# Flavor Physics for Non-Experts: (a Theory) Overview 

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## PLAN OF THE TALK

- General introduction to the Unitary Triangle Fit
- SM Analysis
- Tensions and unknown
- Uncertainties in lattice calculations;
- From simple to complicated;
- Future directions, new/old ideas
- Beyond the SM
- Conclusion


Thanks to
Bona, Ciuchini, Lubicz, Silvestrini,Sachrajda, Tantalo, ...

Impossible to cover all recent developments - a selected list of topics apologies for the interesting work that is not reported here

## STANDARD MODEL UNITARITY <br> TRIANGLE ANALYSIS <br> (Flavor Physics)



- Provides the best determination of the CKM parameters;
- Tests the consistency of the SM ("direct" vs "indirect" determinations) @ the quantum level;
- Provides predictions for SM observables (in the past for example sin $2 \beta$ and $\Delta m_{S}$ )
- It could lead to new discoveries (CP violation, Charm, !?)

The fundamental issue is to find signatures of new physics and to unravel the underlying theoretical structure;

Precision Flavor physics is a key tool, complementary to the large energy searches at the LHC;

If the LHC discovers new elementary particles BSM, then precision flavor physics will be necessary to constrain the underlying framework;
The discovery potential of precision flavor physics should not be underestimated.

The extraordinary progress of the experimental measurements requires accurate theoretical predictions
Precision flavour physics requires the control of hadronic effects for which lattice QCD simulations are essential.

## $Q^{E X P}=V_{C K M}\langle F| \hat{O}|I\rangle$

$$
Q^{E X P}=\sum_{i} C_{S M}^{i}\left(M_{W}, m_{t}, \alpha_{s}\right)\langle F| \hat{O}_{i}|I\rangle+\sum_{i^{\prime}} C_{B e y o n d}^{i}\left(\tilde{m}_{\beta}^{\prime}, \alpha_{s}\right)\langle F| \hat{O}_{i^{i}|I\rangle}
$$

## Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and $\varnothing$ violation originate, is determined by the coupling of the Higgs boson to fermions.

$$
\mathcal{L}^{\text {quarks }}=\mathcal{L}^{\text {kinetic }}+\mathcal{L}^{\text {gauge }}+\mathcal{L}^{\text {Yukawa }}
$$



CP and symmetry breaking are striclty correlated

$$
\mathcal{L}\left(\Lambda_{\text {Fermi }}\right)=\mathcal{L}\left(\Lambda, H, H^{\dagger}\right)+\mathcal{L}^{\text {kin }}+\mathcal{L}_{S M}^{\text {gauge }}+\mathcal{L}_{S M}^{Y \text { ukawa }}+\frac{\mathcal{L}_{5}}{\Lambda}+\frac{\mathcal{L}_{6}}{\Lambda^{2}}+\ldots
$$

EWSB has many accidental simmetries

Absence of FCNC at tree level (\& GIM suppression of FCNC @loop level)

Almost no CP violation at tree level
Flavour Physics is extremely sensitive to New Physics (NP)

In competition with Electroweak Precision Measurements

## WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

## Proton decay

baryon and lepton number conservation
$\mu$-> e $+\gamma$
lepton flavor number
$\nu_{i} \quad->\quad v_{k}$ found!


$$
\mathcal{B}(\mu \rightarrow e \gamma) \sim \alpha \frac{m_{\nu}^{4}}{m_{W}^{4}} \sim 10^{-52}
$$

## RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

## FCNC:

$$
\mathrm{q}_{\mathrm{i}} \rightarrow \mathrm{q}_{\mathrm{k}}+v \bar{v}
$$ these decays occur only via loops because of GIM and

$$
\mathrm{q}_{\mathrm{i}} \rightarrow \mathrm{q}_{\mathrm{k}}+\mathrm{l}^{+} \mathrm{l}^{-}
$$ are suppressed by CKM

## THUS THEY ARE SENSITIVE TO NEW PHYSICS

## CP Violation in the Standard Model

In the Standard Model the quark mass matrix, from which the CKM Matrix and EP originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs

$$
\mathcal{L}^{\text {quarks }}=\mathcal{L}^{\text {kinetic }}+\mathcal{L}^{\text {weak int }}+\mathcal{L}^{\text {yukawa }}
$$

## CP invariant

CP and symmetry breaking are closely related !

QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY

## BREAKING

$$
H=\binom{\phi^{+}}{\phi^{0}}, \quad H^{C}=i \tau_{2} H^{*}
$$

$$
\begin{aligned}
& \phi^{+} \rightarrow 0 \quad \phi^{0} \rightarrow \frac{V}{\sqrt{2}} \text { Charge +2/3 } e \\
& \mathcal{S}^{\text {yukawa }} \equiv \sum_{\mathrm{i}, \mathbf{k}=1, \mathbf{N}}\left[\mathrm{Y}_{\mathrm{i}, \mathbf{k}}\left(\mathrm{q}_{\mathrm{L}}^{\mathbf{i}} \mathrm{H}^{\mathrm{C}}\right) \mathbf{U}_{\mathbf{R}}^{\mathrm{k}}\right.
\end{aligned}
$$

$$
\left.+X_{i, k}\left(q_{L}^{i} H\right) D_{R}^{k}+\text { h.c. }\right]
$$

Charge -1/3

$$
\begin{aligned}
& \sum_{i, k=1, N}\left[\mathrm{~m}^{\mathrm{u}}{ }_{i, k}\left(\bar{u}_{\mathrm{L}}^{\mathrm{i}} \mathrm{u}^{\mathrm{k}}{ }_{\mathrm{R}}\right)\right. \\
& \quad+\mathrm{m}_{\mathrm{i}, \mathrm{k}}^{\mathrm{d}} \overline{\left.\left(\mathrm{~d}_{\mathrm{L}}^{\mathrm{i}} \mathrm{~d}_{\mathrm{k}}^{\mathrm{k}}\right)+\mathrm{h} . \mathrm{c} .\right]}
\end{aligned}
$$

## Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$$
\begin{aligned}
& \mathrm{u}_{\mathrm{L}}^{\mathrm{i}} \rightarrow \mathbf{U}_{\mathrm{L}}^{\mathrm{ik}} \mathrm{u}_{\mathrm{L}}^{\mathrm{k}} \\
& \mathrm{u}_{\mathrm{R}}^{\mathrm{i}} \rightarrow \mathrm{U}^{\mathrm{ik}}{ }_{\mathrm{R}} \mathrm{u}^{\mathrm{k}}{ }_{\mathrm{R}} \\
& \mathbf{M}^{\prime}=\mathbf{U}^{\dagger}{ }_{\mathrm{L}} \mathbf{M} \mathbf{U}_{\mathrm{R}} \\
& \left(\mathbf{M}^{\prime}\right)^{\dagger}=\mathrm{U}^{\dagger}{ }_{\mathrm{R}}(\mathbf{M})^{\dagger} \mathrm{U}_{\mathrm{L}} \\
& \mathcal{L}^{\text {mass }} \equiv \mathrm{m}_{\mathrm{up}}\left(\overline{\mathrm{u}}_{\mathrm{L}} \mathrm{u}_{\mathrm{R}}+\overline{\mathrm{u}}_{\mathrm{R}} \mathrm{u}_{\mathrm{L}}\right)+\mathrm{m}_{\mathrm{ch}}\left(\overline{\mathrm{c}}_{\mathrm{L}} \mathrm{c}_{\mathrm{R}}+\overline{\mathrm{c}}_{\mathrm{R}} \mathrm{c}_{\mathrm{L}}\right) \\
& +\mathrm{m}_{\text {top }}\left(\overline{\mathrm{t}}_{\mathrm{L}} \mathrm{t}_{\mathrm{R}}+\overline{\mathrm{t}}_{\mathrm{R}} \mathrm{t}_{\mathrm{L}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& L_{C C}^{\text {weakint }}=\frac{g_{W}}{\sqrt{2}}\left(J_{\mu}^{-} W_{\mu}^{+}+\text {h.c. }\right) \\
& \quad \rightarrow \frac{g_{W}}{\sqrt{2}}\left(\bar{u}_{L} \mathbf{V}^{C K M} \gamma_{\mu} d_{L} W_{\mu}^{+}+\ldots\right)
\end{aligned}
$$

$\mathrm{N}(\mathrm{N}-1) / 2$
angles
and
( $\mathrm{N}-1$ )( $\mathrm{N}-2$ )/2 phases
$\mathrm{N}=3 \quad 3$ angles +1 phase $K M$ the phase generates complex couplings i.e. $C P$ violation;
6 masses +3 angles +1 phase $=10$ parameters


$$
=\left[\begin{array}{ccc}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta_{13}} \\
-s_{12} c_{23}-c_{12} s_{23} s_{13} e^{i \delta_{13}} & c_{12} c_{23}-s_{12} s_{23} s_{13} e^{i \delta_{13}} & s_{23} c_{13} \\
s_{12} s_{23}-c_{12} c_{23} s_{13} e^{i \delta_{13}} & -c_{12} s_{23}-s_{12} c_{23} s_{13} e^{i \delta_{13}} & c_{23} c_{13}
\end{array}\right]
$$

> NO Flavour Changing Neutral Currents (FCNC) at Tree Level
> (FCNC processes are good candidates for observing NEW PHYSICS)

## CP Violation is natural with three quark generations (Kobayashi-Maskawa)

With three generations all CP phenomena are related to the same unique parameter ( $\delta$ )


Quark masses \& Generation Mixing


## Textures

There is a clear correlation between mixings and masses

```
m
```

$m_{d} \sim 8 \mathrm{MeV} \quad m_{s} \sim 110 \mathrm{MeV} \quad m_{b} \sim 4.3 \mathrm{GeV}$
Orízontal $\mathcal{U}(2) \quad: \quad \psi_{\mathrm{L}} \quad \psi_{\mathrm{L}}{ }^{\mathrm{c}}$
$\mathcal{S}_{\text {aiggs }}=$ Y H $\left[\left(\psi_{\mathrm{L}}{ }^{\mathrm{a}}\right)\left(\psi_{\mathrm{L}}{ }^{\mathrm{b}}\right)^{\mathrm{c}} \mathrm{S}^{\mathrm{ab}}+\left(\psi_{\mathrm{L}}{ }^{\mathrm{a}}\right)\left(\psi_{\mathrm{L}}{ }^{\mathrm{b}}\right)^{\mathrm{c}} \mathrm{A}^{\mathrm{ab}}\right]$

Symmetric tensor

Antisymmetric tensor

$$
\begin{aligned}
& M^{d}=M\left(\begin{array}{cc}
0 & -\sqrt{x} \\
\sqrt{x} & 1+x
\end{array}\right) \quad \begin{array}{c}
\sin \theta_{\mathrm{c}} \sim V_{m_{d}} / m_{s} \\
\text { R.Gatto } 70
\end{array} \\
& \operatorname{diag}(M)=M(x, 1) \quad x=m_{d} / m_{s} \\
& V_{1}=\binom{1}{\sqrt{x}} \quad \lambda_{1}=M x \begin{array}{l}
\text { Masses \& }
\end{array} \\
& V_{2}=\left(\begin{array}{c}
\text { Mixings } \\
\text { (including the } \\
\text { CP phases ) } \\
\text { are related !! } \\
1
\end{array}\right) \quad \lambda_{2}=M \quad
\end{aligned}
$$

## The Wolfenstein Parametrization

| $1-1 / 2 \lambda^{2}$ | $\lambda$ | $\mathrm{~A} \lambda^{3}(\rho-\mathrm{i} \eta)$ |
| :---: | :---: | :---: | $\mathbf{V}_{\mathbf{u b}}$

$\mathbf{V}_{\mathrm{td}}$
$\lambda \sim 0.2 \quad$ A $\sim 0.8$
$\sin \theta_{12}=\lambda$
$\operatorname{Sin} \theta_{23}=A \lambda^{2}$
$\sin \theta_{13}=A \lambda^{3}(\rho-i \eta)$
$\eta \sim 0.2 \quad \rho \sim 0.3$

## The Bjorken-Jarlskog Unitarity Triangle


$\left|\mathrm{V}_{\mathrm{ij}}\right|$ is invariant under phase rotations

$$
\begin{aligned}
& a_{1}=V_{11} \mathbf{V}_{12}{ }^{*}=V_{u d} V_{u s}^{*} \\
& a_{2}=V_{21} \mathbf{V}_{22}{ }^{*} a_{3}=V_{31} V_{32}^{*}
\end{aligned}
$$

$$
\begin{aligned}
& a_{1}+a_{2}+a_{3}=0 \\
& \left(b_{1}+b_{2}+b_{3}=0 \text { etc. }\right)
\end{aligned}
$$

Only the orientation depends $a_{3}$ on the phase convention


## STRONG CP VIOLATION

$$
\begin{aligned}
& \mathcal{L}_{\theta}=\theta \tilde{\mathrm{G}}^{\mu v a} \mathrm{G}_{\mu \nu}^{\mathrm{a}} \\
& \mathcal{L}_{\theta} \sim \theta \overrightarrow{\mathrm{E}^{\mathrm{a}}} \cdot \overrightarrow{\overrightarrow{\mathrm{~B}}^{\mathrm{a}}}
\end{aligned}
$$

$$
\tilde{G}^{\mathrm{a}}{ }_{\mu \nu}=\varepsilon_{\mu \nu \rho \sigma} \mathrm{G}^{\mathrm{a}}{ }_{\rho \sigma}
$$

This term violates $C P$ and gives a contribution to the electric dipole moment of the neutron

$$
e_{n}<310^{-26} \mathrm{e} \mathrm{~cm}
$$



## Neutron electric dipole moment in SuperSymmetry


$\mathcal{L}^{\Delta \mathrm{F}=0}=-\mathrm{i} / 2 \mathrm{C}_{\mathrm{e}} \psi \sigma_{\mu \nu} \gamma_{5} \psi \mathrm{~F}^{\mu \nu}$
$-\mathrm{i} / 2 \mathrm{C}_{\mathrm{C}} \psi \sigma_{\mu \nu} \gamma_{5} \mathrm{t}^{\mathrm{a}} \psi \mathrm{G}^{\mu v a}$
$-1 / 6 \mathrm{C}_{\mathrm{g}} \mathrm{f}_{\mathrm{abc}} \mathrm{G}_{\mu \rho}^{\mathrm{a}} \mathrm{G}^{\mathrm{b} \mathrm{\rho}}{ }_{\nu} \mathrm{G}_{\lambda \sigma}^{\mathrm{c}} \varepsilon^{\mu \nu \lambda \sigma}$


## www.utfit.org


C. Alpigiani, A. Bevan, M.B., M. Ciuchini, D. Derkach, E. Franco, V. Lubicz, G. Martinelli, F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini, A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni

Other UT analyses exist, by:
CKMfitter (http://ckmfitter.in2p3.fr),
Laiho\&Lunghi\&Van de Water (http://latticeaverages.org/)
Lunghi\&Soni (1010.6069)

$$
\begin{array}{ccc}
\text { Measure } & V_{C K M} & \text { Other NP parameters } \\
\Gamma(b \rightarrow u) / \Gamma(b \rightarrow c) & \bar{\rho}^{2}+\bar{\eta}^{2} & \bar{\Lambda}, \lambda_{1}, F(1), \ldots \\
\varepsilon_{K} & \eta[(1-\bar{\rho})+\ldots] & B_{K} \\
\Delta m_{d} & (1-\bar{\rho})^{2}+\bar{\eta}^{2} & f_{B_{d}}^{2} B_{B_{d}} \\
\Delta m_{d} / \Delta m_{1} & (1-\bar{\rho})^{2}+\bar{\eta}^{2} & \xi \\
A_{C P}\left(B_{d} \rightarrow J / \psi K_{s}\right) & \sin 2 \beta & -
\end{array}
$$

$$
Q^{E X P}=V_{C K M} \times\left\langle H_{F}\right| \hat{O}\left|H_{I}\right\rangle
$$

## For details see: <br> UTfit Collaboration

http://www.utfit.org


## DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible theor. uncertainties

$$
\begin{gathered}
A_{C P}\left(B \rightarrow J / \psi K_{s}\right) \underset{K^{0} \rightarrow \pi^{0} v \bar{v}}{\gamma \operatorname{from}} B \rightarrow D K \\
K^{2}
\end{gathered}
$$

2) Second class quantities, with theoretical errors of $\mathrm{O}(10 \%)$ or less that can be reliably estimated

$$
\begin{aligned}
\varepsilon_{K} & \Delta M_{d s} \\
\Gamma(B \rightarrow c, u), & K^{+} \rightarrow \pi^{+} v \bar{v}
\end{aligned}
$$

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)
In case of discrepacies we cannot tell whether is new physics or we must blame the model

$$
\begin{array}{ll}
B \rightarrow K \pi & B \rightarrow \pi^{0} \pi^{0} \\
B \rightarrow \phi K_{s} &
\end{array}
$$

## Quantities used in the Standard UT Analysis


levels @ 68\% (95\%) CL
$\Delta \mathbf{m}_{\mathrm{d}} / \Delta \mathbf{m}_{\mathrm{s}}$


Inclusive vs Exclusive Opportunity for lattice QCD

## Other Quantities used in the UT Analysis

## UT-ANGLES



Several new determinations of UT angles are now available, thanks to the results coming from the B-Factory experiments


New Constraints from B and K rare decays (not used yet)

New bounds are available from rare B and K decays. They do not still have a strong impact on the global fit and they are not used at present.




Winter 2018 results

$$
\bar{\rho}=0.145 \pm 0.014 \bar{\eta}=0.349 \pm 0.010
$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation


$$
\begin{gathered}
\alpha=(88.2 \pm 0.1)^{0} \\
\sin 2 \beta=0.699 \pm 0.068 \\
\beta=(22.3 \pm 0.66)^{0} \\
\gamma=(65.8 \pm 2.2)^{0} \\
\boldsymbol{A}=\mathbf{0 . 8 2 5} \pm \mathbf{0 . 0 1 2} \\
\boldsymbol{\lambda}=\mathbf{0 . 2 2 5 0 2} \pm \mathbf{0 . 0 0 0 5 3}
\end{gathered}
$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

## CKM Matrix in the SM Winter 2018

$V_{C K M}=\left(\begin{array}{ccc}(0.97431 \pm 0.00012) & (0.22517 \pm 0.00052) & (0.00365 \pm 0.00010) e^{i(-67.222 .1)^{0}} \\ (-0.22504 \pm 0.000555) e^{i(0.0351 \pm 0.0010)^{0}} & (0.97344 \pm 0.00012) e^{i(-.001880 \pm 0.000552)^{\circ}} & (-) \\ (0.00871 \pm 0.00014) e^{i(-222330.63)^{\circ}} & (-0.04111 \pm 0.00056) e^{i(1.059 \pm 0.032)^{\circ}} & (0.999115 \pm 0.000024)\end{array}\right)$

Standard Parametrization (PDG)
$\operatorname{Sin} \theta_{12}=0.22502 \pm 0.00053$
$\operatorname{Sin} \theta_{23}=0.04190 \pm 0.00058$
$\operatorname{Sin} \theta_{13}=0.003669 \pm 0.000096 \quad \delta=67.3 \pm 2.2$
Wolfenstein Parametrization (PDG)

$$
\lambda=0.22502 \pm 0.00053 \quad \mathrm{~A}=0.825 \pm 0.0 .12
$$

## PROGRESS SINCE 1988

Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)


## Experimental progress so impressive

- the given constraint from the fit

| Observables | Measurement | Prediction | Pull (\#б) |
| :---: | :---: | :---: | :---: |
| $B_{\mathrm{K}}$ | $0.740 \pm 0.029$ | $0.81 \pm 0.07$ | $<1$ |
| $\mathrm{f}_{\mathrm{Bs}}$ | $0.226 \pm 0.005$ | $0.220 \pm 0.007$ | 0.843 |
| $\mathrm{f}_{\mathrm{Bs}} / \mathrm{f}_{\mathrm{Bd}}$ | $1.203 \pm 0.013$ | $1.210 \pm 0.030$ | $<$ Pull $=+1.2$ |
| $\mathrm{~B}_{\mathrm{Bs}} / \mathrm{B}_{\mathrm{Bd}}$ | $1.032 \pm 0.036$ | $1.07 \pm 0.05$ | $<1$ |
| $\mathrm{~B}_{\mathrm{Bs}}$ | $1.35 \pm 0.08$ | $1.30 \pm 0.07$ | $<1$ |

in general: average the $\mathrm{Nf}=2+1+1$ and $\mathrm{Nf}=2+1$ FLAG averages, through eq.(28) in arXiv:1403.4504
for $\mathrm{Bk}, \mathrm{fBs}, \mathrm{fBs} / \mathrm{fBd}$ :
FLAG Nf=2+1+1 (single result) and $\mathrm{Nf}=2+1$ average
for $\mathrm{B}_{\mathrm{bs}}, \mathrm{B}_{\mathrm{bs}} / \mathrm{B}_{\mathrm{bd}}$ :
update w.r.t. the $\mathrm{Nf}=2+1$ FLAG average (no Nf=2+1+1 results yet) updating the FNAL/MILC result to FNAL/MILC 2016 (1602.03560)

## Do we still care? Tensions and Unknowns

1) A "classical" example $B->\tau v$
2) $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$ inclusive vs exclusive
3) $I V_{c b} I$, $B$ mixing and $\varepsilon_{K}$
4) D-mixing
5) $\mathrm{R}(\mathrm{D})$ and $\mathrm{R}\left(\mathrm{D}^{*}\right)$
6) $\mathrm{B}->\mathrm{K}^{*} 11$
7) Physics BSM ?

- What can be computed and what cannot be computed



## The Simplest Example


$B R\left(B^{-} \rightarrow \tau^{-} \bar{v}_{\tau}\right)=f_{B}^{2}\left|V_{u b}\right|^{\frac{G_{F}^{2}}{2} m_{B} m_{\tau}^{2}} \frac{8 \pi}{\left(1-\frac{m_{\tau}^{2}}{m_{B}^{2}}\right)^{2} \tau_{B}, ~}$

$$
f_{B}^{2}\left|V_{u b}\right|^{2}
$$

$$
\langle 0| \bar{b} \gamma_{\mu} \gamma_{5} d\left|B^{0}(p)\right\rangle=i f_{B} p_{\mu}
$$

COULD WE COMPUTE THIS PROCESS WITH SUFFICIENT COMPUTER POWER?


IT IS NOT ONLY A QUESTION OF COMPUTER POWER BECAUSE THERE ARE COMPLICATED FIELD THEORETICAL PROBLEMS

Euclide vs Minkowski


Leptonic ( $\pi, K, D, B$ )


Semileptonic $(K, D, B)^{+}$,

(some) Radiative and Rare long distance effects
(also $K->\pi l^{+} l^{-}$)


Non-leptonic $B->\pi \tau, K \pi$, etc. No!
but only below the inelastic threshold (may be also 3 body decays)


Neutral meson mixing (local)


+ some long distance contributions to $K$ and $D$ neutral meson mixing + short distance contributions to $B->K^{(*)} l^{+} l^{-}$

Radiative corrections to weak amplitudes important for hadron masses, leptonic and semileptonic decays, $I \mathrm{~V}_{\mathrm{us}} \mathrm{I}$, but also for D and B decays


FIG. 5: Connected diagrams contributing at $O(\alpha)$ contribution to the amplitude for the decay $\pi^{+} \rightarrow \ell^{+} \nu_{l}$.

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking and em effects cannot be neglected anymore:

## FLAG Collaboration, arXiv:1607.00299

$$
\begin{array}{lc}
\mathrm{N}_{\mathrm{f}}=2+1 \mathrm{~m}_{\mathrm{ud}}=3.37(8) \mathrm{MeV} & \mathrm{~m}_{\mathrm{s}}=92.0(2.1) \mathrm{MeV} \\
\mathrm{~m}_{\mathrm{s}} / \mathrm{m}_{\mathrm{ud}}=27.43(31) & \varepsilon=3 \%-6 \% \\
\mathrm{~N}_{\mathrm{f}}=2+1+1 & \\
\mathrm{~m}_{\mathrm{ud}}=3.70(17) \mathrm{MeV} & \mathrm{~m}_{\mathrm{s}}=93.9(1.1) \mathrm{MeV} \\
\mathrm{~m}_{\mathrm{s}} / \mathrm{m}_{\mathrm{ud}}=27.30(34) & \\
\mathrm{f}_{\pi}=130.2(1.4) \mathrm{MeV} \quad \mathrm{f}_{\mathrm{K}}=155.36(0.4) \mathrm{MeV} \varepsilon=0.26 \% \\
\mathrm{f}_{\mathrm{K}} / \mathrm{f}_{\pi}=1.1933(29) \varepsilon=0.24 \% & \mathrm{~F}^{\mathrm{K} \pi}(0)=0.9704(32) \varepsilon=0.34 \%
\end{array}
$$

STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (FLAG)


- $\left|V_{u d}\right|^{2}+\left|V_{u s}\right|^{2}+\left|V_{u b}\right|^{2}=0.9998(5)$ or $0.9999(6)$ from semileptonic and leptonic respectively

Relevant also in $\mathbf{D}$ and $B$ meson decays

## ATTENTION TO THE QUOTED ERRORS

significant differences in estimates of fit and systematic uncertainties in otherwise very similar computations
well-known example from light-quark physics (both computations use MILC ensembles, relatively minor differences)

MILC 13

$$
f_{K^{ \pm}} /\left.f_{\pi^{ \pm}}\right|_{N_{\mathrm{f}}=2+1+1}=1.1947(26)(33)(17)(2)
$$

HPQCD 13

$$
f_{K^{ \pm}} /\left.f_{\pi^{ \pm}}\right|_{N_{\mathrm{f}}=2+1+1}=1.1916(15)(12)(1)(10)
$$

+ perturbative renormalization courtesy of C. Pena



## Do we still care? Tensions and Unknowns

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3) $I V_{c b} I$, $B$ mixing and $\varepsilon_{K}$
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6) $\mathrm{B}->\mathrm{K}^{*} 11$
7) Physics BSM ?

# CKM-TRIANGLE ANALYSIS State of The Art 2015 

|  | Measurement | Fit | Prediction | Pull |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $(92.7 \pm 6.2)^{\circ}$ | $(90.1 \pm 2.7)^{o}$ | $(88.3 \pm 3.4)^{o}$ | 0.6 |
|  | 6.7 \% | 2.9 \% | 3.8 \% |  |
| $\overline{\sin 2 \beta}$ | $0.680 \pm 0.024$ | $0.696 \pm 0.022$ | $0.747 \pm 0.039$ | 1.8 |
|  | 3.5 \% | 2.6 \% | 5.2 \% |  |
| $\gamma$ | $(71.4 \pm 6.5)^{o}$ | $(67.4 \pm 2.8)^{o}$ | $(66.7 \pm 3.0)^{\circ}$ | 0.7 |
|  | 9.1 \% | 4.2 \% | 4.5 \% |  |
| $\overline{\left\|V_{u b}\right\| \times 10^{3}}$ | $3.81 \pm 0.40$ | $3.66 \pm 0.12$ | $3.64 \pm 0.12$ | 0.5 |
|  | $10 \%$ | 3.3 \% | 3.3 \% |  |
| $\overline{\left\|V_{c b}\right\| \times 10^{2}}$ | $4.09 \pm 0.11$ | $4.206 \pm 0.053$ | $4.240 \pm 0.062$ | 0.9 |
|  | 2.6 \% | 1.2 \% | 1.4 \% |  |
| $\varepsilon_{K} \times 10^{3}$ | $2.228 \pm 0.011$ | $2.227 \pm 0.011$ | $2.08 \pm 0.18$ | 0.8 |
|  | 0.5 \% | 0.5 \% | 8.7 \% |  |
| $\overline{\Delta m_{s}\left(\mathrm{ps}^{-1}\right)}$ | $17.761 \pm 0.022$ | $17.755 \pm 0.022$ | $17.3 \pm 1.0$ | 0.2 |
|  | 0.1 \% | 0.1\% | 5.7 \% |  |
| $\overline{B R}(B \rightarrow \tau \nu) \times 10^{4}$ | $1.06 \pm 0.20$ | $0.83 \pm 0.07$ | $\begin{aligned} & 0.81 \pm 0.7 \\ & 8.2 \% \end{aligned}$ | $1.3 \longrightarrow$ |
|  | 18.9 \% | 7.9 \% |  |  |
| $\bar{B} R\left(B_{s} \rightarrow \mu \mu\right) \times 10^{s}$ | 2.9 $\ddagger$ \% 0.7 | $5.00 \pm 0.15$ | 2.04 10.10 | ew corrections not included |
|  | 24.1 \% | 3.8 \% | 4.0 \% |  |
| $\overline{B R\left(B_{d} \rightarrow \mu \mu\right) \times 10^{9}}$ | $0.39 \pm 0.15$ | $0.1098 \pm 0.0057$ | $0.1103 \pm 0.0058$ | 1.9 <br> ew corrections not included |
|  | 38.5 \% | 5.2 \% | 5.2 \% |  |
| $\overline{\beta_{s}}$ | $(0.97 \pm 0.95)^{\circ}$ | $(1.056 \pm 0.039)^{\circ}$ | $(1.056 \pm 0.039)^{\circ}$ | 0.1 not included in the fit |
|  | $98 \%$ | 4.4 \% | 4.1 \% |  |

$$
\mathrm{B}(\mathrm{~B} \rightarrow \tau v)_{\mathrm{Old}}=(1.67 \pm 0.30) 10^{-4}
$$

## LATTICE PARAMETERS (2017)

It does not make sense to improve the precision on $B_{K}$ if we do not control long distance effects; Similarly for $f_{\pi}$ or $f_{K}$ without radiative corrections

Observables Measurement Prediction Pull (\#б)

| $\mathrm{B}_{\mathrm{K}}$ | $0.740 \pm 0.029$ | $0.81 \pm 0.07$ | $<1$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\text {BS }}$ | $0.226 \pm 0.005$ | $0.220 \pm 0.007$ | $<1$ |
| $\mathrm{f}_{\text {BS }} / \mathrm{f}_{\text {Bd }}$ | $1.203 \pm 0.013$ | $1.210 \pm 0.030$ | $<1$ |
| $\mathrm{~B}_{\text {Bs }} / \mathrm{B}_{\text {Bd }}$ | $1.032 \pm 0.036$ | $1.07 \pm 0.05$ | $<1$ |
| $\mathrm{~B}_{\text {Bs }}$ | $1.35 \pm 0.08$ | $1.30 \pm 0.07$ | $<1$ |

## Long Distance Effects in Neutral Meson Mixing

N.H.Christ, T.Izubuchi, CTS, A.Soni \& J.Yu (RBC-UKQCD), arXiv:1212.5931
Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni \& J.Yu (RBC-UKQCD), arXiv:1406.0916
Z.Bai (RBC-UKQCD), arXiv:1411.3210

$$
\begin{array}{l|l|l|}
\begin{array}{l}
\text { exp } \\
\Delta m_{K} \equiv m_{K_{L}}-m_{K_{S}}=3.483(6) \times 10^{-12} \mathrm{MeV} .
\end{array} \begin{array}{l}
\text { 3.19(41)(96) } \\
\text { lattice unphysical } \\
\text { masses }
\end{array} \\
\hline
\end{array}
$$

- Historically led to the prediction of the energy scale of the charm quark.

Mohapatra, Rao \& Marshak (1968); GIM (1970); Gaillard \& Lee (1974)

- Tiny quantity $\Rightarrow$ places strong constraints on BSM Physics.
- Within the standard model, $\Delta m_{K}$ arises from $K^{0}-\bar{K}^{0}$ mixing at second order in the weak interactions:

$$
\Delta M_{K}=2 \mathcal{P} \sum_{\alpha} \frac{\left\langle\bar{K}^{0}\right| H_{W}|\alpha\rangle\langle\alpha| H_{W}\left|K^{0}\right\rangle}{m_{K}-E_{\alpha}},
$$

New project: $64^{3} \times 128, a^{-1}=2.36 \mathrm{GeV}, m_{c}=1.2 \mathrm{GeV}, m_{\pi}=136 \mathrm{MeV}$

- Based on 59 configurations: $\Delta M_{K}=5.5(1.7) \times 10^{-12} \mathrm{MeV}$

Lattice 2017 [C. Sachrajda's talk, Wednesday 12:30@Seminarios 6+7]

## Long Distance Effects in Neutral Meson Mixing



- $\Delta m_{K}$ is given by

$$
\Delta m_{K} \equiv m_{K_{L}}-m_{K_{S}}=2 \mathcal{P} \sum_{\alpha} \frac{\left\langle\bar{K}^{0}\right| \mathcal{H}_{W}|\alpha\rangle\langle\alpha| \mathcal{H}_{W}\left|K^{0}\right\rangle}{m_{K}-E_{\alpha}}=3.483(6) \times 10^{-12} \mathrm{MeV} .
$$

- The above correlation function gives $\left(T=t_{B}-t_{A}+1\right)$

$$
\begin{aligned}
C_{4}\left(t_{A}, t_{B} ; t_{i}, t_{f}\right)=\left|Z_{K}\right|^{2} e^{-m_{K}\left(t_{f}-t_{i}\right)} & \sum_{n} \frac{\left\langle\bar{K}^{0}\right| \mathcal{H}_{W}|n\rangle\langle n| \mathcal{H}_{W}\left|K^{0}\right\rangle}{\left(m_{K}-E_{n}\right)^{2}} \times \\
& \left\{e^{\left(M_{K}-E_{n}\right) T}-\left(m_{K}-E_{n}\right) T-1\right\}
\end{aligned}
$$

- From the coefficient of $T$ we can therefore obtain

$$
\Delta m_{K}^{\mathrm{FV}} \equiv 2 \sum_{n} \frac{\left\langle\bar{K}^{0}\right| \mathcal{H}_{W}|n\rangle\langle n| \mathcal{H}_{W}\left|K^{0}\right\rangle}{\left(m_{K}-E_{n}\right)} .
$$

## Long Distance Effects in Neutral Meson Mixing

- The general formula can be written:
N.H.Christ, G.Martinelli \& CTS, arXiv:1401.1362 N.H.Christ, X.Feng, G.Martinelli \& CTS, arXiv:1504.01170

$$
\Delta m_{K}=\Delta m_{K}^{\mathrm{FV}}-2 \pi_{V}\left\langle\bar{K}^{0}\right| H\left|n_{0}\right\rangle_{V}\left\langle n_{0}\right| H\left|K^{0}\right\rangle_{V}\left[\cot \pi h \frac{d h}{d E}\right]_{m_{K}},
$$

where $h(E, L) \pi \equiv \phi(q)+\delta(k)$.

- This formula reproduces the result for the special case when the volume is such that there is a two-pion state with energy $=m_{K}$.
N.H.Christ, arXiv:1012.6034
- Increasing the volumes keeping $h=n / 2$ and thus avoiding the power corrections is an intriguing possibility.

Within reasonable approximations can be extended to $D$ meson mixing M. Ciuchini,V. Lubicz, L. Silvestrini, S. Simula (progresses made by M. T. Hansen \& S. Sharpe,1204.0826v4,1409.7012v,1504.04248v1) Also CPV in D $->\pi \pi$ or KK

$C_{L}(E, \vec{P})=\bigcirc+\bigcirc+0+0+0+\cdots$


## D MIXING

- D mixing is described by:
- Dispersive $D \rightarrow \bar{D}$ amplitude $M_{12}$

SM: long-distance dominated, not calculable

- NP: short distance, calculable w. lattice
- Absorptive $D \rightarrow \bar{D}$ amplitude $\Gamma_{12}$
- SM: long-distance, not calculable
- NP: negligible
- Observables: $\left|M_{12}\right|,\left|\Gamma_{12}\right|, \Phi_{12}=\arg \left(\Gamma_{12} / M_{12}\right)$

Let us assume that the Standard Model contributions to $M_{12}$ and $\Gamma_{12}$ are real

## "REAL SM" APPROXIMATION

- Define $\left|D_{S, L}\right|=p\left|D^{0}\right| \pm q\left|D^{0}\right|$ and $\delta=\left(1-|q / p|^{2}\right) /\left(1+|q / p|^{2}\right)$. All observables can be written in terms of $x=\Delta m / \Gamma, y=\Delta \Gamma / 2 \Gamma$ and $\delta$
- Introduce $\phi=\arg (q / p)=\arg (y+i \delta x)$
- $|q / p| \neq 1 \Leftrightarrow \phi \neq 0$ clear signals of NP
- Combine all available data with the assumption of real decay amplitudes and real $\Gamma_{12}$
- Preliminary winter18 combination


## D mixing fit results



## Do we still care? Tensions and Unknowns

1) A"classical" example B $->\tau v$
2) $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$ inclusive vs exclusive
3) $I V_{c b} I$, $B$ mixing and $\varepsilon_{K}$
4) D-mixing
5) $R(D)$ and $R\left(D^{*}\right)$
6) $\mathrm{B}->\mathrm{K}^{*} 11$
7) Physics BSM ?

2016
$\left|V_{u b}\right|,\left|V_{c b}\right|$


Vub Exclusive $=0.00369 \pm 0.00015$
Vcb Exclusive $=0.0392 \pm 0.0007$
Vub/Vcb Exclusive $=0.083 \pm 0.006$
Vub Inclusive $=0.00441 \pm 0.00022$
Vcb Inclusive $=0.0422 \pm 0.0007$
Belle $=0.04247 \pm 0.00100$

New HFAG (HFLAV) @CKM16

$$
\left|\mathrm{V}_{\mathrm{cb}}\right|(\mathrm{excl})=(38.88 \pm 0.60) 10^{-3}
$$

$$
\left|\mathrm{V}_{\text {cb }}\right|(\text { incl })=(42.19 \pm 0.78) 10^{-3}
$$

New HFAG @CKM16 -3.3o discrepancy

New HFAG @CKM16

$$
\mid \mathrm{V}_{\mathrm{ub}}(\mathrm{exc} \mid)=(3.65 \pm 0.14) 10^{-3}
$$

$$
\left|\mathrm{V}_{\mathrm{ub}}\right|(\text { incl })=(4.50 \pm 0.20) 10^{-3}
$$

New HFAG @CKM16
~3.4 $\sigma$ discrepancy
$\left|\mathrm{V}_{\mathrm{ub}} / \mathrm{V}_{\mathrm{cb}}\right|(\mathrm{LHCb})=(8.0 \pm 0.6) 10^{-2}$
Updated value

## updated for LHPC17

2D average inspired by D'Agostini skeptical procedure (hep-ex/9910036) with $\sigma=1$. Very similar results obtained from a 2D a la PDG procedure.

$$
\left|\mathrm{V}_{\mathrm{cb}}\right|=(40.5 \pm 1.1) 10^{-3}
$$

$$
\text { uncertainty ~ } 2.4 \%
$$

$$
\left|\mathrm{V}_{\mathrm{ub}}\right|=(3.74 \pm 0.23) 10^{-\beta}
$$

uncertainty ~ 5.6\%
Incl


Excl
${ }_{\left|V_{\text {ob }}\right|}=(42.1 \pm 0.6) 10^{-3}$
UTfit predictions

$$
\rightarrow\left|\mathrm{V}_{\mathrm{ub}}\right|=(3.68 \pm 0.11) 10^{-3}
$$

## exclusives vs inclusives



$$
\begin{aligned}
& \sin 2 \beta_{\text {exp }}=0.745 \pm 0.050 \\
& \sin 2 \beta_{\text {exp }}=0.729 \pm 0.019 \\
& \sin 2 \beta_{\text {exp }}=0.796 \pm 0.026 \\
& \sin 2 \beta_{\text {exp }}=0.680 \pm 0.023 \\
& \sin 2 \beta_{\text {UTfit }}=0.737 \pm 0.031
\end{aligned}
$$

## UT-fit Preliminary


smallest 99.7\% interval(s) smallest 95.5\% interval(s) smallest 68.3\% interval(s) global mode
mean and standard deviation

- $\varepsilon_{K}$ large Vcb
- B mixing with large lattice matrix elements smaller Vcb




## Model-Independent Extraction of $\left|V_{c b}\right|$ from $\bar{B} \rightarrow D^{*} \ell \bar{\nu}$, cont'd

- New Belle analysis released:
- Unfolded data, full correlation matrix
- Large dataset, energy and angular distributions
- CLN: $\left|V_{c b}\right|=(37.4 \pm 1.3) \times 10^{-3}$
- Two independent analyses using BGL:
- Very consistent fits:

$$
\begin{array}{ll}
\left|V_{c b}\right|=\left(41.7_{-2.1}^{+2.0}\right) \times 10^{-3} & \text { Bigi, Gambino \& Schacht, } 1703.06124 \\
\left|V_{c b}\right|=\left(41.9_{-1.9}^{+2.0}\right) \times 10^{-3} & \text { BG \& Kobach, 1703.08170 }
\end{array}
$$

- Robust: different numerical inputs
- Likely culiprit: independent form factors (no HiQET symmerty)

$$
\begin{aligned}
\left\langle D^{*}\left(\varepsilon, p^{\prime}\right)\right| \bar{c} \gamma^{\mu} b|\bar{B}(p)\rangle & =i g \epsilon^{\mu \nu \alpha \beta} \varepsilon_{\nu}^{*} p_{\alpha} p_{\beta}^{\prime} \\
\left\langle D^{*}\left(\varepsilon, p^{\prime}\right)\right| \bar{c} \gamma^{\mu} \gamma^{5} b|\bar{B}(p)\rangle & =f \varepsilon^{* \mu}+\left(\varepsilon^{*} \cdot p\right)\left[a_{+}\left(p+p^{\prime}\right)^{\mu}+a_{-}\left(p-p^{\prime}\right)^{\mu}\right]
\end{aligned}
$$

Recall: BGL introduced $z$-parametrization, eg,

$$
g(z)=\frac{1}{P_{g}(z) \phi_{g}(z)} \sum_{n=0}^{N} a_{n} z^{n} \quad \text { with } \quad \sum_{n} a_{n}^{2} \leq 1 \quad \text { and } \quad 0 \leq z \leq z_{\max }=0.056
$$

with calculable outer function $\phi$ and Blaschke factor $P$

- CLN uses BGL technique, but imposes HQET conditions

Work ahead:

- Experiments: release unfolded data
- Experiments' next best alternative: do BGL fits
- Global analysts: do BGL fits, others (e.g., polynomail in $q^{2}$ )?
- Theorists: $\Lambda / m_{c}$ effects?
- Theorists: Is BGL better than polynomial for independent form factors?
- Can this affect $B \rightarrow D^{(*)} \tau \nu$
- LATTICE !!

If I may be so bold: problem solved

- Retrospect: What went wrong?
- The probelm was sociological!

Also: FF calculations only on MILC configurations $\Rightarrow$ need confirmation with different methods


# Universal Unitarity Triangle 2016 and the Tension Between $\Delta M_{s, d}$ and $\varepsilon_{K}$ in CMFV Models 

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#### Abstract

Motivated by the recently improved results from the Fermilab Lattice and MILC Collaborations on the hadronic matrix elements entering $\Delta M_{s, d}$ in $B_{s, d}^{0}-\bar{B}_{s, d}^{0}$ mixing, we determine the Universal Unitarity Triangle (UUT) in models with Constrained Minimal Flavour Vi-


$$
\begin{equation*}
F_{B_{s}} \sqrt{\hat{B}_{B_{s}}}, \quad F_{B_{d}} \sqrt{\hat{B}_{B_{d}}}, \quad \hat{B}_{K} \tag{2}
\end{equation*}
$$

Fortunately, during the last years these uncertainties decreased significantly. In particular, concerning $F_{B_{s}} \sqrt{\hat{B}_{B_{s}}}$ and $F_{B_{d}} \sqrt{\hat{B}_{B_{d}}}$, an impressive progress has recently been made by the Fermilab Lattice and MILC Collaborations (Fermilab-MILC) that find [3]

$$
\begin{equation*}
F_{B_{s}} \sqrt{\hat{B}_{B_{s}}}=(274.6 \pm 8.8) \mathrm{MeV}, \quad F_{B_{d}} \sqrt{\hat{B}_{B_{d}}}=(227.7 \pm 9.8) \mathrm{MeV} \tag{3}
\end{equation*}
$$

with uncertainties of $3 \%$ and $4 \%$, respectively. An even higher precision is achieved for the ratio

$$
\begin{equation*}
\xi=\frac{F_{B_{s}} \sqrt{\hat{B}_{B_{s}}}}{F_{B_{d}} \sqrt{\hat{B}_{B_{d}}}}=1.206 \pm 0.019 \tag{4}
\end{equation*}
$$

## CKM Uncertainties

$$
\begin{aligned}
& \operatorname{Br}\left(\mathrm{K}^{+} \rightarrow \pi^{+} v \bar{v}\right)=(8.39 \pm 0.30) \cdot 10^{-11}\left[\frac{\left|\mathrm{~V}_{\mathrm{cb}}\right|}{0.0407}\right]^{2.8}\left[\frac{\gamma}{73.2^{\circ}}\right]^{0.71} \\
& \operatorname{Br}\left(\mathrm{~K}_{\mathrm{L}} \rightarrow \pi^{0} v \bar{v}\right)=(3.36 \pm 0.09) \cdot 10^{-11}\left[\frac{\left|\mathrm{~V}_{\mathrm{ub}}\right|}{3.88 \cdot 10^{-3}}\right]^{2}\left[\frac{\left|\mathrm{~V}_{\mathrm{cb}}\right|}{0.0407}\right]^{2}\left[\frac{\sin \gamma}{\sin (73.2)}\right]^{2}
\end{aligned}
$$

$$
\operatorname{Br}\left(\mathrm{K}^{+} \rightarrow \pi^{+} v \bar{v}\right)=(65.3 \pm 3.1)\left[\overline{\mathrm{Br}}\left(\mathrm{~B}_{\mathrm{s}} \rightarrow \mu^{+} \mu^{-}\right)\right]^{1.4}\left[\frac{\gamma}{70^{\circ}}\right]^{0.71}\left[\frac{227 \mathrm{MeV}}{\mathrm{~F}_{\mathrm{B}_{\mathrm{s}}}}\right]^{2.8}
$$

A. Buras , Buttazzo, Girrbach-Noe, Knegjens 1503.02693

For $B_{s} \rightarrow \mu^{+} \mu^{-}$we use the formula from [56], slightly modified in [2]

$$
\overline{\mathcal{B}}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)_{\mathrm{SM}}=(3.65 \pm 0.06) \cdot 10^{-9}\left[\frac{m_{t}\left(m_{t}\right)}{163.5 \mathrm{GeV}}\right]^{3.02}\left[\frac{\alpha_{s}\left(M_{Z}\right)}{0.1184}\right]^{0.032} R_{s}
$$

where

$$
R_{s}=\left[\frac{F_{B_{s}}}{227.7 \mathrm{MeV}}\right]^{2}\left[\frac{\tau_{B_{s}}}{1.516 \mathrm{ps}}\right]\left[\frac{0.938}{r\left(y_{s}\right)}\right]\left[\frac{\left|V_{t s}\right|}{41.5 \cdot 10^{-3}}\right]^{2} .
$$

Now,

$$
\left|V_{t d}\right|=\left|V_{u s}\right|\left|V_{c b}\right| R_{t}, \quad\left|V_{t s}\right|=\eta_{R}\left|V_{c b}\right|
$$

with $R_{t}$ being one of the sides of the unitarity triangle (see Fig. 11) and

$$
\eta_{R}=1-\left|V_{u s}\right| \xi \sqrt{\frac{\Delta M_{d}}{\Delta M_{s}}} \sqrt{\frac{m_{B_{s}}}{m_{B_{d}}}} \cos \beta+\frac{\lambda^{2}}{2}+\mathcal{O}\left(\lambda^{4}\right)=0.9825,
$$

## Do we still care? Tensions and Unknowns

1) A "classical" example B $->\tau v$
2) $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$ inclusive vs exclusive
3) $I V_{c b} I, B$ mixing and $\varepsilon_{K}$
4) D-mixing (already discussed)
5) $R(D)$ and $R\left(D^{*}\right)$ (and Vcb of course)
6) $\mathrm{B}->\mathrm{K}^{(*)} 11$
7) Physics BSM ?

## B semileptonic decay: $\left|V_{c b}\right|$



$$
\begin{gathered}
\frac{\mathrm{d} \Gamma\left(B_{(s)} \rightarrow P l \nu\right)}{\mathrm{d} q^{2}}=\frac{G_{\mathrm{F}}^{2}\left|V_{c b}\right|^{2}}{24 \pi^{3}} \frac{\left(q^{2}-m_{l}^{2}\right)^{2} \sqrt{E_{P}^{2}-m_{P}^{2}}}{q^{4} m_{B_{(s)}}^{2}}\left[\left(1+\frac{m_{l}^{2}}{2 q^{2}}\right) m_{B_{(s)}}^{2}\left(E_{P}^{2}-m_{P}^{2}\right)\left|f_{+}\left(q^{2}\right)\right|^{2}\right. \\
e, \mu \text { suppressed }
\end{gathered}
$$

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

## B semileptonic decay: $\left|V_{c b}\right|$



$$
\begin{gathered}
\frac{\mathrm{d} \Gamma\left(B \rightarrow D l \nu_{l}\right)}{\mathrm{d} w}=\frac{G_{\mathrm{F}}^{2}}{48 \pi^{3}}\left(m_{B}+m_{D}\right)^{2}\left(w^{2}-1\right)^{3 / 2}\left|\eta_{\mathrm{Ew}}\right|^{2}\left|V_{c b}\right|^{2}|\mathcal{G}(w)|^{2}+\mathcal{O}\left(\frac{m_{l}^{2}}{q^{2}}\right) \\
\frac{\mathrm{d} \Gamma\left(B \rightarrow D^{*} l \nu_{l}\right)}{\mathrm{d} w}=\frac{G_{\mathrm{F}}^{2}}{4 \pi^{3}}\left(m_{B}-m_{D^{*}}\right)^{2}\left(w^{2}-1\right)^{1 / 2}\left|\eta_{\mathrm{Ew}}\right|^{2} \chi(w)\left|V_{c b}\right|^{2}|\mathcal{F}(w)|^{2}+\mathcal{O}\left(\frac{m_{l}^{2}}{q^{2}}\right)
\end{gathered}
$$

$$
w=\frac{p_{B} \cdot p_{D^{(*)}}}{m_{B} m_{D^{(*)}}} \quad \mathcal{G}(w)=\frac{4 \frac{m_{D}}{m_{B}}}{1+\frac{m_{D}}{m_{B}}} f_{+}\left(q^{2}\right) \quad \text { etc }
$$

Low recoil region ( $w=1$ ) accessible to lattice calculations

B $\rightarrow$ D-D*
same lattice configurations used $m_{b} a \approx 1.1$ in the best case

|  | FNAL/MILC* | FNAL/MILC | HPQCD |
| :--- | :---: | :---: | :---: |
| process | $B \rightarrow D^{*} \ell \nu$ | $B \rightarrow D l \nu$ | $B \rightarrow D l \nu$ |
| kinematics | $w=1$ | MILC | MILC |
| ensembles | $2+1$ | $2+1$ | MILC |
| $N_{\mathrm{f}}$ | $5 / 0.045-0.15$ | $4 / 0.045-0.12$ | $2+1$ |
| $(\mathrm{fm})$ | 260 | 220 | $2 / 0.09,0.12$ |
| $M_{\pi}^{\text {min }}[\mathrm{Mev}]$ | 3.8 | 200 |  |
| $M_{\pi}^{\text {min }} L$ | 3.8 | asqtad | 3.8 |
| $I$ quarks | asqtad | RHQ (Fermilab) | HISQ |
| $c$ quark | RHQ (Fermilab) | RHQ (Fermilab) | NRQCD |
| $b$ quark | RHQ (Fermilab) | $[1403.0635]$ |  |
| reference |  |  |  |

(* full publication of $B \rightarrow D^{*}$ results, no changes wrt proceedings value quoted in FLAG)

## new results for $B \rightarrow D l \nu$

Form factors from single and double ratios of lattice correlation functions


$$
R(D)=\frac{\mathcal{B}(B \rightarrow D \tau \nu)}{\mathcal{B}(B \rightarrow D \ell \nu)}=0.299(11)
$$

Form factors from direct fit

0.300(8)

## HPQCD June 132016




FIG. 1. Average of $R\left(D^{(*)}\right)$ measurements, displayed as red filled ellipses ( $68 \%$ CL and $95 \%$ CL). The SM prediction is shown as a black ellipse ( $95 \%$ CL), and the individual measurements as continuous contours ( $68 \%$ CL): Belle (blue ellipse and horizontal bands), BaBar (green ellipse), and LHCb (horizontal orange band).

## $\left|\mathrm{V}_{\mathrm{ub}}\right| \&\left|\mathrm{~V}_{\mathrm{cb}}\right|$ inclusive vs exclusive and all that

1) On the long run exclusive decays based on non-perturbative (lattice) determination of the relevant form factors will win;
2) The precision of the theoretical predictions for inclusive decays cannot be improved (are the present quoted errors reliable?);
3) Still (much) more work is needed, and different lattice approaches to the physical B should be used and compared;
4) $R(D)$ and $R\left(D^{*}\right)$ is an open problem; more lattice collaborations should work on these calculations. A comparison with Bs and Bc decays fundamental;
5) Theoretical calculations and experimental analyses should not be biased by the HQFT - after all $\Lambda_{\mathrm{QCD}} / \mathrm{m}_{\mathrm{C}} \approx \mathrm{O}(1)$;
6) I hope to be wrong, but the possibility of new physics in tree level b-> c decays looks to me quite remote.

## Do we still care? Tensions and Unknowns

1) A"classical" example B $->\tau v$
2) $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$ inclusive vs exclusive
3) $I V_{c b} I, B$ mixing and $\varepsilon_{K}$
4) D-mixing (already discussed)
5) $R(D)$ and $R\left(D^{*}\right)$ (and Vcb of course)
6) $\mathrm{B}->\mathrm{K}^{*} 11$
7) Physics BSM ?

## Is the present picture showing a Model Standardissimo?

An evidence, an evidence, my kingdom for an evidence
From Shakespeare's Richard III and A. Stocchi

## 1) Fit of $N P-\Delta F=2$ parameters in a Model "independent" way

2) "Scale" analysis in $\Delta F=2$

## BSM



VERY GOOD CONSISTENCY WITHIN THE SM!


## NP ANALYSIS: RESULTS

$$
\begin{aligned}
& \bar{\rho}=0.125 \pm 0.025 \\
& \bar{\eta}=0.381 \pm 0.028
\end{aligned}
$$ to be compared $w$. $\bar{\rho}=0.145 \pm 0.014$ $\bar{\eta}=0.349 \pm 0.010$ in the SM



## RBC-UK QCD NEW PHYSICS IN KAON DECAYS?

$$
\varepsilon^{\prime} / \varepsilon=(1.4 \pm 7.0) \cdot 10^{-4}
$$

$$
\left(\frac{\operatorname{Re} \mathrm{A}_{0}}{\operatorname{Re} \mathrm{~A}_{2}}\right)=31.0 \pm 6.6
$$

$$
\left(\varepsilon^{\prime} / \varepsilon\right)_{\exp }=(16.6 \pm 2.3) \cdot 10^{-4}
$$

$$
\left(\frac{\operatorname{Re~} \mathrm{A}_{0}}{\operatorname{Re} \mathrm{~A}_{2}}\right)_{\mathrm{exp}}=22.4
$$

## Courtesy by A. Buras

## Results for $\operatorname{Re}\left[A_{0}\right], \operatorname{Im}\left[A_{0}\right]$ and $\operatorname{Re}\left[\epsilon^{\prime} / \epsilon\right]$

## Xu Feng Lattice 2017

## [RBC-UKQCD, PRL115 (2015) 212001]

- Determine the $K \rightarrow \pi \pi(I=0)$ amplitude $A_{0}$
- Lattice results

$$
\begin{aligned}
& \operatorname{Re}\left[A_{0}\right]=4.66(1.00)_{\text {stat }}(1.26)_{\text {syst }} \times 10^{-7} \mathrm{GeV} \\
& \operatorname{Im}\left[A_{0}\right]=-1.90(1.23)_{\text {stat }}(1.08)_{\text {syst }} \times 10^{-11} \mathrm{GeV}
\end{aligned}
$$

- Experimental measurement

$$
\begin{aligned}
& \operatorname{Re}\left[A_{0}\right]=3.3201(18) \times 10^{-7} \mathrm{GeV} \\
& \operatorname{Im}\left[A_{0}\right] \text { is unknown }
\end{aligned}
$$

- Determine the direct $C P$ violation $\operatorname{Re}\left[\epsilon^{\prime} / \epsilon\right]$

$$
\begin{array}{ll}
\operatorname{Re}\left[\epsilon^{\prime} / \epsilon\right]=0.14(52)_{\text {stat }}(46)_{\text {syst }} \times 10^{-3} & \text { Lattice } \\
\operatorname{Re}\left[\epsilon^{\prime} / \epsilon\right]=1.66(23) \times 10^{-3} & \text { Experiment }
\end{array}
$$

$2.1 \sigma$ deviation $\quad \Rightarrow \quad$ require more accurate lattice results

## Four dominant contributions to $\varepsilon^{\prime} / \varepsilon$ in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)


Assumes that $\operatorname{ReA}_{0}$ and $\operatorname{ReA}_{2}$ ( $\Delta \mathrm{I}=1 / 2$ Rule) fully described by SM (includes isospin breaking corrections)
$\varepsilon^{\prime} / \varepsilon$ from RBC-UKQCD
Calculate all contributions directly (no isospin breaking corrections)

$$
\left[-(6.5 \pm 3.2)+25.3 \cdot B_{6}^{(1 / 2)}+(1.2 \pm 0.8)-10.2 \cdot B_{8}^{(3 / 2)}\right]
$$

## $\varepsilon^{\prime} / \varepsilon$ from RBC-UKQCD

Anatomy: AJB, Gorbahn, Jäger, Jamin (2015)
Calculate all contributions directly


EW penguins in full agreement with BGJJ but


$$
\begin{aligned}
& \text { + third term } \\
& \text { very similar to BGJJ } \\
& \left(\text { ReA }_{2}\right)_{\text {Lattice }} \approx\left(\operatorname{ReA}_{2}\right)_{\exp }
\end{aligned}
$$

$$
\left[\frac{\left(\operatorname{Re} A_{0}\right)}{\left(\operatorname{Re} A_{0}\right)_{\exp }} \approx 1.4\right] \Rightarrow
$$

| The negative |
| :--- |
| contribution of |
| $\mathbf{Q}_{4}$ overestimated |

## Anatomy of $\varepsilon^{\prime} / \varepsilon$ - A new flavour anomaly?

AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx

## RBC-UKQCD

$$
\varepsilon^{\prime} \varepsilon=(1.4 \pm 7.0) \cdot 10^{-4}
$$

$$
(3.2 \sigma) \quad \varepsilon^{\prime} / \varepsilon=(2.2 \pm 3.8) \cdot 10^{-4}
$$

$$
\varepsilon^{\prime} / \varepsilon=(6.3 \pm 2.5) \cdot 10^{-4}
$$

$$
\text { large } N \text { bounds (AJB, Gérard) }
$$

$$
\varepsilon^{\prime} / \varepsilon=(9.1 \pm 3.3) \cdot 10^{-4}
$$

$$
\begin{aligned}
& \text { RBC-QCD values } \\
& B_{6}^{(1 / 2)}=0.57 \pm 0.15 \\
& B_{8}^{(3 / 2)}=0.76 \pm 0.05
\end{aligned}
$$

large $N$ bounds (AJB, Gérard)

$$
B_{6}^{(1 / 2)}=B_{8}^{(3 / 2)}=0.76
$$

large $N$ bounds (AJB, Gérard)

$$
B_{6}^{(1 / 2)}=B_{8}^{(3 / 2)}=1.0
$$

$\exp : \quad \varepsilon^{\prime} / \varepsilon=(16.6 \pm 3.3) \cdot 10^{-4}$

## Angular Analyses

> First full angular analysis of $\mathrm{B}^{0} \rightarrow \mathrm{~K}^{*} \rho \mu$ : measured all CP-averaged angular terms and CP-asymmetries
> Can construct less form-factor dependent ratios of observables


## New analysis from Belle



# Reminder: $\mathrm{R}_{\mathrm{K}}=\mathrm{B}\left(\mathrm{B}^{+} \rightarrow \mathrm{K}^{+} \mu^{+} \mu^{-}\right) / \mathrm{B}\left(\mathrm{B}^{+} \rightarrow \mathrm{K}^{+} \mathrm{e}^{+} \mathrm{e}^{-}\right)$ 

- Test of lepton universality : $\mathrm{R}_{\mathrm{K}} \sim 1$ in SM , with negligible theoretical uncertainties

- Compatible with SM at 2.6o
- Experimentally challenging
- lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test: $\mathrm{B}^{0} \rightarrow \mathrm{~K}^{* 0} l^{+} l^{-}, \mathrm{B}_{\mathrm{s}} \rightarrow \phi l^{+} l^{-}, \Lambda_{\mathrm{B}} \rightarrow \Lambda l^{+} l^{-}$


## AND NOW:

The hint that the loop induced decays $b \rightarrow$ sll can break lepton flavor universality (1) was corroborated by the most recent LHCb results [4],

$$
\begin{align*}
& R_{K^{*}}^{\text {low }}=\frac{\mathcal{B}\left(B \rightarrow K^{*} \mu \mu\right)_{q^{2} \in[0.045,1.1] \mathrm{GeV}^{2}}}{\mathcal{B}\left(B \rightarrow K^{*} e e\right)_{q^{2} \in[0.045,1.1] \mathrm{GeV}^{2}}}=0.660 \pm_{0.070}^{0.110} \pm 0.024, \\
& R_{K^{*}}^{\text {central }}=\frac{\mathcal{B}\left(B \rightarrow K^{*} \mu \mu\right)_{q^{2} \in[1.1,6] \mathrm{GeV}^{2}}}{\mathcal{B}\left(B \rightarrow K^{*} e e\right)_{q^{2} \in[1.1,6] \mathrm{GeV}^{2}}}=0.685 \pm_{0.069}^{0.113} \pm 0.047, \tag{2}
\end{align*}
$$

## VERY DIFFICULT TO EXPLAIN WITH HADRONIC UNCERTAINIES!!

Heavy to light semileptonic ।
New results after Lattice 2015:

|  | Fermilab/MILC | Fermilab/MILC | Detmold and Meinel |
| :---: | :---: | :---: | :---: |
| process | $B \rightarrow K I I$, | $B \rightarrow \pi I I$ | $\Lambda_{b} \rightarrow \Lambda$ |
| kinematics | full $q^{2}$ | full $q^{2}$ | full $q^{2}$ |
| ensembles | MILC asqtad | MILC asqtad | RBC/UKQCD DWF |
| $N_{f}$ | $2+1$ | $2+1$ | $2+1$ |
| $a$ | $4 / 0.045-0.12$ | $4 / 0.045-0.12$ | $2 / 0.09-0.12$ |
| $M_{\pi}^{\text {min }}$ | 260 | 260 | 227 |
| light quark | asqtad | asqtad | DWF |
| $b$ quark | Fermilab | Fermilab | RHQ |
| Ref. | PRD. 93.025026 | PRL.115.152002 | PRD. 93.074501 |

- PRD. 93.034005 (Fermilab/MILC, $B$ rare decay pheno)
- PRD.94.013007 (Meinel and van Dyk, $\wedge_{b}$ rare decay pheno)
- PRD.88.054509, PRL.111.162002 (HPQCD, $B \rightarrow K / l$ ff and pheno), PRD.89.094501, PRL.112.212003 ( $B \rightarrow K^{*} \| l$ ff and pheno)


## Standard Model predictions of $B$ rare decays




- Standard-Model predictions of the differential decay rate in $B \rightarrow \pi / l$ and $B \rightarrow K / l$ process (PRL.115.152002, PRD.93.034005).

There are good chances that the lattice calculation of the most important long distance contributions via a charm loop is possible M. Ciuchini, V.Lubicz, G.M., L. Silvestrini, S. Simula


## RADIATIVE/RARE KAON DECAYS

G. Isidori, G. M., and P. Turchetti, Phys.Lett. B633, 75 (2006), arXiv:hep-lat/0506026
N.H. Christ X. Feng A. Portelli and C.T. Sachrajda Phys.Rev. D92 (2015) no.9, 094512 10.1103/PhysRevD.92.094512 *

$$
K \rightarrow \pi l^{+} l^{-} \quad K \rightarrow \pi \nu \bar{\nu}
$$

Conserved currents and GIM important
$2.1 K \rightarrow \pi \ell^{+} \ell^{-}$
G. Isidori, G. M., and P. Turchetti

The main non-perturbative correlators relevant for these decays are those with the electromagnetic current. In particular, the relevant $T$-product in Minkowski space is [7, 8]

$$
\begin{align*}
\left(\mathcal{T}_{i}^{j}\right)_{\mathrm{em}}^{\mu}\left(q^{2}\right) & =-i \int d^{4} x e^{-i q \cdot x}\left\langle\pi^{j}(p)\right| T\left\{J_{\mathrm{em}}^{\mu}(x)\left[Q_{i}^{u}(0)-Q_{i}^{c}(0)\right]\right\}\left|K^{j}(k)\right\rangle  \tag{11}\\
J_{\mathrm{em}}^{\mu} & =\frac{2}{3} \sum_{q=u, c} \bar{q} \gamma^{\mu} q-\frac{1}{3} \sum_{q=d, s} \bar{q} \gamma^{\mu} q \tag{12}
\end{align*}
$$

for $i=1,2$ and $j=+, 0$. Thanks to gauge invariance we can write

$$
\begin{equation*}
\left(\mathcal{T}_{i}^{j}\right)_{\mathrm{em}}^{\mu}\left(q^{2}\right)=\frac{w_{i}^{j}\left(q^{2}\right)}{(4 \pi)^{2}}\left[q^{2}(k+p)^{\mu}-\left(m_{k}^{2}-m_{\pi}^{2}\right) q^{\mu}\right] \tag{13}
\end{equation*}
$$

The normalization of (13) is such that the $O(1)$ scale-independent low-energy couplings $a_{+, 0}$ defined in [8] can be expressed as

$$
\begin{equation*}
a_{j}=\frac{1}{\sqrt{2}} V_{u s}^{*} V_{u d}\left[C_{1} w_{1}^{j}(0)+C_{2} w_{2}^{j}(0)+\frac{2 N_{j}}{\sin ^{2} \theta_{W}} f_{+}(0) C_{7 V}\right] \tag{14}
\end{equation*}
$$

A detailed analysis of the extraction of the amplitude from lattice correlators by N.H. Christ X. Feng A. Portelli and C.T. Sachrajda

## $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}:$ Experiment vs Standard model


$K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ : largest contribution from top quark loop, thus theoretically clean

$$
\mathcal{H}_{\text {eff }} \sim \frac{G_{F}}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\mathrm{EM}}}{2 \pi \sin ^{2} \theta_{W}} \lambda_{t} X_{t}\left(x_{t}\right)}_{\mathcal{N} \sim 2 \times 10^{-5}} \cdot(\bar{s} d)_{V-A}(\bar{\nu} \nu)_{V-A}
$$

Probe the new physics at scales of $\mathcal{N}^{-\frac{1}{2}} M_{W}=O(10 \mathrm{TeV})$

Past experimental measurement is 2 times larger than SM prediction

$$
\begin{array}{ll}
\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}\right)_{\text {exp }}=1.73_{-1.05}^{+1.15} \times 10^{-10} & {[\text { BNL E949, '08] }} \\
\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}\right)_{\mathrm{SM}}=9.11 \pm 0.72 \times 10^{-11} & {[\text { Buras et. al., '15] }}
\end{array}
$$

but still consistent with $>60 \%$ exp. error

## Results for charm quark contribution

Charm quark contribution $P_{c}$

$$
P_{c}=P_{c}^{\mathrm{SD}}+\delta P_{c, u}
$$

NNLO QCD [Buras, Gorbahn, Haisch, Nierste, '06]:

$$
P_{c}^{\mathrm{SD}}=0.365(12)
$$

Phenomenological ansatz [Isidori, Mescia, Smith, '05]

$$
\delta P_{c, u}=0.040(20)
$$

Lattice results @ $m_{\pi}=420 \mathrm{MeV}, m_{c}=860 \mathrm{MeV}$
[RBC-UKQCD, arXiv:1701.02858]

$$
\begin{aligned}
P_{c} & =0.2529( \pm 13)_{\text {stat }}( \pm 32)_{\text {scale }}(-45)_{\mathrm{FV}} \\
P_{c}-P_{c}^{\mathrm{SD}} & =0.0040( \pm 13)_{\text {stat }}( \pm 32)_{\text {scale }}(-45)_{\mathrm{FV}}
\end{aligned}
$$

- As a smaller $m_{c}$ is used, $P_{c}$ is also smaller
- Cancellation in $W-W$ and $Z$-exchange diag. leads to small $P_{c}-P_{c}^{\text {SD }}$
- Important to perform the calculation at physical $m_{\pi}$ and $m_{c}$


## $K \rightarrow \pi \ell^{+} \ell^{-}: C P$ conserving chanel

$C P$ conserving decay: $K^{+} \rightarrow \pi^{+} \ell^{+} \ell^{-}$and $K_{S} \rightarrow \pi^{0} \ell^{+} \ell^{-}$

- Involve both $\gamma$ - and $Z$-exchange diagram, but $\gamma$-exchange is much larger

- Unlike $Z$-exchange, the $\gamma$-exchange diagram is LD dominated
- By power counting, loop integral is quadratically UV divergent
- EM gauge invariance reduces divergence to logarithmic
- $c-u$ GIM cancellation further reduces log divergence to be UV finite


## First exploratory calculation on $K^{+} \rightarrow \pi^{+} \ell^{+} \ell^{-}$

Use $24^{3} \times 64$ ensemble, $N_{\text {conf }}=128$ [RBC-UKQCD, PRD94 (2016) 114516]
$a^{-1}=1.78 \mathrm{GeV}, m_{\pi}=430 \mathrm{MeV}$
$m_{K}=625 \mathrm{MeV}, m_{c}=530 \mathrm{MeV}$
Momentum dependence of $V_{+}(z)$

$$
\begin{aligned}
& V_{+}(z)=a_{+}+b_{+} z \\
& \quad \Rightarrow \quad a_{+}=1.6(7), b_{+}=0.7(8)
\end{aligned}
$$


$K^{+} \rightarrow \pi^{+} e^{+} e^{-}$data + phenomenological analysis: $a_{+}=-0.58(2), b_{+}=-0.78(7)$
[Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399]

$$
V_{j}(z)=a_{j}+b_{j} z+\underbrace{\frac{\alpha_{j} r_{\pi}^{2}+\beta_{j}\left(z-z_{0}\right)}{G_{F} M_{K}^{2} r_{\pi}^{4}}}_{K \rightarrow \pi \pi \pi} \underbrace{\left[1+\frac{z}{r_{V}^{2}}\right]}_{F_{V}(z)} \underbrace{\left[\phi\left(z / r_{\pi}^{2}\right)+\frac{1}{6}\right]}_{\text {loop }}, \quad j=+, S
$$

- Experimental data only provide $\frac{d \Gamma}{d z} \Rightarrow$ square of form factor $\left|V_{+}(z)\right|^{2}$
- Need phenomenological knowledge to determine the sign for $a_{+}, b_{+}$

TESTING THE NEW PHYSICS SCALE Effective Theory Analysis $\Delta \mathrm{F}=\mathbf{2}$
Effective Hamiltonian in the mixing amplitudes

$$
C_{j}(\Lambda)=\frac{L F_{j}}{\Lambda^{2}} \Rightarrow \Lambda=\sqrt{\frac{L F_{j}}{C_{j}(\Lambda)}}
$$

$$
\begin{array}{cc}
H_{e f f}^{\Delta B=2}=\sum_{i=1}^{S} C_{i}(\mu) Q_{i}(\mu)+\sum_{i=1}^{3} \widetilde{C}_{i}(\mu) \widetilde{Q}_{i}(\mu) \\
Q_{1}=\bar{q}_{L}^{\alpha} y_{\mu} b_{L}^{\alpha} \bar{q}_{L}^{\beta} y^{\mu} b_{L}^{\beta} & (\mathrm{SM} / \mathrm{MFV}) \\
Q_{2}=\bar{q}_{R}^{\alpha} b_{L}^{\alpha} \bar{q}_{R}^{\beta} b_{L}^{\beta} & Q_{3}=\bar{q}_{R}^{\alpha} b_{L}^{\beta} \bar{q}_{R}^{\beta} b_{L}^{\beta} \\
Q_{4}=\bar{q}_{R}^{\alpha} b_{L}^{\alpha} \bar{q}_{L}^{\beta} b_{R}^{\beta} & Q_{5}=\bar{q}_{R}^{\alpha} b_{L}^{\beta} \bar{q}_{L}^{\beta} b_{R}^{\beta} \\
\widetilde{Q}_{1}=\bar{q}_{R}^{\alpha} y_{\mu} b_{R}^{\alpha} \bar{q}_{R}^{\beta} y^{\mu} b_{R} & \\
\widetilde{Q}_{2}=\bar{q}_{L}^{\alpha} b_{R}^{\alpha} \bar{q}_{L}^{\beta} b_{R}^{\beta} & \bar{Q}_{3}=\bar{q}_{L}^{\alpha} b_{R}^{\beta} \bar{q}_{L}^{\beta} b_{R}^{\beta}
\end{array}
$$

$$
\begin{aligned}
& \mathbf{F}_{1}=F_{\mathrm{SM}}=\left(\mathbf{V}_{\mathrm{tq}} \mathbf{V}_{\mathrm{tb}} *\right)^{2} \\
& \mathbf{F}_{\mathrm{j}=1}=\mathbf{0}
\end{aligned}
$$

$\left|\mathbf{F}_{\mathrm{j}}\right|=\mathrm{F}_{\mathrm{SM}}$
arbitrary phases
$\left|\mathrm{F}_{\mathrm{j}}\right|=\mathbf{1}$
arbitrary phases
$L$ is loop factor and should be :
$L=1$ tree/strong int. NP
$L=\alpha^{2}$ or $\alpha^{2}{ }_{w}$ for strong/weak perturb. NP

LATTICE
CALCULATIONS
MFV ESSENTIAL IN THIS CASE !!

## NMFV

Flavour generic

## Resolution of the discrepancy for $B_{4}, B_{5}$

$N_{f}=2+1$ DWF, $a=0.08,0.11 \mathrm{fm}, m_{\pi}=300 \mathrm{MeV}[$ RBC-UKQCD, JHEP11(2016)001]
open question
$B_{4}$



- Use both $\mathrm{RI} / \mathrm{MOM}$ and $\mathrm{SMOM} \Rightarrow$ the former is significantly smaller
- Use two RI/SMOM schemes, $(\phi, \phi)$ and $\left(\gamma_{\mu}, \gamma_{\mu}\right) \Rightarrow$ consistent results
- $\mathrm{RI} /(\mathrm{S}) \mathrm{MOM}$ result compatible with previous $\mathrm{RI} /(\mathrm{S}) \mathrm{MOM}$ calculation

Study suggests $\mathrm{RI} / \mathrm{MOM}$ suffers from large IR artifacts $\Rightarrow$ discrepancy
On-going project: [J. Kettle's talk, Wednesday 11:30@Seminarios 6+7]

- $64^{3}$ and $48^{3}$ ensembles with physical $m_{\pi}$ and finer lattice spacing


## results from the Wilson coefficients

Generic: $C(\Lambda)=\alpha / \Lambda^{2}, F_{i} \sim 1$, arbitrary phase
$\alpha \sim 1$ for strongly coupled NP


Lower bounds on NP scale (in TeV at $95 \%$ prob.)
Non-perturbative NP $\Lambda>5.010^{5} \mathrm{TeV}$

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by $\alpha_{s}(\sim 0.1)$ or by $\alpha_{w}(\sim 0.03)$.
$\alpha \sim \alpha_{w}$ in case of loop coupling through weak interactions NP in $\alpha_{w}$ loops

$$
\Lambda>1.510^{4} \mathrm{TeV}
$$

Best bound from $\varepsilon_{k}$ dominated by CKM error CPV in charm mixing follows, exp error dominant
Best CP conserving from $\Delta \mathrm{m}_{\kappa}$, dominated by long distance
$B_{d}$ and $B_{s}$ behind, errors from both CKM and B-parameters

## NMFV STRONGLYINTERACTING NP

- If new chiral structures present, $\varepsilon_{k}$ still gives best constraint
- $B_{d}$ and $B_{s}$ most powerful if no new operators arise
- Non-perturbative NMFV NP (e.g. composite Higgs)
- $\Lambda>94 \mathrm{TeV}$
- Weakly interacting:

- $\Lambda>3 \mathrm{TeV}$
absence says more than presence FRANK HERBERT
(Dune)

THANKS FOR YOUR ATTENTION


