



Neural networks for Arca

Master Project

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KM3NET outing

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Introduction into neural networks

A network of neurons

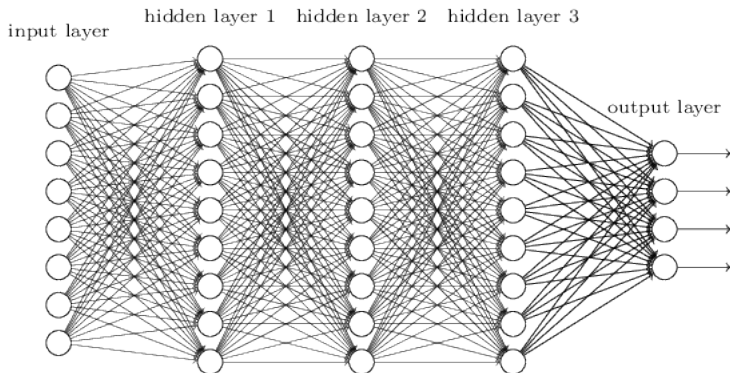


Figure 1: The common way to represent a neural network

$$f(W\vec{x} + \vec{B})$$

With

x input data vector

W weight matrix

B bias vector

f a non-linear "activation" function like \tanh or $\frac{1}{1+e^{-x}}$, that often

$f : \mathbb{R} \rightarrow (-1, 1)$

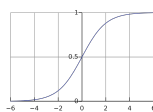
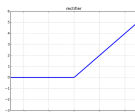


Figure 2: Relu- and the Sigmoid function

Building a "network"

$$f(W_2 f(W_1 f(W_0 x + B_0) + B_1) + B_2) = y$$

If x has n data point W_0 is a $n * m$ matrix and B_0 is a m vector.

W_1 is an $m * z$ matrix and B_1 a z vector, ect.

f can be a different function each layer.

y is the output vector

Outputs

For classification one-hot encoding is common. Some activation functions normalise the output.

$$\hat{y} = \begin{bmatrix} \nu_e \\ \nu_\mu \\ {}^{40}\text{K} \\ \vdots \\ \mu_{atmos} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad y = \begin{bmatrix} 0.11 \\ .8 \\ 0 \\ \vdots \\ .09 \end{bmatrix}$$

Where \hat{y} is the true label and y the output of the neural network. The label with the highest output is chosen.

Outputs

For regression / fitting the activation function of the last layer is left out so the output is continue. It is possible to predict one ore more values.
For example.

$$\hat{y} = \begin{bmatrix} E \\ dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} 10\text{GeV} \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad y = \begin{bmatrix} 8\text{GeV} \\ .1 \\ .1 \\ .8 \end{bmatrix}$$

Cost function

The outputs are compared with each other with a cost function. This cost should be minimised by training. Common cost functions are square error- and cross entropy function.

$$C(y, \hat{y}) = \sum_i (y_i - \hat{y}_i)^2$$

$$C(y, \hat{y}) = - \sum_i \hat{y}_i \log(y_i)$$

Update the weights proportional to the gradient of the cost function.

$$\vec{W} = \vec{W} - \alpha \nabla C(\vec{W})$$

Where ∇C are the derivatives of C with respect to all matrix elements of the weights and biases. This is calculated with the chain-rule and means that all the activation functions should have a derivative.

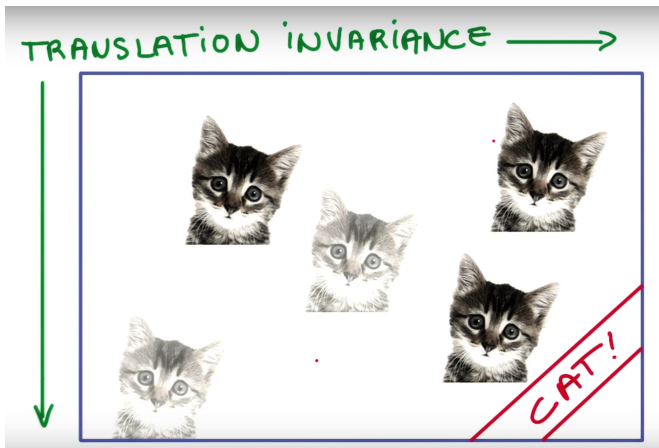


Figure 3: A Cat on different places in the picture. 1978 by 1330 pixels, total of 2.6×10^6 pixels

Convolutional neural network

Too much weights needed to be computational efficient.
Picture shifted by one pixel can give total different output.

Convolutional networks are introduced to solve this problem and to take advantage of translation symmetry.

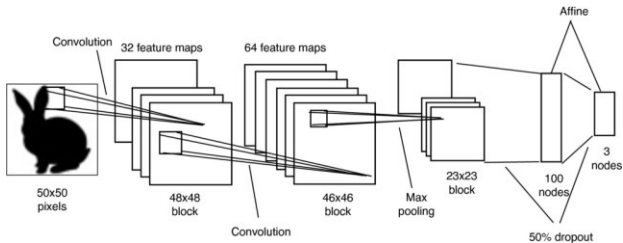


Figure 4: Convolutional neural network for classifying animals on pictures

Convolution

(Filter * Picture with rabbit) = new picture

$$\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 & \dots \\ 4 & 5 & 6 & \\ 7 & 8 & 9 & \\ \vdots & & & \ddots \end{bmatrix} \right) [1, 1]$$

=

$$(i \cdot 1) + (h \cdot 2) + (g \cdot 3) + (f \cdot 4) + (e \cdot 5) + (d \cdot 6) + (c \cdot 7) + (b \cdot 8) + (a \cdot 9) + B$$

Max pooling

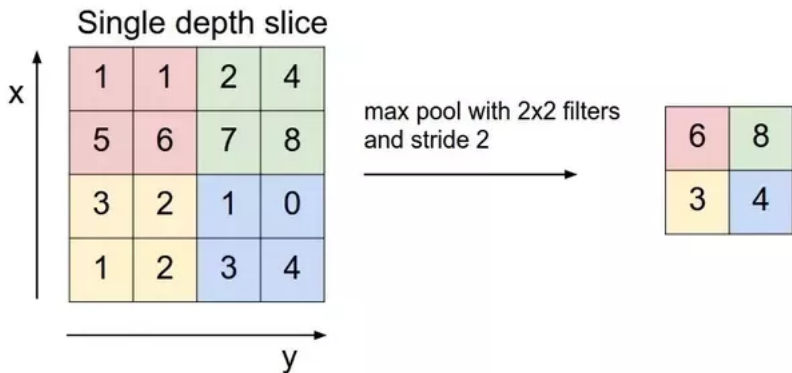


Figure 5: The max pooling of a matrix

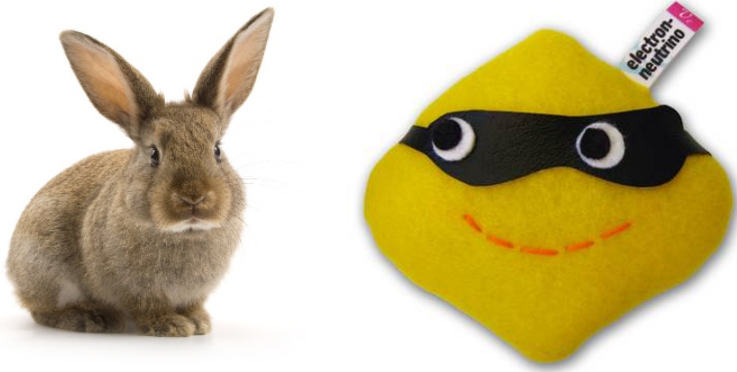


Figure 6: Rabbit and a electron neutrino

Classification between showers, tracks and background

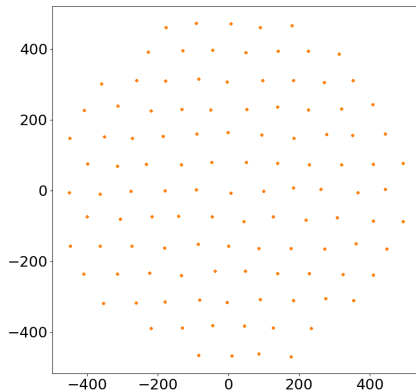


Figure 7: Top view of arca. meters

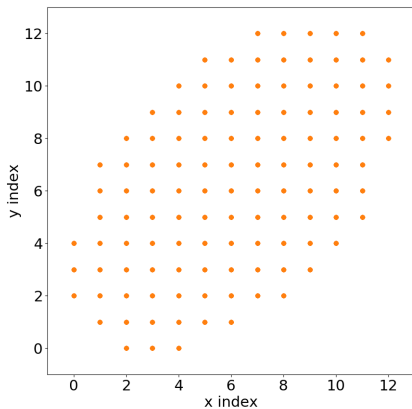


Figure 8: Matrix representation of Arca. Top view. index

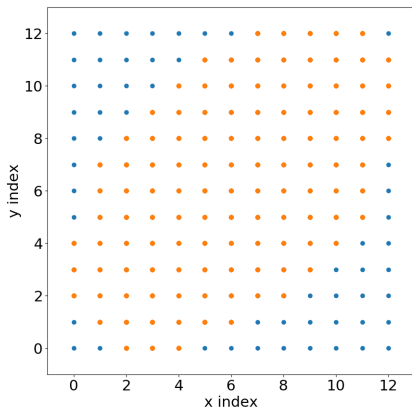


Figure 9: Matrix representation of Arca with padding. Top view. index

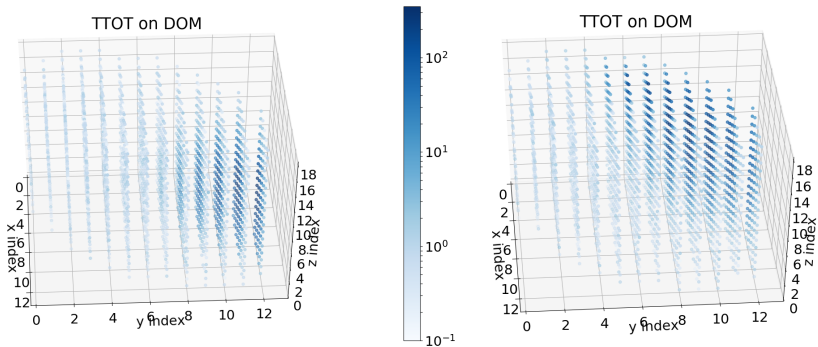


Figure 10: High energy shower and track (5×10^6 GeV). Time integrated over ~ 12000 ns

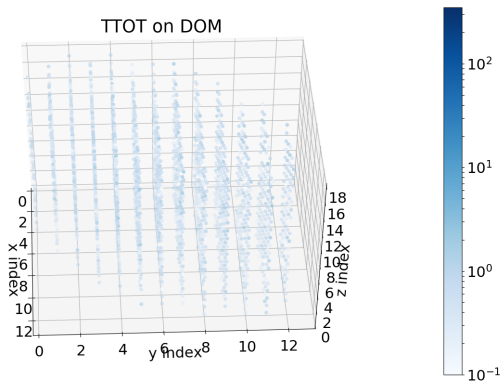


Figure 11: Only K40. Time integrated over 12000 ns

input 13, 13, 18, 1 event matrix

Layer 1

35 filters of 6, 6, 6

Activation function Relu and max pooling

output 7, 7, 9, 35

Layer 2

60 filters of 3, 3, 3

Relu and max pooling

output 4, 4, 5, 60

Layer 3

15 filters of 2, 2, 2

Relu and max pooling

output 2, 2, 3, 15

reshape to 180 vector

Layer 4

Normal neural network layer of 180 by 1028 matrix
Activation function Sigmoid

Layer 5

Normal neural network layer of 1028 by 60 matrix
Sigmoid

Layer 6

Normal neural network layer of 60 by 3 matrix
Activation function Softmax

output 3 vector "probability distribution for the classes"
Costfunction is cross entropy

Confusion matrix

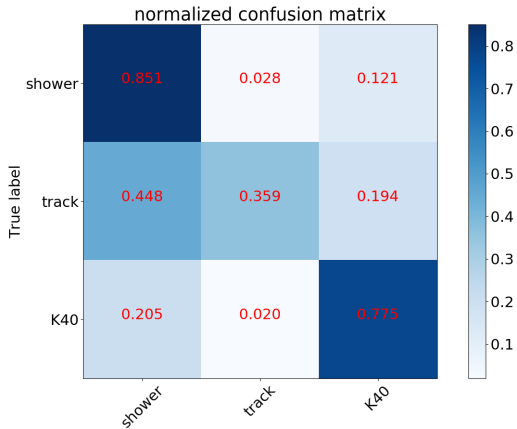


Figure 12: Confusion matrix

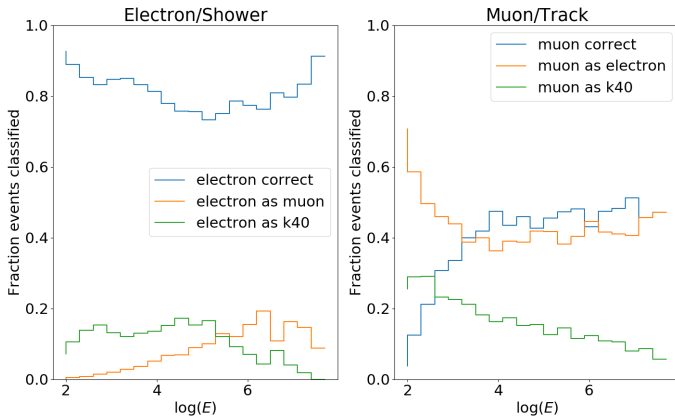


Figure 13: Energy distribution of classified events

Number of hits

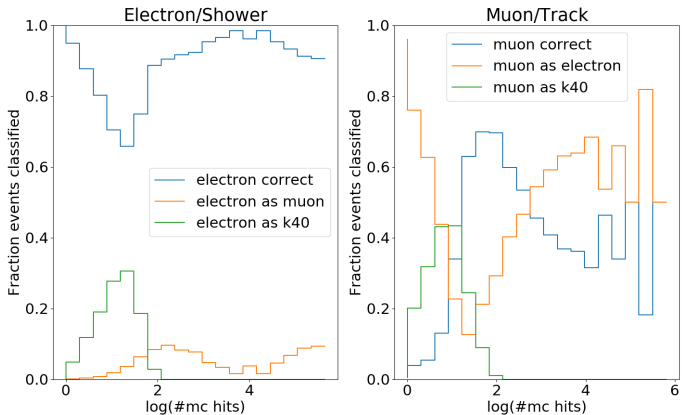


Figure 14: Number of monte carlo hits distribution of classified events

Next step: time

LSTM Network

LSTM Cell stands for Long short term memory cell and are often used for sequential data like text but can also be used for pictures. Output becomes input.

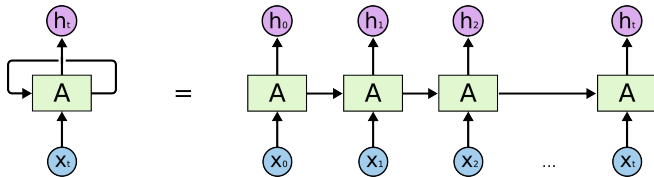


Figure 15: Simple LSTM network

Next time the mathematics, let's use them first.

Layer 7

Break events up in time slices and use each slide as input for KM3NET. Then take the outputs as inputs for a LSTM network.

Shower

Track

K40