On the empirical verification of the black hole no-hair conjecture from gravitational-wave observations

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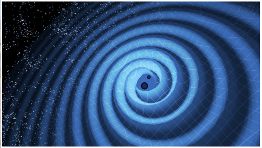


Gregorio Carullo, Laura van der Schaaf, Lionel London, Peter T.H.Pang, Otto A. Hannuksela, Jeroen Meidam, Michalis Agathos, Anuradha Samajdar, Archisman Ghosh, Tjonnie G.F.Li, Walter Del Pozzo, Chris Van Den Broeck

Introduction

Gravitational waves have enabled first tests of genuinely strong-field dynamics of General Relativity (GR).
So far 5 announced binary black holes
(and 1 binary neutron star with EM counterpart!).

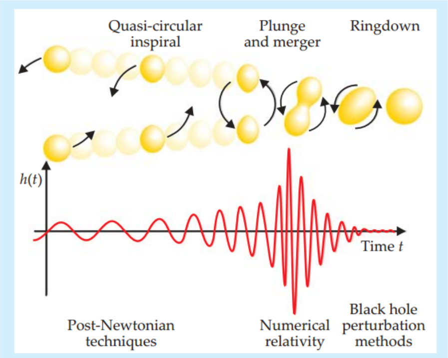
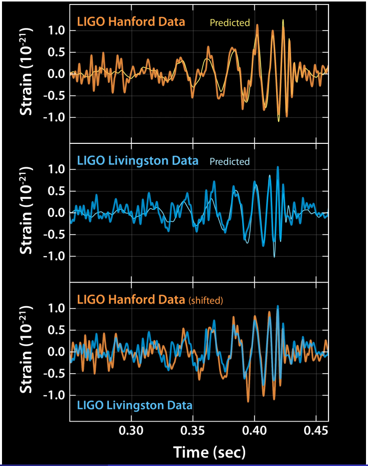


Image: Baumgarte and Shapiro 2011

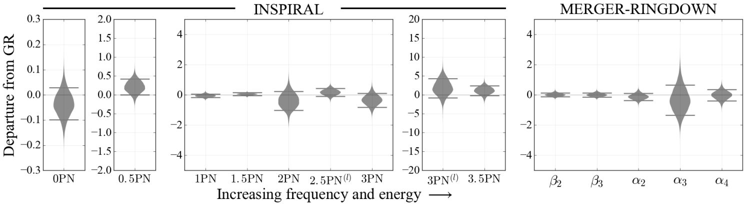


Testing the dynamics of binary merger

Phase evolution in frequency domain is

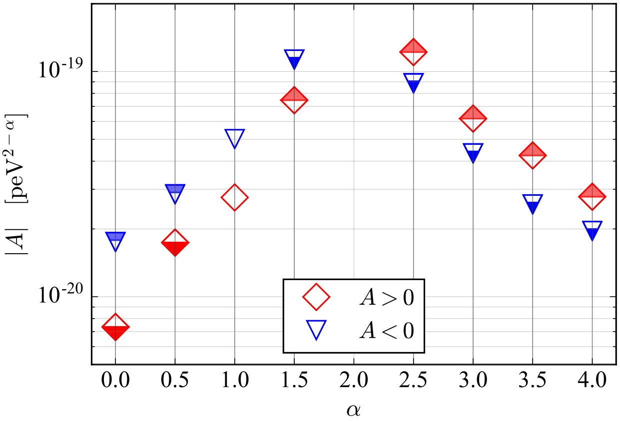
$$\Phi(v) = (\frac{v}{c})^{-5} \sum_{i=0}^{7} (\varphi_i + \varphi_i^{(l)} \log \frac{v}{c})(\frac{v}{c})^i$$

where φ_i are the post-Newtonian expansion coefficients.
If we introduce a relative shift on these coefficients, $\varphi_i \to \varphi_i(1 + \delta\varphi_i)$, we can test General Relativity.



Testing the propagation of gravitational wave

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha$$



$\alpha = 0$ corresponds to massive graviton.

Our current upper bound of massive graviton is $m_g < 7.7 \times 10^{-23}\mathrm{eV}/\mathrm{c}^2$.

Probing the nature of black holes

Theorem 1: No-hair theorem

In GR, a stationary isolated black hole is determined uniquely by its mass M_f, angular momentum a_f, and electric charge Q.

$$h(t) = \frac{M_f}{r} \sum_{lmn} A_{lmn} \ {}_{-2}S_{lmn} e^{i\tilde{\omega}_{lmn} t}$$

1. $A_{lmn}(q)$ is complex mode amplitude
2. ${}_{-2}S_{lmn}(a_f \tilde{\omega}_{lmn}, \iota, \psi)$ is spin-weighted spheroidal harmonics
3. $\tilde{\omega}_{lmn}(M_f, a_f) = \omega_{lmn} + i/\tau_{lmn}$,
   Quasi-Normal Modes frequencucies and decay time

For black holes in GR, $\tilde{\omega}_{lmn}$ depends on M_f and a_f only as a manifestation of no-hair theorem.

Current status

Previous studies

1. Einstein telescope (ET), a third-generation gravitational wave observatory, is able to test no-hair conjecture with an accuracy of a few percent by combining $\mathcal{O}(10)$ ringdown signals from black holes with masses in the range $500 - 1000 M_\odot$ at distance up to 50 Gpc.
   Meidam et al. Phys.Rev.D.90.064009 (2014)

Our work

1. The existing Advanced LIGO and Virgo interferometric detectors, operating at design sensitivity, will be capable of testing the no-hair conjecture with an accuracy of a few percent by combining $\mathcal{O}(5)$ ringdown signals from stellar-mass black holes at distance up to 1 Gpc.

Setup

Method

1. Bayesian inference for all analysis

Ringdown waveform London et al. PhysRevD.90.124032 (2014)

1. Include fundamental mode ($n = 0$) and overtones ($n > 0$)
2. Include relative phase shifts
3. Calibrated with 68 numerical relativity (NR) simulations with mass ratio ranging from 1 to 15
4. Non-spinning black hole binaries

Assessing the method

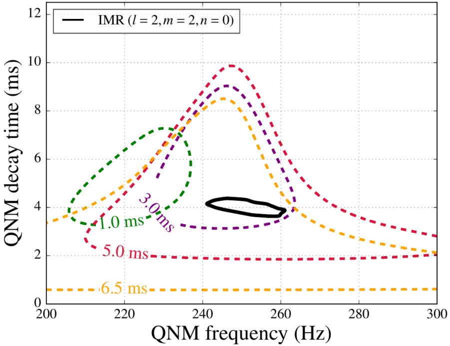
1. NR waveforms with mass ratio ranging from 1 to 3
2. Injected total mass uniformly distributed in the interval [50,90] $M_{\odot}$
3. Total signal-to-noise ratio (SNR) in the ringdown ~ 15
   GW150914 will have a SNR of 17 in the ringdown assuming the Advanced LIGO-Virgo network at design sensitivity.

When does the ringdown start?

Theoretically, it should be $10 - 20M$ after merger.

Systematic vs Statistical

- If we isolate the ringdown too early, it introduces systematic uncertainty
- If we isolate the ringdown too late, it introduces statistical uncertainty



GW150914 QNM

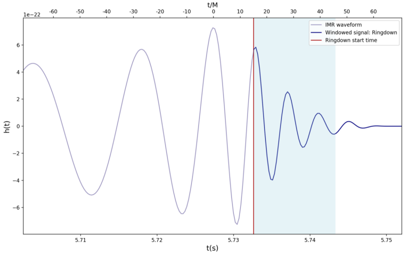
Abbott et al. Phys.Rev.Lett.116.221101 (2016)

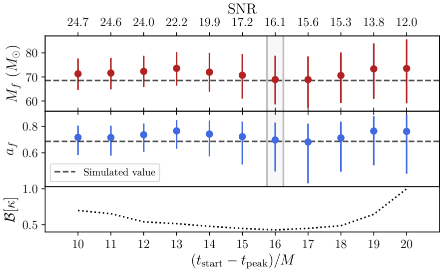
Effective ringdown start time

- We can isolate the ringdown signal by a Planck window to avoid Gibbs phenomena.
- Isolating the ringdown signal at 16M after the peak strain is found to be the best for parameter estimation by minimizing the combination of systematic and statistical uncertainties.

$$\mathcal{B}(\kappa) = \sqrt{\Delta\vec{x}(\kappa)\mathrm{C}^{-1}(\kappa)\Delta\vec{x}(\kappa) + \det\ \mathrm{C}(\kappa)}$$

$$\Delta\vec{x}(\kappa) = \left(\frac{\bar{M}_f(\kappa) - M_f}{M_\odot}, \bar{a}_f - a_f\right)$$





Test of No-hair conjecture

Introducing testing parameter

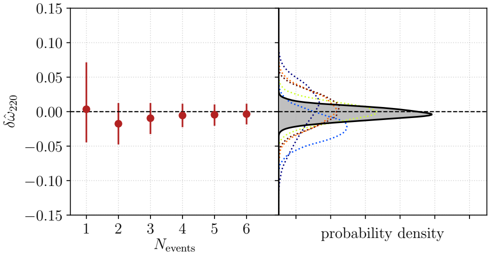
$$\omega_{lmn}(M_f, a_f) \rightarrow (1 + \delta\omega_{lmn})\ \omega_{lmn}(M_f, a_f)$$
$$\tau_{lmn}(M_f, a_f) \rightarrow (1 + \delta\tau_{lmn})\ \tau_{lmn}(M_f, a_f)$$

Sampling is done over 10 basic parameters

1. Intrinsic parameter (M_f, a_f, q)
2. Sky location (α, δ)
3. Source orientation (ι, ψ)
4. Distance (r)
5. reference time (t_c) and phase (ϕ_c)

and 1 testing parameter

1. Relative shift on QNM freq. $(\delta\omega_{220})$



Conclusion

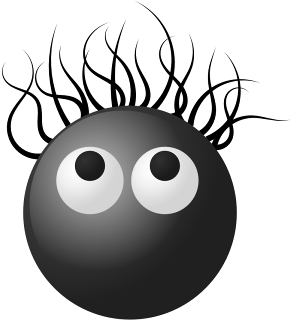
No-hair conjecture can be tested with second-generation detector network at design sensitivity by combining $\mathcal{O}(5)$ ringdown signals from stellar-mass black hole binaries.

Future improvements

- Study the effective ringdown start time with varying signal-to-noise ratio
- Extend to model with aligned-spin progenitor binary black holes
- Extend to higher mass ratio

Thank you!

The End



Backup: Recovery Prior

1. Final mass M_f, uniform in [5, 200]$M_\odot$
2. Final spin a_f, uniform in [-1, 1]
3. Mass ratio q, uniform in [1, 15]
4. Sky location α, δ, constant number density in comoving volumn
5. Source orientation (ι, ψ), uniform on the sphere
6. Distance r, uniform in [1, 1000] Mpc
7. Coalescence time t_c, uniform in $[t_{\mathrm{trigger}} - 0.1, t_{\mathrm{trigger}} + 0.1]$s