
  - AdS/CFT for QCD
  - Deformations of AdS/CFT
  - Bottom-up approach to AdS/CFT
  - Improved Holographic QCD

- **Lecture II:**
  - Glueball Spectrum
  - Thermodynamics
  - Transport
  - Jet quenching
  - Langevin diffusion
  - Thermalization
  - Outlook
Heavy ion collisions

- RHIC: Au + Au at $\sqrt{s} = 200$ GeV per nucleon; about $T = 200 - 300$ MeV.
- LHC: Pb + Pb at $\sqrt{s} = 2.76$ TeV/n about $T = 300 - 400$ MeV.
Heavy ion collisions

- **RHIC**: Au + Au at $\sqrt{s} = 200$ GeV per nucleon; about $T = 200 - 300$ MeV.
- **LHC**: Pb + Pb at $\sqrt{s} = 2.76$ TeV/n about $T = 300 - 400$ MeV.
- The quark-gluon plasma forms at $\sim 1$ fm and exists for $5 - 10$ fm.
- Cools down as it expands $\Rightarrow$ and hadronizes around $T = 170$ MeV.
What can we learn from Holography

- Phase diagram of QCD at finite $\mu$, $T$ and $B$
- Transport coefficients: Viscous relativistic hydrodynamics account for the observed $v_2$, $v_3$, etc quite well
  ⇒ Calculate the viscosities $\eta/s$ and $\zeta/s +$ higher order coefficients, other transport coefficients.

- Energy loss in hard probes
  Basic mechanisms: Gluon brehmstrahlung and Langevin diffusion
  ⇒ Calculate the jet-quenching parameter $\hat{q}$ and momentum diffusion parameters $\kappa$.

- Anomalous transport: Chial Magnetic Effect, Chiral Magnetic Wave, etc. ⇒ Calculate chiral conductivities

- Thermalization
  Non-equilibrium physics, formation of QGP ⇒ black-hole formation by collapsing matter
Large-N approximation?
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$$N_c \rightarrow \infty, \quad g^2 \rightarrow 0, \quad \lambda = g^2 N = \text{fixed}$$

- Only planar Feynman diagrams
- Quarks in loops suppressed by $N_f/N_c$
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This is what we will assume in the rest of the lectures...
N-dependence of thermodynamic quantities

Panero ’09

- About 10 % deviation in the hadron spectra
- Thermodynamic observables very close to each other
Holographic dual of QCD?

Top-bottom approach: holography from two different descriptions of D-branes.
Holographic dual of QCD?

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1. **Open strings** \(\Rightarrow\) **Gauge theory in** \(d\) **dimensions**
2. **Closed strings** \(\Rightarrow\) **GR in** \(d + 1\) **dimensions**

At low energy 1. and 2. decouple and become equivalent!
Holographic dual of QCD?

Top-bottom approach: holography from two different descriptions of D-branes. They couple to:

1. Open strings ⇒ Gauge theory in $d$ dimensions
2. Closed strings ⇒ GR in $d + 1$ dimensions

At low energy 1. and 2. decouple and become equivalent!

Very hard to deal with in practice...
Phenomenological approach

Da Rold, Pomarol; Erlich et al ’05
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Da Rold, Pomarol; Erlich et al ’05

- Construct a consistent GR set-up in the most economic fashion:
  - Dimensions of QFT + 1 (energy scale)
  - Symmetries of QFT in the bulk
  - One bulk field for each relevant + marginal operator
  - Realization of dynamical phenomena (e.g. spontaneous symmetry breaking)
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- Check this by calculations
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- Check this by calculations
- For generic GR set-ups ⇒ universal lessons
Some details of the duality
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Domain-wall type geometries with boundary

Minimal metric:
\[ ds^2 = b(r)^2 (dr^2 + dx_d^2) \]
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Fundamental relation:
\[ \exp(-S_G[\phi(x, r) \rightarrow \phi_0(x)]) = \langle \exp(\int \mathcal{O}\phi_0) \rangle \]

Computes n-point functions \( \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle \) of QFT.
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   \]
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2. Finite temperature in the QFT \( \leftrightarrow \) black-hole in the geometry.
The minimal holographic dual at zero $T$
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- Consider pure $SU(N)$ at large $N$, at strong coupling ⇒
  classical Einstein’s GR
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  1. $Mink^4 +$ energy scale $\Rightarrow 5$ dimensions
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  1. $Mink^4$ + energy scale ⇒ 5 dimensions
  2. Only relevant/marginal operators the stress-tensor $T_{\mu\nu}$ and the gluon condensate $\text{Tr} F^2$

  ⇒ Need metric $g_{\mu\nu} \Leftrightarrow T_{\mu\nu}$ and dilaton $\phi \Leftrightarrow \text{Tr} F^2$
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- Most economic set-up:
  1. $Mink^4 +$ energy scale $\Rightarrow$ 5 dimensions
  2. Only relevant/marginal operators the stress-tensor $T_{\mu\nu}$ and the gluon condensate $\text{Tr} F^2$
     $\Rightarrow$ Need metric $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$ and dilaton $\phi \leftrightarrow \text{Tr} F^2$
  3. Running coupling extremely important for correct thermodynamics $\Rightarrow$ non-conformally invariant background
     with $e^\phi \propto g^2 N$ a function of r: $\phi = \phi(\Lambda r)$ with $\Lambda \Rightarrow$
     dynamically generated QCD scale.
Improved HQCD  U.G, Kiritsis; U.G. Kiritsis, Nitti ’07
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- Gravitational dual in $2\partial$ effective GR theory:

$$S = M_p^3 N_c^2 \int d^5 x \sqrt{g} \left\{ R - \frac{4}{3} (\partial \phi)^2 - V(\phi) \right\}$$

- Look for domain-wall type solutions of the Einstein-dilaton eqs:

$$ds^2 = b^2(r) (dr^2 - dt^2 + dx_3^2), \quad \lambda = \lambda(r) \equiv \exp(\phi(r))$$
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Dictionary: Geometry vs. QFT:

- Scale factor $b_0(r)$ is the energy scale in the field theory $E$,
- Dilaton $\lambda(r) \propto \lambda_t(E)$ running ’t Hooft coupling,
- Dilaton potential $V(\phi) \Leftrightarrow \beta(\lambda_t)$ the beta-function of the QFT.
Quark potential and confinement

Linear quark potential from flux tube:

\[ V_{q\bar{q}}(L) = \sigma_s L + \cdots \]
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K.G. Wilson ’74  \[ \langle W[C] \rangle = \langle \text{Tr} P e^{- \oint_C A_\mu dx^\mu} \rangle = e^{-V_{q\bar{q}}(L)T} \]
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Gauge-gravity duality:

\[ W[C] \Leftrightarrow \text{string world-sheet ending on } C \]

\[ \langle W[C] \rangle = e^{-S[\text{string}; C]} \]

J. Maldacena ’98; S. Rey, J. Yee ’98
Linear quark potential $\iff \exists$ minimum of $b_s$
This constrains large $\lambda$ asymptotics of the dilaton potential $V(\lambda)$. 
• Requirement of a marginal deformation $\text{Tr} F^2$ fixes the UV asymptotics as
  \[ V(\lambda) = v_0 + v_1 \lambda + \cdots, \quad \lambda \to 0 \]

• Requirement of linear color confinement fixes the IR asymptotics as
  \[ V(\lambda) \propto \lambda^{\frac{4}{3}} \log^{\frac{1}{2}} \lambda + \cdots, \quad \lambda \to \infty \]

• Then 1) mass gap 2) first order $T_c$ is automatic

• Spectrum of glueballs can be computed with no IR ambiguity
IR asymptotics
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In terms of the potential:

\[ V(\phi) \rightarrow e^{\frac{4}{3} \phi} \phi^{\frac{\alpha - 1}{\alpha}} + \cdots \]

(we will eventually set \( \alpha = 2 \))
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IR asymptotics of the background:

\[ b(r) \sim e^{-\left(\frac{r}{L}\right)^\alpha}, \quad \lambda(r) \sim e^{3/2(\frac{r}{L})^\alpha} \left(\frac{r}{L}\right)^{\frac{3}{4}(\alpha-1)} \quad \text{as} \quad r \rightarrow \infty \]
Parameters of the theory
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- The dilaton potential:

\[ V = \frac{12}{\ell^2} \left\{ 1 + V_0 \lambda + V_1 \lambda^{4/3} \log \left( 1 + V_2 \lambda^{4/3} + V_3 \lambda^2 \right)^{1/2} \right\} \]

- Parameters in the action: \( V_0, V_2 \) fixed by scheme independent \( \beta \)-function coefficients \( (b_0 \text{ and } b_1) \), \( V_1, V_3 \) fixed by the latent heat \( L_h \) and \( S(2T_c) \) (lattice)
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- The Planck scale \( M_p \) also by thermodynamics. matching high T asymptotics of QCD free energy: \[ M_p = \left( 45\pi^2 \right)^{-\frac{1}{3}} \ell^{-1} \]
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- The string length \( \ell_s \) by lattice string \( \sigma_s \): \( \frac{\ell_{AdS}}{\ell_s} \approx 6.5 \)
  This measures how good the two-derivative approximation is!
The spectra of the theory
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Spectrum of 4D glueballs ⇔ Spectrum of normalizable fluctuations of the bulk fields.

- Spin 2: $h_{\mu \nu}^{TT}$;  Spin 0: mixture of $h_\mu^\mu$ and $\delta \Phi$;
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  where $V_s = V_s[b(r), \lambda(r)]$

- Both mass gap and discrete spectra $m^2$ follows if $V_s$ has a well-shape ⇔ linear quark potential!
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Quadratic action for fluctuations:

$$S \sim \frac{1}{2} \int d^4 x d r e^{2B(r)} \left[ \ddot{\zeta}^2 + (\partial_\mu \zeta)^2 \right]$$

$$\ddot{\zeta} + 3\dot{B}\dot{\zeta} + m^2 \zeta = 0, \quad \partial_\mu \partial_\mu \zeta = -m^2 \zeta$$
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- Scalar: $B(r) = 3/2 A(r) + \log(\dot{\Phi}/\dot{A})$
- Tensor: $B(r) = 3/2 A(r)$
Comparison with one lattice study  Meyer, ’02

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>Lattice (MeV)</th>
<th>Our model (MeV)</th>
<th>Mismatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^{++}</td>
<td>1475 (4%)</td>
<td>1475</td>
<td>0</td>
</tr>
<tr>
<td>2^{++}</td>
<td>2150 (5%)</td>
<td>2055</td>
<td>4%</td>
</tr>
<tr>
<td>0^{+++}</td>
<td>2755 (4%)</td>
<td>2753</td>
<td>0</td>
</tr>
<tr>
<td>2^{+++}</td>
<td>2880 (5%)</td>
<td>2991</td>
<td>4%</td>
</tr>
<tr>
<td>0^{++++}</td>
<td>3370 (4%)</td>
<td>3561</td>
<td>5%</td>
</tr>
<tr>
<td>0^{+++++}</td>
<td>3990 (5%)</td>
<td>4253</td>
<td>6%</td>
</tr>
</tbody>
</table>

0^{++} : Tr $F^2$; 2^{++} : Tr $F_{\mu\rho}F^{\rho}_{\nu}$. 
Thermodynamics: results

- Fix the dilaton potential:

\[ V = \frac{12}{\ell^2} \left\{ 1 + V_0 \lambda + V_1 \lambda^{4/3} \log \left( 1 + V_2 \lambda^{4/3} + V_3 \lambda^2 \right)^{1/2} \right\} \]
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- Two sol’ns with AdS asymptotics \( ds^2 = e^{A(r)} \left( dt^2 f(r) + dx_3^2 + \frac{dr^2}{f(r)} \right) \):
  - Thermal Gas \( \Leftrightarrow \) thermal gas of glueballs.
  - Black-hole \( \Leftrightarrow \) quark-gluon plasma.
  - Hawking-Page transition \( \Leftrightarrow \) deconfinement transition at \( T_c \).
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- Free energy from \( S_{BH} - S_{TG} \).

- Parameter fixing: \( V_0, V_2 \) fixed by scheme independent \( \beta \)-function coefficients \( (b_0 \text{ and } b_1) \), \( V_1, V_3 \) fixed by the latent heat \( L_h \) and \( S(2T_c) \) (lattice).
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- Deconfinement transition at \( T_c = 247 \text{ MeV} \) (lattice: \( T_c = 260 \text{ MeV} \)). Comparison to Boyd et al. '96
The free energy:

\[ \frac{F}{N_c^2 T_c^4 V_3} \]

\[ \begin{array}{c}
\text{0.01} \\
\text{0} \\
\text{-0.01} \\
\text{-0.02} \\
\text{-0.03}
\end{array} \]

\[ \frac{T}{T_c} \]

\[ \begin{array}{c}
1 \\
1.1 \\
1.2
\end{array} \]
The free energy:

\[
\frac{F}{N_c^2 T_c^4 V_3}
\]

\[
\frac{T}{T_{\text{min}}}
\]

\[
\frac{T}{T_c}
\]

\[
\lambda_h
\]
iHQCD Thermodynamics continued

The free energy:

- Big and Small black-hole solutions, like $N = 4$ on $R^3$
- Existence of $T_{min} \Leftrightarrow$ phase transition at $T_c > T_{min}$
Survey of thermodynamical quantities I

Comparison to Panero ’09

Entrophy density

$\frac{s}{T^3}$, normalized to the SB limit

SU(3)  
SU(4)  
SU(5)  
SU(6)  
SU(8)  

improved holographic QCD model

$T/T_c$
Comparison to Panero ’09
Survey of thermodynamical quantities III

Comparison to Panero ’09
Survey of thermodynamical quantities IV

Comparison to Panero ’09

Trace of the energy-momentum tensor

$\Delta / T^4$, normalized to the SB limit of $p / T^4$

- SU(3)
- SU(4)
- SU(5)
- SU(6)
- SU(8)

improved holographic QCD model
Survey of thermodynamic quantities V

Comparison to Boyd et al. ’96 Thermodynamic functions and the speed of sound:

$$\frac{e}{T^4 N_c^2}, \frac{3s}{4 T^3 N_c^2}, \frac{3p}{T^4 N_c^2}$$
Dissipation in relativistic hydrodynamics
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- \( T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu} \), what is \( \Pi^{\mu\nu} \) ?
Dissipation in relativistic hydrodynamics

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Landau reference frame.
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\[
T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} \\
+ P^{\mu\alpha} P^{\nu\beta} \left[ \eta \left( \partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \partial \cdot u \right) + \zeta g_{\alpha\beta} \partial \cdot u \right] \\
+ O(\partial u)^2; \quad P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu
\]

- $\eta$: “shear viscosity”; $\zeta$: “bulk viscosity"
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Another exercise: Navier-Stokes and continuity eqs. follow from $\partial_\mu T^{\mu\nu} = 0$. 
Calculation of viscosities in QFT
Calculation of viscosities in QFT

- Kubo’s linear response theory:
  \[ \mathcal{L} \rightarrow \mathcal{L} + \int \mathcal{O}^A \delta \phi_A, \]
  then \[ \langle \mathcal{O}^B \rangle = G^{BA}_R \delta \phi_A \]
  where \[ G_R(\omega, \vec{k}) = -i \int d^4x e^{-i\vec{k} \cdot \vec{x}} \theta(t) \langle [\mathcal{O}^A(t, \vec{x}), \mathcal{O}^B(0, \vec{0})] \rangle \]
Calculation of viscosities in QFT

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- **Viscosities:** response of \( T^{\mu \nu} \) to \( g_{\alpha \beta} \).
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- **Viscosities:** response of \( T^{\mu\nu} \) to \( g_{\alpha\beta} \).

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Calculation of viscosities in QFT

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- Read off \( \eta \) from the \( xy \) component, and \( \zeta \) from the \( 11 + 22 + 33 \) component.
Hydrodynamics at first order
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- Relativistic fluid with 4-velocity $u^\mu$, energy density $\epsilon$ and pressure $p$.

- Navier-Stokes & continuity equations from the energy-momentum tensor:

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The characteristic parameters of the fluid at $\mathcal{O}(\partial u)$

- Shear viscosity $\eta$: For all 2$\partial$ theories $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$

Buchel and Liu '03
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The characteristic parameters of the fluid at $\mathcal{O}(\partial u)$

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  Buchel and Liu ’03

- **Bulk viscosity $\zeta$:** What is already known from field theory and lattice?
Holographic computation

- Kubo’s linear response theory:
  \[ \zeta = -\frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G_R(\omega, 0) \]

- More complicated than shear because \( h_{ii} \) mix with dilaton fluctuations \( \delta \phi \).

- Derive the fluctuation equations for \( h_{ii} \), pick up the gauge \( \delta \phi = 0 \),

- Fluctuations decouple in the smart gauge! Gubser et al ’08: Define \( X = \phi' / 3A' \)

- \( h''_{ii} + \left( 3A' + \frac{2X'}{X} + \frac{f'}{f} \right) h'_{ii} + \left( \frac{\omega^2}{f^2} - \frac{f'X'}{fX} \right) h_{ii} = 0 \)

- Boundary conditions:
  - \( h_{ii}(\phi = -\infty) = 1 \) and,
  - In-falling wave at horizon \( h_{ii} \to c_b (r_h - r)^{-\frac{i\omega}{4\pi T}} \)

- Read off \( c_b(\omega, T) \)
Results I: Comparison to Meyer '08
Results I: Comparison to Meyer '08

- Near UV, vanishes as expected: ideal gluon gas at high T
- Near $T_c$ Peak, much smaller than lattice expectations!
- Agreement with another holographic model Gubser et al. 08
Jet quenching

Back-to-back jet production is highly suppressed at RHIC:
Jet quenching

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The first direct signals of jet-quenching - November 2010!
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A clear signal of strongly-coupled plasma.
Quantification of jet quenching Baier et al ’96

What is known: recoiling hadrons are suppr

Compare to d+Au: suppression is final-state

M. van Hees, JINR, JINR
High-pT at SPS, RHIC and LHC
Quantification of jet quenching  

Baier et al ’96

What is known: recoiling hadrons are suppressed

\[ \frac{1}{N_{\text{Trigger}}} \frac{dN}{d(\Delta \phi)} \]

\( \Delta \phi \) (radians)

Compare to d+Au: suppression is final-state

\( \text{QGP} \)

quenched
Quantification of jet quenching  

Baier et al '96

What is known: recoiling hadrons are suppr

Compare to d+Au: suppression is final-state

Average transverse momentum lost into the media in a flight of distance $D$.

$$\hat{q} = \frac{\langle p_{\perp}^2 \rangle}{D}$$

Weak-coupling computation does not explain the data.
Energy loss of a heavy quark

- Highly energetic partons produced in head-on nuclei collisions are very important probes
Energy loss of a heavy quark

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- In weakly coupled QGP: main source of energy loss is collisions with thermal gluons and quarks.
  
  D. Teaney ’03

- What happens in a strongly coupled plasma?
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  Combination of two distinct mechanisms:
Energy loss of a heavy quark

- Highly energetic partons produced in head-on nuclei collisions are very important probes.
- **In weakly coupled QGP**: main source of energy loss is collisions with thermal gluons and quarks.
  
  D. Teaney ’03

- What happens in a strongly coupled plasma? Combination of two distinct mechanisms:
  1. Energy loss by Langevin diffusion process
  2. Energy loss by gluon Brehmstahlung
Langevin diffusion process
Langevin diffusion process

- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau) F^\mu(\tau)$
  
  $S_0$: free quark action, $F(\tau)$: drag force—summarizes the d.o.f of the plasma

- EOM of the hard probe:
Langevin diffusion process

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  \[
  \frac{\delta S_0}{\delta X_i(t)} = \int_{-\infty}^{+\infty} d\tau \theta(\tau) C^{ij}(\tau) X_j(t - \tau) + \xi^i(t), \quad i = 1, 2, 3
  \]
  with \( \langle \xi^i(t)\xi^j(t') \rangle = A^{ij}(t - t') \)
Langevin diffusion process

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  with $\langle \xi^i(t)\xi^j(t') \rangle = A^{ij}(t - t')$

- The entire information is stored in:
  $C^{ij}(t) \equiv -i\langle [F^i(t), F^j(0)] \rangle$,
  $A^{ij}(t) \equiv -\frac{i}{2} \langle \{F^i(t), F^j(0)\} \rangle$. 

AdS/CFT and Heavy Ion Physics – p.37
Local approximation
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- Suppose correlations vanish for $t - t' \gg \tau_c$:
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- Suppose correlations vanish for $t - t' \gg \tau_c$:

$$A^{ij}(t - t') \approx \kappa^{ij} \delta(t - t'), \quad C^{ij}(t) = \frac{d}{dt} \gamma^{ij}(t)$$
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\]

with

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\kappa^{ij} = \lim_{\omega \to 0} A^{ij}(\omega),
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- For QGP at equilibrium at temperature \( T \):
  \[
  A^{ij}(\omega) = - \coth \left( \frac{\omega}{2T} \right) \text{Im} G^{ij}_R(\omega), \quad C^{ij}(\omega) = \text{Im} G^{ij}_R(\omega)
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- Thus, it is sufficient to calculate
  \[ G_R(\omega) = -i \int dt e^{-i\omega t} \theta(t) \langle [F^i(t), F^j(0)] \rangle \]
Momentum broadening
Momentum broadening

For $\Delta t \ll \tau_c$, short-time solution to the EOM for the hard-probe:

(in momentum space, around linear trajectory: $\vec{p} \simeq p_0 \vec{v}/v + \delta \vec{p}$.
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$$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$$
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Solution with initial conditions $\delta \vec{p}(t = 0) = 0$:

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$$p^\perp(t) = \int_0^t dt' e^{\eta^\perp (t'-t)} \xi^\perp (t'),$$

Compute the noise-average of fluctuations:

$$\langle (p^\perp)^2 \rangle = \int_0^t dt' \int_0^t dt'' e^{\eta^\perp (t'+t''-2t)} \langle \xi^\perp (t') \xi^\perp (t'') \rangle$$
Momentum broadening

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Use $\langle \xi(t') \xi(t'') \rangle = \kappa \delta(t' - t'')$, for $t\eta^\perp \ll 1$:

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Momentum broadening

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$$\langle (p^\perp)^2 \rangle = 2\kappa^\perp t$$

thus jet-quenching parameter:

$$\hat{q}^\perp = \frac{\langle (p^\perp)^2 \rangle}{vt} = 2\frac{\kappa^\perp}{v}.$$
How to calculate in the bulk dual?

Recall \( S[X(t)] = S_0 + \int d\tau X_\mu(\tau) F^\mu(\tau) \)

To calculate \( \langle \{ F^\perp(t), F^\perp(0) \} \rangle = O(0) + \langle \{ \xi^\perp(t), \xi^\perp(0) \} \rangle \)

We need to calculate the fluctuations \( \delta X^\perp(t) \).
**Dual picture**

**Herzog et al; Gubser ’06**

**Holography:** Represent the (infinitely) heavy quark with a trailing string moving with constant $v$: 

![Diagram showing holography](image-url)
Holography: Represent the (infinitely) heavy quark with a trailing string moving with constant $v$:

Drag force on a heavy quark in a hot wind:

$$ F = \frac{dp}{dt} = \frac{1}{v} \frac{dE}{dt} = -\mu p + \zeta(t) $$

Ignore stochastic force $\zeta(t)$ in this talk $\Leftrightarrow$ fluctuations of the trailing string $\Rightarrow$ diffusion constant.
Standard calculation:

- Pick up the static gauge: $\sigma^0 = t, \sigma^1 = r$.
- String ansatz $x^1 = vt + \delta(r)$
- Minimize the area (in the string frame!)
- Compute the WS momentum flowing into the BH horizon
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$$ F = \frac{1}{v} \frac{dE}{dt} = -\frac{1}{2\pi \ell_s^2} v e^{2A(r_s)} \lambda(r_s)^{\frac{4}{3}}, \quad r_s \text{ defined by } f(r_s) = v^2. $$
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Relativistic limit, $v \to 1$: $F = -\frac{\ell^2}{\ell_s^2} \sqrt{\frac{45}{4N_c^2} \frac{T_s(T)}{s(T)}} \frac{v}{\sqrt{1-v^2}} \left( -\frac{\beta_0}{4} \log[1-v^2] \right)^4 + \cdots$

Non-relativistic limit $v \to 0$: $F = -\frac{\ell^2}{\ell_s^2} \left( \frac{45\pi}{N_c^2} s(T) \right)^{\frac{2}{3}} \frac{\lambda(r_h)}{2\pi} \frac{4}{3} \frac{4}{3} v + \cdots$
Comparison to conformal case

The AdS result: 
\[ F_{\text{conf}} = \frac{\pi}{2} \sqrt{\lambda T^2} \frac{v}{\sqrt{1-v^2}} \]

Fix \( \ell_s \) in our model by the lattice string tension
Fix \( \lambda = 5.5 \) in \( \mathcal{N} = 4 \) SYM:
Comparison to conformal case

The AdS result: $F_{con.f} = \frac{\pi}{2} \sqrt{\lambda T^2} \frac{v}{\sqrt{1-v^2}}$

Fix $\ell_s$ in our model by the lattice string tension
Fix $\lambda = 5.5$ in $N = 4$ SYM:

We clearly see the effects of asymptotic freedom!
Comparison schemes
Comparison schemes

- An important detail: How to compare to QCD?
- Direct scheme: $T_{QGP} = T_{our}$
Comparison schemes

- **An important detail:** How to compare to QCD?
- **Direct scheme:** $T_{QGP} = T_{our}$

In the range $1.5T_c < T < 3T_c$ $E_{QGP} \propto E_{GP} \propto T^4$

- **Alternative schemes:** $E_{QGP} = E_{our}$ or $s_{QGP} = s_{our}$
- We try all possible schemes.
Predictions for experiments
Predictions for experiments

Equilibration times for **charm** and **bottom**:

**Solid**: direct, **dashed**: energy, **dot-dashed** entropy schemes.
Equilibration times for charm and bottom:

Solid: direct, dashed: energy, dot-dashed entropy schemes.

Some experimental studies + models PHENIX col. ’06, van Hees et al ’05:
For \( p = 10 \) GeV, \( \tau_e \approx 4.5 \) fm (charm)

We have \( 3 < \tau_e < 5.5 \) fm
Diffusion constants
Diffusion constants

- In Fourier space
  \[ \kappa = \lim_{\omega \to 0} G_{sym}(\omega) = \lim_{\omega \to 0} \coth\left(\frac{\omega}{4T_s}\right) \text{Im} G_R(\omega) \]
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Diffusion constants

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- There is a "horizon" on the world-sheet:
  \[
  ds^2 = b^2 \left[ -(f(r) - v^2)d\tau^2 + \frac{dr^2}{f-v^2b^4(r_s)/b^4(r)} \right]
  \]
  WS horizon at \( f(r_s) = v^2 \).

- \( \kappa_\perp = \frac{2}{\pi \ell_s^2} b^2(r_s) T_s, \quad \kappa_\parallel = \frac{32\pi}{\ell_s^2} \frac{b^2(r_s)}{f'(r_s)^2} T_s^3 \)
Physical picture

rs: WS horizon
rh: BH horizon
Physical picture

- A black-hole horizon on the WS at $r_s$:
  Fluctuations on the string fall into the horizon $\Rightarrow$ energy loss
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  Fluctuations on the string fall into the horizon $\Rightarrow$ energy loss

- However, there is Hawking radiation at $r_s$ towards the boundary
  $\Rightarrow$ momentum broadening.
Numerical results for jet quenching
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- $\hat{q}_\perp = 5.2$ (direct), 12 (energy), 13.13 (entropy) $\text{GeV}^2/fm$,

for a charm quark traveling at $p = 10\text{GeV}$ at $T = 250 \text{ MeV}$.
Jet quenching, non-perturbative

Non-perturbative def. of $\hat{q}$:

$\langle W(C) \rangle \approx \exp \left[ -\frac{1}{8\sqrt{2}} \hat{q} L - L^2 \right]$.  

Wiedemann '00
Jet quenching, non-perturbative

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Holographic computation Liu, Rajagopal, Wiedemann '06: $$\langle W(C) \rangle = e^{iS}$$

Pick up gauge: $x^- \equiv x_1 - t = \tau, x_2 = \sigma$, Compute minimal area:

$$\hat{q} = \frac{\sqrt{2}}{\pi \ell_s^2} \int_0^{r_h} \frac{dr}{e^{2A_s} \sqrt{f(1-f)}}$$
### Results

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<thead>
<tr>
<th>$T_{QGP}, MeV$</th>
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Close to AdS somewhat smaller than pQCD + fit to data \(\text{Eskola et al '05}\)

$\hat{q}_{\text{expect}} \sim 5 - 12 \text{ GeV}^2/\text{fm}$
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