Non-standard electroweak phase transitions in extensions to the standard model: Monopoles[1] and Scale invariance[2]

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[1] SA & A. Kobakhidze, 1702.04068 Eur. Phys. J. C (2017) 77: 444
[2] SA, A. Kobakhidze, C.Lagger, S. Liang and A.Zhou 1709.10322 PLB 776 (2018)

Outline

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Motivation

- There is a significant asymmetry between the matter and antimatter abundance in the universe.
- Sakharov conditions:
  - Baryon number violation
  - C and CP violation
  - Out of equilibrium processes
- One possible mechanism is electroweak baryogenesis
- Baryon asymmetry is generated via sphaleron mediated scattering processes which violate B + L.
- Requires a first order electroweak phase transition for departure from equilibrium.
- Needs to be strong enough to suppress sphaleron processes in the broken phase.

The Electroweak Phase transition

- SM high temperature effective potential:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 - \frac{1}{4}\lambda_T\phi^4$$

- curvature at the origin changes at $T = T_0$
- the nature of the transition depends on the values of the SM parameters.

First order phase transition

- The minima become degenerate before T_0
- Bubbles of the broken phase form
- collisions lead to gravitational waves, baryogenesis etc.

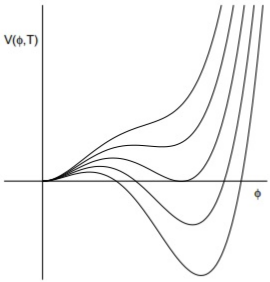


Figure: First order phase transition (Petropoulos,2003)

Second order phase transition

- the universe rolls homogeneously into th broken phase
- predicted by SM parameters

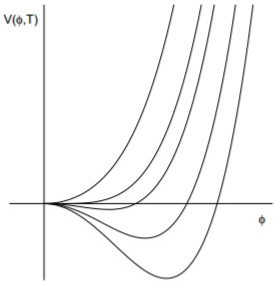


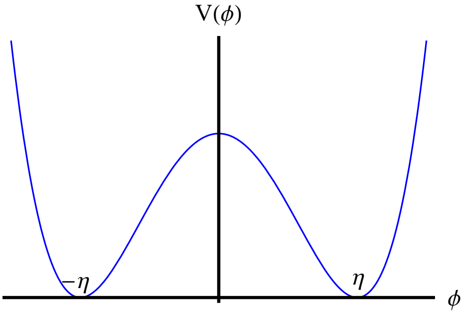
Figure: Second order phase transition (Petropoulos,2003)

Outline

1-D topological defects

- Consider a 1D potential:

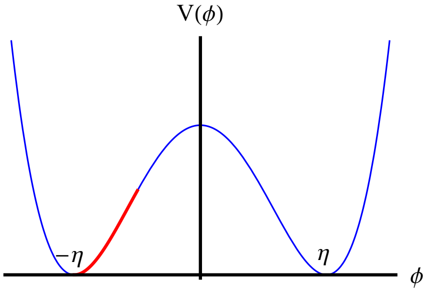
$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$



- For $\int_{-\infty}^{\infty} V(\phi)dx < \infty, \phi(\pm\infty) \to \pm\eta$

1-D topological defects

- Suppose $\phi(\infty) = \phi(-\infty) = -\eta$



- Decays to the constant solution

1-D topological defects

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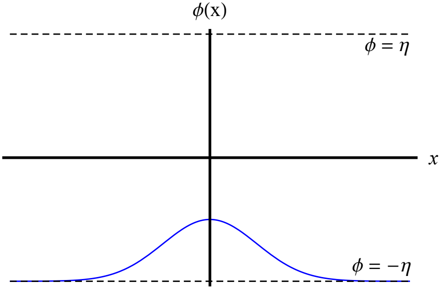

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1-D topological defects

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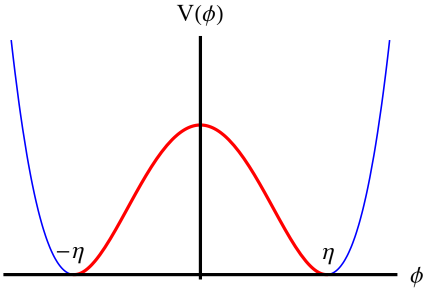

- Heuristically requires an infinite amount of energy to transition to constant solution.
- Topological stability from disconnected vacuum manifold
- $\pi_0(M_{vac}) \neq 0$.

1-D topological defects

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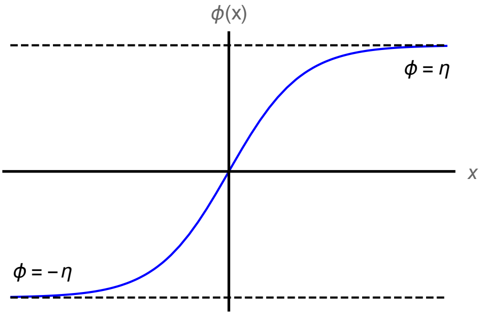

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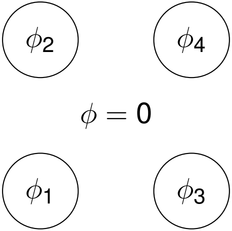
Monopoles

- Consider an extension of this idea to 3 spatial dimensions
- Spatial infinity is described by a 2-sphere
- Finite energy requires $\phi : S^2_{\infty'} \to M_{vac}$.
- Topologically non-trivial solutions exist when $\pi_2(M_{vac}) \neq 0$
- Monopoles are spherically symmetric solutions that derive their stability from such topologies

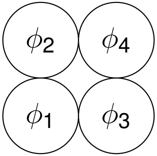
The Kibble Mechanism

- At $T = T_c$, domains of the broken phase will appear
- The higgs field in each domain takes independent directions on the vacuum manifold



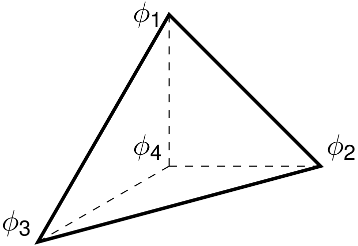
The Kibble mechanism

- As the Higgs field is continuous, it must be interpolated at the intersections.
- Consider an intersection of four of these domains:



The Kibble mechanism

- In field space, these points form the vertices of a tetrahedron.
- This tetrahedron should be shrunk to a point at the intersection.
- If these cannot be shrunk to a point continuously, a topological defect in the form of a monopole which continuously joins the two minima.
- The tetrahedron is homotopically equivalent to S^2.
- Therefore, $\pi_2(M_{vac}) \neq 0$ implies the existence of monopoles



The standard model

- For the standard model, $M_{\text{vac}} = (SU(2) \times U(1)_Y)/U(1)_Q$
- $\pi_2(M_{\text{vac}}) = \pi_2(S^3) = 0$
- No electroweak monopoles?

The Ansatz

- Cho and Maison (1997) found electroweak monopoles through the ansatz:

$$\phi = \frac{1}{\sqrt{2}}\rho\xi$$

$$\rho = \rho(r)$$

$$\xi = i\begin{pmatrix}\sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2)\end{pmatrix}$$

$$A_\mu = \frac{1}{g}A(r)\partial_\mu t\hat{r} + \frac{1}{g}(f(r)-1)\hat{r}\times\partial_\mu\hat{r}$$

$$B_\mu = -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi$$

Why is this stable?

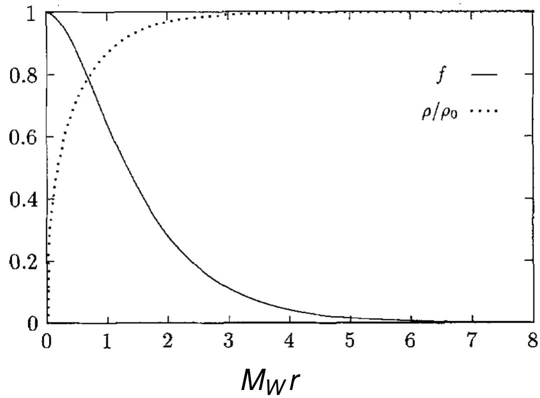
$$\xi = i \begin{pmatrix} \sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

$$B_\mu = -\frac{1}{g'}(1 - \cos\theta)\partial_\mu \varphi$$

- Singularity in the hypercharge field at $\theta = \pi$
- Can be removed using a Wu-Yang construction
- Introduces singularities to the Higgs manifold.
- $M_{\text{vac}} = S^3 \setminus \{2 \text{ points}\} \cong S^2$
- $\pi_2(M_{\text{vac}}) = \mathbb{Z}$

Solution

- Simple solution: $A = B = 0$ (Cho & Maison, 1997)
- $h = \frac{4\pi}{e}$



The energy

$$E = E_0 + E_1$$

$$E_0 = 4\pi \int_0^\infty \frac{dr}{2r^2} \left\{ \frac{1}{g'^2} + \frac{1}{g^2}(f^2 - 1)^2 \right\}$$

$$E_1 = 4\pi \int_0^\infty dr \left\{ \frac{1}{2}(r\dot{\rho})^2 + \frac{1}{g^2}\left(\dot{f}^2 + \frac{1}{2}(r\dot{A})^2 + f^2 A^2 \right) \right.$$
$$+ \frac{1}{2g'^2}(r\dot{B})^2 + \frac{\lambda r^2}{8}(\rho^2 - \rho_0^2)^2$$
$$\left. + \frac{1}{4} f^2 \rho^2 + \frac{r^2}{8}(B - A)^2 \rho^2 \right\}$$

- The first term of E_0 is divergent at the origin.

Regularisation

- Cho, Kim and Yoon(2015) proposed a regularisation of the form:

$$g' \to \frac{g'}{\sqrt{\epsilon}}$$

$$\epsilon = \left(\frac{\phi}{\phi_0}\right)^n$$

- However, g' becomes non-peturbative as $\phi \to 0$.
- This is undesirable in an EFT framework.
- We instead propose a Born-Infeld modification for the $U(1)_Y$ kinetic term.

Born-Infeld modification

- We regularise the $U(1)_Y$ kinetic term by replacing it with:

$$\beta^2 \left[1 - \sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{\beta}B_{\mu\nu}\right)}\right]$$

$$= \beta^2 \left[1 - \sqrt{1 + \frac{1}{2\beta^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{16\beta^4}(B_{\mu\nu}\tilde{B}^{\mu\nu})^2}\right]$$

- As $\beta \to \infty$, the SM is recovered.
- The corresponding energy is

$$\int_0^\infty dr\,\beta^2 \left[\sqrt{(4\pi r^2)^2 + \left(\frac{4\pi}{g'\beta}\right)^2} - 4\pi r^2\right]$$

$$= \frac{4\pi^{5/2}}{3\Gamma\left(\frac{3}{4}\right)^2}\sqrt{\frac{\beta}{g'^3}}$$

- Hence, β acts as a mass parameter for the monopoles.

The electroweak phase transition

- The Gibbs free energy:

$$G_u = V(0)$$
$$G_b = V(\phi_c(T)) + E_{\text{monopoles}}$$

- At the critical temperature:

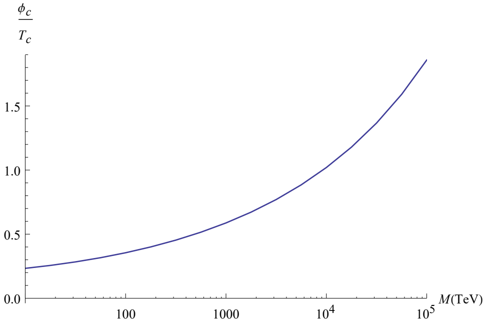
$$V(0) = V(\phi_c(T_c)) + E_{\text{monopoles}}$$

- Assuming $T << M$,the monopoles are decoupled and $E_{monopoles} = M \times n_M$

The initial density

- $n_M \approx \frac{1}{d^3}$ where d is the separation of two uncorrelated monopoles.
- This is chosen to be the Coulomb capture distance.
- Hence, $n_M \approx \left(\frac{4\pi}{h^2}\right)^3 T^3$

Results



- Sphaleron processes are suppressed for $M > 0.9 \cdot 10^4$ TeV.

The constraint

- The monopole density should not dominate the universe at the time of helium synthesis. This implies:
- $\left.\frac{n}{T^3}\right|_{T=1\text{MeV}} < \frac{1\text{MeV}}{M}$
- Hence, the evolution of the number density over time must be considered.

The number density at lower temperatures

- Consider monopoles drifting towards anti monopoles in a plasma of charged fermions.
- Scattering cross-section: $\sigma_{q_i M} = (hq_i/4\pi)^2 T^{-2}$
- After $\sim \frac{M}{T}$ collisions, the monopole is scattered at a large angle and drifts towards the antimonopole.
- This yields a mean free path of:

$$\lambda \approx \frac{v_{\text{drift}}}{\sum_i n_i \sigma_i} \frac{M}{T} \approx \frac{1}{B} \left(\frac{M}{T^3} \right)^{1/2}$$

- $B = \frac{3}{4\pi^2} \zeta(3) \sum_i (hq_i/4\pi)^2$

- Annihilation ends when $\lambda \approx \frac{h^2}{4\pi T}$, the Coulomb capture radius.
- This occurs at $T_f \approx \left(\frac{4\pi}{h^2}\right)^2 \frac{M}{B^2}$
- For $T < T_f$, the monopole density simply dilutes as $n \propto T^3$.

Nucleosynthesis constraint

- Solving the Boltzmann equation, one obtains (Preskill, 1979)

$$\frac{n}{T^3} = \frac{1}{Bh^2}\left(\frac{4\pi}{h^2}\right)^2 \frac{M}{CM_{pl}}, \; (T > T_f)$$

- $C = (45/4\pi^3 N)^{1/2}$
- This constrains the mass of the monopole to $M \lesssim 2.3 \cdot 10^4$ TeV.

Summary and future work

- Sphaleron mediated processes can be made ineffective in the broken phase while remaining under the nucleosynthesis constraints.
- This occurs for monopoles with a mass of $(0.9 - 2.3) \cdot 10^4$TeV.
- This could lead to a new mechanism for baryogenesis.

Outline

Motivation

- Scale invariance is an attractive framework for addressing the problem of the origin of mass and hierarchies of mass scales.
- Quantum fluctuations result in a mass scale via dimensional transmutation
- Dimensionless couplings are responsible for generating mass hierarchies.
- Scale (conformal) invariance is an essential symmetry in string theory
- What is the nature of the EWPT in this framework?

The model

- Consider the SM as a low energy Wilsonian effective theory with cutoff Λ:

$$V(\Phi^\dagger\Phi) = V_0(\Lambda) + \lambda(\Lambda)\left[\Phi^\dagger\Phi - v_{ew}^2(\Lambda)\right]^2$$

- Assume the fundamental theory exhibits conformal invariance which is spontaneously broken down to Poincare invariance
- Promote dimensionful parameters to the dilaton field, the scalar Goldstone boson.

$$\Lambda \to \alpha\chi, \quad v_{ew}^2(\Lambda) \to \frac{\xi(\alpha\chi)}{2}\chi^2, \quad V_0(\Lambda) \to \frac{\rho(\alpha\chi)}{4}\chi^4 \,, \qquad (1)$$

The model

- Impose the following conditions:
  - $\frac{dV}{d\phi}\Big|_{\Phi=v_{ew},\chi=v_\chi} = \frac{dV}{d\chi}\Big|_{\Phi=v_{ew},\chi=v_\chi} = 0$ (Existence of the electroweak vev)
  - $V(v_{ew}, v_\chi) = 0$ (Cosmological constant)
- Implications:
  - $\rho(\alpha v_\chi) = \beta_\rho(\alpha v_\chi) = 0$
  - $\xi(\alpha v_\chi) = \frac{v_{ew}^2}{v_\chi^2}$
  - $m_\chi^2 \simeq \frac{\beta'_\rho(\Lambda)}{4\xi(\Lambda)} v_{ew}^2 \simeq (10^{-8}\text{eV})^2$ for $\alpha\chi \sim M_P$

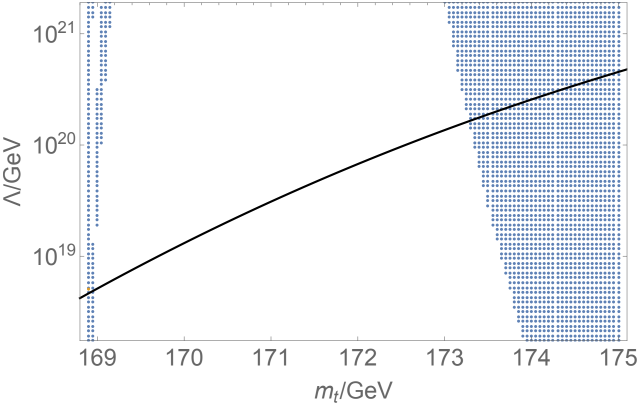


Figure: Plot of the allowed range of parameters (shaded region) with $m_\chi^2(v_{ew}) > 0$, i.e., the electroweak vacuum being a minimum. The solid line displays the cut-off scale Λ as function of the top-quark mass m_t for which the conditions are satisfied.

The thermal effective potential

- At high temperatures:

$$V_T(h,\chi) = \frac{\lambda(\Lambda)}{4}\left[h^2 - \frac{v_{ew}^2}{v_\chi^2}\chi^2\right]^2 + c(h)\pi^2 T^4 - \frac{\lambda(\Lambda)}{24}\frac{v_{ew}^2}{v_\chi^2}\chi^2 T^2 + \frac{1}{48}\left[6\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2 T^2$$

- Minimising this potential w.r.t. χ :

$$\chi^2 \approx \frac{v_\chi^2}{v_{ew}^2}\left(h^2 + \frac{T^2}{12}\right)$$

Thermal effective potential

The effective potential in this direction is given by:

$$V_T(h, \chi(h)) = \left[c(h)\pi^2 - \frac{\lambda(\Lambda)}{576}\right] T^4 + \frac{1}{48}\left[4\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2 T^2$$

Standard model

- SM high temperature effective potential:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 - \frac{1}{4}\lambda_T\phi^4$$

- curvature at the origin changes at $T = T_0$
- the nature of the transition depends on the values of the SM parameters.

The scale-invariant model

- Along the flat direction, $T_0 = 0$
- Furthermore, the minima are degenerate only at $T = 0$
- No phase transition???

Chiral phase transition

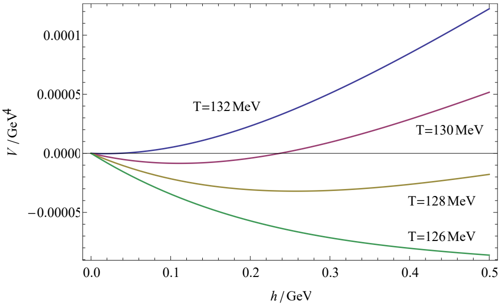
- Consider the Yukawa term:

$$y \langle \bar{q}q \rangle_T \phi$$

- At $T \sim 132$MeV, chiral condensates form.
- This term is given by (Gasser & Leutwyler, 1987):

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle \left[1 - (N^2 - 1) \frac{T^2}{12 N f_\pi^2} + \mathcal{O}\left(T^4\right) \right]$$

The electroweak phase transition

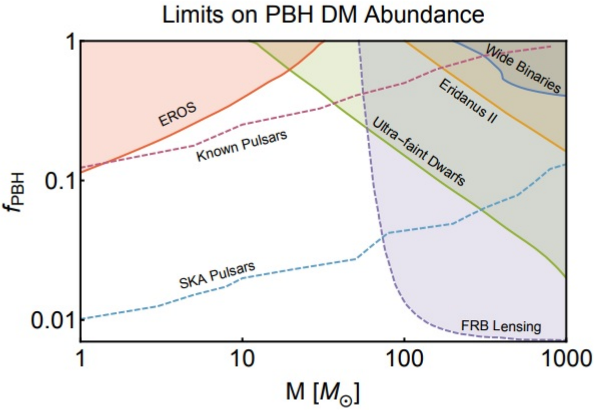


- The linear term shifts the minimum from the origin
- at $T \sim 127$MeV, the minimum disappears and the EWPT is triggered
- The EWPT is 2nd order.

Implications

- 6 relativistic quarks at the critical temperature indicates a 1st order chiral PT. (Pisarski& Wilczek, 1983)
- Gravitational waves with peak frequency $\sim 10^{-8}$ Hz, potentially detectable by means of pulsar timing (EPTA, SKA...)

- Production of primordial black holes with mass $M_{BH} \sim M_{\odot}$



(Schutz & Liu, 2016)

Implications

- Scale invariant theories predict a light feebly coupled dilaton.
- Electroweak phase transition driven by the QCD chiral phase transition and occurs at $T \sim 130$ MeV.
- QCD phase transition could be strongly first order $\Longrightarrow$ gravitational waves, black holes, QCD baryogenesis.
- Detection of a light scalar particle + the above astrophysical signatures will provide strong evidence for the fundamental role of scale invariance in particle physics and cosmology.

Outline

1. Motivation
2. Electroweak monopoles and the electroweak phase transition
3. The Standard Model with hidden scale invariance
4. Conclusion

Conclusion

- Electroweak monopoles
  - Cho-Maison monopoles have infinite energy in the SM
  - This can be be regularised using a Born-Infeld extension
  - Production increases the energy of the broken phase at the EWPT
  - Results in a strong electroweak phase transition while consistent with BBN results.
  - Monopole Baryogenesis(?)
- Scale Invariance
  - Chiral phase transition occurs before EWPT
  - 6 massless quarks and therefore it is first order
  - Potentially leads to GW, primordial black holes etc.)