Method for searching for and testing general relativity with supernovae gravitational-wave signals

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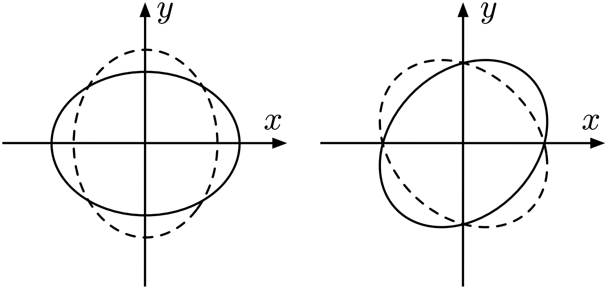
Nikhef

What is a gravitational wave?

Ripple of space-time first predicted by general relativity



Gravitational waves can be detected by measuring these length changes

What can generate gravitational waves?

- Binary black hole mergers
- Binary neutron star mergers
- Core-collaspe supernovae
- and more ...

Binary black hole mergers

- Component masses: m_1, m_2
- Component spins: $\vec{s}_1, \vec{s}_2$
- Luminosity distance D
- Sky location θ, φ
- Orientation: ι, ϕ
- Polarization angle: ψ
- Time of merger: t_c

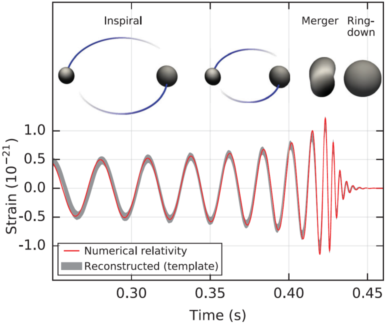
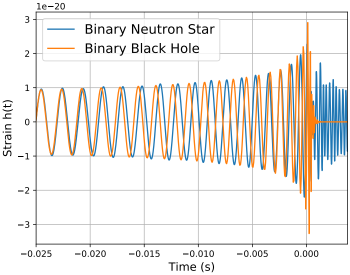
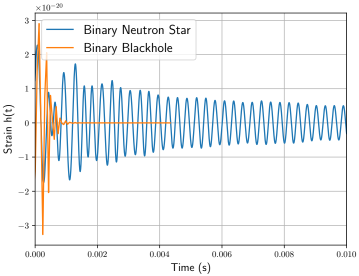


Image taken from [1]

Binary neutron star mergers

- Similar to binary black hole mergers
- Matter tidal effect:
  - Tidal deformability $\Lambda = \lambda/m^5, \lambda = -Q_{ij}/E_{ij}$
  - Tidal coupling constant $\kappa_2^T = 3\left(\frac{q^4}{(1+q)^5}\Lambda_1 + \frac{q}{(1+q)^5}\Lambda_2\right)$
- Various post-merger phase behaviours





Core-collapse supernovae

- Death of a star
- Weaker compared to compact binaries
- Waveform not well modelled

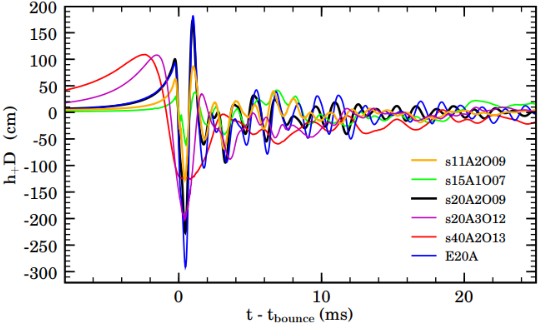


Image taken from [2]

Bayesian Statistics

Parameter estimation

$$\mathrm{P}(\vec{\theta}|d,\mathcal{H}) = \frac{\mathrm{P}(d|\vec{\theta},\mathcal{H})\mathrm{P}(\vec{\theta}|\mathcal{H})}{\mathrm{P}(d|\mathcal{H})} \tag{1}$$

- $\mathrm{P}(\vec{\theta}|d,\mathcal{H})$: Posterior
- $\mathrm{P}(d|\vec{\theta},\mathcal{H})$: Likelihood
- $\mathrm{P}(\vec{\theta}|\mathcal{H})$: Prior
- $\mathrm{P}(d|\mathcal{H})$: Evidence

Multiple detections:

$$\mathrm{P}(\vec{\theta}_c|\{d_i\},\mathcal{H}) = \mathrm{P}(\vec{\theta}_c|\mathcal{H})^{1-N}\prod_i^N \mathrm{P}(\vec{\theta}_c|d_i,\mathcal{H}) \tag{2}$$

Bayesian Statistics

Hypothesis testing

$$\begin{aligned} \frac{\mathrm{P}(\mathcal{H}_1|d)}{\mathrm{P}(\mathcal{H}_2|d)} &= \frac{\mathrm{P}(d|\mathcal{H}_1)}{\mathrm{P}(d|\mathcal{H}_2)} \frac{\mathrm{P}(\mathcal{H}_1)}{\mathrm{P}(\mathcal{H}_2)} \\ \mathcal{O}_2^1 &= \mathcal{B}_2^1 \frac{\mathrm{P}(\mathcal{H}_1)}{\mathrm{P}(\mathcal{H}_2)} \end{aligned} \tag{3}$$

- $\mathcal{O}_2^1$: Odds ratio
- $\mathcal{B}_2^1$: Bayes factor
- $\mathrm{P}(\mathcal{H}_1)/\mathrm{P}(\mathcal{H}_2)$: Prior odds

$\mathcal{O}_2^1 > 1 \rightarrow \mathcal{H}_1$ is more plausible than $\mathcal{H}_2$, vice versa

Signal-to-Noise ratio

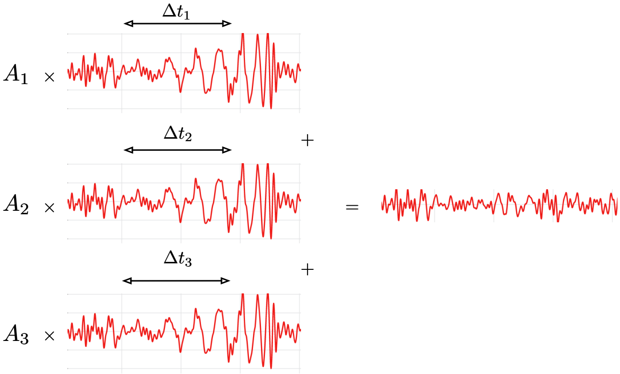
The inner product:

$$\langle a|b\rangle = 4\Re \int \frac{a(f)b^*(f)}{S_n(f)} df \tag{4}$$

$S_n(f)$ is the power spectral density of the noise
The (optimal) signal-to-noise ratio (SNR):

$$\mathrm{SNR} = \sqrt{\langle h|h\rangle} \tag{5}$$

What is a null stream?



The coefficients $\{A_i\}$ and $\{\Delta t_i\}$ are given by the source sky location (θ, ϕ).

$$\text{Null Stream} \rightleftharpoons \text{Sky Location}$$

What is a null stream?

$$\begin{bmatrix} \tilde{d}_{\mathrm{H1}} \\ \tilde{d}_{\mathrm{L1}} \\ \vdots \\ \tilde{d}_{\alpha} \end{bmatrix} = \begin{bmatrix} F_{\mathrm{H1},+} & F_{\mathrm{H1},\times} \\ F_{\mathrm{L1},+} & F_{\mathrm{L1},\times} \\ \vdots & \vdots \\ F_{\alpha,+} & F_{\alpha,\times} \end{bmatrix} \begin{bmatrix} \tilde{h}_{+} \\ \tilde{h}_{\times} \end{bmatrix} + \begin{bmatrix} \tilde{n}_{\mathrm{H1}} \\ \tilde{n}_{\mathrm{L1}} \\ \vdots \\ \tilde{n}_{\alpha} \end{bmatrix} \tag{6}$$

$$d \equiv Fh + n$$

The null stream z is given by

$$z = P_{\mathrm{null}}(\hat{\Omega})d = (I - F(F^{\dagger}F)^{-1}F^{\dagger})d = P_{\mathrm{null}}n \tag{7}$$

if $\hat{\Omega} = \hat{\Omega}_{\mathrm{True}}$, The notion of *Null Energy*. E_{null} was introduced in Sutton *et.al* [4]

$$E_{\mathrm{null}} = z^{\dagger}z \tag{8}$$

- A measure of the energy of the resultant null stream.
- If the data consist of noise only or $F = F_{\mathrm{source}}$
  - $E_{\mathrm{null}} \sim \chi^2, \mathrm{DoF} = 2N_f(N_{\mathrm{IFO}} - 2) = 2N_f(N_{\mathrm{IFO}} - N_{\mathrm{modes}})$
- Likelihood function for Bayesian analysis

Hypotheses

Sutton *et.al* has shown that glitches can create similar effect as signals if we only consider E_{null} [4]
Two hypotheses are introduced for distinguishing glitches and signals

$\mathcal{H}_{\text{coherent}}$:= The resultant strain consists of noise-only when the data are time shifted coherently according to the proposed sky location
$\mathcal{H}_{\text{coherent}} \rightarrow \{\theta, \phi\}$

$\mathcal{H}_{\text{incoherent}}$:= The resultant strain consists of noise-only when the data are time shifted independent to the proposed sky location
$\mathcal{H}_{\text{incoherent}} \rightarrow \{\theta, \phi, \{\Delta t_i\}\}$

Mock data simulation results

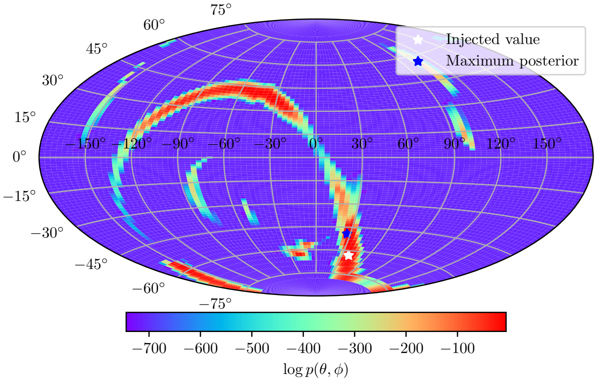
GW150914-like signal is injected

Type	$\log \mathcal{Z}_{\text{coherent}}$	$\log \mathcal{Z}_{\text{incoherent}}$	$\log \mathcal{B}^{\text{coherent}}_{\text{incoherent}}$
noise	-6.04	-6.17	0.13
glitch (H1)	-8.58	-8.68	0.09
glitch (L1)	-8.90	-8.80	-0.10
glitch (V1)	-7.45	-7.42	-0.04
glitch (H1&L1)	-11.75	-13.17	1.42
glitch (H1&V1)	-11.74	-13.17	1.42
glitch (L1&V1)	-11.39	-11.36	-0.03
signal	-12.14	-198.62	186.48

The values of $\log \mathcal{Z}_{\text{coherent}}$ agree with Sutton *et.al*'s finding

Mock data simulation results

Skymap of the source $\log p(\theta, \phi | \mathcal{H}_{\text{coherent}})$



The skymap shows that the sky location is well constrained

Detection Characterization

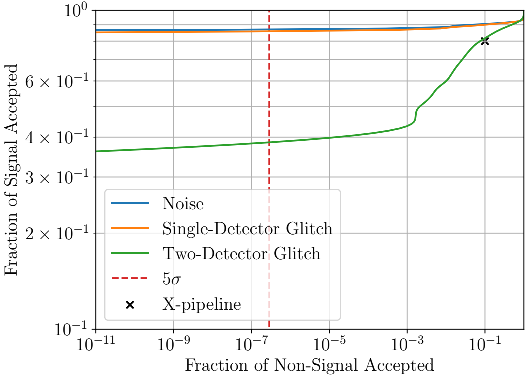
Mock data simulation

- AdLIGO-AdVirgo network is simulated
- Signal: 200 BBH signals are injected uniformly in co-moving volume
- Glitch: Same as signal but only appears in H1, L1, V1, H1&L1, H1&V1, L1&V1 coherently
- Noise: Simulated Gaussian noise with design sensitivity

Detection Characterization

For SNR ≥ 20

- 87%, 86% and 39% of the injections have more than 5σ significance with respect to noise, single-detector glitches and two-detector glitches.

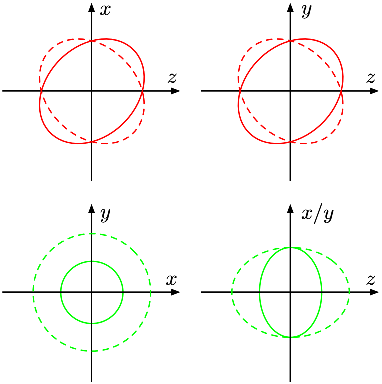


Detection Characterization

For SNR greater than 16 or 20, a significant fraction of detections has a statistical significance greater than 5σ.

SNR	Noise	Single-detector glitch	Two-detector glitch
8	27.3%	26.7%	4.1%
12	58.4%	59.0%	10.7%
16	80.3%	80.6%	21.3%
20	86.7%	85.8%	38.6%

Testing GW Polarization



$$
\begin{aligned}
F_{\mathrm{B}} &= -\frac{1}{2}\sin^2\theta\cos 2\phi = -F_{\mathrm{L}} \\
F_{\mathrm{X}} &= -\sin\theta(\cos\theta\cos 2\phi\cos\psi - \sin 2\phi\sin\psi) \\
F_{\mathrm{Y}} &= -\sin\theta(\cos\theta\cos 2\phi\sin\psi + \sin 2\phi\cos\psi)
\end{aligned} \tag{9}
$$

Testing GW Polarization

By changing the beam pattern function in F and the corresponding DoF of χ^2

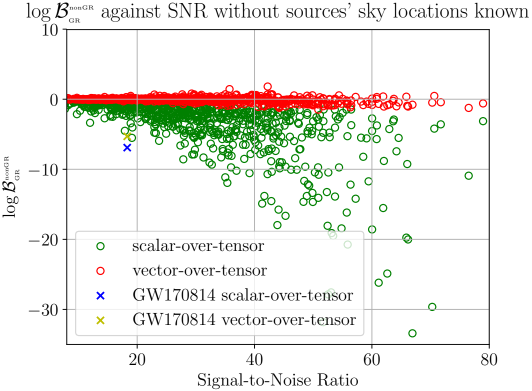
$$F = \begin{bmatrix} F_{\mathrm{H1},+} & F_{\mathrm{H1},\times} \\ F_{\mathrm{L1},+} & F_{\mathrm{L1},\times} \\ \vdots & \vdots \\ F_{\alpha,+} & F_{\alpha,\times} \end{bmatrix} \rightarrow \begin{bmatrix} F_{\mathrm{H1},0} & F_{\mathrm{H1},1} & \dots & F_{\mathrm{H1},N} \\ F_{\mathrm{L1},0} & F_{\mathrm{L1},1} & \dots & F_{\mathrm{L1},N} \\ \vdots & \vdots & \vdots & \vdots \\ F_{\alpha,1} & F_{\alpha,2} & \dots & F_{\alpha,N} \end{bmatrix} \tag{10}$$

$$\mathrm{DoF} = 2N_f(N_{\mathrm{IFO}} - 2) \rightarrow 2N_f(N_{\mathrm{IFO}} - N)$$

The hypothesis changes from $\mathcal{H}_{\text{coherent, GR}} \rightarrow \mathcal{H}_{\text{coherent, nonGR}}$

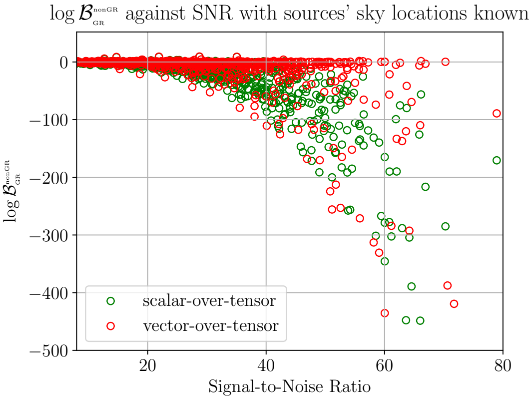
Testing GW Polarization

- Pure tensor signals are injected
- Pure tensor hypothesis outranks pure scalar hypothesis significantly



Testing GW Polarization

- Fix the sky location to the injected value
- Pure tensor hypothesis outranks both pure scalar and pure vector hypotheses



Testing GW Polarization

If we consider the limitation given by a general metric theory to be true:

- 7 possible combinations of polarization modes
    1. pure scalar
    2. pure vector
    3. pure tensor
    4. pure scalar + pure tensor
    5. pure vector + pure tensor
    6. pure scalar + pure vector
    7. pure scalar + pure vector + pure tensor
- These combinations formed an set of exhaustive hypotheses
- A 6-detector network is required

A 3-detector network

- pure tensor, pure vector and pure scalar are testable
- Consider the following to be a set of exhaustive hypotheses
    1. pure scalar + pure tensor
    2. pure vector + pure tensor
    3. pure scalar + pure vector + pure tensor

$$\mathrm{P}(\mathrm{GR}|d) = \frac{1}{1 + \sum_i \mathcal{O}_{\mathrm{GR}}^{\mathcal{H}_i}}, \tag{11}$$

SNR required for GR to be true with 5σ statistical significance ($\mathrm{P}(\mathrm{GR}|d) \geq 1 - 2.87 \times 10^{-7}$):

Set	Without EM counterpart	With EM counterpart
1	~ 40	~ 23
2	–	~ 23
3	–	~ 23

Conclusions

- Bayesian null stream analysis method is introduced
- Successfully distinguish between signal, glitch and noise
- Model-agnostic GW polarization test
- With a signal with electromagnetic counterpart with SNR greater than 23, general relativity can be accepted with 5σ statistical significance

Reference I

Abbott et. al., Phys. Rev. Lett. 116, 061102 (2016)

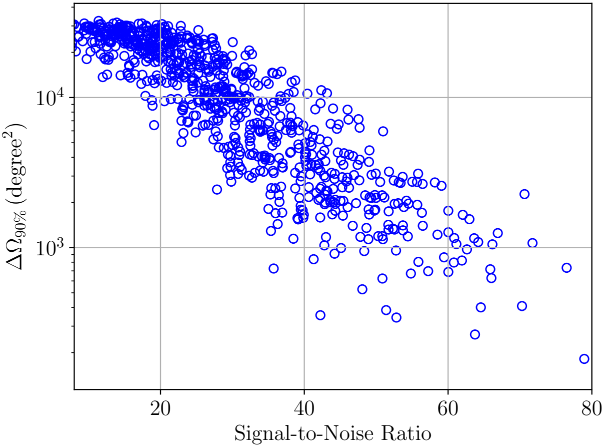
Ott, Classical Quantum Gravity 26, 063001 (2009)

Gursel and Tinto, Phys. Rev. D 40, 3884 (1989)

Sutton *et.al*, arXiv:0908.3665 (2010)

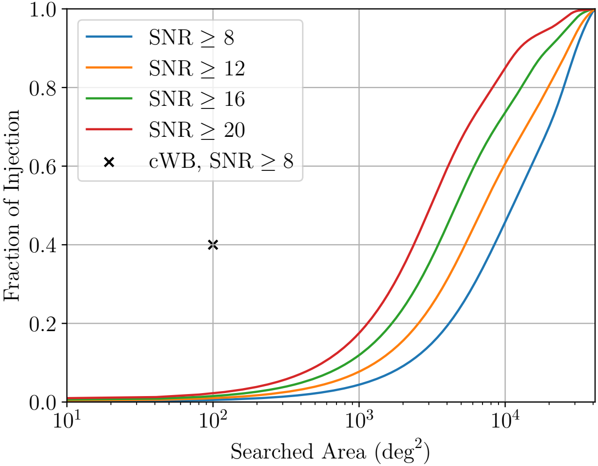
Sky Location Accuracy

- For SNR ≥ 40, the area of 90% credible region is less than 10^4 deg^2
- For template-based parameters inference, the area of 90% credible region spans between 14 to 220 deg^2



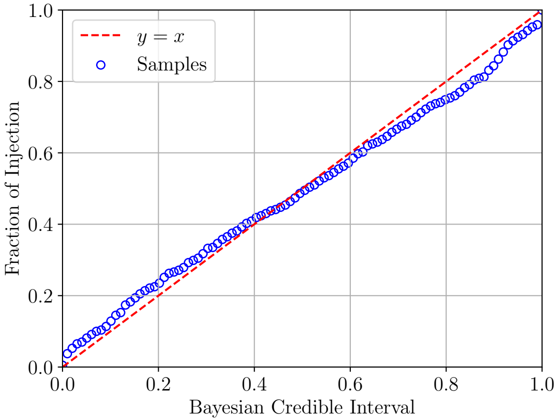
Sky Location Accuracy

Searched Area: Area of the sky with a posterior probability higher than that of the true location



Sky Location Accuracy

P-P plot shows that the skymap is describing the source correctly



Waveform reconstruction

$$h_{\text{recon}} = (F^{\dagger}F)^{-1}F^{\dagger}d \tag{12}$$