Setting confidence levels on the oscillation parameters

KM3NeT Outing

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Outlook

- How does oscillation work
- How is oscillation measured
- Parametrized model
- Fitting and constraining of parameters

Motivation

Goal:

Determine the most probable values for the oscillation parameters

Motivation:

- Fundamental properties of neutrinos
- Oscillation parameters help us determine Neutrino Mass Hierarchy, which is still an undetermined parameter in the Standard Model
- CP violation angle helps in matter/anti-matter understanding
- Helps determine sensitivity to CP-violation, Majorana in new experiments

Neutrino oscillation

- Relation between mass eigen state and flavour eigenstate:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle \quad \alpha = e, \mu, \tau$$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\,|\nu_\alpha\rangle \quad i = 1, 2, 3$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

δ is the CP-violation angle

- θ_{ij} are the neutrino mixing angles
- Left out majorana phases, which are decoupled from oscillation but can be included in the PMNS matrix

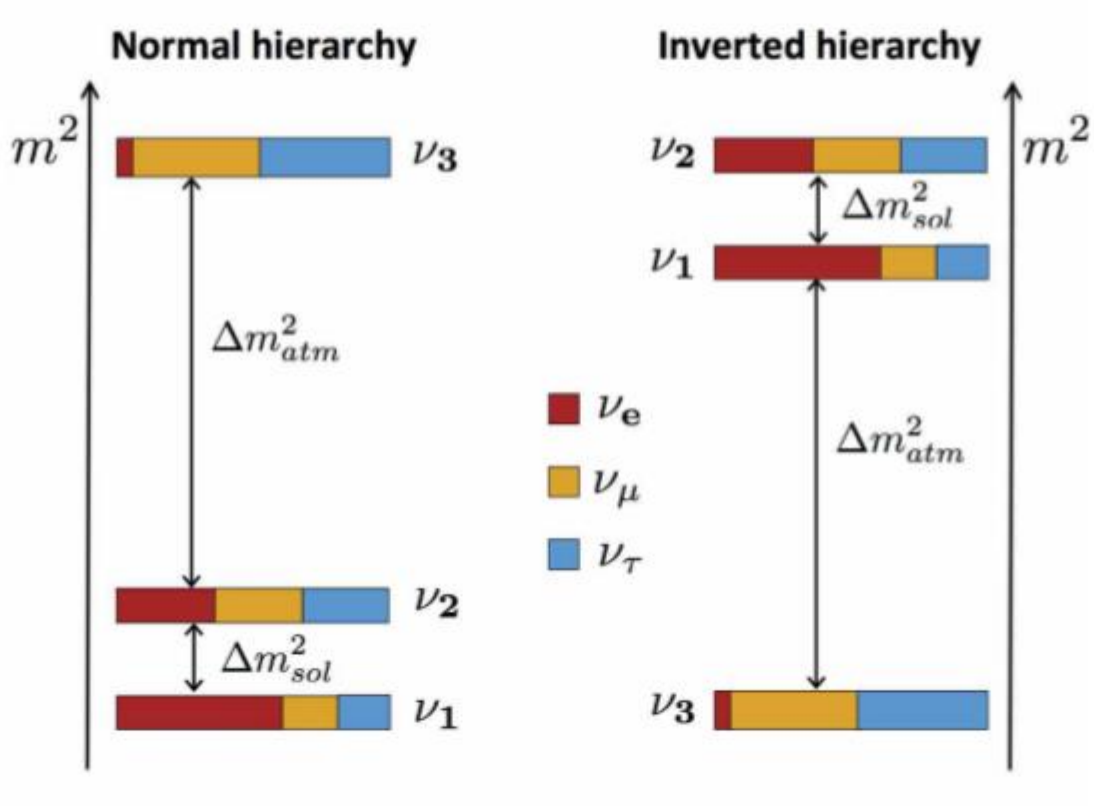
Neutrino mixing and mass hierarchy

- Mass difference:

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

- When $\Delta(m^2)_{sol} = \Delta m_{21}^2 > 0$
  there are two solutions:
  - Normal hierarchy (NH) $m_1 < m_2 < m_3$
  - Inverted hierarchy (IH) $m_3 < m_2 < m_2$

$$\Delta m_{32}^2 \approx \Delta m_{31}^2 = \Delta(m^2)_{atm}$$



Source: KM3NeT Letter of Intent

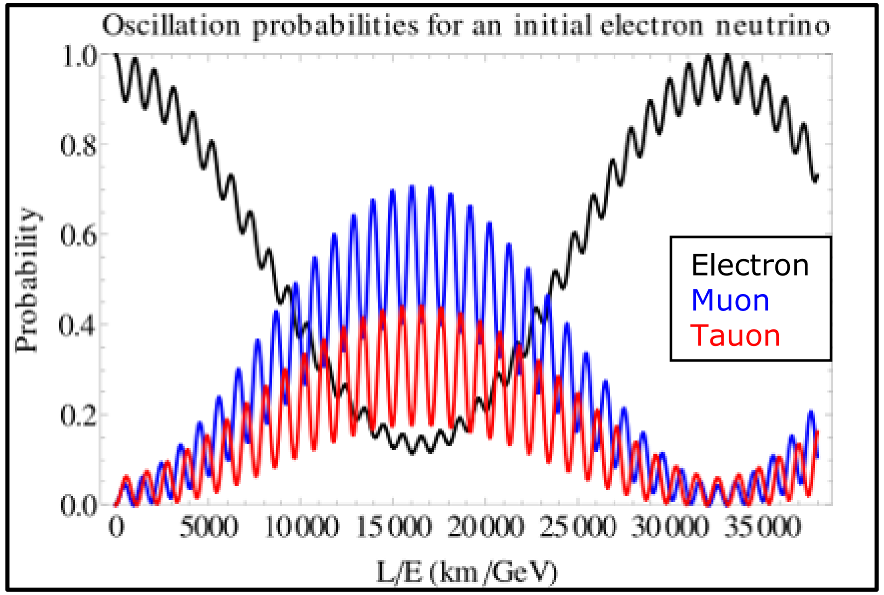
Oscillation probabilities

- Probabilities in vacuum are approx. given by

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \approx \sin^2\theta_{23}\,\sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right)$$

$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4\cos^2\theta_{13}\,\sin^2\theta_{23}\,(1-\cos^2\theta_{13}\,\sin^2\theta_{23})\,\sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right)$$

- Depends on:
  - Parameters: θ_{13}, θ_{23}, Δm^2_{31},
  - E energy of the neutrino, L length travelled



Source: wikipedia.org

Probabilities in matter

- Probabilities in matter are approx. given by

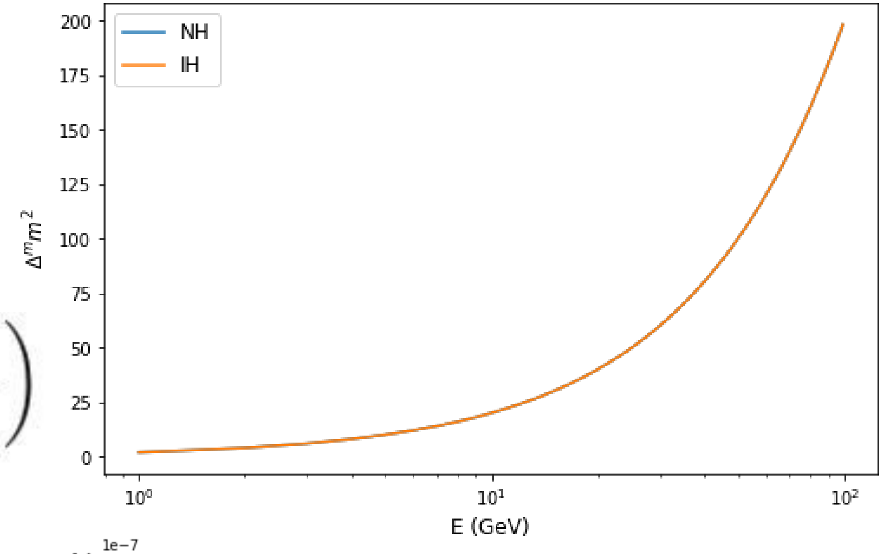
$$P^m_{3\nu}(\nu_\mu \to \nu_e) \approx \sin^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\left(\frac{\Delta^m m^2 L}{4E_\nu}\right)$$

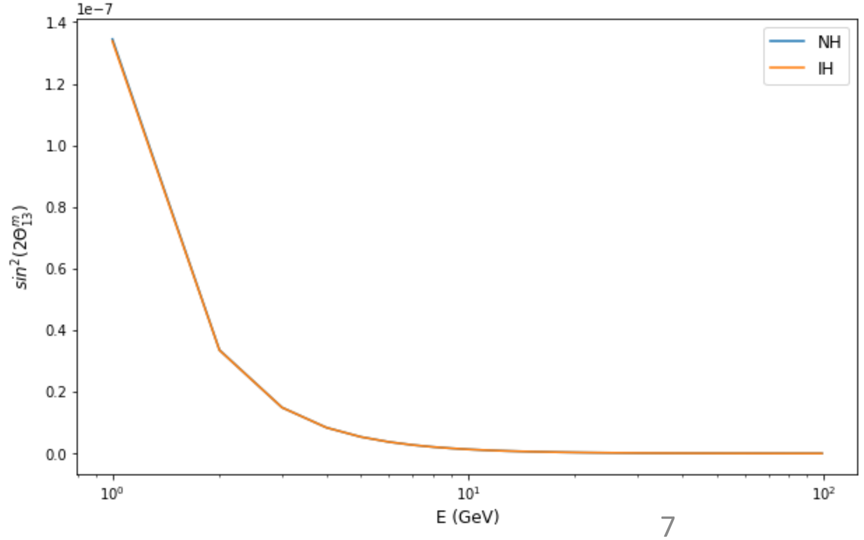
$$\begin{aligned} P^m_{3\nu}(\nu_\mu \to \nu_\mu) &\approx 1 - \sin^2 2\theta_{23}\cos^2\theta^m_{13}\sin^2\left(\frac{(\Delta m^2_{31} + \Delta^m m^2)L}{8E_\nu} + \frac{AL}{4}\right) \\ &- \sin^2 2\theta_{23}\sin^2\theta^m_{13}\sin^2\left(\frac{(\Delta m^2_{31} - \Delta^m m^2)L}{8E_\nu} + \frac{AL}{4}\right) \\ &- \sin^4\theta_{23}\sin^2 2\theta^m_{13}\sin^2\left(\frac{\Delta^m m^2 L}{4E_\nu}\right)\end{aligned}$$

$$\sin^2 2\theta^m_{13} \equiv \sin^2 2\theta_{13}\left(\frac{\Delta m^2_{31}}{\Delta^m m^2}\right)^2$$

$$\Delta^m m^2 \equiv \sqrt{(\Delta m^2_{31}\cos 2\theta_{13} - 2E_\nu A)^2 + (\Delta m^2_{31}\sin 2\theta_{13})^2},$$

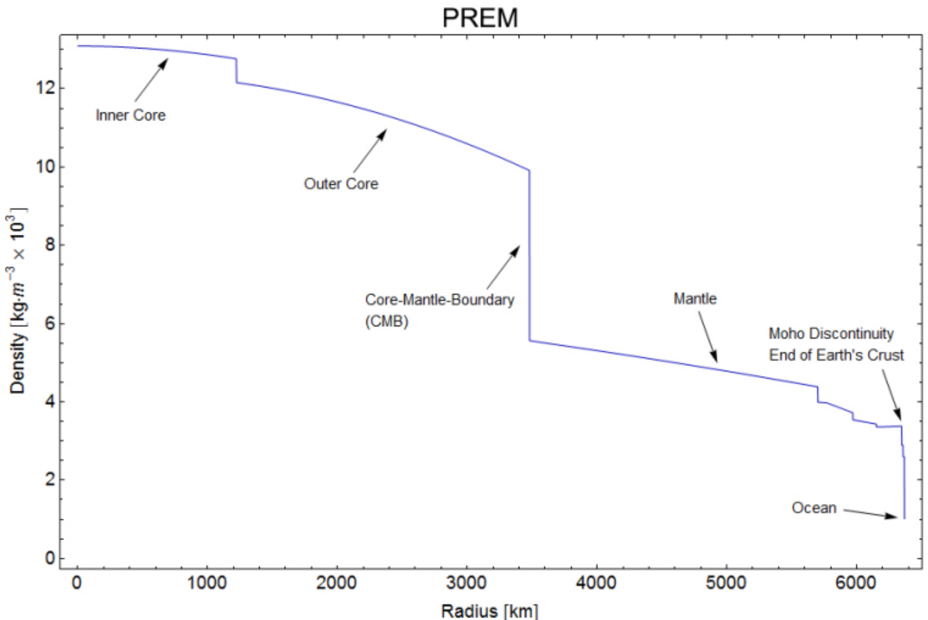
A = +1 for nu
-1 for anu

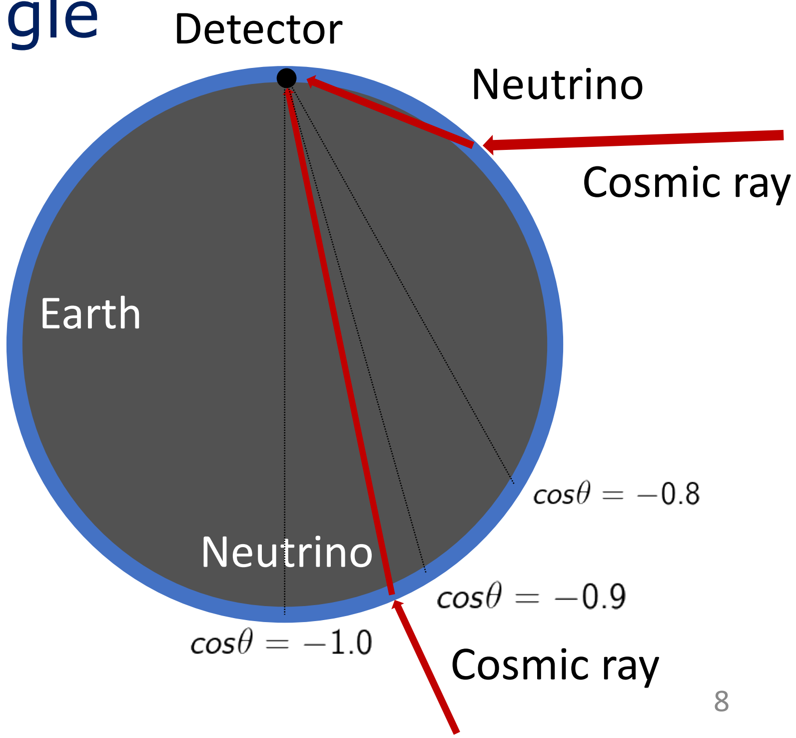




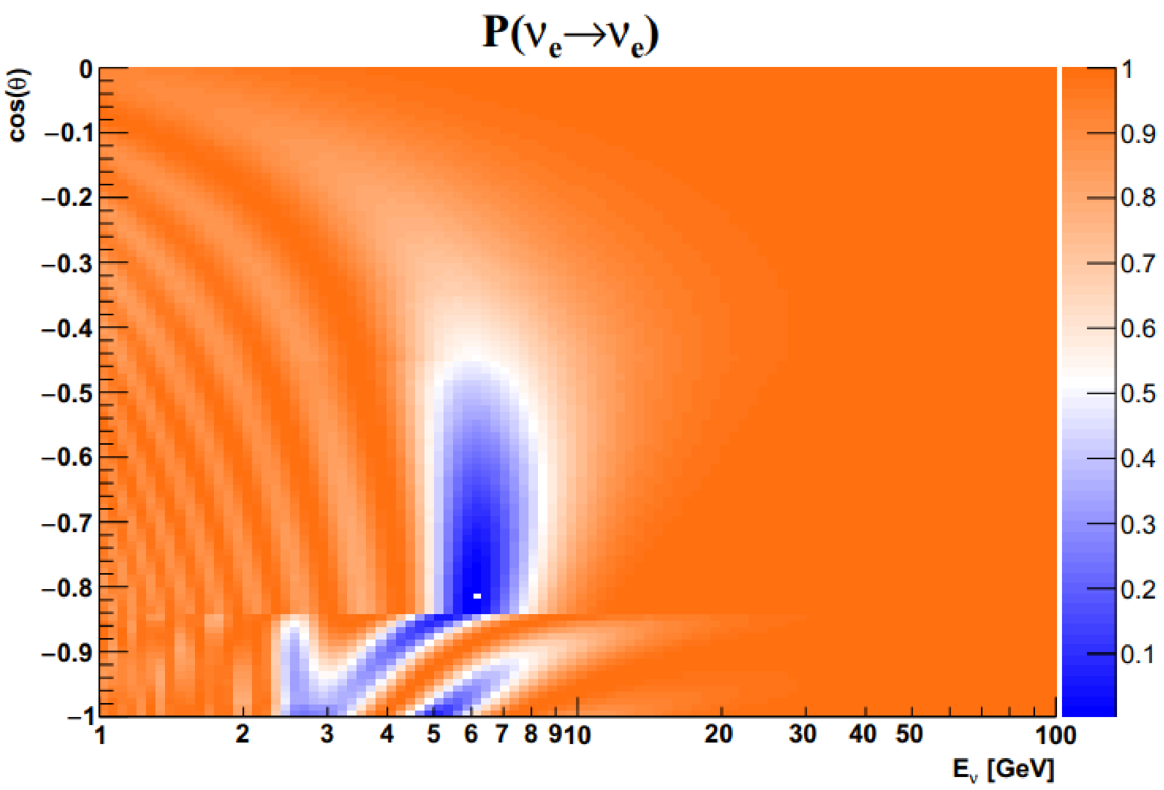
Atmospheric neutrinos

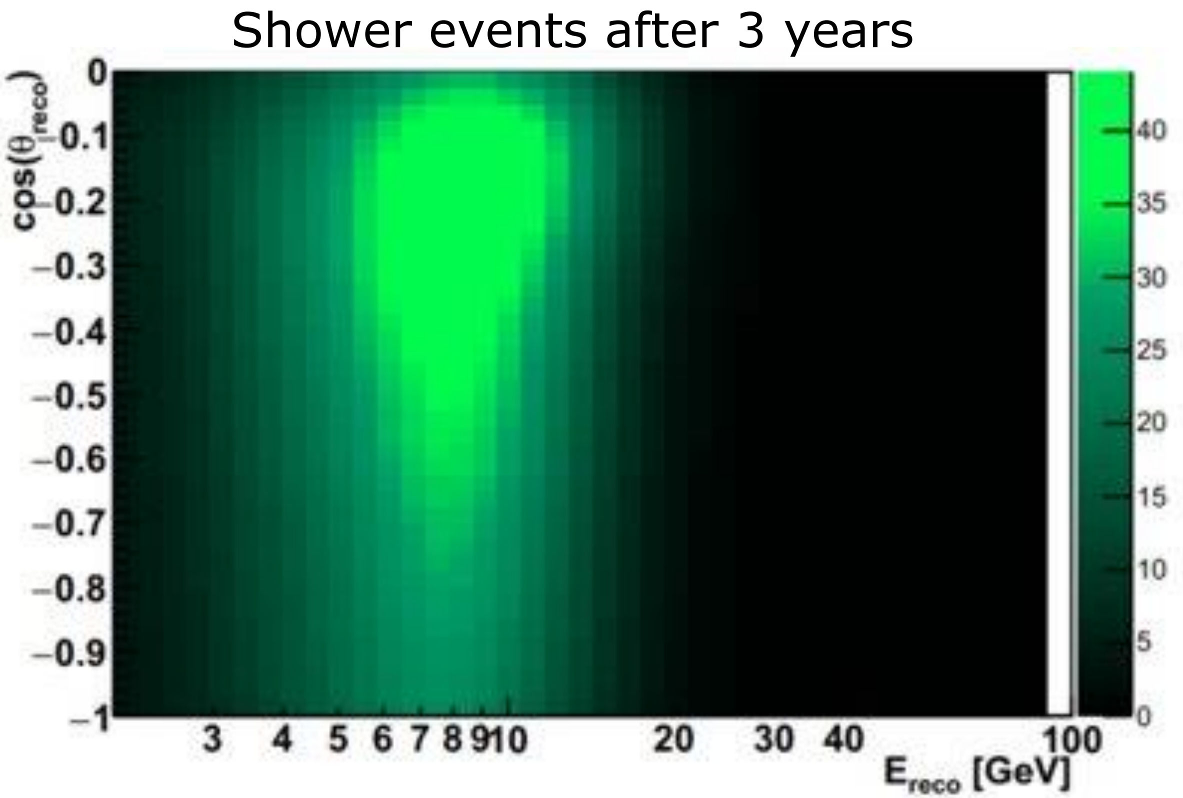
- Neutrino is created by decay process from a cosmic ray interaction in the upper atmosphere
- Symmetry in z-axis: azimuth angle does not matter
- Length travelled only depends on zenith angle
- Matter changes P_{osc}





Measurement





Parameter Values

NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$	$31.42 \to 36.05$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$	$0.418 \to 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$	$40.3 \to 51.5$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$	$0.01981 \to 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$	$8.09 \to 8.98$
$\delta_{CP}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$	$144 \to 374$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$	$\begin{bmatrix} +2.399 \to +2.593 \\ -2.536 \to -2.395 \end{bmatrix}$

Source: JHEP 01 (2017) 087, [arXiv:1611.01514], www.nu-fit.org

Parametrizing the model

$$R_a(E,\theta) = \frac{\rho_{\text{water}}}{m_{\text{nucleon}}} \times \sum_b \sigma_a(E) \times P^{\text{osc}}_{a,b}(E,\theta) \times \Phi^{\text{atm}}_b(E,\theta)$$

$R_a(E,\theta)$	Interaction rate of flavor a
$\frac{\rho_{water}}{m_{nucleon}}$	Particle density of the medium
σ_a	Cross section of flavor a
$P^{osc}_{a,b}(E,\theta)$	Oscillation probability of flavor b → a
$\Phi^{atm}_b(E,\theta)$	Flux of flavor b

Parametrizing the model

$$R_a(E,\theta) = \frac{\rho_{\text{water}}}{m_{\text{nucleon}}} \times \sum_b \sigma_a(E) \times P^{\text{osc}}_{a,b}(E,\theta) \times \Phi^{\text{atm}}_b(E,\theta)$$

Calculated / tabulated:

- Flux: Honda flux tables
- Oscillation probabilities: calculate

Parametrized:

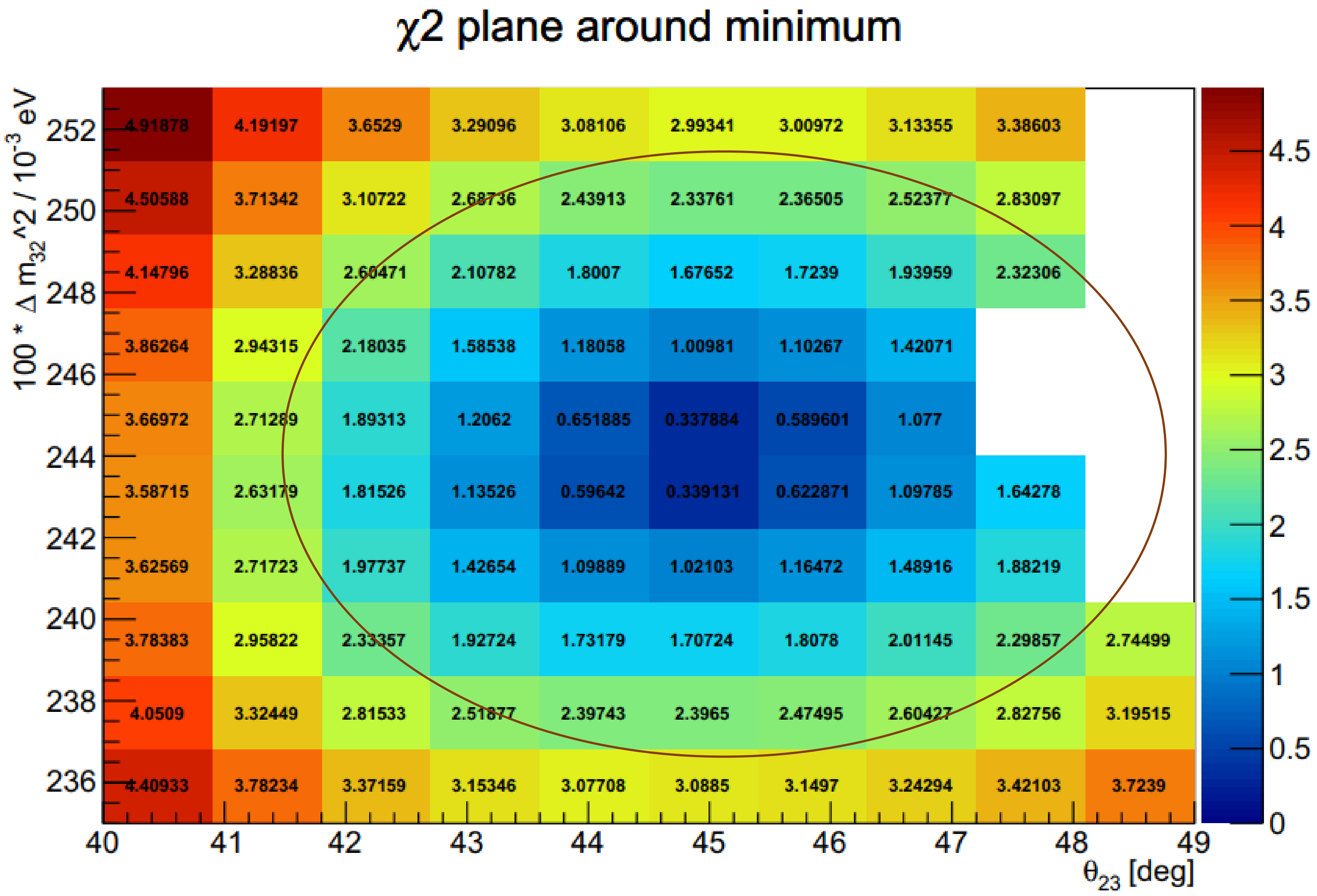
- Cross section: from MC or effective theories
- Water: effective mass calculation (approximate)
- Measurement: detector response, resolution per flavour (shower vs. track)

ParamNMH

ParamNM(H)
Todo dododo

- Quickly (seconds) generate parametrized MC data
- Quickly (seconds) generate model data
- Make plots of n-year neutrino measurement (seconds)
- Fit MC to model (hours) with
  - Determine mass hierarchy sensitivity (fit all parameters, runtime: few hours)
  - Determine parameter confidence levels (fit all but interested parameters, runtime: 1 day)

Results: χ^2-values of contour plot



Results: confidence contour

- 3 sigma levels [nuFit 3.2]:

$$\theta_{23} \in (40.3, 51.5)$$

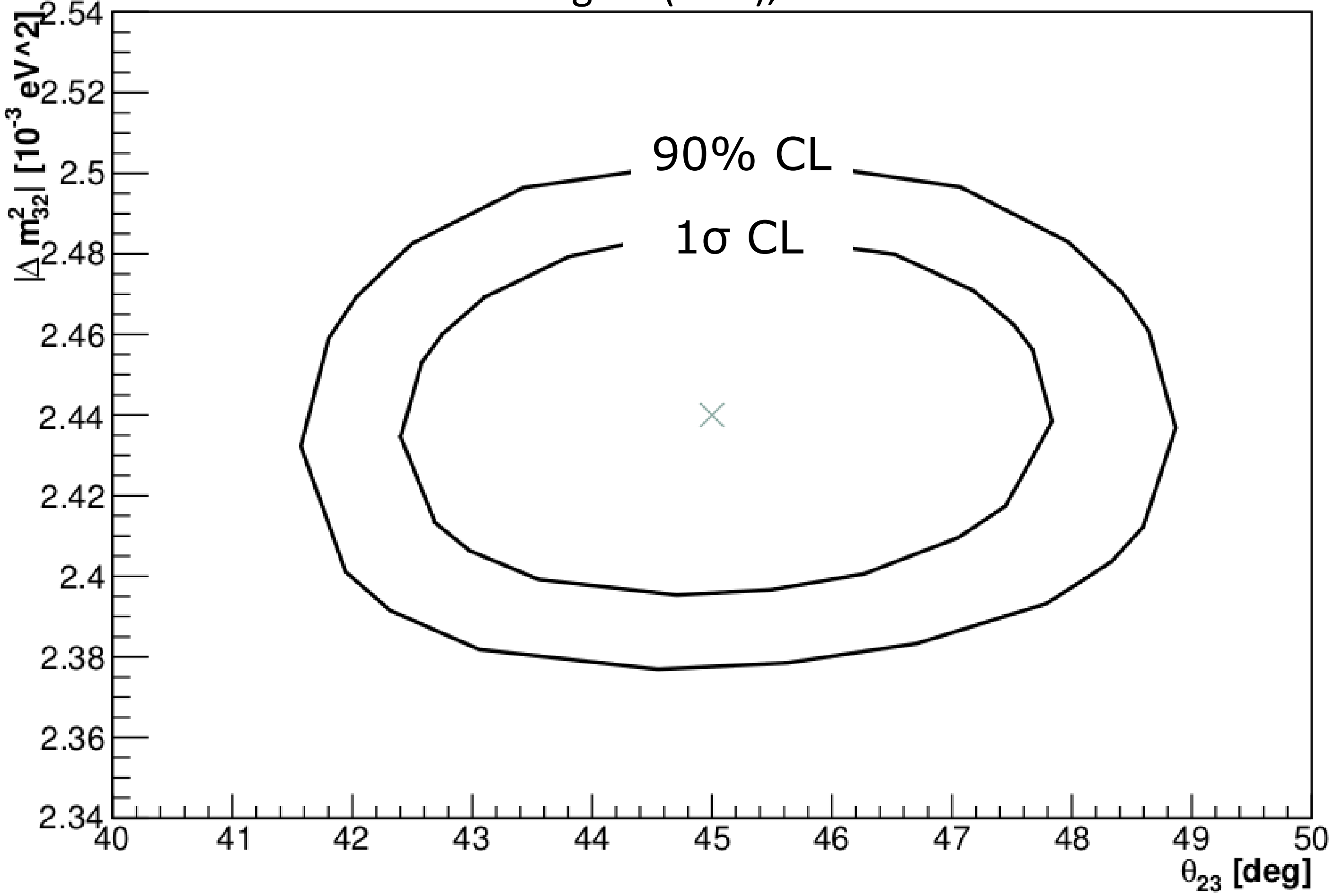
$$\frac{\Delta m^2_{32}}{10^{-3} eV^2} \in (2.399, 2.593)$$

- 3 sigma levels (back of envelope)

$$\theta_{23} \in (39.9, 50.9)$$

$$\frac{\Delta m^2_{32}}{10^{-3} eV^2} \in (2.33, 2.55)$$

Confidence levels w/ 3 years of data taking
1 sigma (68%), 90%



90% CL = 1.64σ

Conclusion

Using parametrized model (and back of envelope calculation) we can already get quite close to the best fitted data available

NuFit 3.2 3 σ limits	ParamNMH 3 σ limits (back of envelope)
$\theta_{23} \in (40.3, 51.5)$	$\theta_{23} \in (39.9, 50.9)$
$\frac{\Delta m_{32}^2}{10^{-3} eV^2} \in (2.399, 2.593)$	$\frac{\Delta m_{32}^2}{10^{-3} eV^2} \in (2.33, 2.55)$

Next steps

- ParamNMH
  - Improve fits / constraints in parametrized model
  - Constrain different parameters (if necessary)
  - Fit against full MC data
  - Maintain codebase (Liam's retiring)
- NMH (Bruno's program)
  - Fit full MC against model