AdS/CFT and Heavy Ion Physics

Umut Gürsoy

Utrecht

NIKHEF - June 16, 2015

References

- Based on the papers
  U.G., E. Kiritsis, L. Mazzanti, F. Nitti arXiv:1006.3261
  U.G., E. Kiritsis, F. Nitti, G. Michalogiorgakis arXiv:0906.1890
  U.G., E. Kiritsis, F. Nitti, L. Mazzanti arXiv:0903.2859
  U.G., E. Kiritsis, F.Nitti arXiv:0707.1349
  U.G., E. Kiritsis arXiv:0707.1324

- Reviews on Heavy Ion Collisions and AdS/CFT
  arXiv:1101.0618 — Exhaustive, emphasis on AdS/CFT
  arXiv:0902.3663—Hydrodynamics for HIC
  arXiv:1102.3010—RHIC/LHC results and elliptic flow
  arXiv:1006.546—Non-conformal holographic QCD approach

Outline

- Lecture I:
  - AdS/CFT for QCD
  - Deformations of AdS/CFT
  - Bottom-up approach to AdS/CFT
  - Improved Holographic QCD

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  - Bottom-up approach to AdS/CFT
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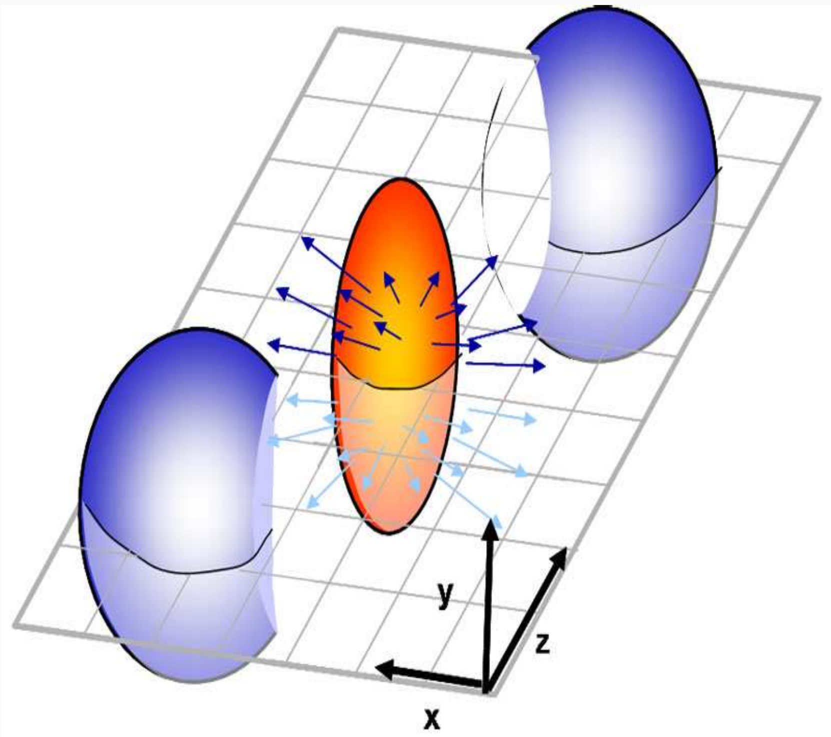
- Lecture II:
  - Glueball Spectrum
  - Thermodynamics
  - Transport
  - Jet quenching
  - Langevin diffusion
  - Thermalization
  - Outlook

Heavy ion collisions



- RHIC: Au + Au at $\sqrt{s} = 200$ GeV per nucleon; about $T = 200 - 300$ MeV .
- LHC: Pb + Pb at $\sqrt{s} = 2.76$ TeV/n about $T = 300 - 400$ MeV .

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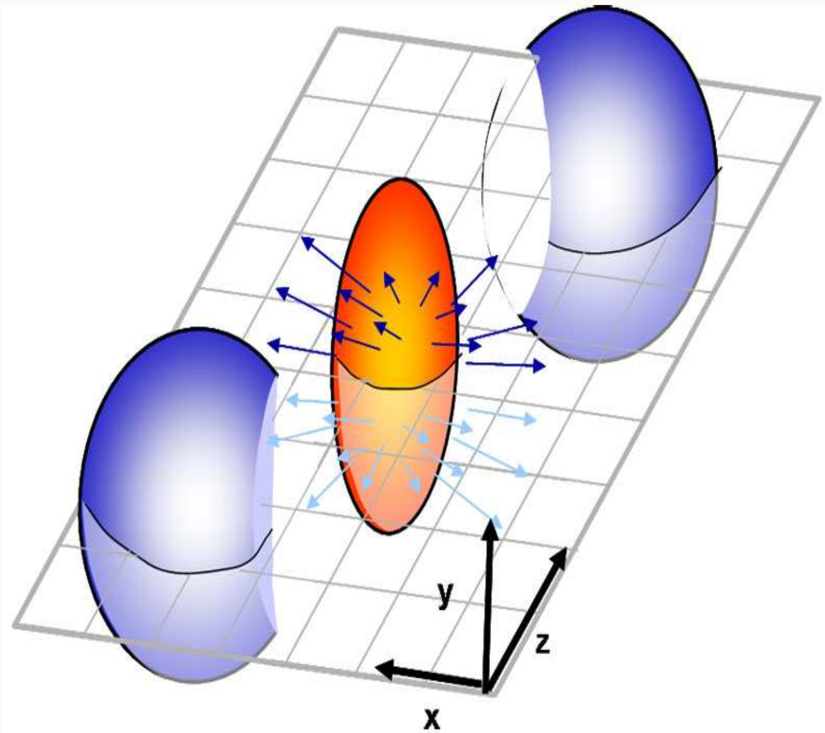

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- LHC: Pb + Pb at $\sqrt{s} = 2.76$ TeV/n about $T = 300 - 400$ MeV .
- The quark-gluon plasma forms at $\sim$ 1fm and exists for $5 - 10$ fm
- Cools down as it expands $\Rightarrow$ and hadronizes around $T = 170$ MeV.

What can we learn from Holography

- Phase diagram of QCD at finite μ, T and B
- Transport coefficients: Viscous relativistic hydrodynamics account for the observed v_2, v_3, etc quite well
  $\Rightarrow$ Calculate the viscosities η/s and ζ/s + higher order coefficients, other transport coefficients.
- Energy loss in hard probes
  Basic mechnamisms: Gluon brehmstrahlung and Langevin diffusion
  $\Rightarrow$ Calculate the jet-quenching parameter $\hat{q}$ and momentum diffusion parameters κ.
- Anomalous trasport: Chial Magnetic Effect, Chiral Magnetic Wave, etc. $\Rightarrow$ Calculate chiral conductivities
- Thermalization
  Non-equilibrium physics, formation of QGP $\Rightarrow$ black-hole formation by collapsing matter

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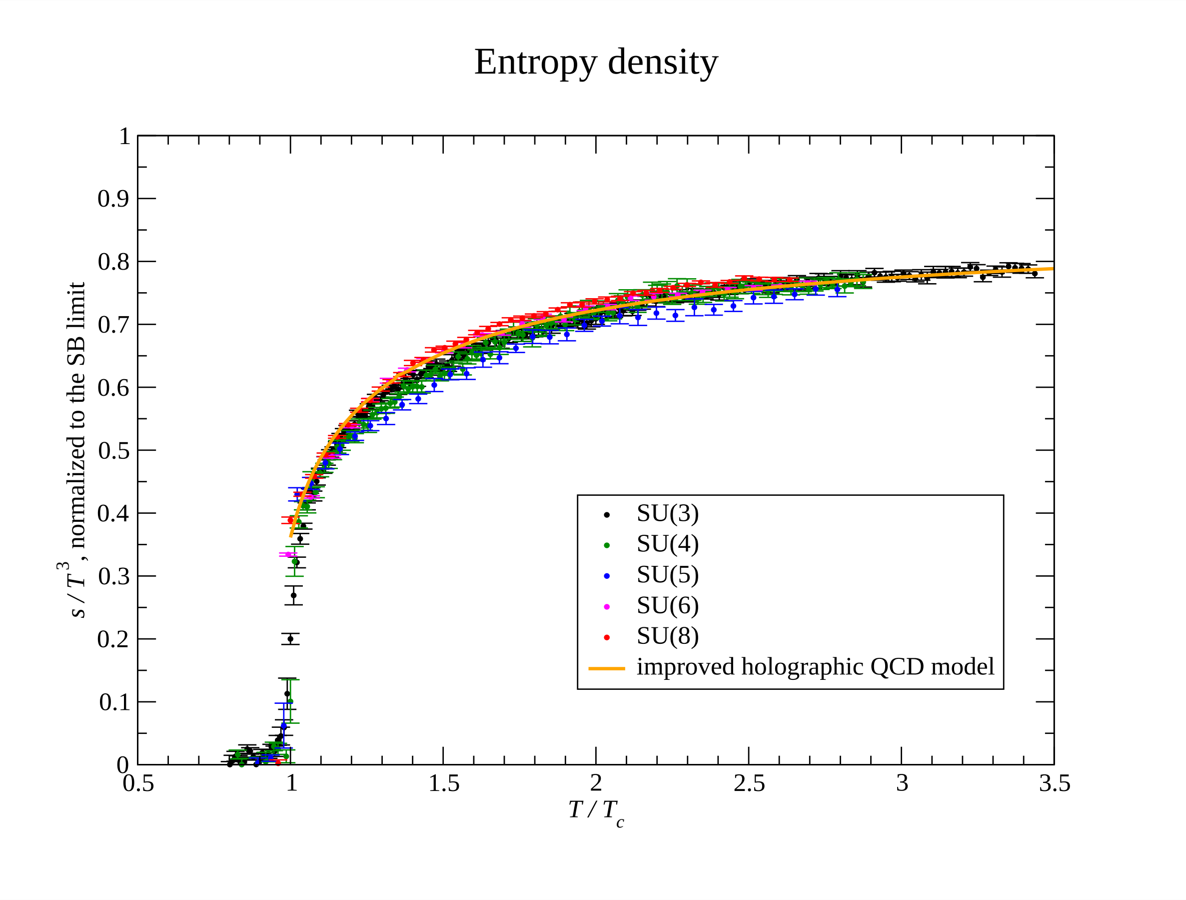
This is what we will assume in the rest of the lectures...

N-dependence of thermodynamic quantities

Panero ’09



- About 10 % deviation in the hadron spectra
- Thermodynamic observables very close to each other

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Very hard to deal with in practice...

Phenomenological approach

Da Rold, Pomarol; Erlich et al ’05

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  - Dimensions of QFT + 1 (energy scale)
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  - One bulk field for each relevant + marginal operator
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- Check this by calculations
- For generic GR set-ups $\Rightarrow$ universal lessons

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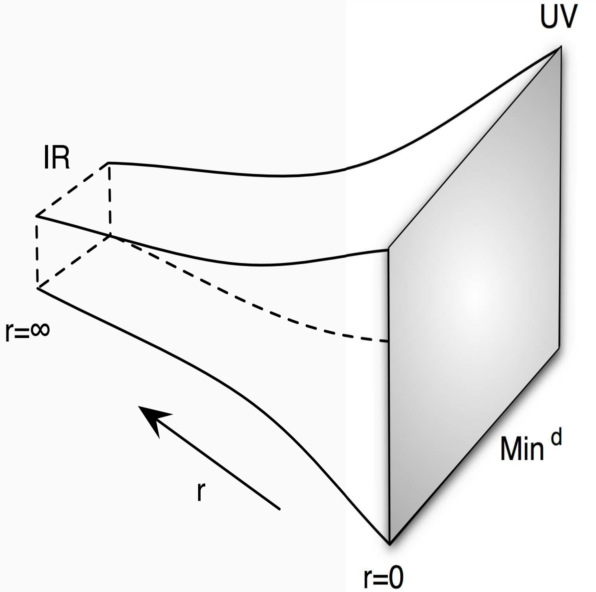

Domain-wall type geometries with boundary

Minimal metric:

$$ds^2 = b(r)^2 \left(dr^2 + dx_d^2\right)$$

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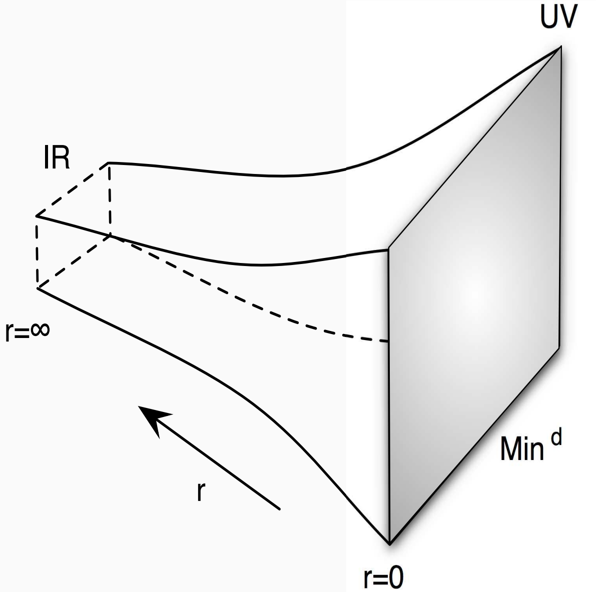

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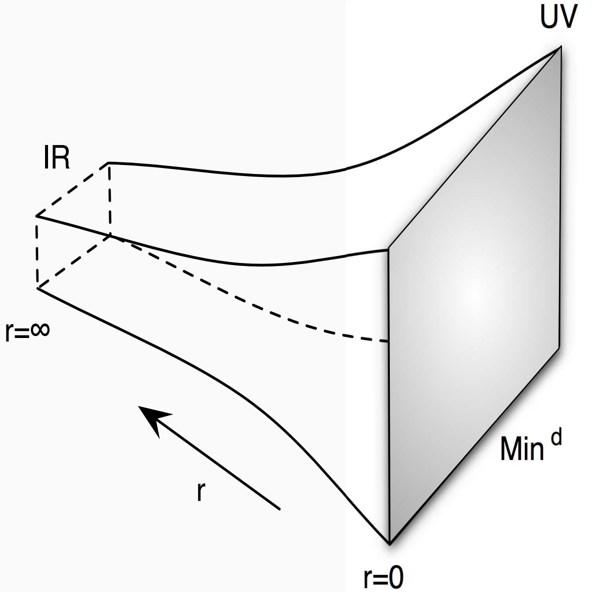

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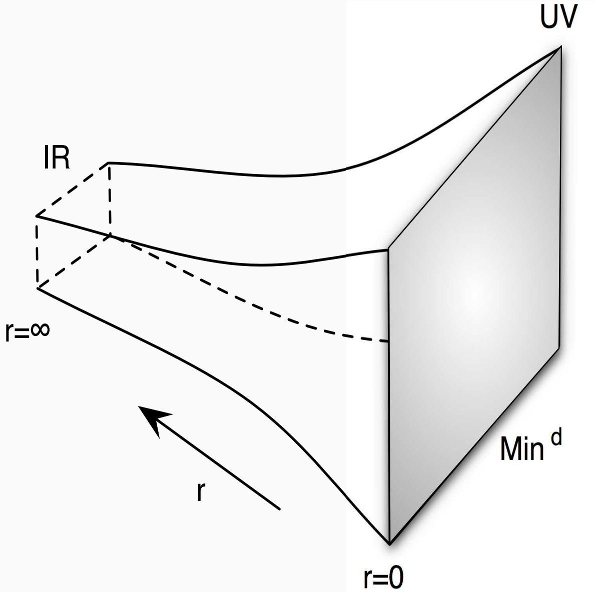

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   Fundamental relation:

   $$\exp(-S_G[\phi(x,r) \to \phi_0(x)]) = \langle \exp(\int \mathcal{O}\phi_0) \rangle$$

   Computes n-point functions $\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle$ of QFT.

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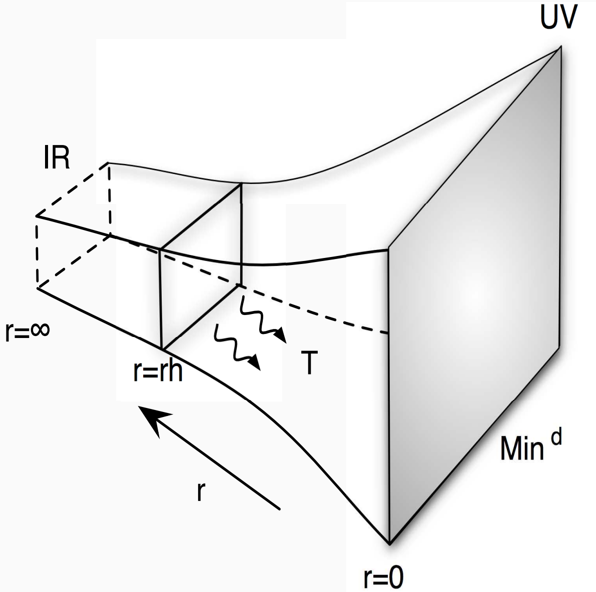

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2. Finite temperature in the QFT $\Leftrightarrow$ black-hole in the geometry.

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  3. Running coupling extremely important for correct thermodynamics $\Rightarrow$ non-conformally invariant background with $e^\phi \propto g^2 N$ a function of r: $\phi = \phi(\Lambda r)$ with $\Lambda \Rightarrow$ dynamically generated QCD scale.

Improved HQCD U.G, Kiritsis; U.G. Kiritsis, Nitti ’07

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- Gravitational dual in 2∂ effective GR theory:

$$S = M_p^3 N_c^2 \int d^5x \sqrt{g} \left\{ R - \frac{4}{3}(\partial\phi)^2 - V(\phi) \right\}$$

- Look for domain-wall type solutions of the Einstein-dilaton eqs:
$ds^2 = b^2(r) \left(dr^2 - dt^2 + dx_3^2\right)$, $\lambda = \lambda(r) \equiv \exp(\phi(r))$

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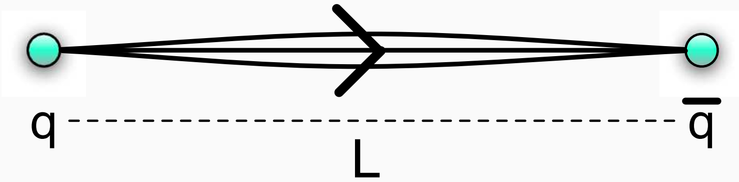
Dictionary: Geometry vs. QFT:

- Scale factor $b_0(r)$ is the energy scale in the field theory E,
- Dilaton $\lambda(r) \propto \lambda_t(E)$ running 't Hooft coupling,
- Dilaton potential $V(\phi) \Leftrightarrow \beta(\lambda_t)$ the beta-function of the QFT.

Quark potential and confinement

Linear quark potential from flux tube:



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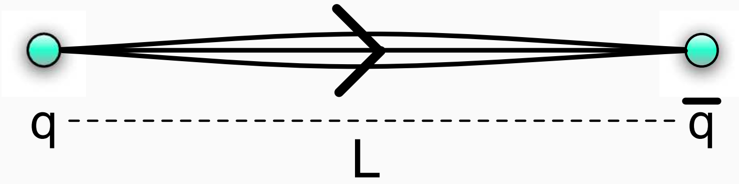

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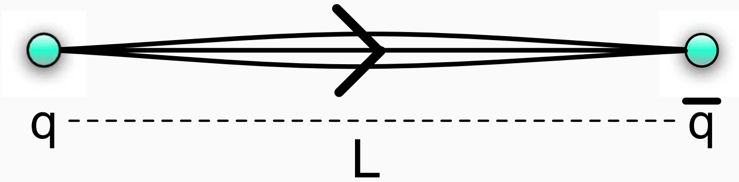

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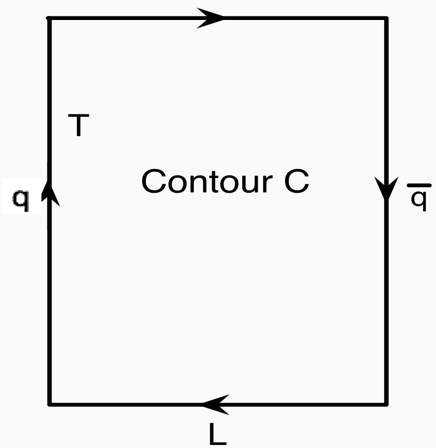

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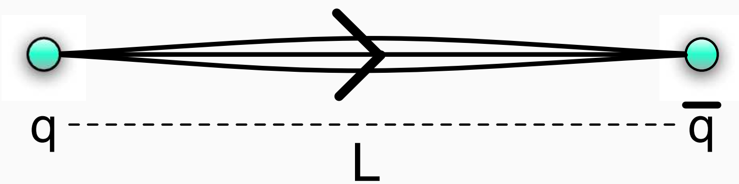

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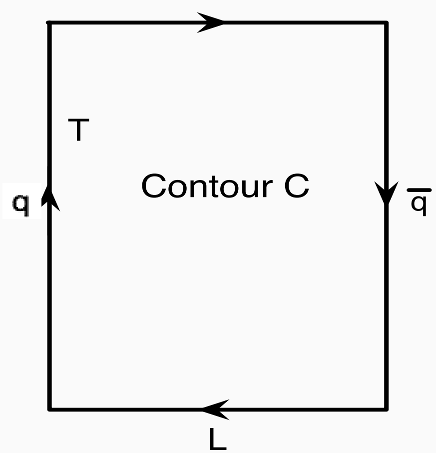


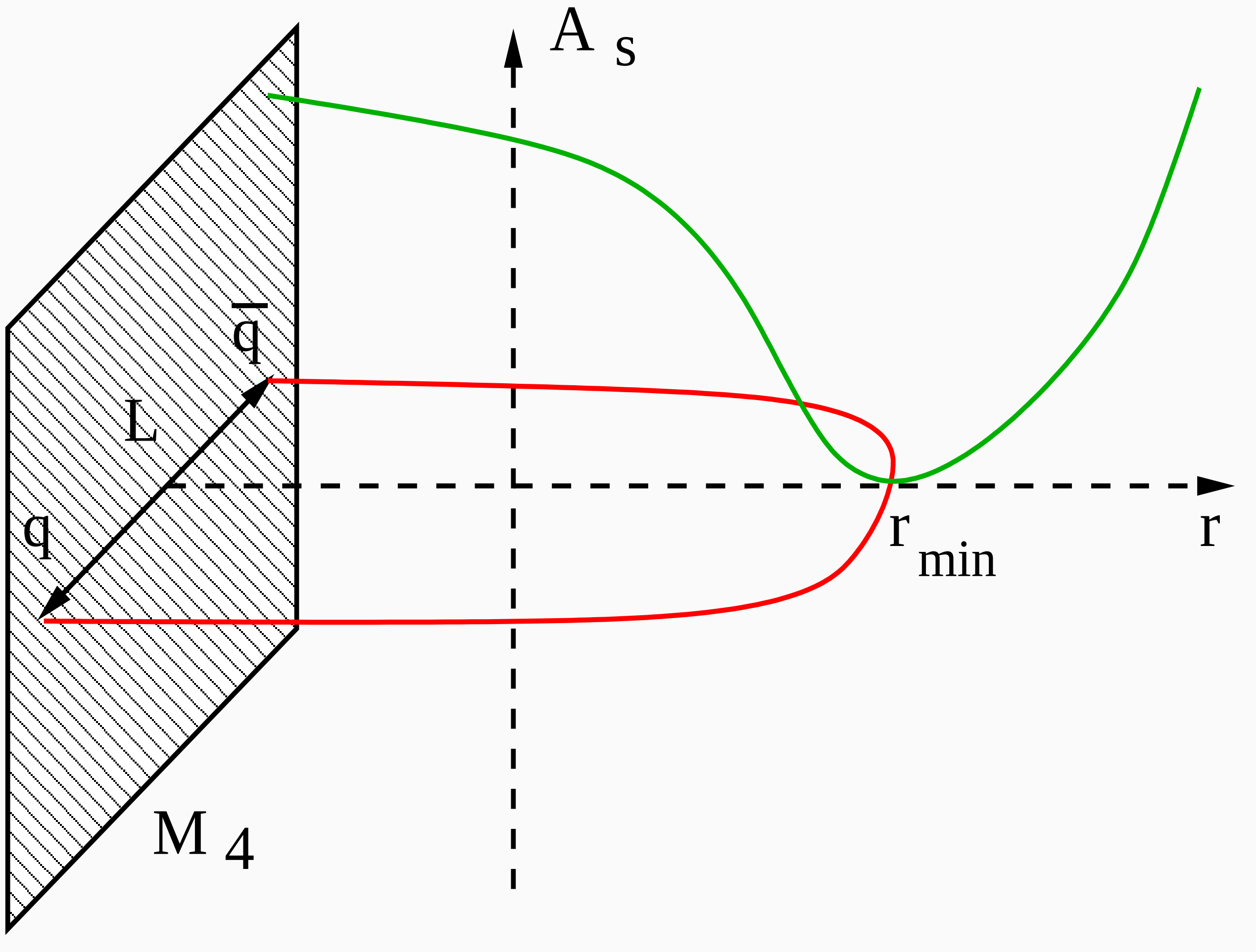
Gauge-gravity duality:

$W[C] \Leftrightarrow$ string world-sheet ending on C

$\langle W[C] \rangle = e^{-S[string;C]}$

J Maldacena '98; S. Rey, J. Yee '98

Color confinement



Linear quark potential $\Leftrightarrow \exists$ minimum of b_s

This constrains large λ asymptotics of the dilaton potential $V(\lambda)$.

- Requirement of a marginal deformation $\mathrm{Tr}F^2$ fixes the UV asymptotics as
$V(\lambda) = v_0 + v_1\lambda + \cdots , \qquad \lambda \to 0$
- Requirement of linear color confinement fixes the IR asymptotics as
$V(\lambda) \propto \lambda^{\frac{4}{3}} \log^{\frac{1}{2}} \lambda + \cdots , \qquad \lambda \to \infty$
- Then 1) mass gap 2) first order T_c is automatic
- Spectrum of glueballs can be computed with no IR ambiguity

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IR asymptotics of the background:

$$b(r) \sim e^{-\left(\frac{r}{L}\right)^{\alpha}}, \qquad \lambda(r) \sim e^{3/2\left(\frac{r}{L}\right)^{\alpha}} \left(\frac{r}{L}\right)^{\frac{3}{4}(\alpha-1)}, \qquad r \to \infty$$

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$$V = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\log\left(1 + V_2\lambda^{\frac{4}{3}} + V_3\lambda^2\right)^{\frac{1}{2}}\right\}$$

- Parameters in the action: V_0, V_2 fixed by scheme independent β-function coefficients (b_0 and b_1), V_1, V_3 fixed by the latent heat L_h and $S(2T_c)$ (lattice)

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- The string length ℓ_s by lattice string σ_s: $\frac{\ell_{AdS}}{\ell_s} \approx 6.5$
  This measures how good the two-derivative approximation is!

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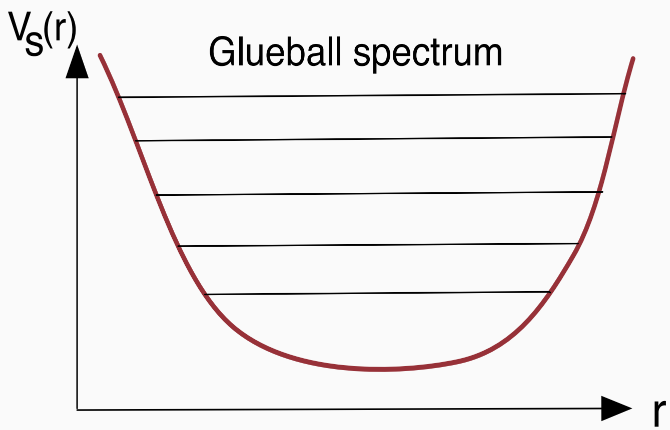

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- Scalar : $B(r) = 3/2A(r) + \log(\dot{\Phi}/\dot{A})$
- Tensor : $B(r) = 3/2A(r)$

Comparison with one lattice study Meyer, '02

J^{PC}	Lattice (MeV)	Our model (MeV)	Mismatch
0^{++}	1475 (4%)	1475	0
2^{++}	2150 (5%)	2055	4%
0^{++*}	2755 (4%)	2753	0
2^{++*}	2880 (5%)	2991	4%
0^{++**}	3370 (4%)	3561	5%
0^{++***}	3990 (5%)	4253	6%

$$0^{++} : TrF^2; \qquad 2^{++} : TrF_{\mu\rho}F^{\rho}_{\nu}.$$

Thermodynamics: results

- Fix the dilaton potential:

$$V = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\log\left(1 + V_2\lambda^{\frac{4}{3}} + V_3\lambda^2\right)^{\frac{1}{2}}\right\}$$

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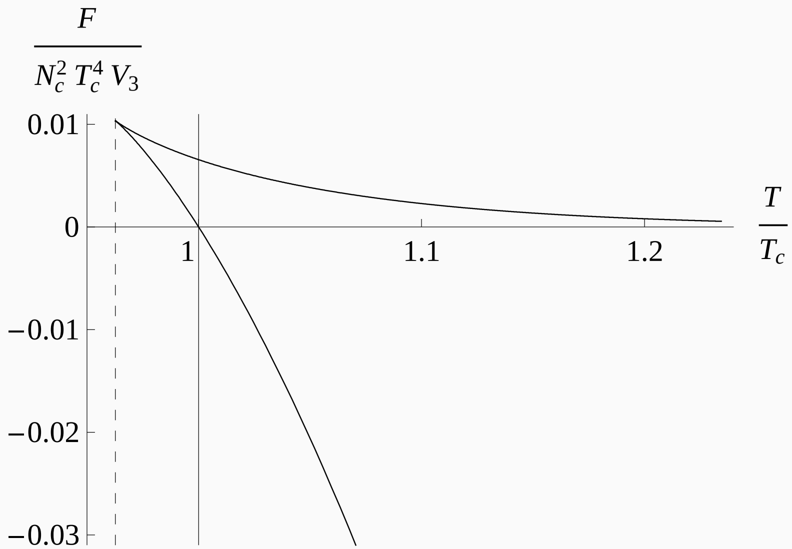
iHQCD Thermodynamics continued

The free energy:



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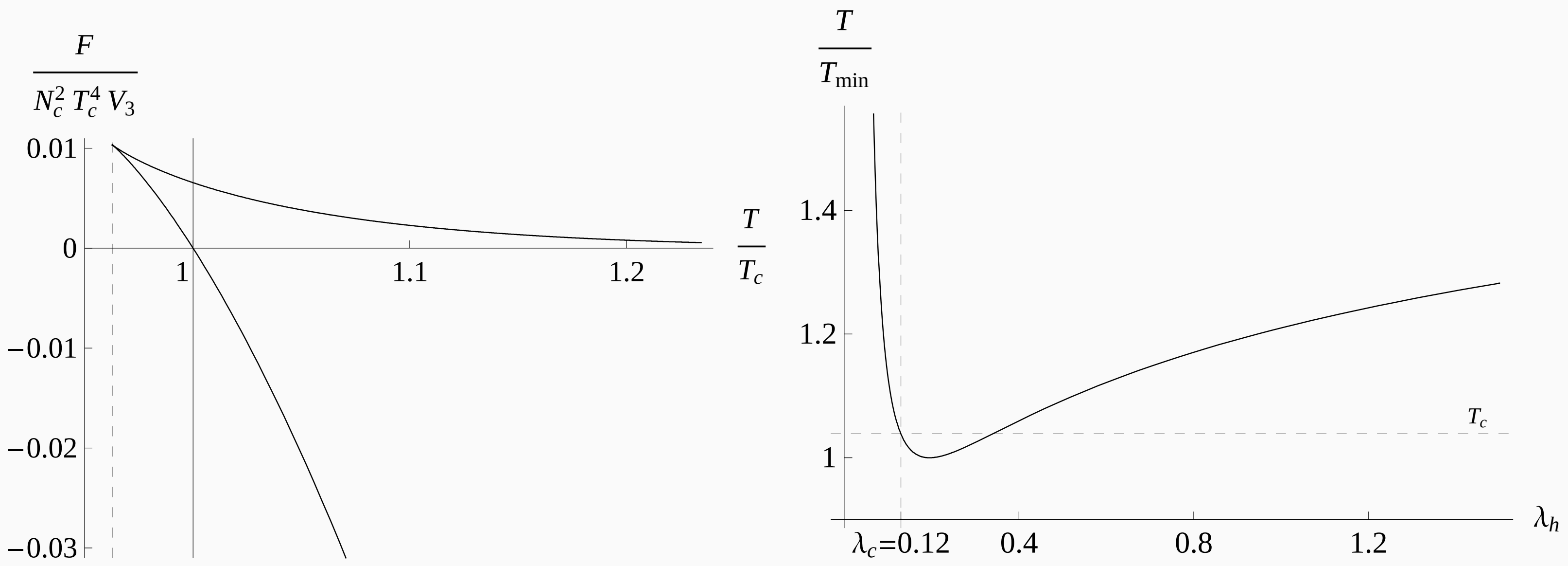

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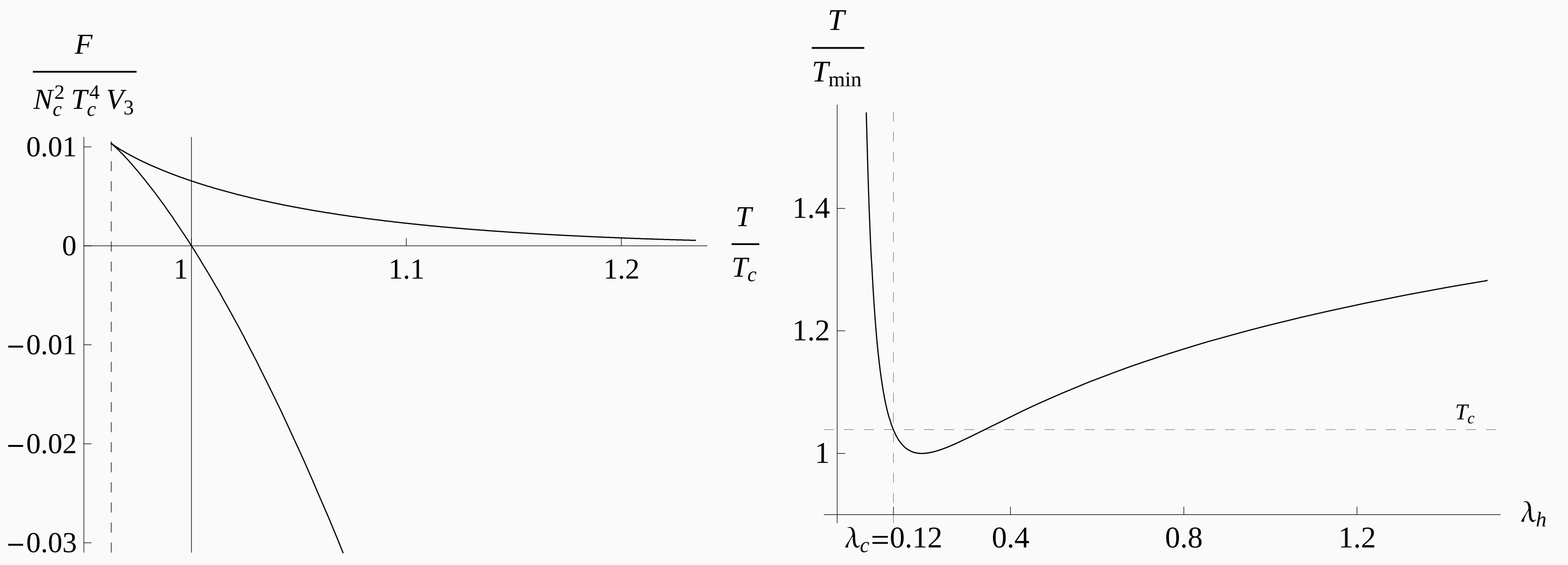
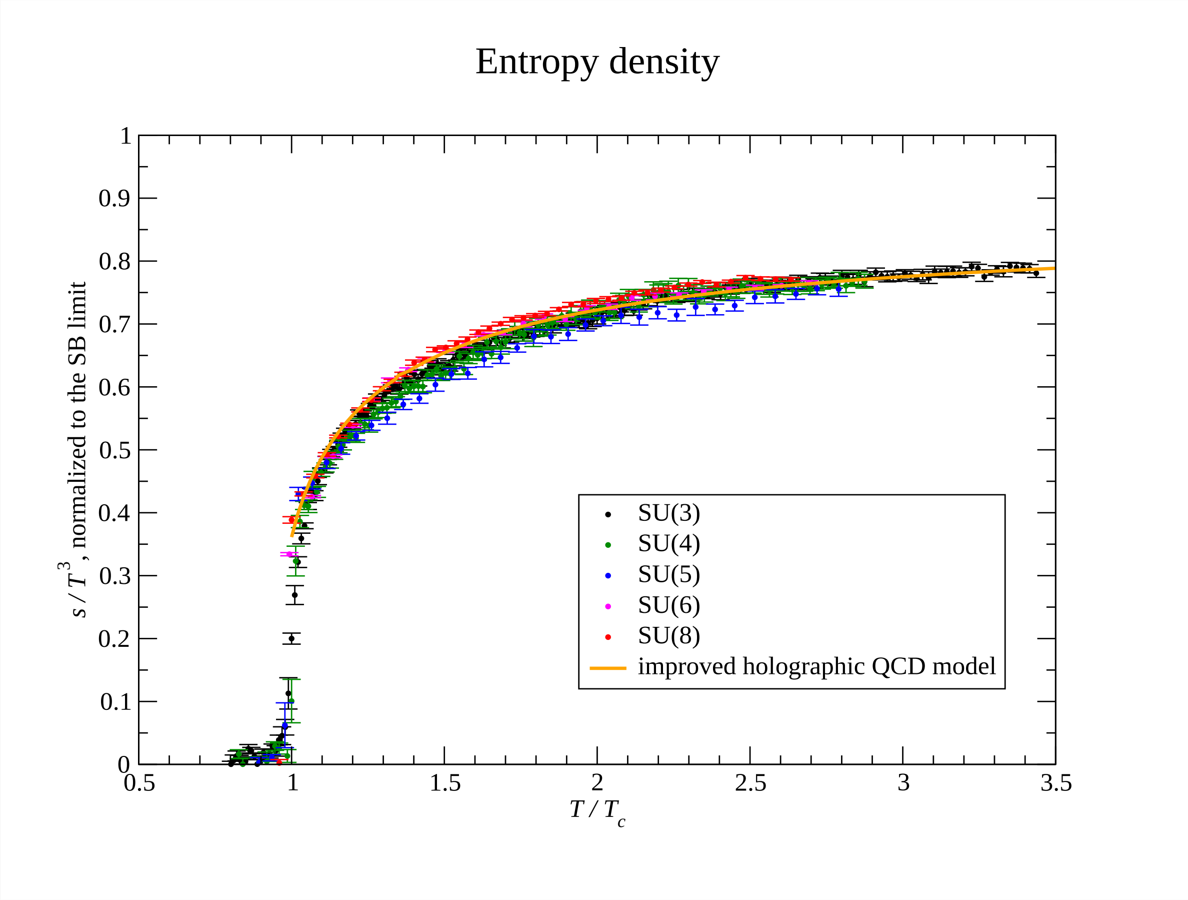


- Big and Small black-hole solutions, like $\mathcal{N} = 4$ on R^3
- Existence of $T_{min} \Leftrightarrow$ phase transition at $T_c > T_{min}$

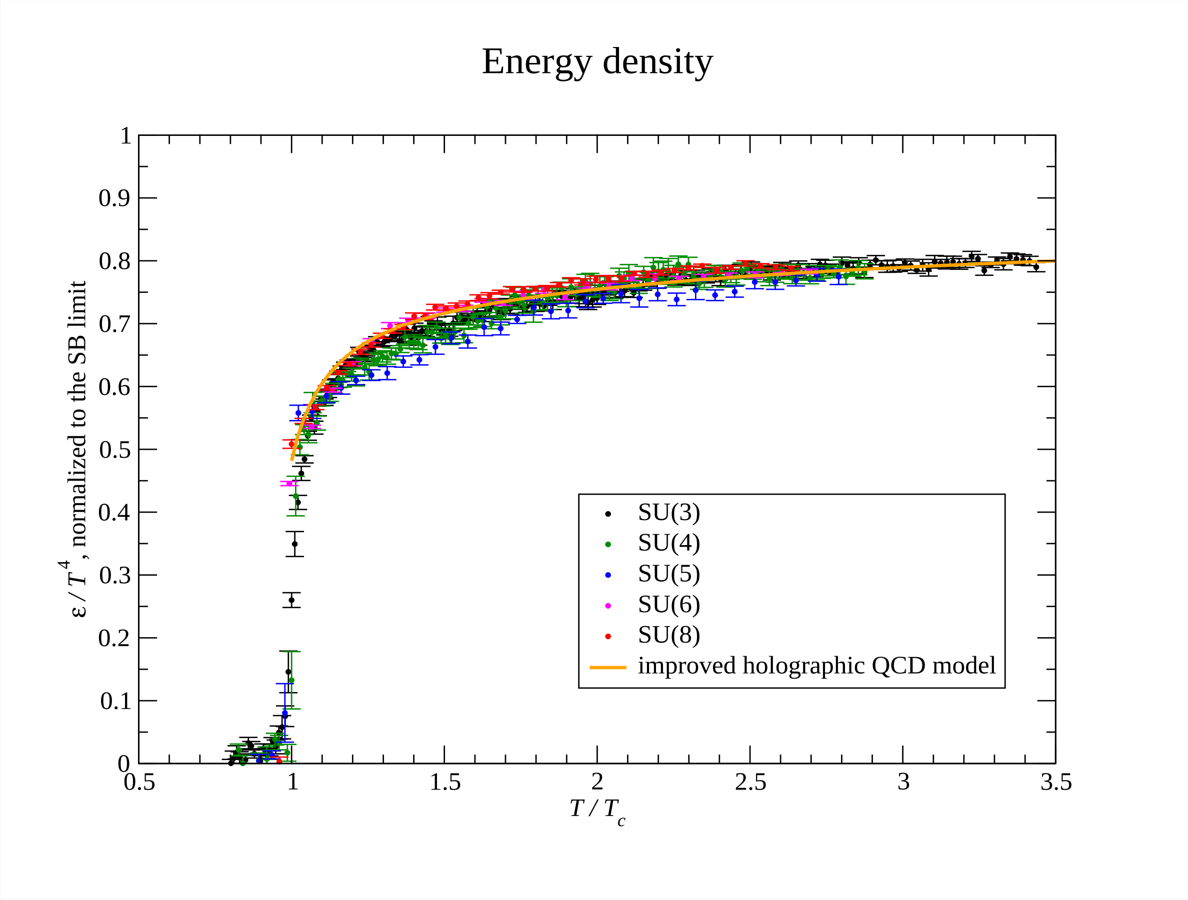
Survey of thermodynamical quantities I

Comparison to Panero ’09



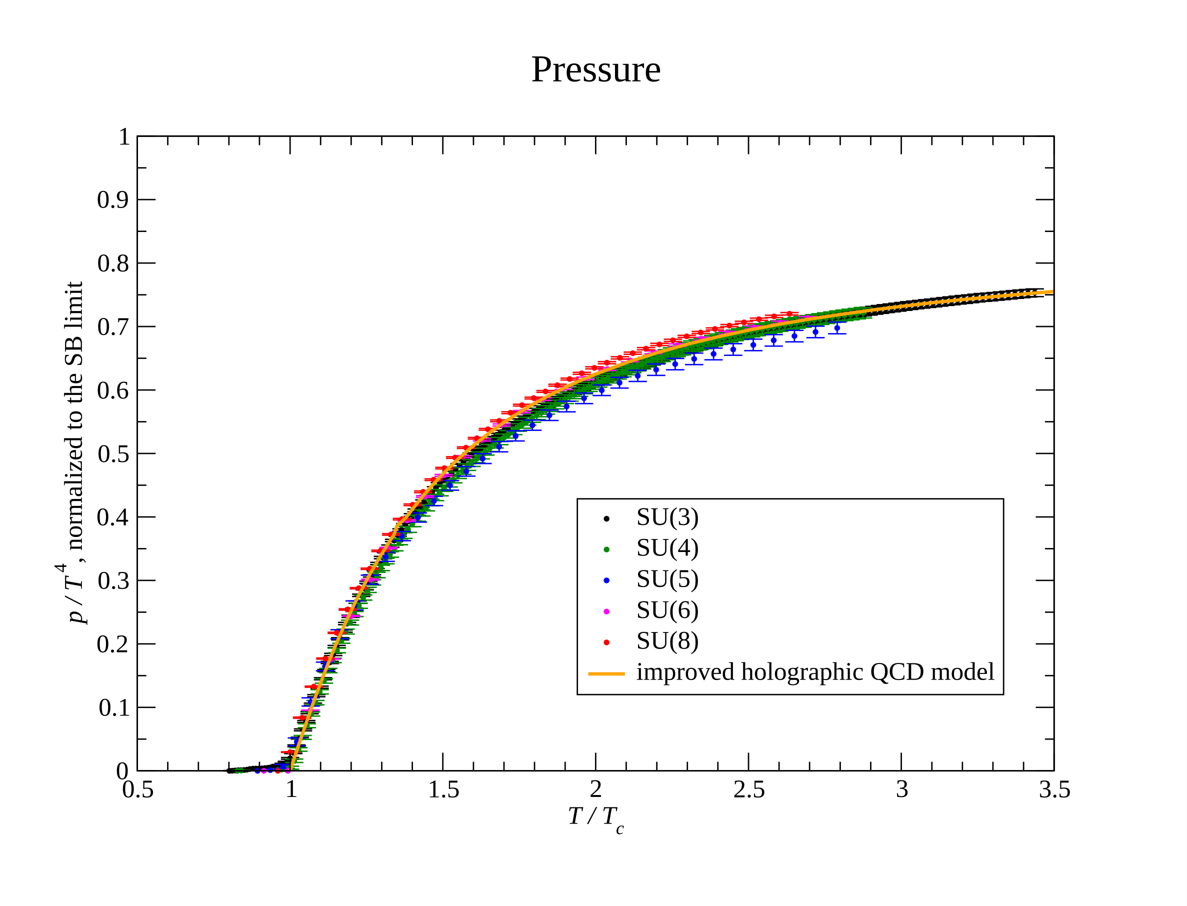
Survey of thermodynamical quantities II

Comparison to Panero '09



Survey of thermodynamical quantities III

Comparison to Panero ’09



Survey of thermodynamical quantities IV

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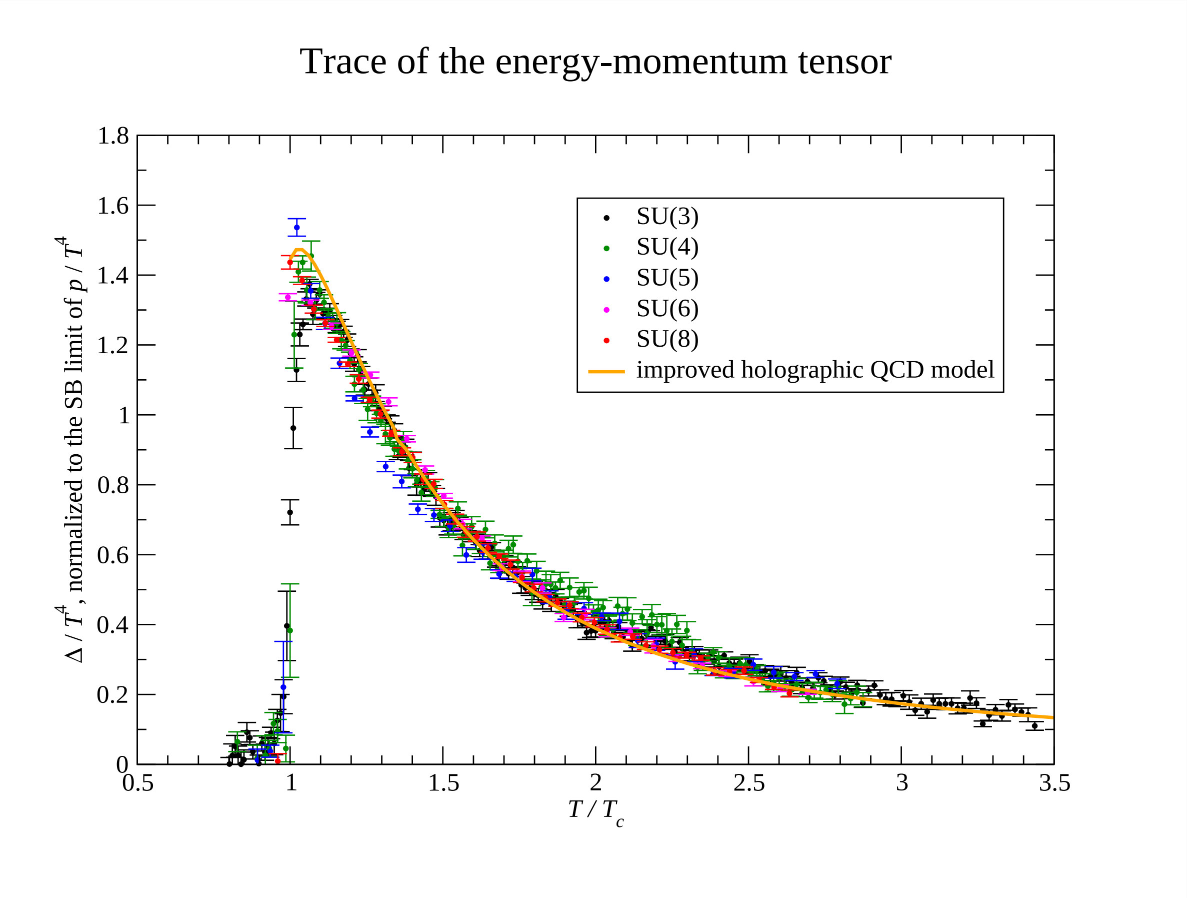
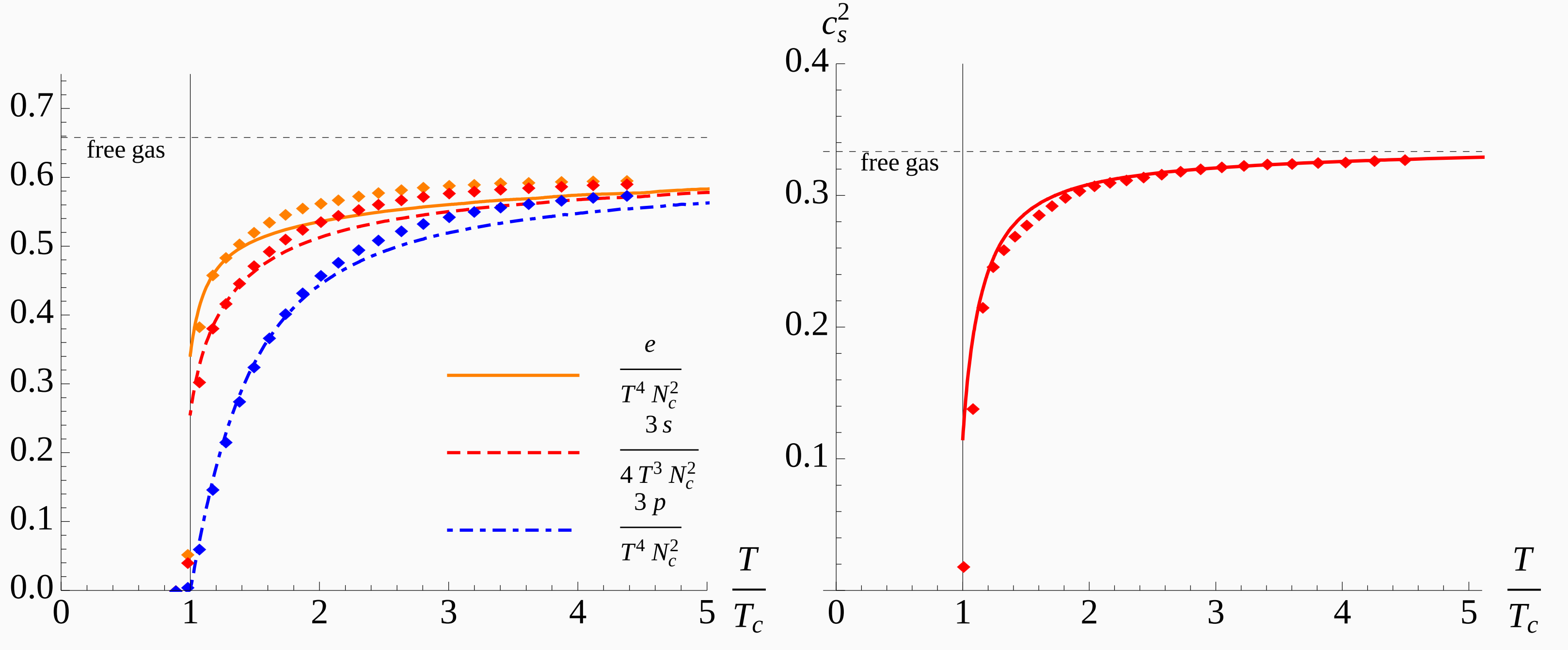


Survey of thermodynamic quantities V

Comparison to Boyd et al. ’96 Thermodynamic functions and the speed of sound:



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Another exercise: Navier-Stokes and continuity eqs. follow from $\partial_\mu T^{\mu\nu} = 0$.

Calculation of viscosities in QFT

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- Kubo's linear response theory:
  $\mathcal{L} \to \mathcal{L} + \int \mathcal{O}^A \delta\phi_A$,
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  where $G_R(\omega, \vec{k}) = -i \int d^4x e^{-ik\cdot x} \theta(t) \langle [\mathcal{O}^A(t, \vec{x}), \mathcal{O}^B(0, \vec{0})] \rangle$

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- Read off η from the xy component, and ζ from the $11 + 22 + 33$ component.

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- Relativistic fluid with 4-velocity u^μ, energy density ϵ and pressure p.
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The characteristic parameters of the fluid at $\mathcal{O}(\partial u)$

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- Bulk viscosty ζ: What is already known from field theory and lattice ?

Holographic computation

- Kubo's linear response theory:
  $\zeta = -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im G_R(w,0)$
- More complicated than shear because h_{ii} mix with dilaton fluctuations $\delta\phi$.
- Derive the fluctuation equations for h_{ii}, pick up the gauge $\delta\phi = 0$,
- Fluctuations decouple in the smart gauge! Gubser et al '08: Define $X = \phi'/3A'$
- $h''_{ii} + \left(3A' + 2\frac{X'}{X} + \frac{f'}{f}\right) h'_{ii} + \left(\frac{\omega^2}{f^2} - \frac{f'X'}{fX}\right) h_{ii} = 0$
- Boundary conditions:
  - $h_{ii}(\phi = -\infty) = 1$ and,
  - In-falling wave at horizon $h_{ii} \to c_b (r_h - r)^{-\frac{i\omega}{4\pi T}}$
- Read off $c_b(\omega, T)$

Results I: Comparison to Meyer ’08

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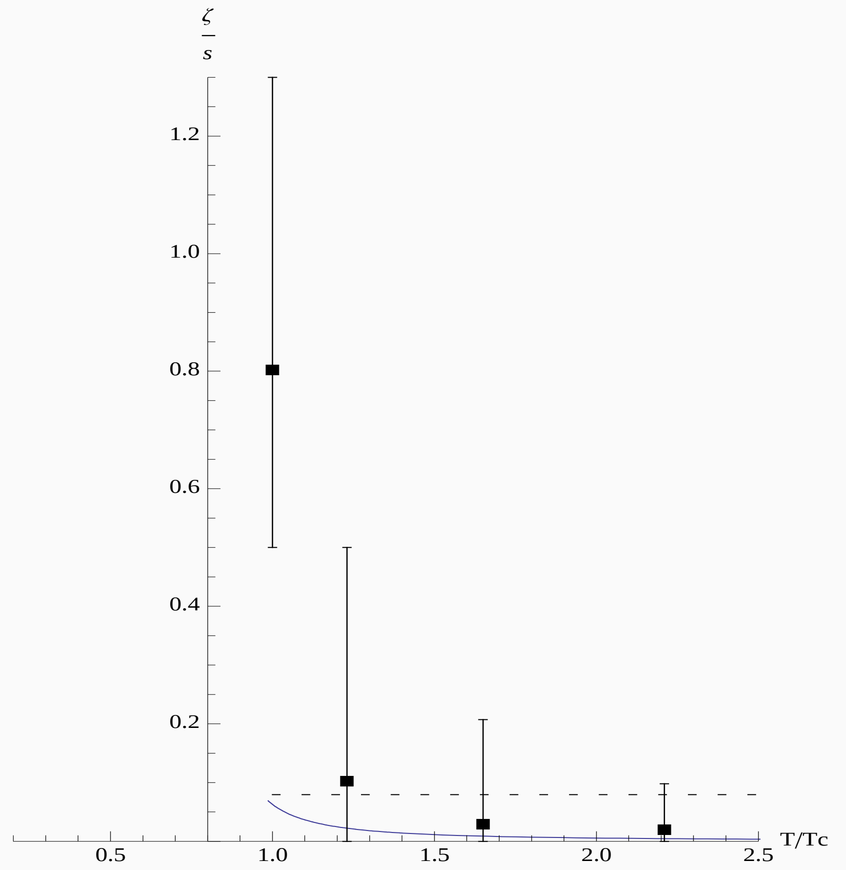
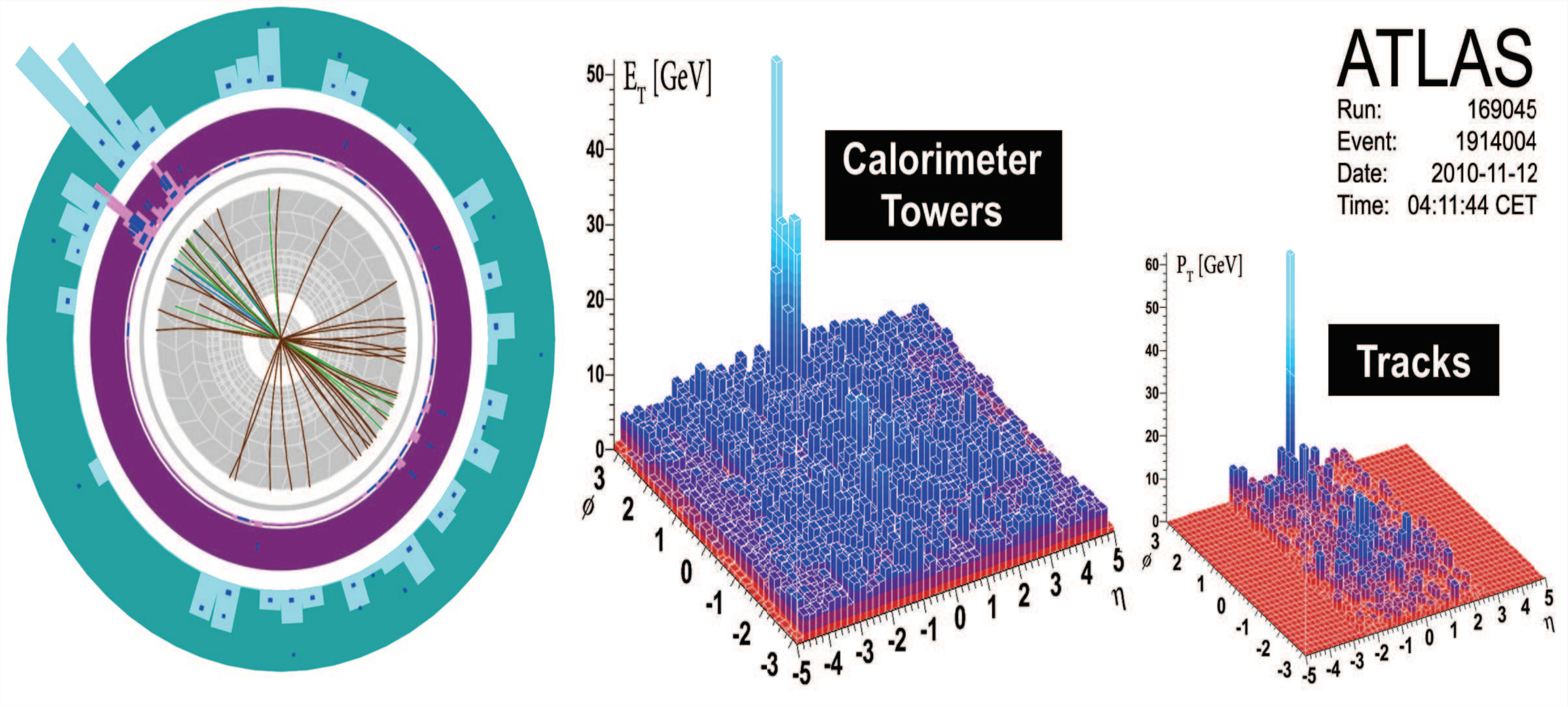


- Near UV, vanishes as expected: ideal gluon gas at high T
- Near T_c Peak, much smaller than lattice expectations!
- Agreement with another holographic model Gubser et al. 08

Jet quenching

Back-to-back jet production is highly suppressed at RHIC:



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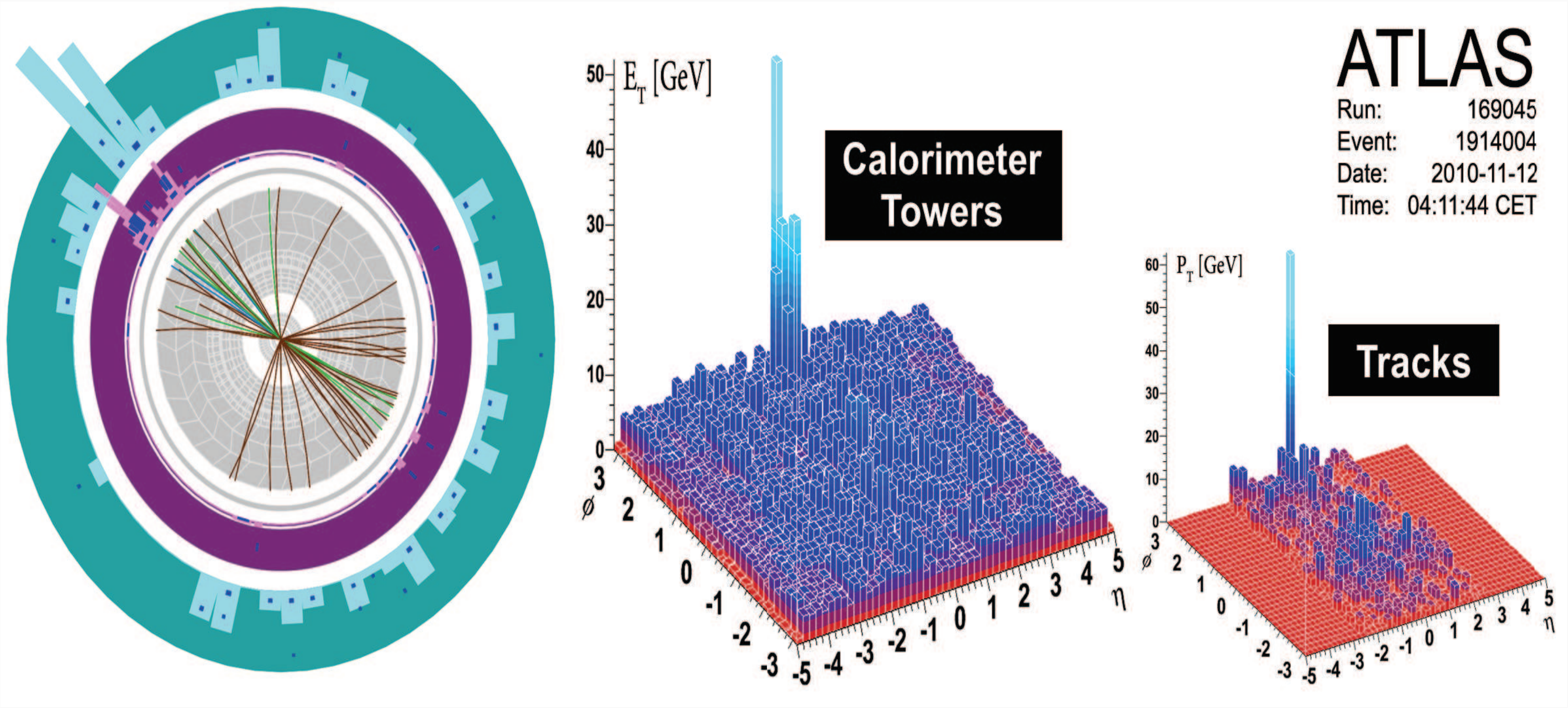

The first direct signals of jet-quenching - November 2010!

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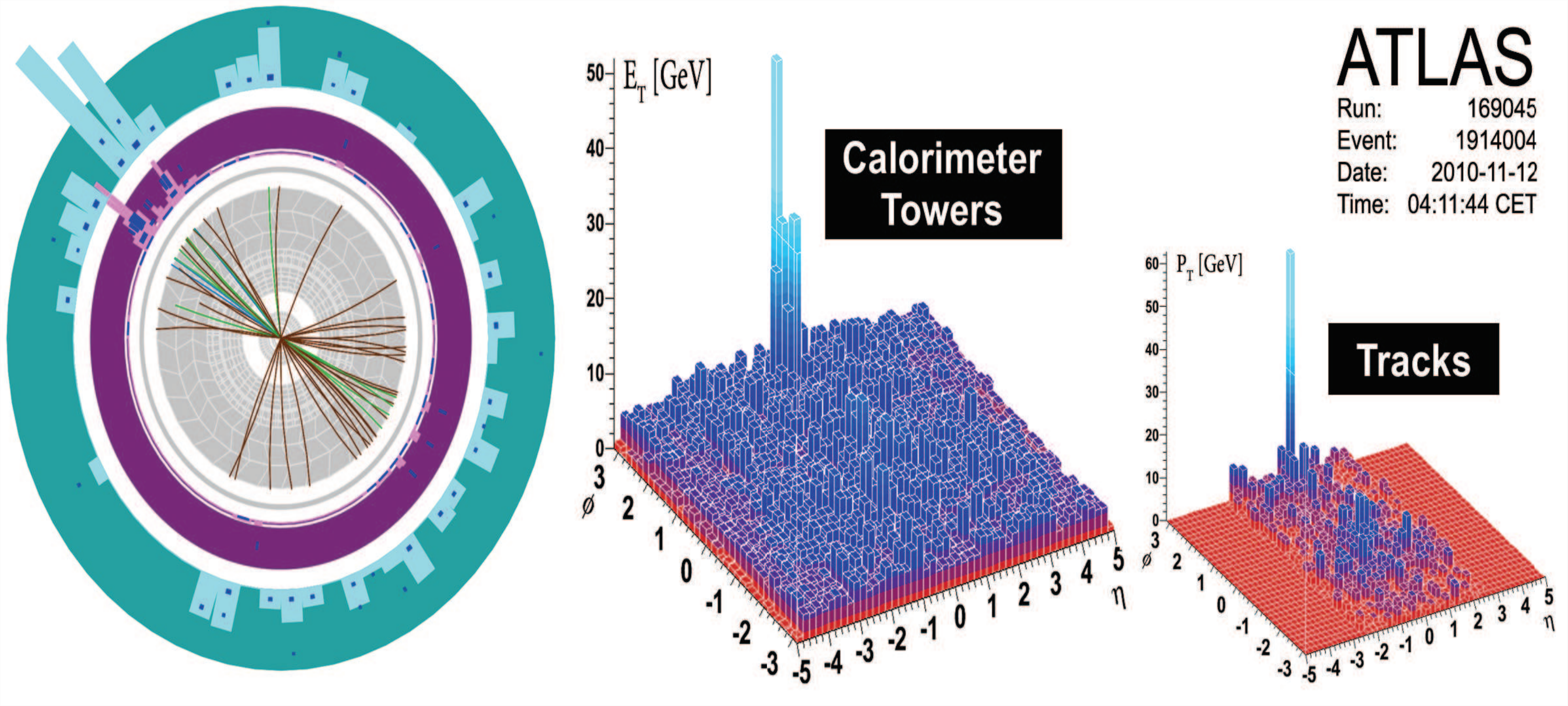

The first direct signals of jet-quenching - November 2010!

A clear signal of strongly-coupled plasma.

Quantification of jet quenching Baier et al ’96

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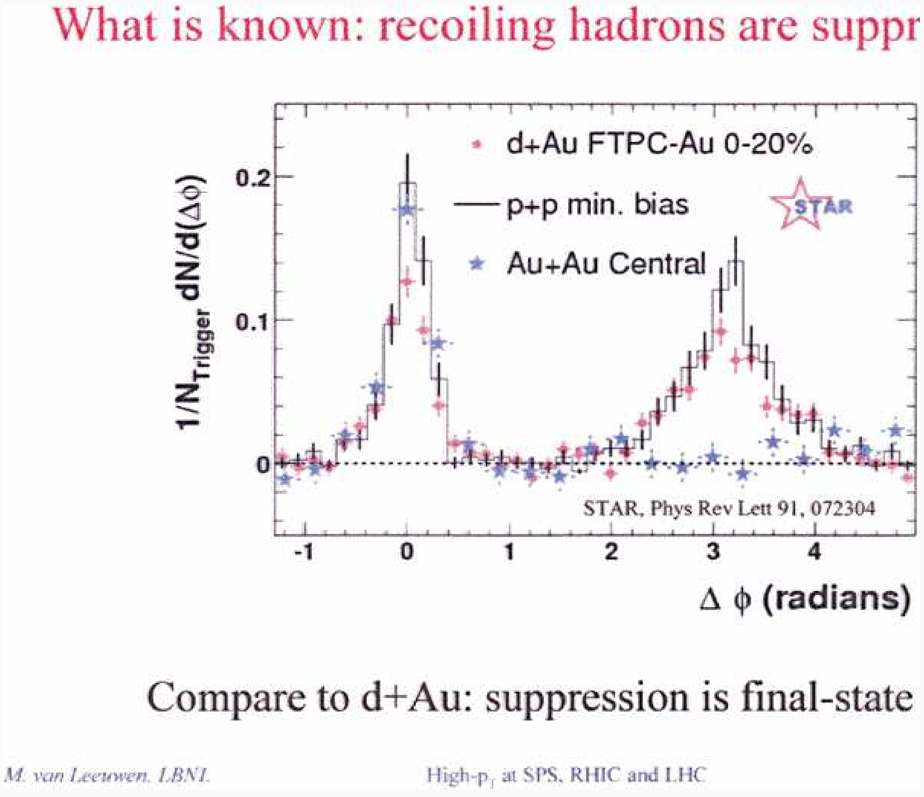

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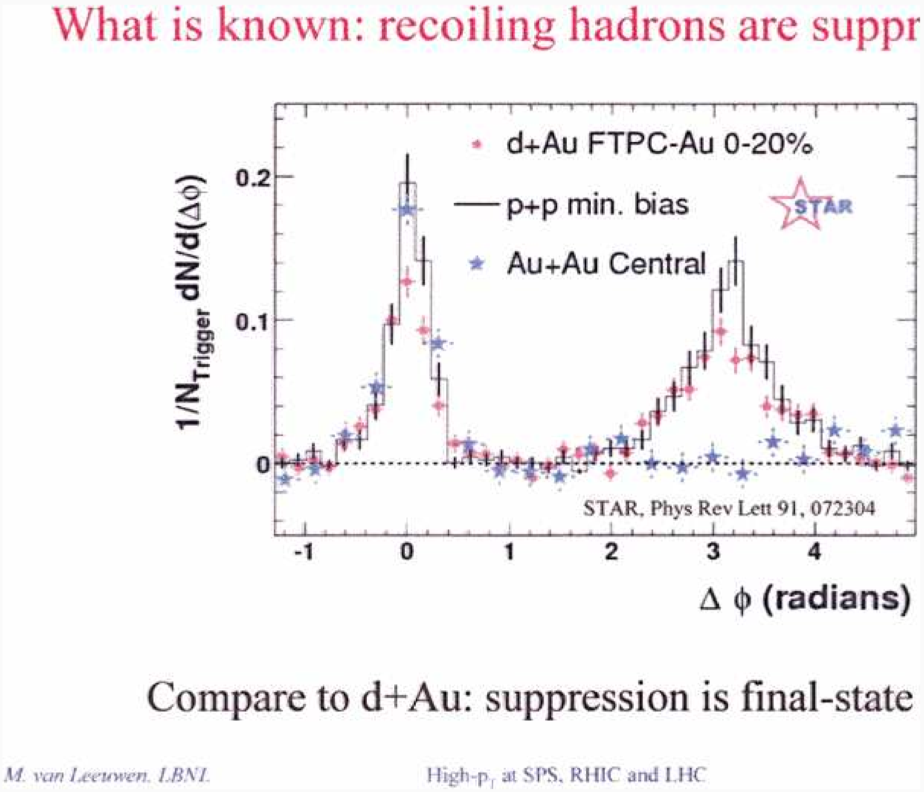

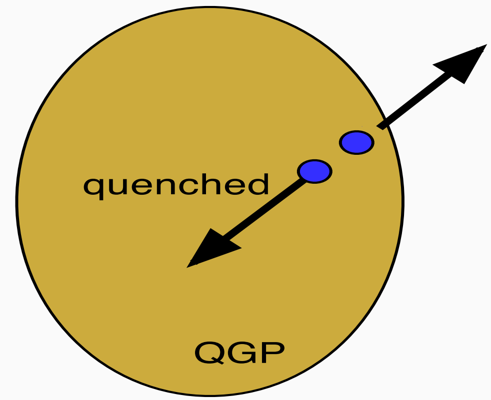

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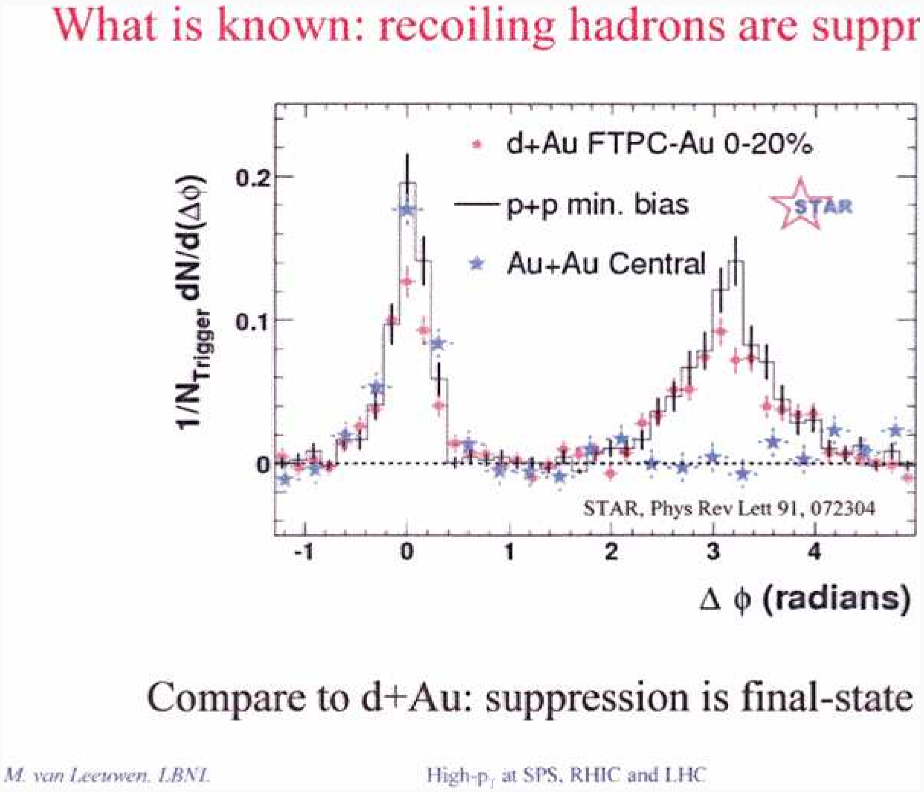

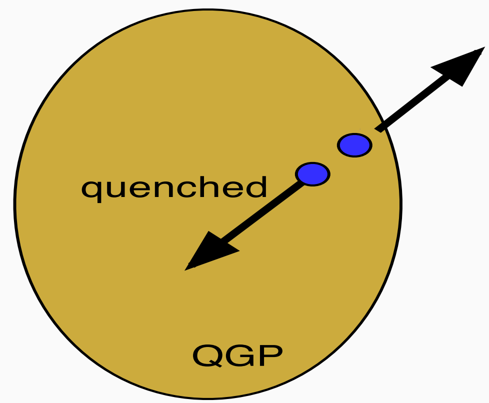

Average transverse momentum lost into the media in a flight of distance D.

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{D}$$

Weak-coupling computation does not explain the data.

Energy loss of a heavy quark

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  Combination of two distinct mechanisms:

  1. Energy loss by Langevin diffusion process
  2. Energy loss by gluon Brehmstahlung

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- Hard probe moving in QGP: $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$
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  with $\langle \xi^i(t)\xi^j(t')\rangle = A^{ij}(t-t')$
- The entire information is stored in:
  $C^{ij}(t) \equiv -i\langle\left[\mathcal{F}^i(t), \mathcal{F}^j(0)\right]\rangle,$
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- Thus, it is sufficient to calculate
  $G_R(\omega) = -i\int dt e^{-i\omega t}\theta(t)\langle[F^i(t), F^j(0)]\rangle$

Momentum broadening

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$\frac{d\delta p^\perp}{dt} = -\eta^\perp \delta p^\perp + \xi^\perp$

Solution with initial conditions $\delta\vec{p}(t=0) = 0$:

$p^\perp(t) = \int_0^t dt'\, e^{\eta^\perp (t'-t)} \xi^\perp(t')$,

Compute the noise-average of fluctuations:

$\langle (p^\perp)^2 \rangle = \int_0^t dt' \; \int_0^t dt''\, e^{\eta^\perp (t'+t''-2t)} \langle \xi^\perp(t') \xi^\perp(t'') \rangle$

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thus jet-quenching parameter:

$\hat{q}^{\perp} = \frac{\langle (p^{\perp})^2 \rangle}{vt} = 2\frac{\kappa^{\perp}}{v}.$

How to calculate in the bulk dual?

Recall $S[X(t)] = S_0 + \int d\tau X_\mu(\tau)\mathcal{F}^\mu(\tau)$

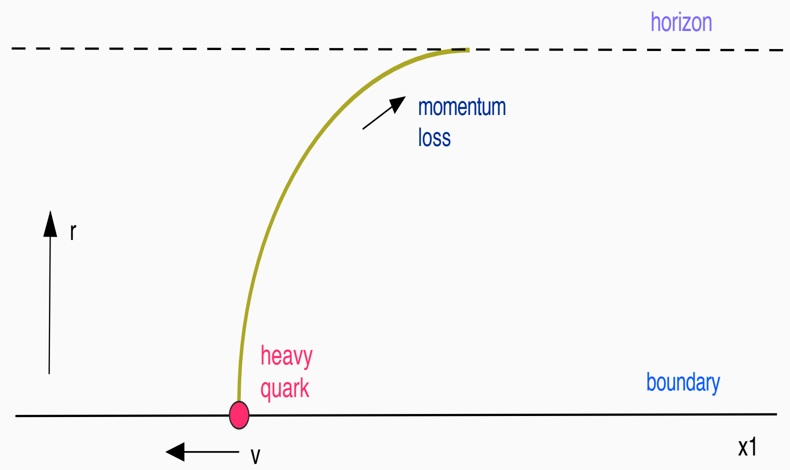
To calculate $\langle\left\{\mathcal{F}^\perp(t), \mathcal{F}^\perp(0)\right\}\rangle = \mathcal{O}(0) + \langle\left\{\xi^\perp(t), \xi^\perp(0)\right\}\rangle$

We need to calculate the fluctuations $\delta X^\perp(t)$.

Dual picture

Herzog et al; Gubser ’06

Holography: Represent the (infinitely) heavy quark with a trailing string moving with constant v:



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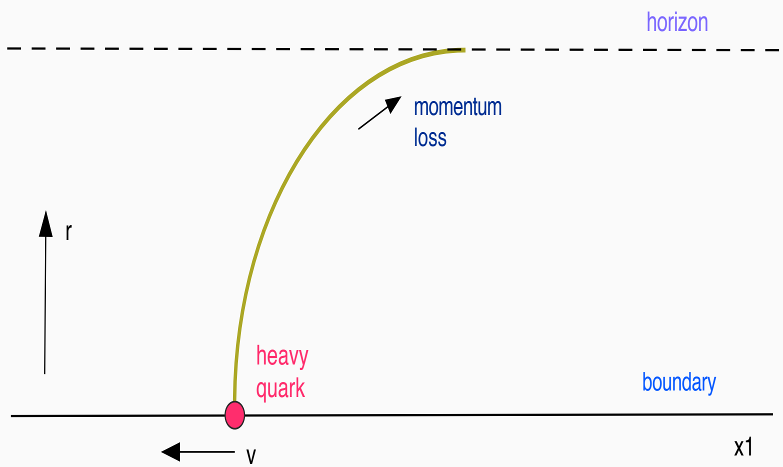

Drag force on a heavy quark in a hot wind:

$$F = \frac{dp}{dt} = \frac{1}{v}\frac{dE}{dt} = -\mu p + \zeta(t)$$

Ignore stochastic force $\zeta(t)$ in this talk $\Leftrightarrow$ fluctuations of the trailing string $\Rightarrow$ diffusion constant.

Standard calculation:

- Pick up the static gauge: $\sigma^0 = t$, $\sigma^1 = r$.
- String ansatz $x^1 = vt + \delta(r)$
- Minimize the area (in the string frame!)
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Relativistic limit, $v \to 1$: $F = -\frac{\ell^2}{\ell_s^2}\sqrt{\frac{45\ Ts(T)}{4N_c^2}}\frac{v}{\sqrt{1-v^2}\left(-\frac{\beta_0}{4}\log\left[1-v^2\right]\right)^{\frac{4}{3}}} + \cdots$

Non-relativistic limit $v \to 0$: $F = -\frac{\ell^2}{\ell_s^2}\left(\frac{45\pi\ s(T)}{N_c^2}\right)^{\frac{2}{3}}\frac{\lambda(r_h)^{\frac{4}{3}}}{2\pi}v + \cdots$

Comparison to conformal case

The AdS result: $F_{conf} = \frac{\pi}{2}\sqrt{\lambda}T^2 \frac{v}{\sqrt{1-v^2}}$

Fix ℓ_s in our model by the lattice string tension

Fix $\lambda = 5.5$ in $\mathcal{N} = 4$ SYM:

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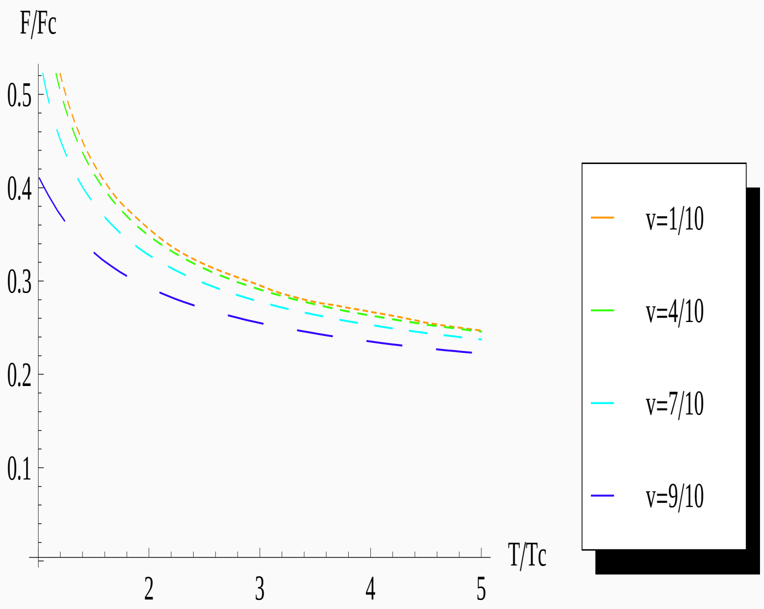

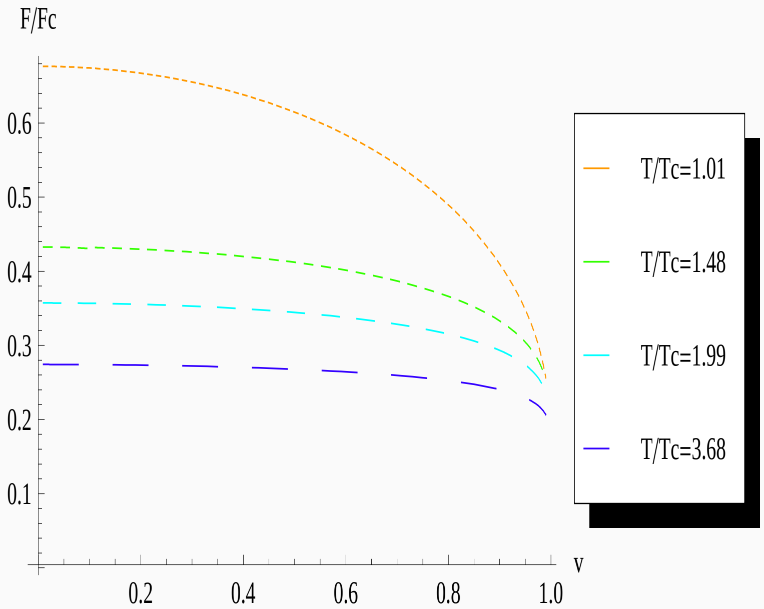

We clearly see the effects of asymptotic freedom!

Comparison schemes

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- Direct scheme: $T_{QGP} = T_{our}$

In the range $1.5T_c < T < 3T_c$ $E_{QGP} \propto E_{GP} \propto T^4$

- Alternative schemes: $E_{QGP} = E_{our}$ or $s_{QGP} = s_{our}$
- We try all possible schemes.

Predictions for experiments

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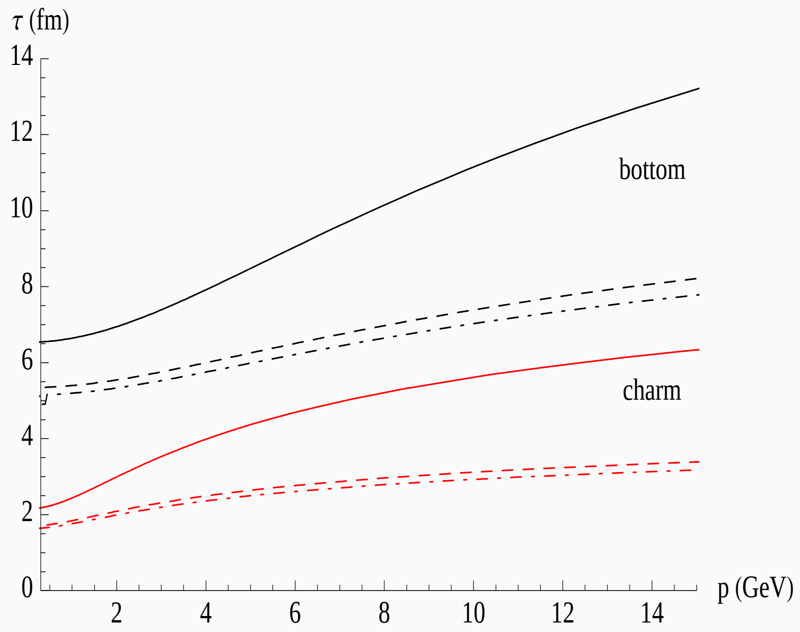
Equilibration times for charm and bottom:



Solid: direct, dashed: energy, dot-dashed entropy schemes.

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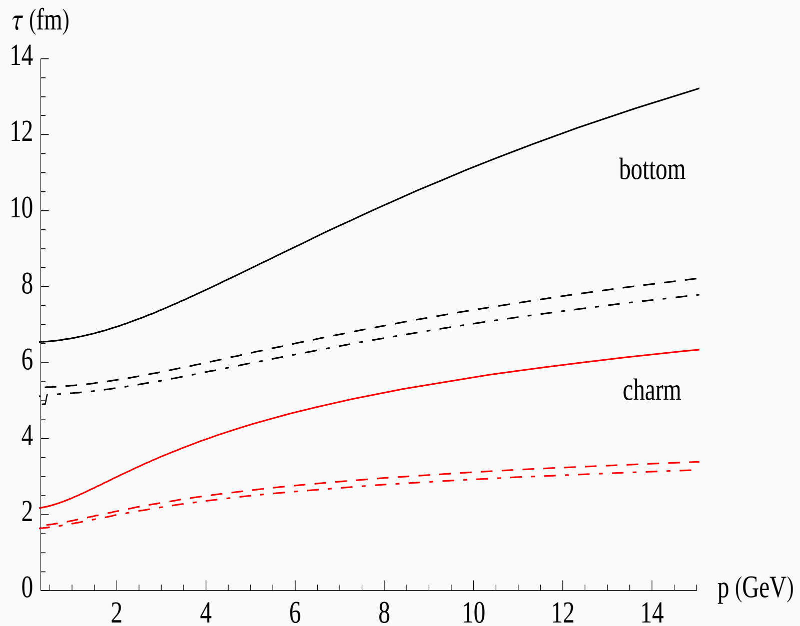

Solid: direct, dashed: energy, dot-dashed entropy schemes.
Some experimental studies + models PHENIX col. ’06, van Hees et al ’05:
For $p = 10$ GeV, $\tau_e \approx 4.5\ fm$ (charm)

We have $3 < \tau_e < 5.5\ fm$

Diffusion constants

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- In Fourier space
  $\kappa = \lim_{\omega\to 0} G_{sym}(\omega) = \lim_{\omega\to 0} \coth(\frac{\omega}{4T_s}) Im\, G_R(\omega)$
  where T_s is the world-sheet temperature.
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- G_R extracted from fluctuations on the trailing string solution:
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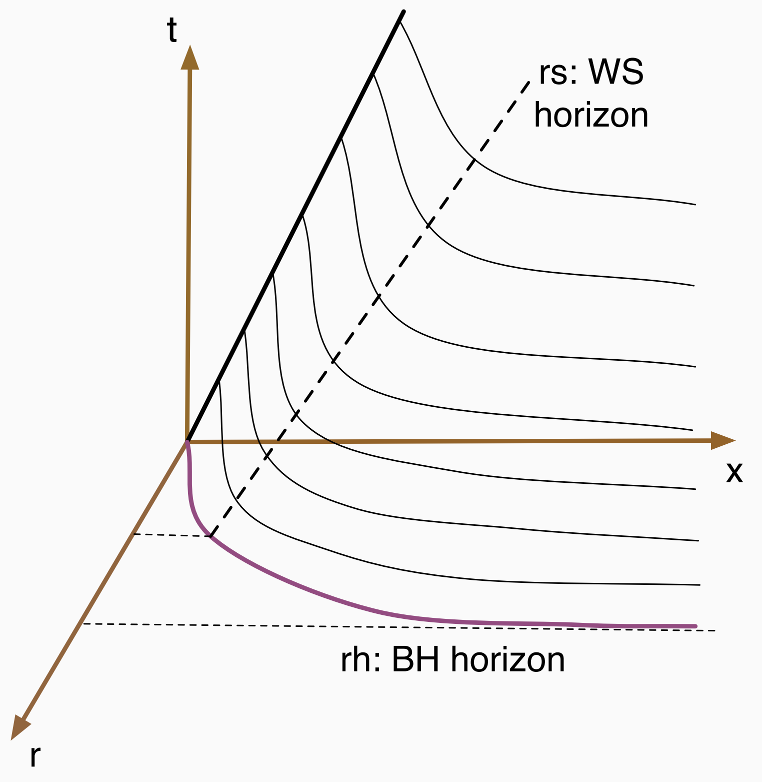
- There is a "horizon" on the world-sheet:
  $ds^2 = b^2 \left[-(f(r) - v^2) d\tau^2 + \frac{dr^2}{f - v^2 b^4(r_s)/b^4(r)} \right]$
  WS horizon at $f(r_s) = v^2$.
- $\kappa_\perp = \frac{2}{\pi \ell_s^2} b^2(r_s) T_s, \qquad \kappa_\| = \frac{32\pi}{\ell_s^2} \frac{b^2(r_s)}{f'(r_s)^2} T_s^3$

Physical picture



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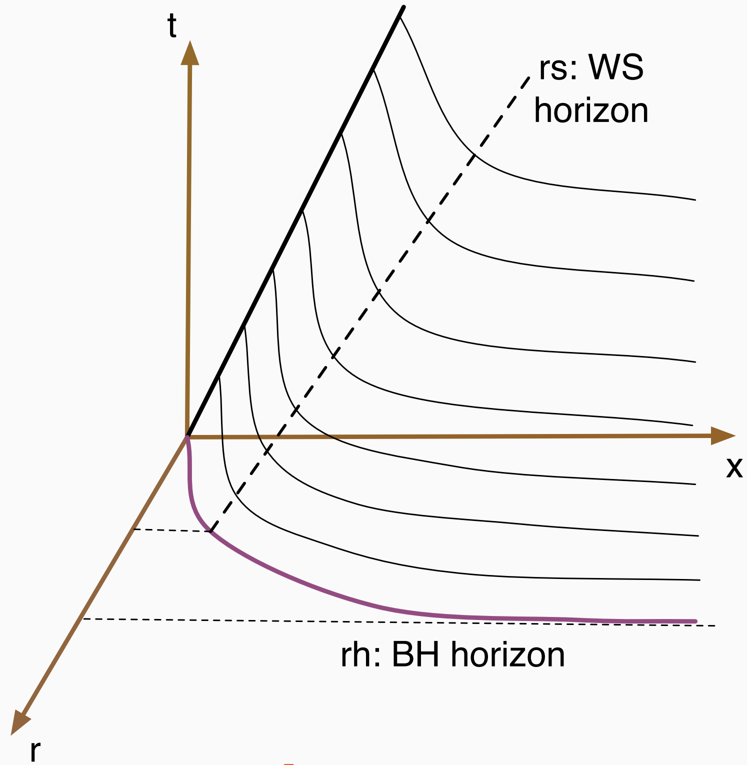

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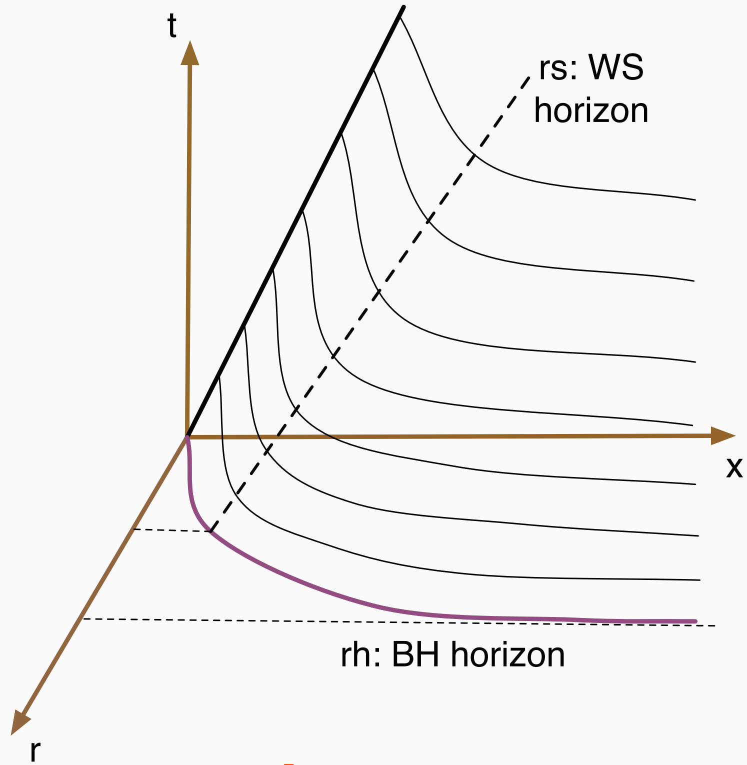

- A black-hole horizon on the WS at r_s:
  Fluctuations on the string fall into the horizon $\Rightarrow$ energy loss
- However, there is Hawking radiation at r_s towards the boundary
  $\Rightarrow$ momentum broadening.

Numerical results for jet quenching

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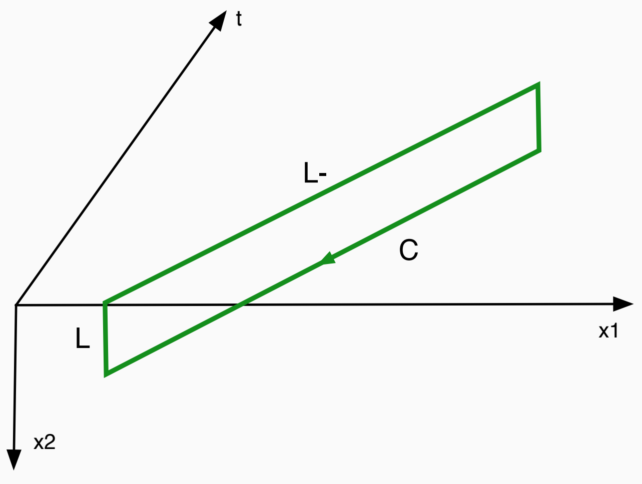
- $\boxed{\hat{q}_\perp = 5.2 \text{ (direct)}, 12 \text{ (energy)}, 13.13 \text{ (entropy)}\ GeV^2/fm,}$

  for a charm quark traveling at $p = 10 GeV$ at $T = 250$ MeV.

Jet quenching, non-perturbative

Non-perturbative def. of $\hat{q}$:



Wiedemann '00

$$\langle W(C)\rangle \approx \exp\left[-\frac{1}{8\sqrt{2}}\hat{q}L^{-}L^{2}\right].$$

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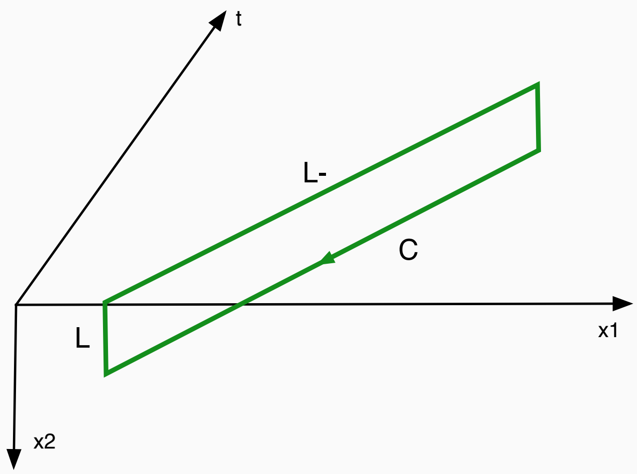

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Holographic computation Liu, Rajagopal, Wiedemann '06: $\langle W(C)\rangle = e^{iS}$

Pick up gauge: $x^{-} \equiv x_1 - t = \tau$, $x_2 = \sigma$, Compute minimal area:

- $\hat{q} = \frac{\sqrt{2}}{\pi \ell_s^2} \frac{1}{\int_0^{r_h} \frac{dr}{e^{2A_s}\sqrt{f(1-f)}}}$

Results

T_{QGP}, MeV	$\hat{q}\ (GeV^2/fm)$ (direct)	$\hat{q}\ (GeV^2/fm)$ (energy)	$\hat{q}\ (GeV^2/fm)$ (entropy)
220	-	0.89	1.01
250	0.53	1.21	1.32
280	0.79	1.64	1.73
310	1.07	2.14	2.21
340	1.39	2.73	2.77
370	1.76	3.37	3.42
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Close to AdS somewhat smaller than pQCD + fit to data Eskola et al '05

$\hat{q}_{expect} \sim 5 - 12\ GeV^2/fm$

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