# Effective field theories for lepton dipole moments

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# Standard Model and open issues

The SM does not take into account the following observations:

- neutrino oscillations;
- dark matter observation;
- baryogenesis;
- gravity.

It does not provide a convincing explanation for:

- hierarchy problem;
- flavour puzzle;
- QCD theta term;
- gauge couplings unification.

# Lepton dipole moments: a conceptual challenge

The Dim-4 SM provides an accidental flavour symmetry:

- it holds in QCD and EM interactions;
- in the quark sector, it's broken by EW interactions.

The lepton sector strictly conserves flavour and CP.

At the same time, we have remarkable phenomenological evidences of FV in the neutrino sector, but...

...No evidence of lepton CP violation and of:

- $l_h^\pm \to \gamma + l_i^\pm$ where $h, i = e, \mu, \tau$,
- $l_h^\pm \to l_i^\pm l_j^\pm l_k^\mp$ where $h, i, j, k = e, \mu, \tau$,
- $Z \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$,
- $H \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$.

# Lepton flavour and CP violation *are* new physics

Leptons come in three generations and mix: CPV is expected.

Neutral sector: neutrino mass generation mechanism

$\nu$ oscillation *is* a BSM signal, but what is the underlying picture?

Charged sector: lepton flavour and CP puzzle

cLFV & CPV are severely constrained, why BSM is so elusive?

## The handhold: leptonic electric dipole moment

"The KM phase in the quark sector can induce a lepton EDM via a diagram with a closed quark loop, but a non-vanishing result appears first at the four-loop level and therefore is even more suppressed, below the level of

$$d_e^{\rm CKM} \leq 10^{-38} e \text{ cm},$$

and so small that the EDMs of paramagnetic atoms and molecules would be induced more efficiently by e.g. Schiff moments and other CP-odd nuclear momenta. [...] The electron EDM is not the best way to probe CP violation in the lepton sector.

M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119

# A selection of limits on leptonic observables

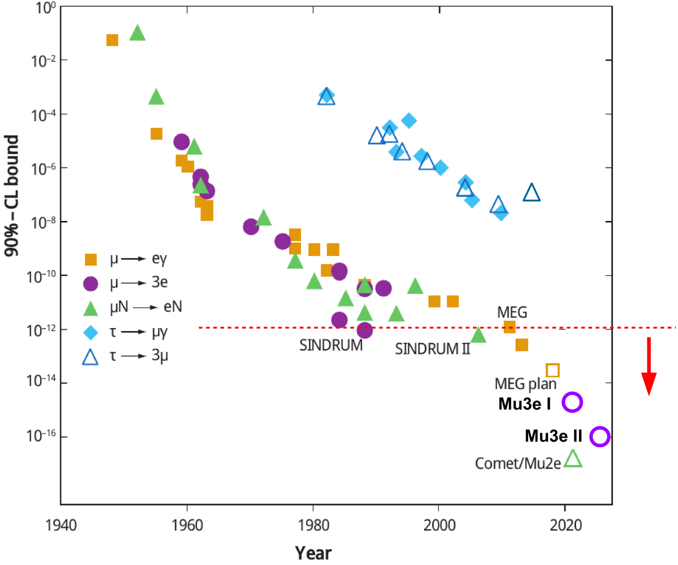
Lepton EDMs:

- $d_e < 0.87 \times 10^{-28} e$cm at the $90\%$ C.L.
  ACME Collaboration, Science **343** (2014) 269;
- $d_\mu < (-0.1 \pm 0.9) \times 10^{-19} e$cm at the $90\%$ C.L.
  Muon $(g-2)$ Collaboration, Phys. Rev. D **80** (2009) 052008;
- $-0.22 \times 10^{-16} e$cm$< d_\tau < 0.45 \times 10^{-16} e$cm at the $95\%$ C.L.
  Belle Collaboration, Phys. Lett. B **551** (2003) 16.

cLFV in the muon sector, the "golden" channels:

- BR$(\mu \to 3e) < 1.0 \times 10^{-12}$ at the $90\%$ C.L.
  SINDRUM collaboration, Nucl. Phys. B **299** (1988) 1;
- $\sigma(\mu^- \to e^-)/\sigma(capt.)\big|_{\rm Au} < 7.0 \times 10^{-13}$ at the $90\%$ C.L.
  SINDRUM II collaboration, Eur. Phys. J. C **47** (2006) 337;
- BR$(\mu \to \gamma + e) < 4.2 \times 10^{-13}$ at the $90\%$ C.L.
  MEG collaboration, Eur. Phys. J. C **76** (2016) 434;

# Golden channels: Past, present and future



# Recent developments

One can contribute in two ways:

1. performing precise calculations for backgrounds;
2. interpreting properly the current absence of signals.

1) Typical low-energy cLFV background computations:

- radiative decays, $l_1 \to l_2 + \gamma + 2\nu$;
- rare decays, $l_1 \to 3l_2 + 2\nu$, $l_1 \to 2l_2 + l_3 + 2\nu$.

2) Typical interpretive approaches:

- bottom-up, effective field theoretical formulations;
- top-down, UV-complete extensions of the SM.

# Precise calculations for cLFV backgrounds

Leptonic radiative and rare decays are known at the Next-to-leading order in the Fermi Theory.

- $l_1 \to l_2 + \gamma + 2\nu$

  M. Fael, L. Mercolli and M. Passera, JHEP **07** (2015) 153

  GMP, A. Signer and Y. Ulrich, Phys. Lett. B **772** (2017) 452

- $l_1 \to 3l_2 + 2\nu$

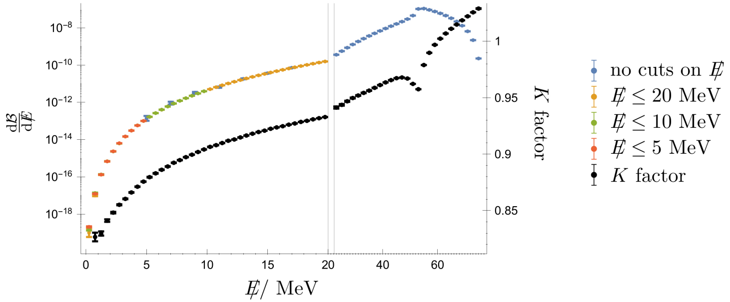
  M. Fael, C. Greub, JHEP **1701**, 084 (2017)

  GMP, A. Signer and Y. Ulrich, Phys. Lett. B **765** (2017) 280

Fully differential NLO Monte Carlo for the radiative/rare decays of a polarised lepton is now available.

Predictions can be tailored on future experiments, arbitrary kinematic cuts can be implemented!

# Rare decay at the NLO: the outcome




The differential decay distribution w.r.t. the invisible $\not{E}$ at NLO in blue, orange, green and red (see text) and the $K$ factor NLO/LO in black. To emphasize the low energy tail, the scaling is broken at $\not{E} = 20$ MeV. The error bars indicate the numerical error of the Monte Carlo integration.

There are substantially fewer background events than expected from the naive tree-level simulations.

# Extending the interactions of the SM

Assumptions: SM is merely an effective theory, valid up to some scale $\Lambda$. It can be extended to a field theory that satisfies the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
- all the SM degrees of freedom must be incorporated;
- at low energies (i.e. when $\Lambda \to \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

# Dimension-five operator

Only one dimension 5 operator is allowed by gauge symmetry:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n \;\equiv\; (\widetilde{\varphi}^\dagger l_p)^T C(\widetilde{\varphi}^\dagger l_r).$$

After the EW symmetry breaking, it can generate neutrino masses and mixing (no other operator can do the job).

Its contribution to LFV has been studied since the late 70s:

- in the context of higher dimensional effective realisations;
  S. T. Petcov, Sov. J. Nucl. Phys. **25** (1977) 340 [Yad. Fiz. **25** (1977) 641]
- in connection with the "see-saw" mechanism.
  P. Minkowski, Phys. Lett. B **67**, 421 (1977)

"[. . . ] This effect is beyond the reach of presently planned experiments."
J. P. Archambault, A. Czarnecki and M. Pospelov, Phys. Rev. D **70** (2004) 073006

# Dimension-six operators

**2-leptons**

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu};$$
$$Q_{eB} = (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}.$$
$$Q^{(1)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$$
$$Q^{(3)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$
$$Q_{\varphi e} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$$
$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$$

**4-leptons**

$$Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$
$$Q_{ee} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$
$$Q_{le} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

**4-fermions**

$$Q^{(1)}_{lq} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$
$$Q^{(3)}_{lq} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$
$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{lu} = (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ld} = (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$$
$$Q_{ledq} = (\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$$
$$Q^{(1)}_{lequ} = (\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$$
$$Q^{(3)}_{lequ} = (\bar{l}^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)$$

# Leptonic tensorial current at the tree level

One dimension-six operator can produce tensorial current:

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

Working in the physical basis, we consider:

$$C_{eB} \to C_{e\gamma} c_W - C_{eZ} s_W,$$
$$C_{eW} \to -C_{e\gamma} s_W - C_{eZ} c_W,$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle.

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi F_{\mu\nu} + \text{h.c.}$$

# Lepton dipole moments

Dimension-six operators contribute to the Wilson coefficients $C_{TL}$ and $C_{TR}$ of the dipole interaction:

$$V^\mu = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left( C_{TL}(p_\gamma^2)\,\omega_L + C_{TR}(p_\gamma^2)\,\omega_R \right) (p_\gamma)_\nu \,.$$

Anomalous magnetic and electric-dipole moments:

$$\begin{aligned} a_l &\propto \Re(C_{TR} + C_{TL})\big|_{p_\gamma^2 \to 0} \qquad \text{CPC} \\ d_l &\propto \Im(C_{TR} - C_{TL})\big|_{p_\gamma^2 \to 0} \qquad \text{CPV} \end{aligned}$$

In terms of effective coefficients:

$$a_l = \frac{2}{e} \frac{2^{1/4} m_l}{\sqrt{G_F}\Lambda^2} \Re \mathcal{C}_{e\gamma}^{ll}, \qquad d_l = \frac{2^{1/4}}{\sqrt{G_F}\Lambda^2} \Im \mathcal{C}_{e\gamma}^{ll}.$$

If flavour is not diagonal, then the momenta are “transitional”.

# Low-energy LFV observables

## Neutrinoless radiative decay

$$\mathrm{Br}\left(\mu \to e\gamma\right) = \frac{\alpha_e m_\mu^5}{\Lambda^4 \Gamma_\mu} \left( \left| C_L^D \right|^2 + \left| C_R^D \right|^2 \right) .$$

## Neutrinoless three-body decay

$$\begin{aligned} \mathrm{Br}(\mu \to 3e) \quad = \quad & \frac{\alpha_e^2 m_\mu^5}{12\pi \Lambda^4 \Gamma_\mu} \left( \left| C_L^D \right|^2 + \left| C_R^D \right|^2 \right) \left( 8 \log \left[ \frac{m_\mu}{m_e} \right] - 11 \right) \\ + \quad & \frac{m_\mu^5}{3(16\pi)^3 \Lambda^4 \Gamma_\mu} \left( \left| C_{ee}^{S\,LL} \right|^2 + 16 \left| C_{ee}^{V\,LL} \right|^2 + 8 \left| C_{ee}^{V\,LR} \right|^2 \right. \\ & \left. + \left| C_{ee}^{S\,RR} \right|^2 + 16 \left| C_{ee}^{V\,RR} \right|^2 + 8 \left| C_{ee}^{V\,RL} \right|^2 \right) . \end{aligned}$$

## Coherent conversion in nuclei

$$\Gamma_{\mu \to e}^N = \frac{m_\mu^5}{4\Lambda^4} \left| e\, C_L^D\ D_N + 4 \left( G_F m_\mu m_p \tilde{C}_{(p)}^{SL} S_N^{(p)} + \tilde{C}_{(p)}^{VR}\ V_N^{(p)} + p \to n \right) \right|^2 + L \leftrightarrow R.$$

# High-energy LFV observables

Flavour-violating $Z$ decays can be parametrised at the tree level by means of the following four operators:

$$\Gamma(Z \to l_1^{\pm} l_2^{\mp}) = \frac{m_Z^3 v^2}{12\pi\Lambda^4} \left( \left|C_{eZ}^{12}\right|^2 + \left|C_{eZ}^{21}\right|^2 + \left|C_{\varphi e}^{12}\right|^2 + \left|C_{\varphi l(1)}^{12}\right|^2 + \left|C_{\varphi l(3)}^{12}\right|^2 \right),$$

and all of their contributions occur at the same order. We have summed over the two possible final states, $l_1^+ l_2^-$ and $l_1^- l_2^+$.

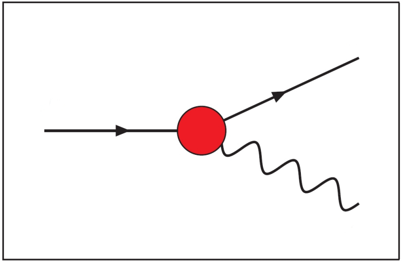
For the Higgs boson decay $H \to l_1^{\pm} l_2^{\mp}$, one has

$$\Gamma(H \to l_1^{\pm} l_2^{\mp}) = \frac{m_H v^4}{16\pi\Lambda^4} \left( \left|C_{e\varphi}^{12}\right|^2 + \left|C_{e\varphi}^{21}\right|^2 \right),$$

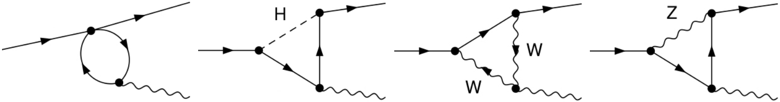
where only one operator contributes at tree level. Again, we have summed over the two possible decays $l_1^+ l_2^-$ and $l_1^- l_2^+$.

# Dimension-six operators: lepton current at one loop

From a point-like interaction. . .



. . . to quantum fluctuations!



# Effective coefficients and energy scale

The effective dipole coefficient can be written as

$$C_T^{(1)} = -\frac{v}{\sqrt{2}} \left( C_{e\gamma} \left( 1 + e^2 c_{e\gamma}^{(1)} \right) + \sum_{i \neq e\gamma} e^2 c_i^{(1)} \, C_i \right) .$$

In general, the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ contain UV singularities, *i.e.* a renormalisation of $C_{e\gamma}$ is required.

Such procedure makes the scale dependence explicit via the *anomalous dimensions* of the coefficient.

At the end of the day, the renormalised effective coefficients and the $C_{TL}$ and $C_{TR}$ are running quantities.

# Renormalisation Group Equations

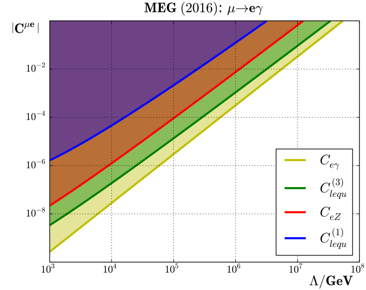
$$16\pi^2 \frac{\partial\, C_{e\gamma}^{ij}}{\partial \log\lambda} \simeq \left(\frac{47e^2}{3} + \frac{e^2}{4c_W^2} - \frac{9e^2}{4s_W^2} + 3Y_t^2\right) C_{e\gamma}^{ij} + 6e^2\left(\frac{c_W}{s_W} - \frac{s_W}{c_W}\right) C_{eZ}^{ij} + 16eY_t\, C_{ijtt}^{(3)},$$

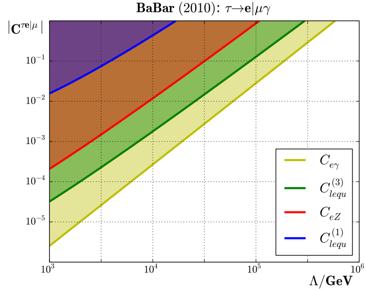
$$16\pi^2 \frac{\partial\, C_{eZ}^{ij}}{\partial \log\lambda} \simeq -\frac{2e^2}{3}\left(\frac{2c_W}{s_W} + \frac{31s_W}{c_W}\right) C_{e\gamma}^{ij} + 2e\left(\frac{3c_W}{s_W} - \frac{5s_W}{c_W}\right) Y_t\, C_{ijtt}^{(3)} + \left(-\frac{47e^2}{3} + \frac{151e^2}{12c_W^2} - \frac{11e^2}{12s_W^2} + 3Y_t^2\right) C_{eZ}^{ij},$$

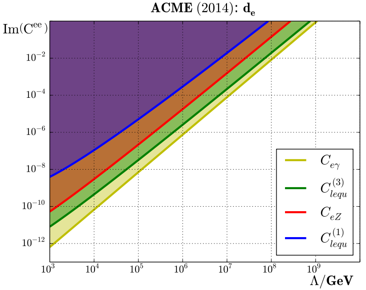
$$16\pi^2 \frac{\partial\, C_{ijtt}^{(3)}}{\partial \log\lambda} \simeq \frac{7eY_t}{3} C_{e\gamma}^{ij} + \frac{eY_t}{2}\left(\frac{3c_W}{s_W} - \frac{5s_W}{3c_W}\right) C_{eZ}^{ij} + \left(\frac{2e^2}{9c_W^2} - \frac{3e^2}{s_W^2} + \frac{3Y_t^2}{2} + \frac{8g_S^2}{3}\right) C_{ijtt}^{(3)} + \frac{e^2}{8}\left(\frac{5}{c_W^2} + \frac{3}{s_W^2}\right) C_{ijtt}^{(1)},$$

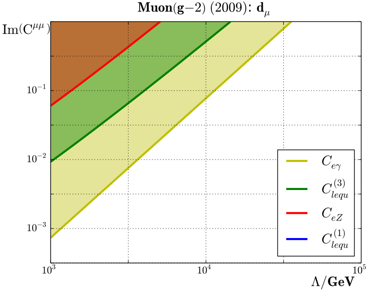
$$16\pi^2 \frac{\partial\, C_{ijtt}^{(1)}}{\partial \log\lambda} \simeq \left(\frac{30e^2}{c_W^2} + \frac{18e^2}{s_W^2}\right) C_{ijtt}^{(3)} + \left(-\frac{11e^2}{3c_W^2} + \frac{15Y_t^2}{2} - 8g_S^2\right) C_{ijtt}^{(1)}.$$

# Experimental limits “reinterpreted” at the EW scale



# cLFV effective contributions to $C_{TL}$ and $C_{TR}$

| Operator | $C_{TL}$ or $C_{TR}(l_2 \longleftrightarrow l_1)$ |
|---|---|
| $Q_{e\gamma}$ | $-C_{e\gamma}\dfrac{\sqrt{2}m_W s_W}{e}$ |
| $Q_{eZ}$ | $-C_{eZ}\dfrac{e m_Z}{16\sqrt{2}\pi^2}\left(3-6c_W^2+4c_W^2\log\left[\dfrac{m_W^2}{m_Z^2}\right]+(12c_W^2-6)\log\left[\dfrac{m_Z^2}{\lambda^2}\right]\right)$ |
| $Q_{\varphi l}^{(1)}$ | $-C_{\varphi l}^{(1)}\dfrac{e m_1\left(1+s_W^2\right)}{24\pi^2}$ |
| $Q_{\varphi l}^{(3)}$ | $C_{\varphi l}^{(3)}\dfrac{e m_1\left(3-2s_W^2\right)}{48\pi^2}$ |
| $Q_{\varphi e}$ | $C_{\varphi e}\dfrac{e m_2\left(3-2s_W^2\right)}{48\pi^2}$ |
| $Q_{e\varphi}$ | $C_{e\varphi}\dfrac{m_W s_W}{48\sqrt{2}m_H^2\pi^2}\left(4m_1^2+4m_2^2+3m_1^2\log\left[\dfrac{m_1^2}{m_H^2}\right]+3m_2^2\log\left[\dfrac{m_2^2}{m_H^2}\right]\right)$ |
| $Q_{lequ}^{(3)}$ | $-\dfrac{e}{2\pi^2}\sum_u m_u\left(C_{lequ}^{(3)}\right)^{21uu}\log\left[\dfrac{m_u^2}{\lambda^2}\right]$ |

| Operator | $C_{TL}$ | $C_{TR}$ |
|---|---|---|
| $Q_{le}$ | $\dfrac{e}{16\pi^2}\left(m_e C_{le}^{2ee1}+m_\mu C_{le}^{2\mu\mu1}+m_\tau C_{le}^{2\tau\tau1}\right)$ | $\dfrac{e}{16\pi^2}(m_e C_{le}^{1ee2}+m_\mu C_{le}^{1\mu\mu2}+m_\tau C_{le}^{1\tau\tau2})$ |

# No correlation: limits from muonic cLFV

GMP and A. Signer
JHEP **1410** (2014) 014

F. Feruglio,
arXiv:1509.08428

GMP and A. Signer
EPJWC **118** (2016) 01031

| Coefficient | MEG ($\mu \to e\gamma$) $BR \leq 5.7 \cdot 10^{-13}$ | ATLAS ($Z \to e\mu$) $BR \leq 7.5 \cdot 10^{-7}$ | SINDRUM ($\mu \to 3e$) $BR \leq 1.0 \cdot 10^{-12}$ |
|---|---|---|---|
| $C_{eZ}^{\mu e}(m_Z)$ | $1.4 \cdot 10^{-13} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $2.8 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{\varphi l}^{(1)}$ | $2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{\varphi l}^{(3)}$ | $2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{\varphi e}$ | $2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | $2.6 \cdot 10^{-11} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{e\varphi}^{\mu e}$ | $2.7 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | | $6.1 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{le}^{eee\mu}$ | $4.2 \cdot 10^{-8} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | | $2.2 \cdot 10^{-11} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{le}^{e\mu\mu\mu}$ | $2.0 \cdot 10^{-10} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | | |
| $C_{le}^{e\tau\tau\mu}$ | $1.2 \cdot 10^{-11} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ | | |
| $C_{ee}^{eee\mu}$ | | | $7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |
| $C_{ll}^{eee\mu}$ | | | $7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$ |

# No correlation: limits from tauonic cLFV

| Coefficient | BaBar ($\tau \to \mu\gamma$) $BR \leq 4.4 \cdot 10^{-8}$ | LEP ($Z \to \tau\mu$) $BR \leq 1.2 \cdot 10^{-5}$ | BELL ($\tau \to 3\mu$) $BR \leq 2.1 \cdot 10^{-8}$ | ATLAS&CMS ($H \to \tau\mu$) $BR \leq 1.85 \cdot 10^{-2}$ |
|---|---|---|---|---|
| $C_{eZ}^{\tau\mu}(m_Z)$ | $1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $6.1 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{\varphi l}^{(1)}$ | $1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{\varphi l}^{(3)}$ | $1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{\varphi e}$ | $1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $9.5 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{e\varphi}^{\tau\mu}$ | $1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$ | | $1.1 \cdot 10^{-5} \frac{\Lambda^2}{[\text{GeV}]^2}$ | $1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$ |
| $C_{le}^{\mu e e\tau}$ | ~~$4.7 \cdot 10^{-4} \frac{\Lambda^2}{[\text{GeV}]^2}$~~ | | | |
| $C_{le}^{\mu\mu\mu\tau}$ | ~~$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$~~ | | $8.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{le}^{\mu\tau\tau\tau}$ | ~~$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$~~ | | | |
| $C_{ee}^{\mu\mu\mu\tau}$ | | | $2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |
| $C_{ll}^{\mu\mu\mu\tau}$ | | | $2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$ | |

# No correlation: limits from EDM

## Many limits on previously unconstrained coefficients

| Coefficient | Limits from $d_e$ | Coefficient | Limits from $d_\mu$ | Coefficient | Limits from $d_\tau$ |
|---|---|---|---|---|---|
| $\Im C_{e\gamma}^{11}$ | $6.1 \times 10^{-15}$ | $\Im C_{e\gamma}^{22}$ | $1.3 \times 10^{-5}$ | $\Im C_{e\gamma}^{33}$ | $3.1 \times 10^{-3}$ |
| $\Im C_{e\varphi}^{11}$ | $8.2 \times 10^{-3}$ | $\Im C_{e\varphi}^{22}$ | N/A | $\Im C_{e\varphi}^{33}$ | N/A |
| $\Im C_{eZ}^{11}$ | $5.1 \times 10^{-12}$ | $\Im C_{eZ}^{22}$ | $1.1 \times 10^{-2}$ | $\Im C_{eZ}^{33}$ | N/A |
| $\Im C_{lequ(3)}^{1122}$ | $6.0 \times 10^{-12}$ | $\Im C_{lequ(3)}^{2222}$ | $1.3 \times 10^{-2}$ | $\Im C_{lequ(3)}^{3322}$ | N/A |
| $\Im C_{lequ(3)}^{1111}$ | $1.3 \times 10^{-9}$ | $\Im C_{lequ(3)}^{2211}$ | N/A | $\Im C_{lequ(3)}^{3311}$ | N/A |

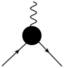
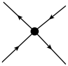
Limits on SMEFT effective coefficients defined at the EW energy scale from leptonic EDMs.

Improving on $d_\tau$: M. Fael *et al.*, JHEP **1603** (2016) 140

# The good old $k$ plot

Due to the **extremely-low** accessible **branching ratios**, CLFV muon channels can strongly **constrain** new physics models and scales.

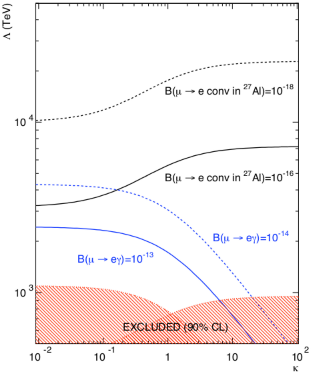
Model independent Lagrangian:

$\frac{m_\mu}{(\kappa+1)\Lambda^2} \times$  $+ \quad \frac{\kappa}{(\kappa+1)\Lambda^2} \times$ 

| Dipole term (*e.g.* SUSY) | Contact term (*e.g.* Z', LQ) |
|---|---|
| $\mu \to e\gamma$ | |

$$\mu \to eee$$
$$\mu - e \text{ conversion}$$

Sensitive to high-mass New Physics!




de Gouvea et Vogel, *ArXiv e-prints* (2013), 1303.4097

# Below the EWSB scale (1)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} + \frac{1}{\Lambda^2} \sum_i C_i Q_i,$$

and the explicit structure of the operators is given by

| Dipole | | | |
|---|---|---|---|
| $Q_{e\gamma}$ | $e m_r (\bar{l}_p \sigma^{\mu\nu} P_L l_r) F_{\mu\nu} + \text{H.c.}$ | | |
| **Scalar/Tensorial** | | **Vectorial** | |
| $Q_S$ | $(\bar{l}_p P_L l_r)(\bar{l}_s P_L l_t) + \text{H.c.}$ | $Q_{VLL}$ | $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_L l_t)$ |
| | | $Q_{VLR}$ | $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_R l_t)$ |
| | | $Q_{VRR}$ | $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{l}_s \gamma_\mu P_R l_t)$ |
| $Q_{Slq(1)}$ | $(\bar{l}_p P_L l_r)(\bar{q}_s P_L q_t) + \text{H.c.}$ | $Q_{VlqLL}$ | $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_L q_t)$ |
| $Q_{Slq(2)}$ | $(\bar{l}_p P_L l_r)(\bar{q}_s P_R q_t) + \text{H.c.}$ | $Q_{VlqLR}$ | $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_R q_t)$ |
| $Q_{Tlq}$ | $(\bar{l}_p \sigma^{\mu\nu} P_L l_r)(\bar{q}_s \sigma_{\mu\nu} P_L q_t) + \text{H.c.}$ | $Q_{VlqRL}$ | $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_L q_t)$ |
| | | $Q_{VlqRR}$ | $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_R q_t)$ |

# Below the EWSB scale (2)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm QCD}$$

$$+ \frac{1}{\Lambda^2}\Big\{ C_L^D O_L^D + \sum_{f=q,\ell} \left( C_{ff}^{V\,LL} O_{ff}^{V\,LL} + C_{ff}^{V\,LR} O_{ff}^{V\,LR} + C_{ff}^{S\,LL} O_{ff}^{S\,LL} \right)$$

$$+ \sum_{h=q,\tau} \left( C_{hh}^{T\,LL} O_{hh}^{T\,LL} + C_{hh}^{S\,LR}\, O_{hh}^{S\,LR} \right) + C_{gg}^L O_{gg}^L + L \leftrightarrow R \Big\} + \text{h.c.},$$

and the explicit structure of the operators is given by

$$\boxed{O_L^D} = e\, m_\mu\, (\bar{e}\sigma^{\mu\nu} P_L \mu)\, F_{\mu\nu},$$

$$O_{ff}^{V\,LL} = (\bar{e}\gamma^\mu P_L \mu)\, (\bar{f}\gamma_\mu P_L f)\,,$$

$$O_{ff}^{V\,LR} = (\bar{e}\gamma^\mu P_L \mu)\, (\bar{f}\gamma_\mu P_R f)\,,$$

$$\boxed{O_{ff}^{S\,LL}} = (\bar{e} P_L \mu)\, (\bar{f} P_L f)\,,$$

$$O_{hh}^{S\,LR} = (\bar{e} P_L \mu)\, (\bar{h} P_R h)\,,$$

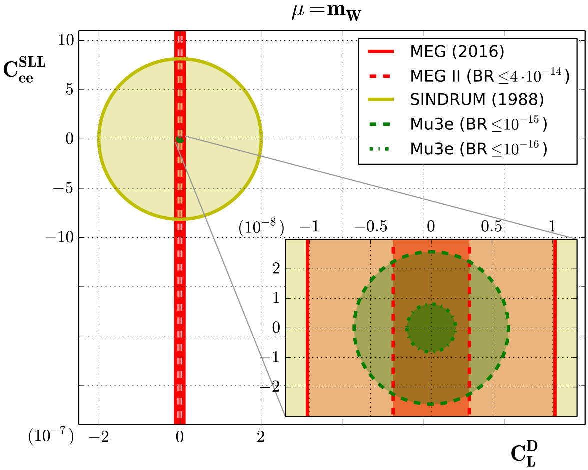
$$O_{hh}^{T\,LL} = (\bar{e}\sigma_{\mu\nu} P_L \mu)\, (\bar{h}\sigma^{\mu\nu} P_L h)\,,$$

$$O_{gg}^L = \alpha_s\, m_\mu G_F\, (\bar{e} P_L \mu)\, G_{\mu\nu}^a G_a^{\mu\nu}\,.$$

# Interplay between $\mu \to e\gamma$ and $\mu \to 3e$

A. Crivellin, S. Davidson, GMP and A. Signer, arXiv:1611.03409 [hep-ph].

Below the EW scale, four-fermion vs dipole:



# Dipole evolution below the EWSB scale

At the two-loop level, in the tHV scheme:

$$\dot{C}_L^D = 16\,\alpha_e\,Q_l^2\,\boxed{C_L^D} - \frac{Q_l}{(4\pi)}\frac{m_e}{m_\mu}\boxed{C_{ee}^{S\,LL}} - \frac{Q_l}{(4\pi)}\boxed{C_{\mu\mu}^{S\,LL}}$$

$$+\sum_h \frac{8Q_h}{(4\pi)}\frac{m_h}{m_\mu}N_{c,h}\boxed{C_{hh}^{T\,LL}}\Theta(\mu - m_h)$$

$$-\frac{\alpha_e Q_l^3}{(4\pi)^2}\left(\frac{116}{9}\boxed{C_{ee}^{V\,RR}} + \frac{116}{9}\boxed{C_{\mu\mu}^{V\,RR}} - \frac{122}{9}\boxed{C_{\mu\mu}^{V\,RL}} - \left(\frac{50}{9} + 8\,\frac{m_e}{m_\mu}\right)\boxed{C_{ee}^{V\,RL}}\right)$$

$$-\sum_h \frac{\alpha_e}{(4\pi)^2}\left(6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9}\right)N_{c,h}\boxed{C_{hh}^{V\,RR}}\Theta(\mu - m_h)$$

$$-\sum_h \frac{\alpha_e}{(4\pi)^2}\left(-6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9}\right)N_{c,h}\boxed{C_{hh}^{V\,RL}}\Theta(\mu - m_h)$$

$$-\sum_h \frac{\alpha_e}{(4\pi)^2}\,4Q_h^2 Q_l N_{c,h}\frac{m_h}{m_\mu}\boxed{C_{hh}^{S\,LR}}\Theta(\mu - m_h) + [\ldots]\,.$$

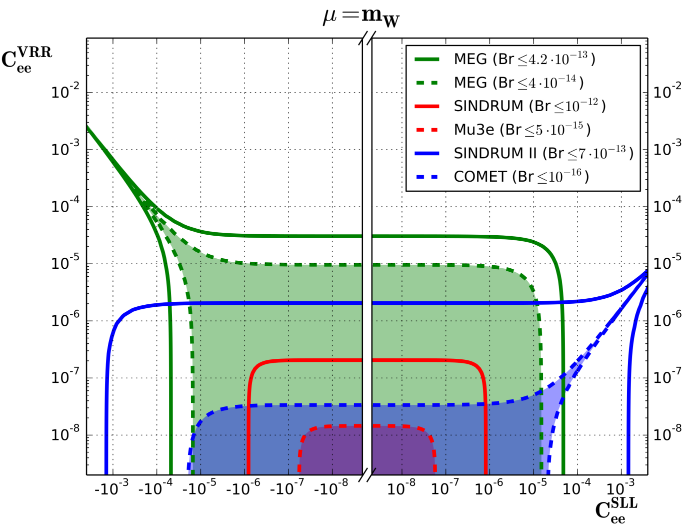
A. Crivellin, S. Davidson, GMP and A. Signer, JHEP **1705** (2017) 117.

# In absence of interplay at the EWSB scale

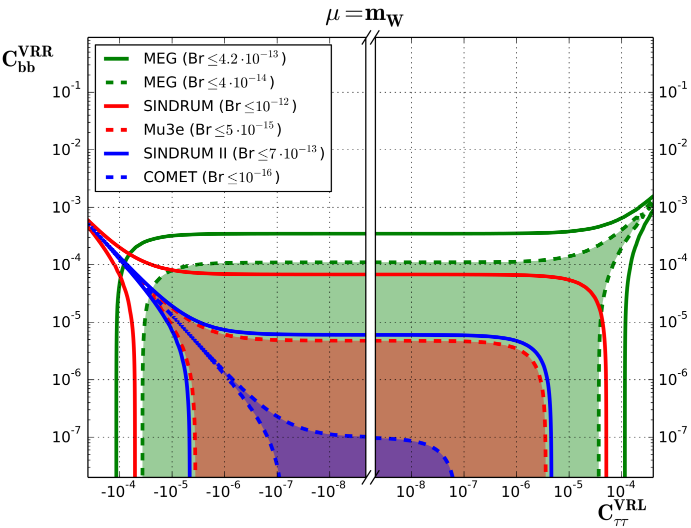
| | $\mathrm{Br}(\mu^+ \to e^+\gamma)$ $4.2\cdot10^{-13}$ | $\mathrm{Br}(\mu^+ \to e^+\gamma)$ $4.0\cdot10^{-14}$ | $\mathrm{Br}(\mu^+ \to e^+e^-e^+)$ $1.0\cdot10^{-12}$ | $\mathrm{Br}(\mu^+ \to e^+e^-e^+)$ $5.0\cdot10^{-15}$ | $\mathrm{Br}^{\mathrm{Au/Al}}_{\mu\to e}$ $7.0\cdot10^{-13}$ | $\mathrm{Br}^{\mathrm{Au/Al}}_{\mu\to e}$ $1.0\cdot10^{-16}$ |
|---|---|---|---|---|---|---|
| $C_L^D$ | $1.0\cdot10^{-8}$ | $3.1\cdot10^{-9}$ | $2.0\cdot10^{-7}$ | $1.4\cdot10^{-8}$ | $2.0\cdot10^{-7}$ | $2.9\cdot10^{-9}$ |
| $C_{ee}^{S\,LL}$ | $4.8\cdot10^{-5}$ | $1.5\cdot10^{-5}$ | $8.1\cdot10^{-7}$ | $5.8\cdot10^{-8}$ | $1.4\cdot10^{-3}$ | $2.1\cdot10^{-5}$ |
| $C_{\mu\mu}^{S\,LL}$ | $2.3\cdot10^{-7}$ | $7.2\cdot10^{-8}$ | $4.6\cdot10^{-6}$ | $3.3\cdot10^{-7}$ | $7.1\cdot10^{-6}$ | $1.0\cdot10^{-7}$ |
| $C_{\tau\tau}^{S\,LL}$ | $1.2\cdot10^{-6}$ | $3.7\cdot10^{-7}$ | $2.4\cdot10^{-5}$ | $1.7\cdot10^{-6}$ | $2.4\cdot10^{-5}$ | $3.5\cdot10^{-7}$ |
| $C_{\tau\tau}^{T\,LL}$ | $2.9\cdot10^{-9}$ | $9.0\cdot10^{-10}$ | $5.7\cdot10^{-8}$ | $4.1\cdot10^{-9}$ | $5.9\cdot10^{-8}$ | $8.5\cdot10^{-10}$ |
| $C_{bb}^{S\,LL}$ | $2.8\cdot10^{-6}$ | $8.6\cdot10^{-7}$ | $5.4\cdot10^{-5}$ | $3.8\cdot10^{-6}$ | $9.0\cdot10^{-7}$ | $1.2\cdot10^{-8}$ |
| $C_{bb}^{T\,LL}$ | $2.1\cdot10^{-9}$ | $6.4\cdot10^{-10}$ | $4.1\cdot10^{-8}$ | $2.9\cdot10^{-9}$ | $4.2\cdot10^{-8}$ | $6.0\cdot10^{-10}$ |
| $C_{ee}^{V\,RR}$ | $3.0\cdot10^{-5}$ | $9.4\cdot10^{-6}$ | $2.1\cdot10^{-7}$ | $1.5\cdot10^{-8}$ | $2.1\cdot10^{-6}$ | $3.5\cdot10^{-8}$ |
| $C_{\mu\mu}^{V\,RR}$ | $3.0\cdot10^{-5}$ | $9.4\cdot10^{-6}$ | $1.6\cdot10^{-5}$ | $1.1\cdot10^{-6}$ | $2.1\cdot10^{-6}$ | $3.5\cdot10^{-8}$ |
| $C_{\tau\tau}^{V\,RR}$ | $1.0\cdot10^{-4}$ | $3.2\cdot10^{-5}$ | $5.3\cdot10^{-5}$ | $3.8\cdot10^{-6}$ | $4.8\cdot10^{-6}$ | $7.9\cdot10^{-8}$ |
| $C_{bb}^{V\,RR}$ | $3.5\cdot10^{-4}$ | $1.1\cdot10^{-4}$ | $6.7\cdot10^{-5}$ | $4.8\cdot10^{-6}$ | $6.0\cdot10^{-6}$ | $1.0\cdot10^{-7}$ |
| $C_{bb}^{RA}$ | $4.2\cdot10^{-4}$ | $1.3\cdot10^{-4}$ | $6.5\cdot10^{-3}$ | $4.6\cdot10^{-4}$ | $1.3\cdot10^{-3}$ | $2.2\cdot10^{-5}$ |
| $C_{bb}^{RV}$ | $2.1\cdot10^{-3}$ | $6.4\cdot10^{-4}$ | $6.7\cdot10^{-5}$ | $4.7\cdot10^{-6}$ | $6.0\cdot10^{-6}$ | $1.0\cdot10^{-7}$ |

Limits on the various coefficients $C_i(m_W)$ from current and future experimental constraints, assuming that (at the high scale $m_W$) only one coefficient at a time is non-vanishing and not including operator-dependent efficiency corrections.

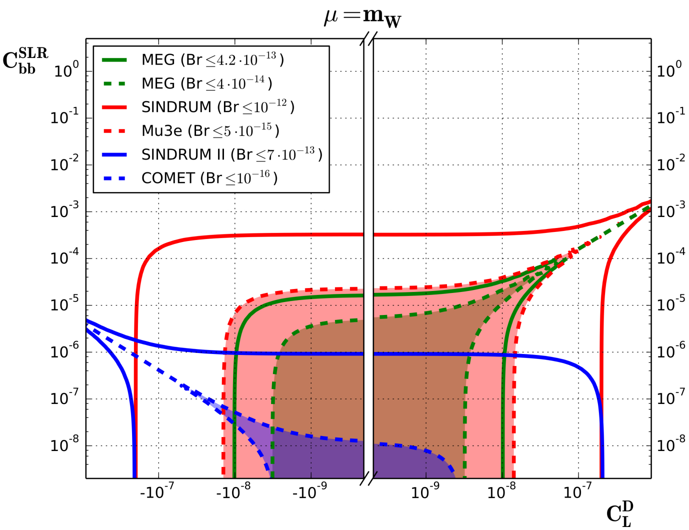
# Interplay at the EWSB scale
## Mu3e money plot



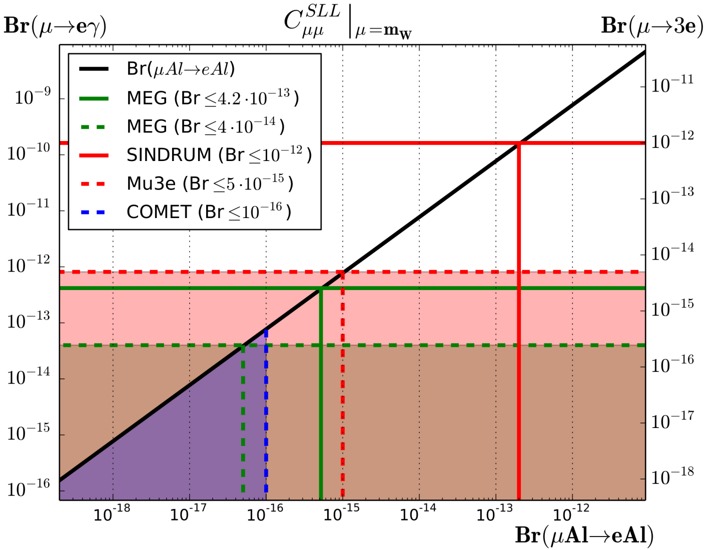
# Interplay at the EWSB scale
# COMET/Mu2e money plot (1)



# Interplay at the EWSB scale COMET/Mu2e money plot (2)



# MEG/MEG-II money plot



# Conclusion

- √ CPV and LFV phenomena are forbidden in the minimal SM
  - Neutrino sector seems to ignore this fact, calling for something more than the minimal theoretical setup
  - Charged sector seems to take the job seriously
- √ If NP lives at very high energy, then consistent EFT techniques can be adopted to extract information of new physics at high scale from low-energy observables
- √ Precise background calculations are important to improve the experimental limits
- √ From limits on leptonic FV and EDM one can gain information on the parameter space of possible UV-complete BSM theories